PART 1

SECTION 1 — INTRODUCTION TO THE WEAPON SYSTEM

CHAPTER 1 — FIRING ZONES AND APPROACH COURSES

Contents

Acquisition and Tracking of the Target

1. The first requirement of a weapon system is to find and acquire the target. In this system acquisition is effected by initially positioning the interceptor, by GCI instructions, at a point within the range of the aircraft radar and from which the chosen approach course can be flown. The radar is then used to search for and lock-on to the target. The locked-on radar beam represents the line of sight and its movement is used to generate the steering instructions to fly the required approach course to a zone within which the missile can be fired successfully.

2. The approach courses selected depend on the weapons in use and the speed of the target. It is arranged that the computers in the system automatically provide steering information to give approach courses as follows:

Weapon	*Target*	*Approach Course (in azimuth)*
Firestreak	All targets	Pure pursuit
Red Top	Slow speed targets	Pure pusuit changing to quasi-lead pursuit
Red Top	Medium speed targets	Quasi-lead pursuit
Red Top	Fast targets	Proportional navigation

These courses are dealt with mathematically in Appendix 1 to this chapter, together with other courses that could have been used.

3. In a pure pursuit course the direction of motion of the fighter is always along the sightline. Any difference between the sightline and the direction of motion is the error signal. Fig. 1 illustrates a pure pursuit course.

4. A quasi or modified lead pursuit course is such that the direction of motion is leading the sightline. The missile is launched with some lead angle which makes its task easier. Fig. 2 demonstrates this.

5. A proportional navigation course (Fig. 3) is a course in which the rate of turn of the fighter is made proportional to the rate of turn of the sightline. It is shown in Appendix 1 that this course tends towards a collision course.

Successful Firing Zones

6. If a missile acquires the target and all indications show that it can be fired, it does not follow that a kill will be achieved or is even likely unless the point of release is within the successful missile firing zone. A firing zone can be constructed for certain target conditions, which gives the greatest chance of a successful kill if the missile is launched within its boundaries. Missile limitations and target and interceptor data define the firing zones.

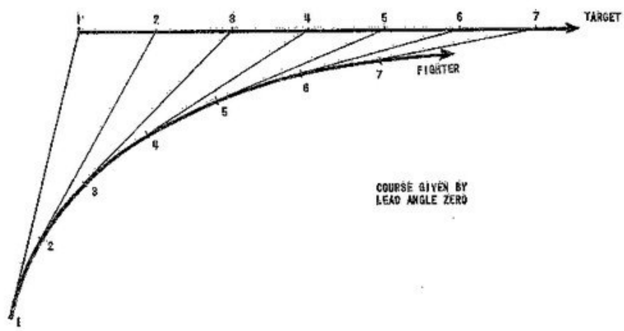
7. A missile firing zone is made up from several limiting parameters; these are:

a. Aerodynamic

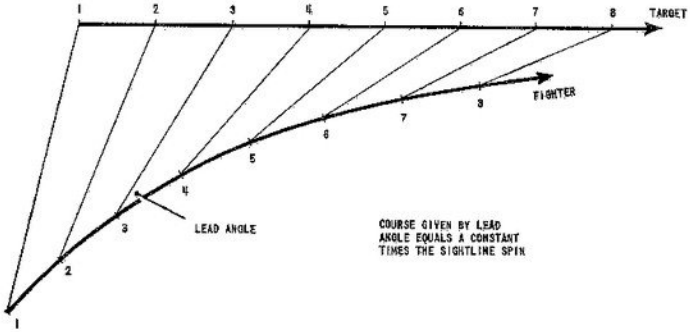
b. Physical

c. Guidance

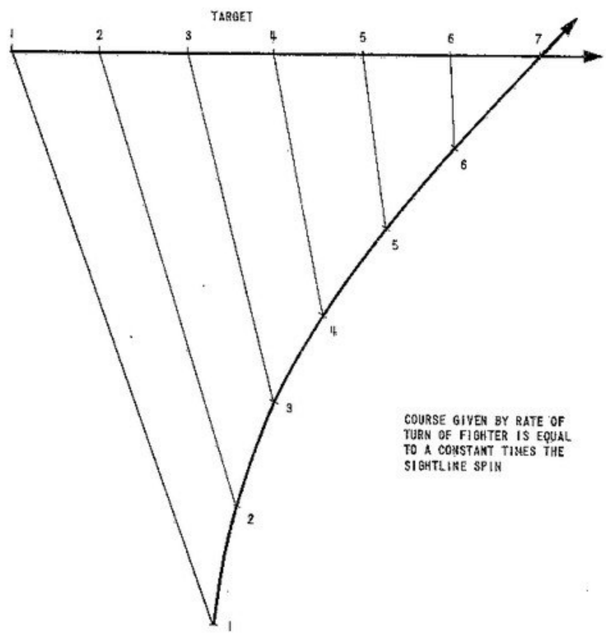
These limits are dependent upon the initial engagement parameters, i.e. fighter speed, fighter height and heading, target speed, target height and heading, range, relative bearing and target manoeuvre.



1—1—1 Fig. 1 Pure Pursuit Course



1—1—1 Fig. 2 Quasi Lead Pursuit Course

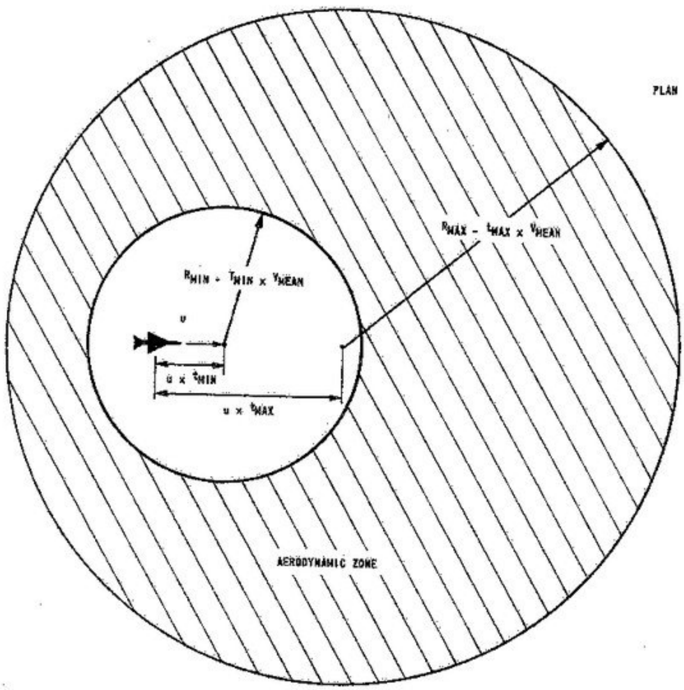


1—1—1 Fig. 3 Proportional Navigation Course

Aerodynamic Boundaries

8. The maximum range of the missile is given by the maximum flight endurance. This may be over-riden by the missile's speed falling into the transonic region or the closure speed reducing below a certain value. In simple terms the maximum range may be regarded as a circle whose centre is on the predicted missile collision point; that is, the distance that the target travels during the missile's flight ahead of the target (see Fig. 4). Additionally, there is a minimum range set by the time necessary for the safety and arming devices to function. These factors, therefore, determine the outer and inner boundaries respectively when considering the aerodynamic aspect.

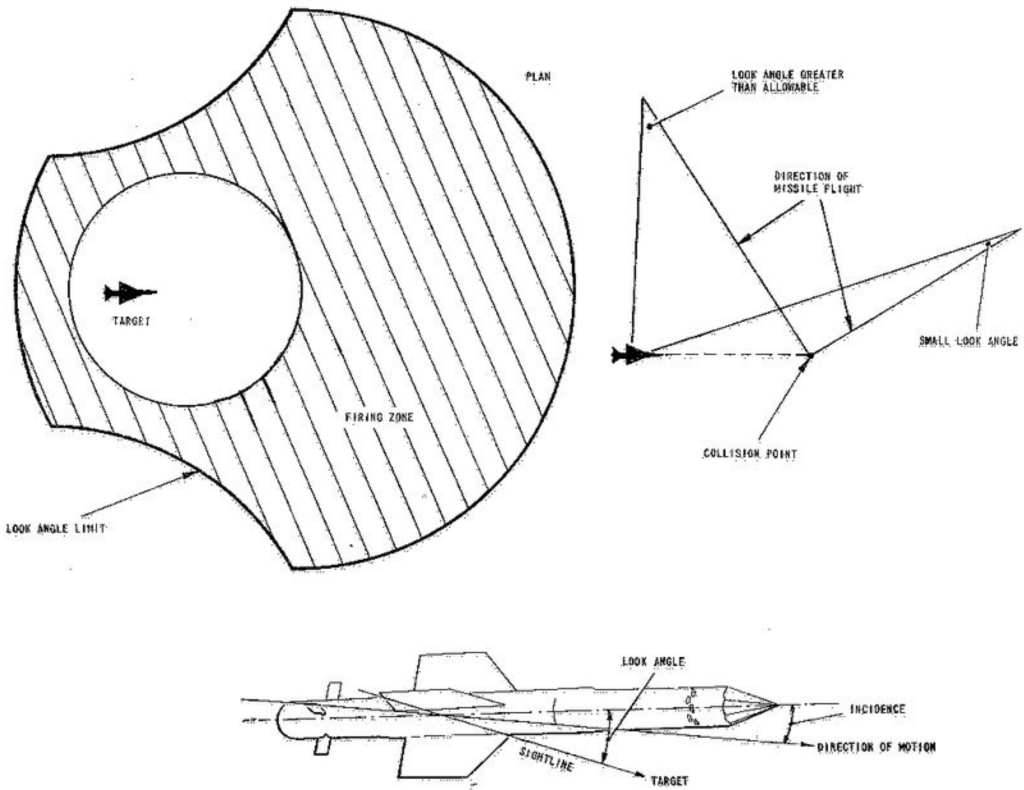
9. The diagrams used to illustrate firing zones assume the target to be stationary, i.e. they are drawn relative to a stationary target. If the target manoeuvres, the zone will be distorted in the direction of target manoeuvre.



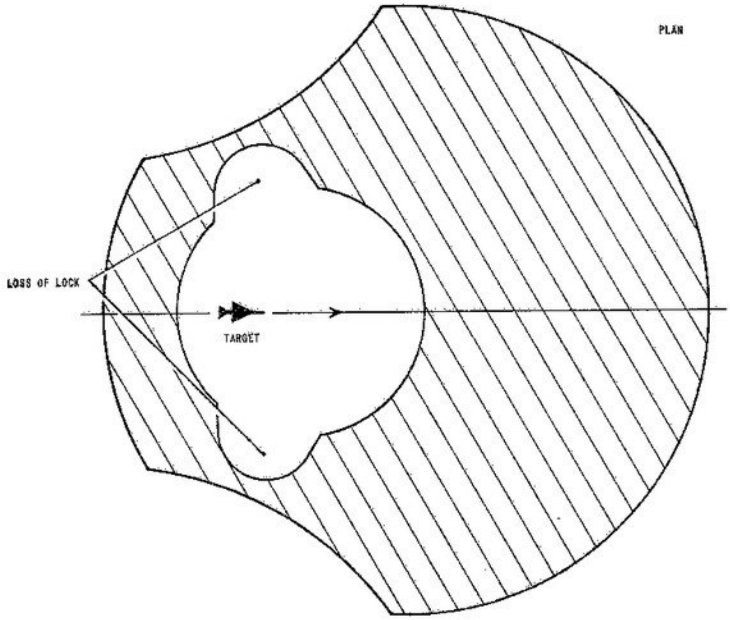
1—1—1 Fig. 4 Aerodynamic Boundaries

Physical Limits

10. During flight towards the target a missile is trying to fly to the collision point whereas it looks at the target. If the angle between the missile's airframe and the sightline exceeds the gimbal angle limitations of the missile then the missile loses lock; this is a look angle failure. The angle between the direction of motion and the sightline at limiting conditions can be considerably less than the physical maximum because the missile may have incidence in order to turn (see Fig. 5). In practice the limiting look angle conditions are most likely to occur at a later stage of an attack when the speed of the missile has reduced. If look angle limits are going to be reached, they always occur at the end of a Red Top attack and may occur earlier in a Firestreak attack.



1—1—1 Fig. 5 Look Angle Limits

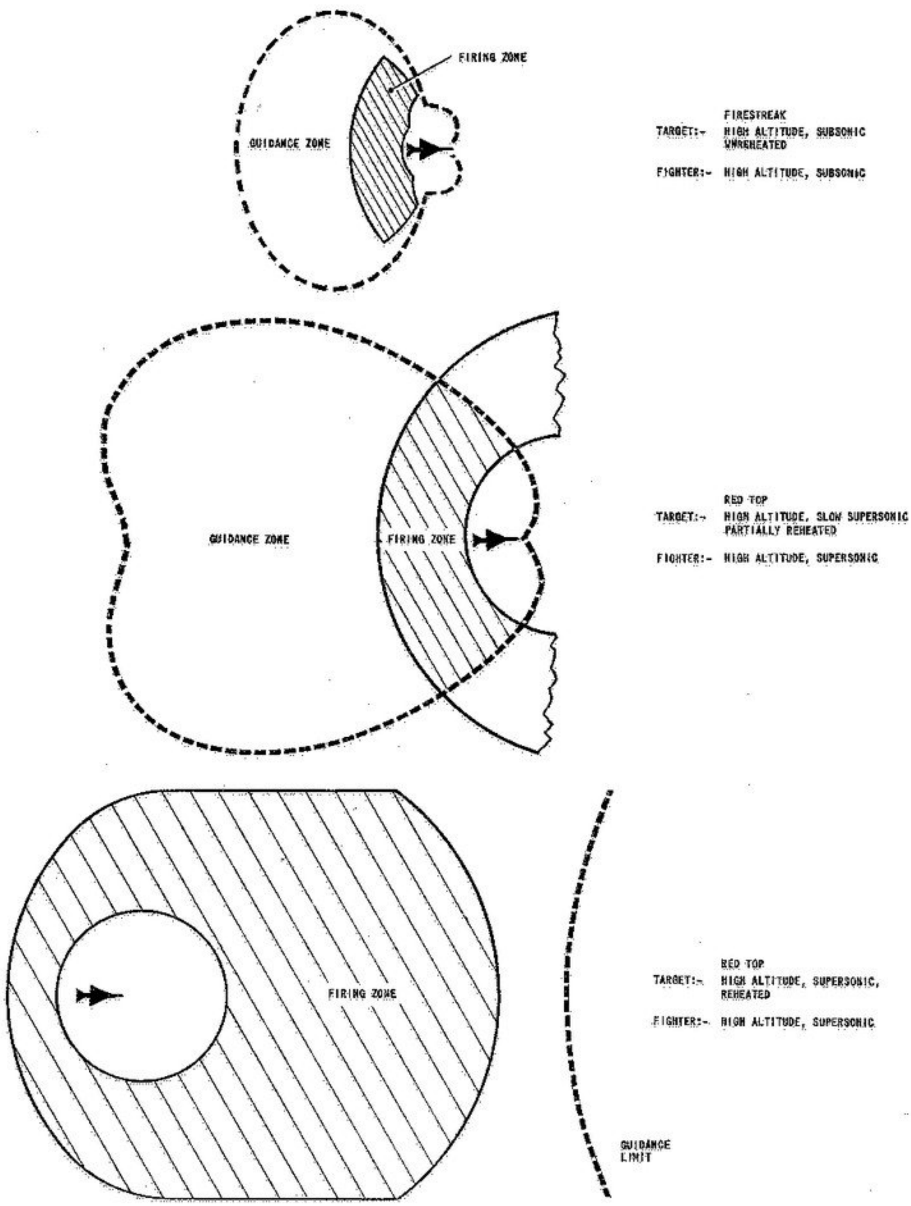


1—1—1 Fig. 6 Sightline Rate Limits

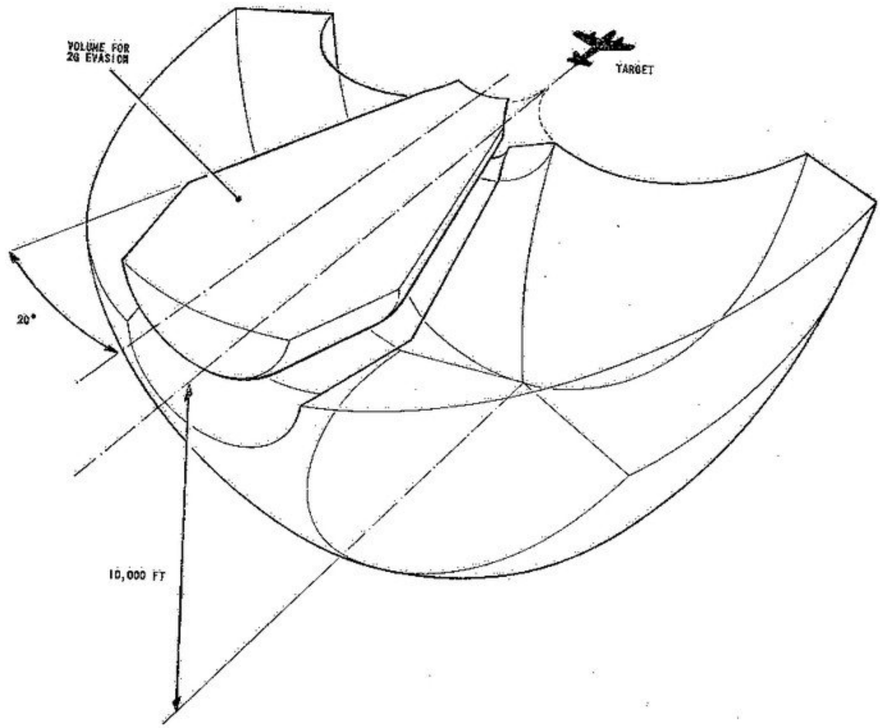
11. In some conditions of launch, particularly when launched at close range on a beam attack and with large lag angles, the sightline rates may increase to such a value that the homing head cannot follow and the missile loses lock. Thus the composite missile launching zone is altered as shown in Fig. 6.

Guidance Limitations

12. Both Firestreak and Red Top rely for their homing on the ability to "see" the IR emitted by the target. This comes mainly from the jet pipe, the hot gases in the jet efflux and radiation from the target skin: the latter is not particularly significant unless the target is supersonic. The amount of IR emitted is dependent on the size of the target and the engine configuration and whether reheat is engaged or not. Typical composite zones with the guidance limits are shown in Fig. 7.



1—1—1 Fig. 7 Typical Overall Missile Launching Zones



1—1—1 Fig. 8 Pictorial Representation of Firing Volume

13. Fig. 7 illustrates only the plan view of the firing zones. They are, however, three dimensional, and Fig. 8 indicates the shape of such a zone for Firestreak. The total volume is the success zone of Firestreak against a non-evading target. The volume is reduced when targets evading at 2G are considered. From the diagram it can be seen that this leads to the "accepted limit" of Firestreak being successful only within 20° angle off the target tail.

14. Examination of the firing zones illustrates that for Firestreak it is essential to have an approach course that tends towards the rear of the target. Furthermore, Firestreak is restrained to search within a small angle relative to the aircraft's axis, so the weapon must be fired from a pursuit course.

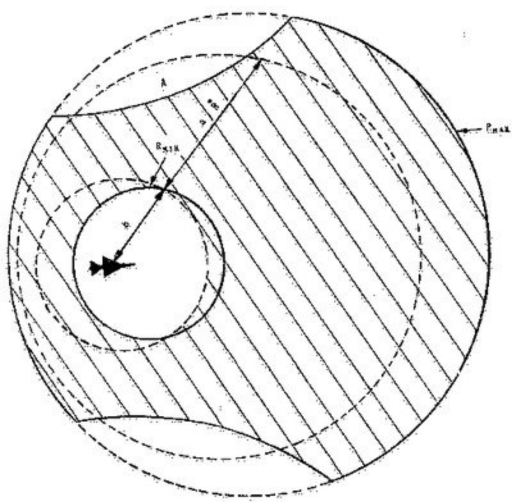
15. Since guidance to subsonic targets tends to be limited to the rear hemisphere, Red Top also requires a course that tends to the target's rear. However, the Red Top homing eye is "slaved" to a greater angle than that for Firestreak and, as the missile benefits from being fired with lead angle, the course is changed from pursuit to lead pursuit at some suitable point.

16. When Red Top is fired against supersonic targets, there is adequate guidance from in front of the target and the firing zone is considerably wider. Here a weapon collision course is desirable but, as the missile zones vary so widely, neither lead pursuit nor straight pass are entirely satisfactory. Both these courses need to know range and, as this may be denied by enemy jamming, a proportional navigation course (see Appendix 1) provides a suitable compromise.

17. The courses arrived at are as listed in para. 2 and are dealt with in greater depth when dealing with the radar and red computer. The set of courses that have been chosen enable the fighter to be steered to within the missile firing zone. Information that the aircraft is within these boundaries must now be derived. Earlier, appropriate missile aerodynamic zones were shown and from these it can be seen that an expression of the form $R = a\dot{R} + b$ would give a good approximation, where R is the range, $\dot{R}$ is its rate of change (i.e. closing rate) and a and b are constants. For any one set of fighter/target conditions a value for a and b can be chosen to give a good fit. However it is necessary to reach a compromise that gives a reasonable fit under all circumstances. This is dealt with more fully in subsequent chapters.

18. With simple range brackets it is possible to have a situation where limits other than aerodynamics, e.g. look angle limits, are not covered. For example, at point A on Fig. 9 the missile is within the range brackets, may acquire, but is not within the firing zone; if launched, therefore, the missile would miss the target.

19. When the missile is released within its firing zone, it navigates towards the target using a proportional navigation course and either strikes the target or passes close to it. If it strikes the target, a contact fuze explodes the warhead. If it misses the target, an IR proximity fuze ensures detonation.



1—1—1 Fig. 9 Firing Zone Derivation

APPENDIX 1

APPROACH COURSES

Contents

General

1. The various types of approach course that can be used are:

a. Pure pursuit
b. Lead or lag pursuit
c. Quasi lead or lag pursuit
d. Collision
e. Proportional navigation
f. Straight pass.

Straight pass has been included in this appendix for completeness although it is not now used in the Lightning.

2. *Notation*

	$*\theta$	=	Lead angle (+lead −lag)
	Ω	=	Sightline spin
	R	=	Range
	k, M	=	Constants
	U	=	Target speed
	V	=	Fighter speed
	ϕ	=	Angle off tail
	ψ	=	Bearing
	T_g	=	Time to go
	F	=	Relative strike range
	β	=	Angle between courses
Suffixes	F	=	Fighter
	S	=	Sightline

The shorthand representation $\dot{\theta}, = \frac{d\theta}{dt}$ = rate of change of θ with respect to time, is frequently used.

*For the purposes of this Chapter, incidence is ignored and lead angle is synonymous with look angle.

Pure Pursuit

3. Pure pursuit is the simplest approach course of all. The direction of motion of the fighter is always along the sightline. Any error between the sightline and the velocity vector must be zeroed to be on course. Once on course the rate of turn of the fighter is equal to the sightline spin.

4. A pursuit course always ends with the fighter directly behind the target. The demanded rate of turn, however, may be high.

Lead Pursuit

5. Lead pursuit courses are courses in which the direction of motion is leading the sightline. Clearly, several types of lead courses are possible. For instance, the fighter may lead the sightline by a fixed angle. More usually the amount of lead is made proportional to the sightline spin. It is of interest to take this a little further.

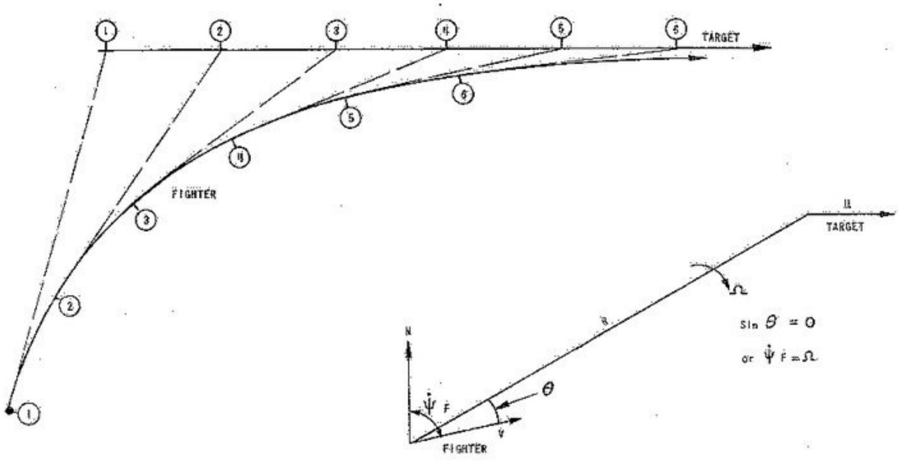
6. If the course is defined by:

$$\text{Sin } \theta = \frac{R}{M} \Omega$$

$$\text{Then } \Omega = \frac{M}{R} \text{ Sin } \theta$$

But by resolving the velocities, perpendicular to the sightline, it can be seen that:

$$\Omega = \frac{U \sin \phi - V \sin \theta}{R}$$



1—1—1 Fig. 10 Pure Pursuit Course

$$\therefore M \sin\theta = U \sin\phi - V \sin\theta$$

$$\therefore \frac{U}{\sin\theta} = \frac{V+M}{\sin\phi}$$

By inspection of triangle B of Fig. 11 and using the Sine rule it can be seen that a missile with a speed increment of M above the fighter speed is on a collision course. Thus a course defined by:

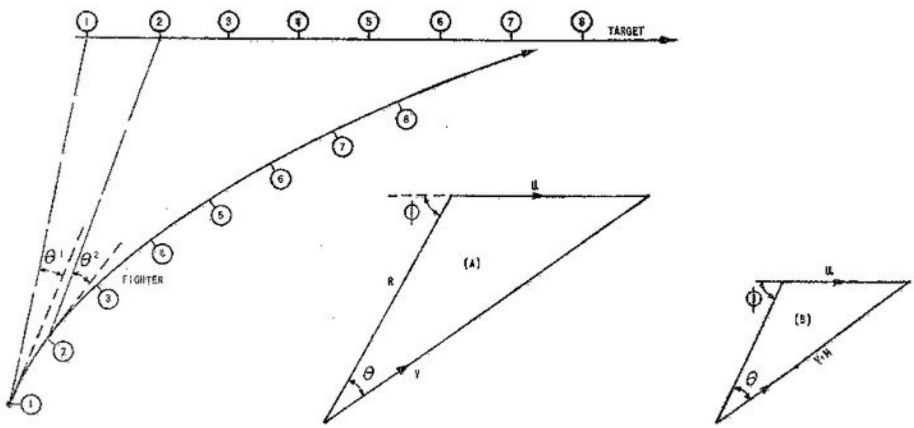
$$\sin\theta = \frac{R}{M}\Omega$$

represents a missile collision course for a missile which has an incremental mean speed of M. Note that the course constant R/M is changing throughout.

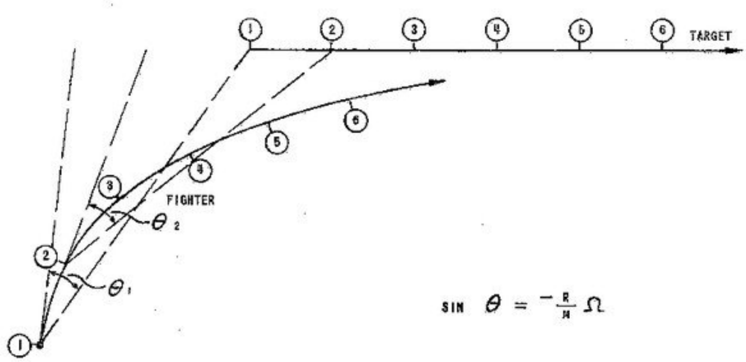
7. In some cases it could be desirable to get rapidly into the rear of the target. A similar type of course may be used but with the direction of motion behind the sightline (lag pursuit course).

Quasi Lead Pursuit

8. A lead pursuit course has a serious disadvantage in that range is needed to compute the course. Range information may be denied by enemy jamming of the radar. A modified lead pursuit course, often called



1—1—1 Fig. 11 Lead Pursuit Course



1—1—1 Fig. 12 Lag Pursuit Course

quasi lead pursuit, is used in which the course constant R/M is replaced by a fixed constant k. The course is now given by:

$$\text{Sin } \theta = k\,\Omega.$$

It is apparant that if $k = \frac{RQ}{M}$ the course would be missile collision at a range of R = RQ only.

Collision Course

9. A collision course is self explanatory; the fighter is always aiming for a point in space where it would collide with the target.

Fig. 13 shows by using the Sine rule, that to satisfy a collision condition:

$$\frac{V}{\text{Sin } \phi} = \frac{U}{\text{Sin } \theta}$$

$$\text{or Sin } \theta = \frac{U}{V} \text{ Sin } \phi$$

Resolving perpendicular to the sightline it can be seen that:

$$\Omega = \frac{U \text{ Sin } \phi - V \text{ Sin } \theta}{R}$$

but $U \text{ Sin } \phi = V \text{ Sin } \theta$

$\therefore \Omega = 0$ for a collision course, i.e. sightline spin is zero.

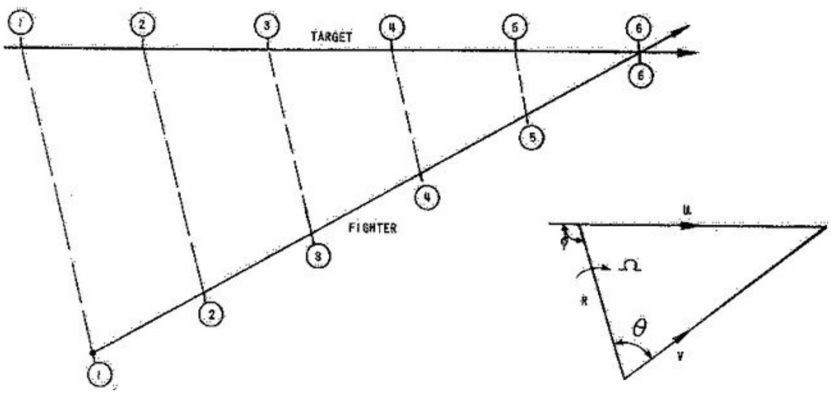
Proportional Navigation

10. A proportional navigation course is often used in weapon system work as it only requires sightline spin to compute and tends towards a collision course. The rate of turn of the fighter is made proportional to the sightline spin.

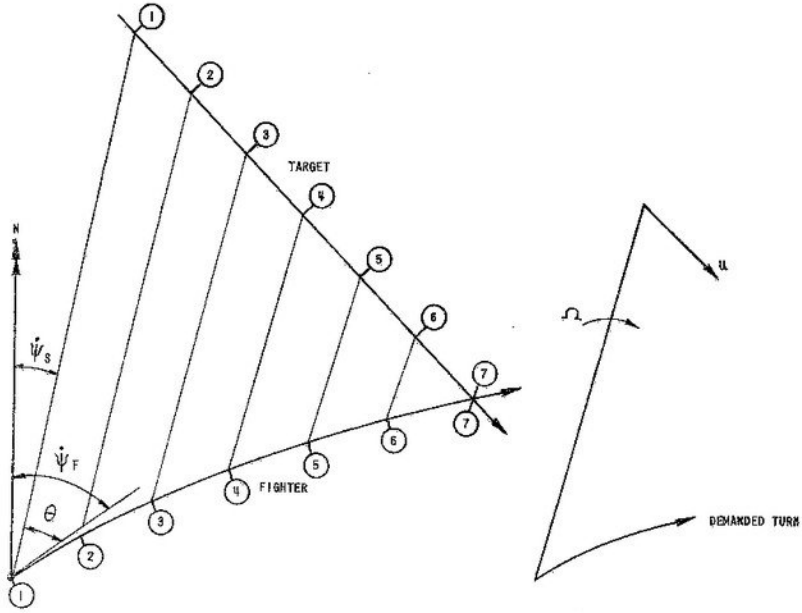
Thus the course is defined by:

Rate of turn of fighter = k x rate of turn of sightline or $\dot{\psi}_F = k\Omega = k\dot{\psi}_S$

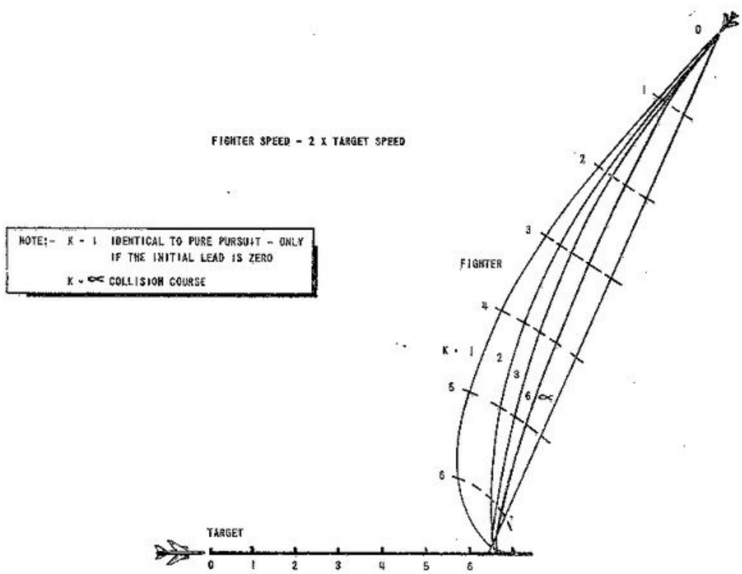
It should be noted that $\dot{\psi}_F$ does not equal $\dot{\theta}$. If the value of k is greater than 1, the fighter turns faster than the sightline and tends to reduce the sightline spin to zero, i.e. tends to a collision course. Pure pursuit is a special case of proportional navigation with k = 1 if the initial lead angle is 0°. Fig. 15 shows the effect of various course constants.



1—1—1 Fig. 13 Collision Course



1—1—1 Fig. 14 Proportional Navigation Course



1—1—1 Fig. 15 Effect of Various Course Constants

Straight Pass

11. As its name implies a straight pass course is a straight trajectory that passes through a point from which the missile is on a collision course. It is particularly favoured for unguided missiles, e.g. rockets, when used in beam attacks.

12. Consider a weapon that has a relative strike range F, i.e. a weapon that strikes the target at a range F in front of the fighter, commonly called the F pole. When the time to go to weapon strike is given by Tg, the diagram at Fig. 16 applies.

The sightline spin is given by:

$$\Omega = \frac{U \sin\phi - V \sin\theta}{R}$$

By applying the Sine rule to the triangle A:

$$\frac{UTg}{\sin\theta} = \frac{V\,Tg + F}{\sin\phi}$$

Rearranging gives:

$$\sin\theta = Tg\,\frac{(U \sin\phi - V \sin\theta)}{F} = \frac{Tg(R\Omega)}{F}$$

Thus, this is similar to a lead pursuit course except that the course constant $\frac{R}{M}$ is replaced by $\frac{R\,Tg}{F}$. In order to compute this course it is necessary to know Tg.

Using the same triangle,

$$R = U\,Tg \cos(180 - \phi) + (V\,Tg + F) \cos\theta$$

$$\text{or } R = (V\,Tg + F)\cos\theta - U\,Tg \cos\phi$$

$$\text{Rearranging, } Tg = \frac{R - F\cos\theta}{V\cos\theta - U\cos\phi}$$

and it can be seen that the denominator is the same as the range rate $\frac{dR}{dT}$ or $\dot{R}$.

$$\text{or } Tg = \frac{R - F\cos\theta}{\dot{R}}$$

Since the course is a straight line course it is possible to calculate the minimum fighter to target separation distance.

Using the Cosine rule,

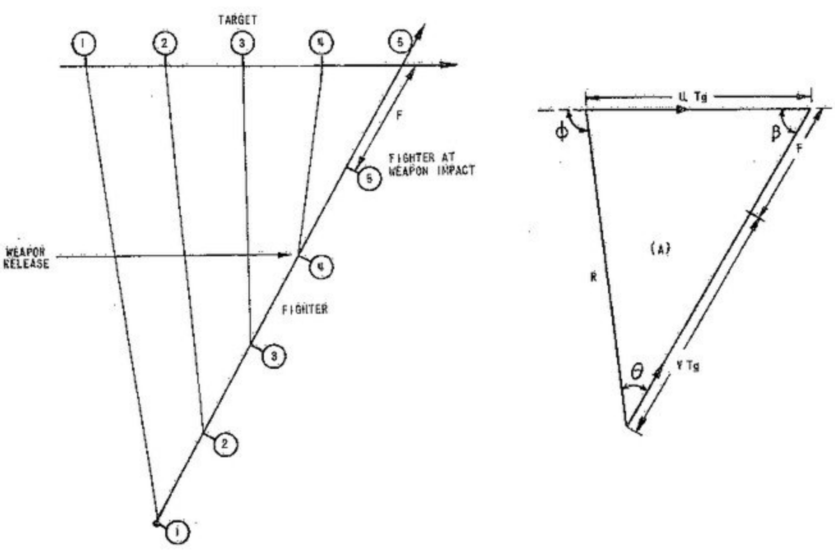
$$R^2 = (UT)^2 + (VT + F)^2 + 2(UT)(VT + F)\cos\beta$$

Rearranging, $R^2 =$

$$(U^2 + V^2 + 2\,UV\cos\beta)\,T^2 + 2F\,(V + U\cos\beta)\,T + F^2$$

In order to obtain a minimum value for R the expression must be differentiated with respect to T

$$2R.\dot{R} = (U^2 + V^2 + 2UV\cos\beta).2T + 2F\,(V + U\cos\beta)$$



1—1—1 Fig. 16 Straight Pass

for R to be minimum, $\dot{R}$ is zero.

$$T = \frac{-F (U + V \cos \beta)}{(U^2 + V^2 + 2UV \cos \beta)}$$

substituting back and rearranging gives

$$R^2_{min} = \frac{F^2U^2 - F^2U^2 \cos^2 \beta}{(U^2 + V^2 + 2UV \cos \beta)}$$

or $$R_{min} = \frac{F U \sin \beta}{\sqrt{U^2 + V^2 + 2UV \cos \beta}}$$

The denominator here is the relative velocity of the fighter to the target which approximates to $\dot{R}$ or

$$R_{min} = \frac{F U \sin \beta}{\dot{R}}$$

Now from the triangle it can be seen that

$$R \sin \theta = U T \sin \beta$$

and

$$\sin \theta = \frac{R \Omega T}{F}$$

$$\therefore \quad R_{min} = \text{pass distance } D = \frac{R^2 \Omega}{\dot{R}} \text{ (approx.)}$$

13. It can be seen that all the above courses may be calculated if range, range rate, sightline spin and look angle are known. It may be desirable to use several of these courses in combination as will be shown later. Similarly, it may be necessary to use different courses in different planes.