CHAPTER II.—MAGNETISM

PERMANENT MAGNETISM

Magnetic polarity

1. Everyone is familiar with the toy horse-shoe magnet, so called from its shape. It has the property of attracting iron filings, pins, needles and in fact, any small pieces of iron or steel. Such a magnet is called a permanent magnet because the attractive force possessed by it is inherent in the magnet itself and does not depend upon any external influence. Actually any permanent magnet slowly loses its magnetic properties, the degree to which they are retained depending upon the retentivity of the material of which the magnet is made.

A more convenient form of magnet for experimental purposes is the bar magnet, which can be made from an ordinary steel knitting needle by stroking it in one direction only with one pole of a permanent magnet. If such a bar magnet is suspended horizontally, as shewn in fig. 1a, it will be found to take up such a position that its axis lies approximately north and south. If one end is marked, it will be found that no matter how the magnet is displaced, the same end eventually comes to rest pointing roughly towards the north. The ends of the magnet are therefore called the north-seeking end (or " pole ") and south-seeking end (or " pole ") respectively.

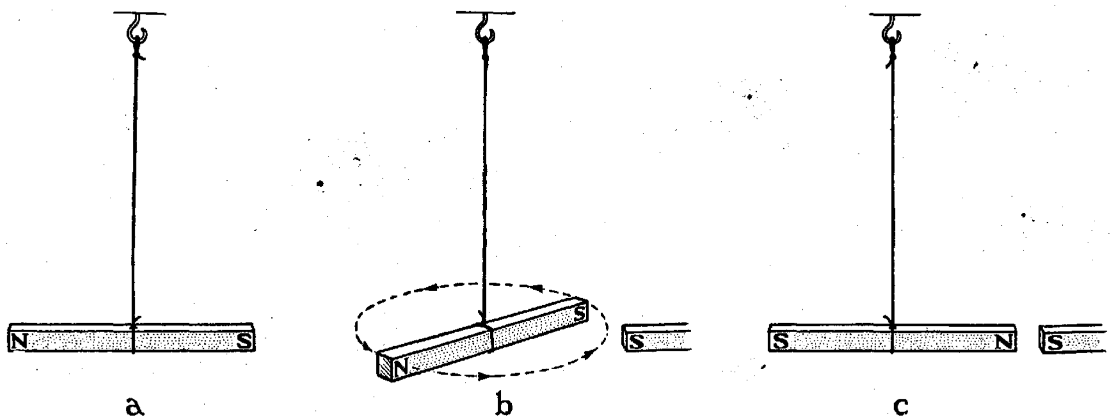


FIG. 1, CHAP. II.—Suspended bar magnets.

This pole-seeking property is due to the fact that the earth itself is a huge magnet, having south-seeking and north-seeking poles situated near, but not coincident with, the geographical poles. The north-seeking pole of a magnet is that which is attracted towards the geographical north region of the earth, but it is impossible to allot north-seeking or south-seeking properties to the earth itself for north or south has no significance except in relation to that body. If instead of the terms north-seeking and south-seeking, the poles are considered to be positive and negative respectively, the geographical north region of the earth is of negative polarity. It is desirable to avoid the use of the terms " north-pole " and " south-pole " when referring to magnets, substituting " N-pole " and " S-pole ", which signify " north-seeking " and " south-seeking " poles. The attractive force of a magnet is most concentrated in the neighbourhood of its poles. This is easily shewn by dipping a magnet into iron filings, and noting in which regions most filings adhere.

The mutual action which takes place between magnets can easily be demonstrated, e.g. if the south-seeking pole of another magnet is brought near to the south-seeking pole of the suspended one, as in fig. 1b, it will be found that repulsion takes place, while if the north-seeking pole is presented to it attraction occurs, fig. 1c. The first law of magnetism is that " like poles repel, and unlike poles attract each other".

2. If a bar magnet is laid upon a table, and a small compass needle—which is merely a pivoted bar magnet—is moved about from point to point in its vicinity, the needle will be found to vary

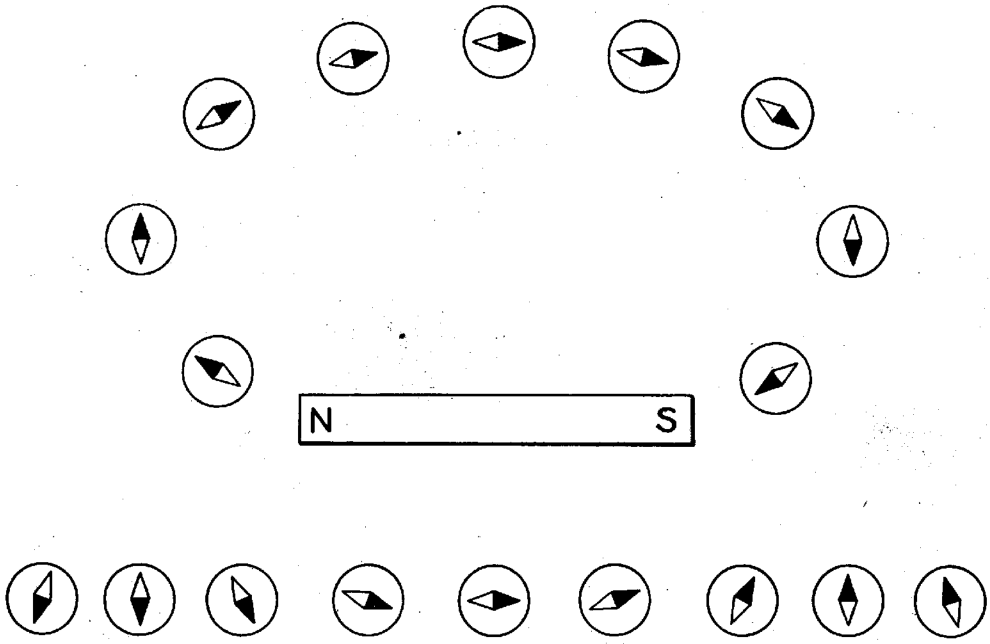


FIG. 2, CHAP. II.—Compass needles in magnetic field.

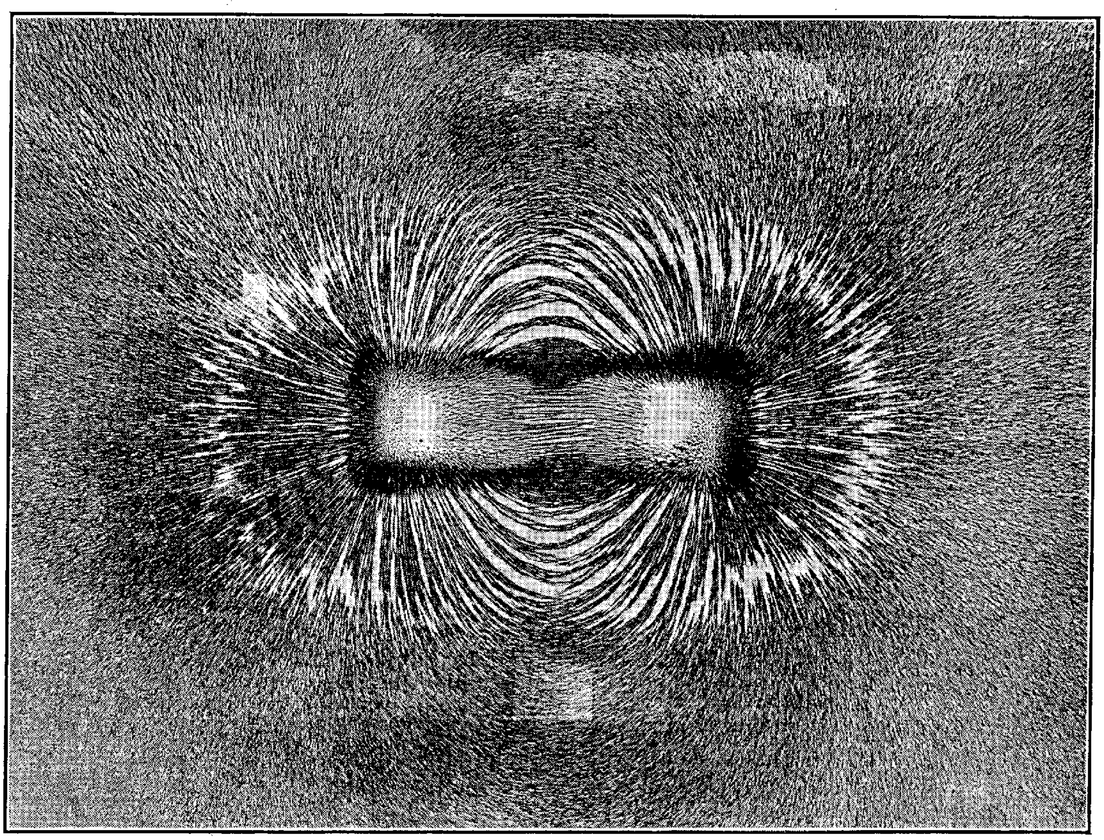


FIG. 3, CHAP. II.—Lines of force shown by iron filings around magnet.

its direction in each individual position. Fig. 2 shews the various directions taken up by the needle in such an experiment. The needle shews the direction of the magnetic force at the point at which it is situated. The magnetic field of a magnet is the region in which its action can be observed, and this field may be said to consist of lines of magnetic force, a line of force being defined as the imaginary line along which the force of a magnet acts. Faraday's conception of the properties of electric lines of force was explained in Chapter I, and it may be assumed that magnetic lines of force behave in an identical manner.

By a long-standing convention the positive direction or sense of the lines of force is taken as that direction in which a north-seeking pole would be urged if it were free to move. Thus it may be considered that in the external magnetic circuit the lines of force flow from north-seeking to south-seeking poles externally, and from south-seeking to north-seeking poles inside the magnet, each magnetic line being a complete closed loop. In diagrams an arrow head is usually placed upon some of the lines to indicate the sense of the field. From this idea of " flowing " we are led to speak of the magnetic flux of a magnet, as synonymous with the total magnetic field.

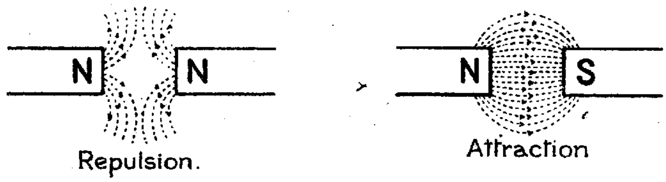


Fig. 4, Chap. II.—Lines of force between adjacent magnetic poles.

3. A map or picture of the magnetic field in any one plane can be made by sifting fine iron filings from a small sieve on to a sheet of tracing paper under which a magnet has been placed. The filings then align themselves in the direction of the lines of force. It will be found advantageous to tap the sheet gently as the filings fall on it. Fig. 3 is reproduced from a photograph of the magnetic field of a magnet, which was produced in this way, while fig. 4 shews qualitatively the distribution of the field in the neighbourhood of (i) two unlike poles and (ii) two like poles, placed adjacent to each other.

Theory of molecular magnetism

4. The process by which a piece of iron or steel becomes magnetised has been explained as follows. The molecules of such substances are themselves minute magnets. In the non-magnetised state these molecular magnets align themselves into closed magnetic chains as shewn in fig. 5, in which the molecules are represented by small rectangles, the black portion being of north-seeking and the white portion of south-seeking polarity. Each little chain forms a

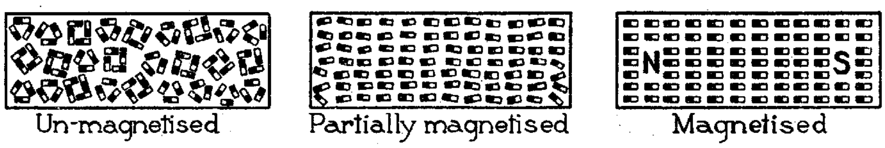


Fig. 5, Chap. II.—Arrangement of molecules in magnetic substance.

complete magnetic circuit and no force is exerted externally. When the substance is magnetised, however, these molecular magnets are dragged into alignment along the axis of the magnet, the lines of force associated with them being now completed through the space surrounding the iron. The shape of the magnetisation curve of iron (*see* para. 22) strongly supports this theory. The difference in the behaviour of iron and steel in this respect compared with most other materials is not yet understood.

Induced magnetism

5. If a piece of soft iron be placed in the field of a permanent magnet, and the resulting field plotted by the iron filings method, the result is as shewn in fig. 6. It will be seen that the

soft iron now appears to have a magnetic field of its own and is said to be magnetised by induction. Consideration of the polarity of the permanent magnet and the soft iron shews that the adjacent poles are unlike and will therefore attract each other. It is in this way that all attraction between magnets and unmagnetised bodies commences, and may be summed up in the statement that " induction always precedes attraction ".

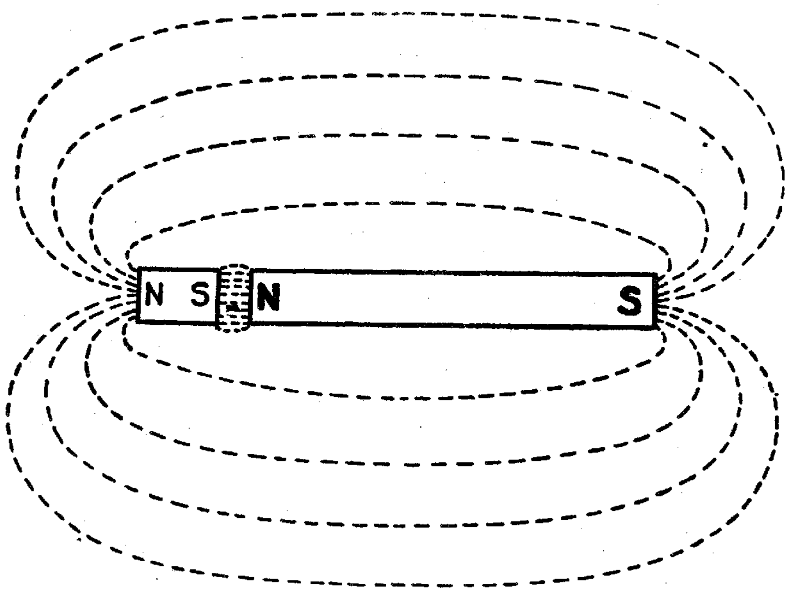


FIG. 6, CHAP. II.—Induced magnetism in soft iron.

ELECTROMAGNETISM

6. In the preceding paragraphs only the phenomena associated with or due to permanent magnets has been considered. The connection between magnetism and electricity was suspected for many years before it was finally established that an electron in motion always produced a magnetic field. Since an electric current consists of a motion of electrons, or possibly a motion of positive and negative ions, each of the latter possessing one or more surplus electrons, we may say that an electric current produces a magnetic field. This field forms circles concentric with the axis of the current-carrying conductor. It is important in certain circumstances (*see* Chapter V) to remember that this field exists both inside and outside the conductor. It can be demonstrated by the use of iron filings as in the case of a permanent magnet, and is shewn diagrammatically in fig. 7a. The direction of the field is shewn conventionally by the arrows, and it will be seen that the direction of the current and the direction of the magnetic field are related in the same way as the thrust and turn respectively of the ordinary corkscrew. This useful mnemonic is usually known as " Maxwell's corkscrew rule".

An immediate practical application of this rule is to indicate the direction of current in a wire. If a conductor carrying a current is laid over and parallel with the compass needle, the needle will be deflected, tending to place itself at right angles to the conductor. The actual angular deflection in any particular case will depend upon the strength of the magnetic field due to the current, and to the controlling force of the earth's magnetic field. If the direction of the current in the wire is from south to north, the north-seeking pole of the needle will be deflected to the left, i.e. toward the west, while if the direction of the current is reversed, the deflection will be toward the east. This is most easily remembered by Ampere's " swimming rule " :—

" Consider a man swimming face downwards with the current in the conductor, over the compass needle. The north-seeking pole of the latter will be deflected towards his left."

7. If two wires carrying currents are placed parallel to each other the resulting magnetic field may be depicted as in figs. 7b and 7c. In fig. 7b the currents are in opposite directions, and in fig. 7c they are in the same direction. The conductors in the former case tend to attract and in the latter to repel each other. An experiment to show this can easily be performed, the apparatus being arranged as in fig. 8. Two light metal rods are suspended from a suitable support,

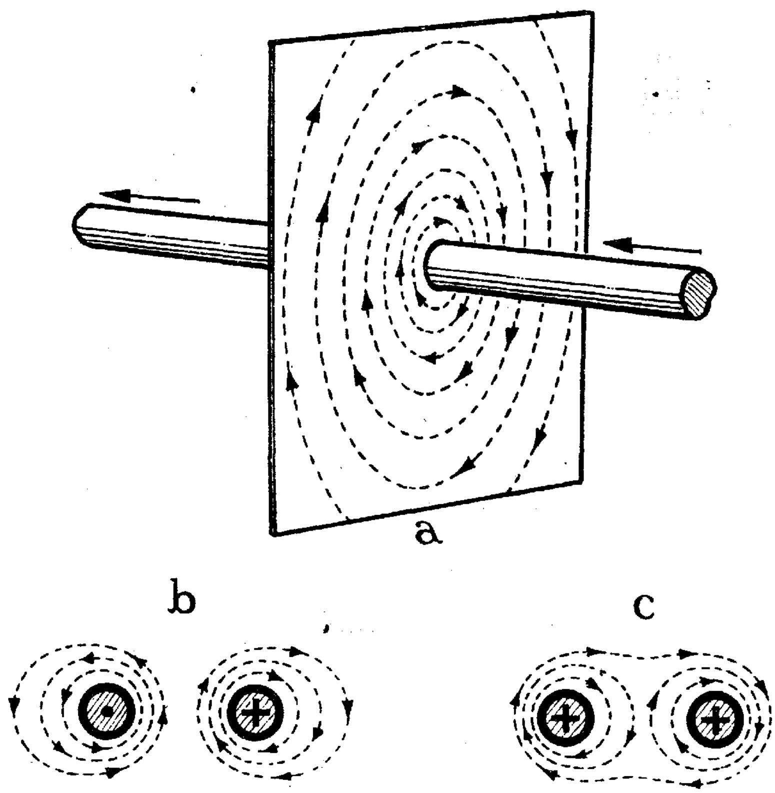


FIG. 7, CHAP. II.—Magnetic fields around conductors.

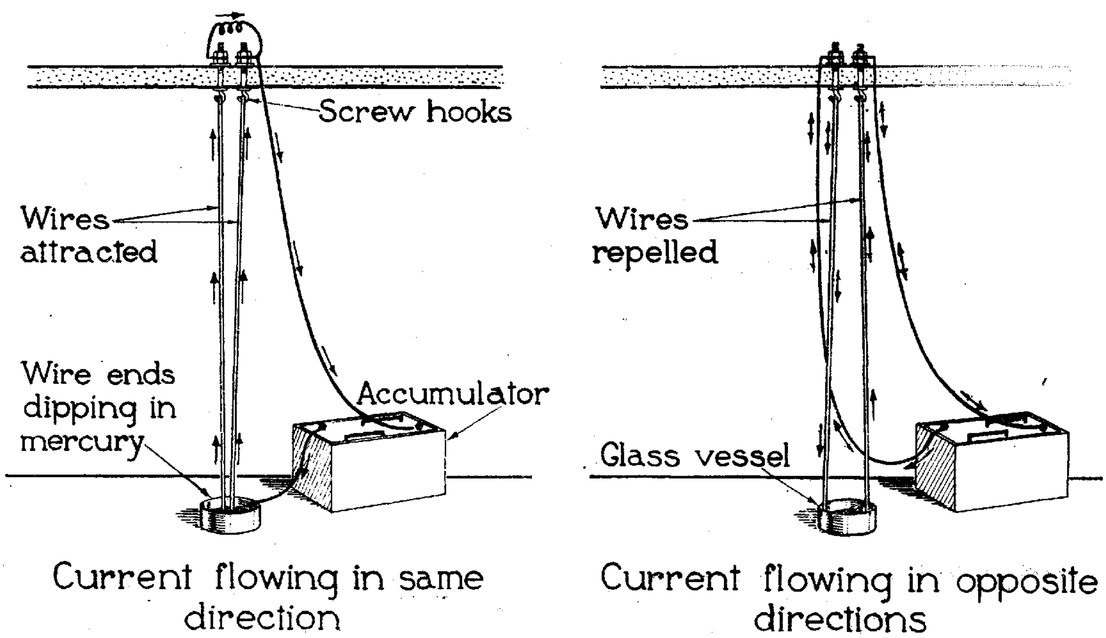


FIG. 8, CHAP. II.—Magnetic action of parallel currents.

which serves also to insulate them from each other. The lower ends of the rods dip into a vessel containing mercury, and electric current is supplied by means of a battery. By making suitable connections, the currents may be made to flow either in the same or in opposite directions in each wire, and the resulting attraction or repulsion may be observed by the creation of ripples upon the surface of the mercury. The magnetic fields caused by the current in each wire tend to remain concentric with the conductors, and the latter move in order to allow this. In practice the foregoing effect has to be allowed for when designing parallel conductors carrying very large currents, e.g., the " bus bars " in a large power station.

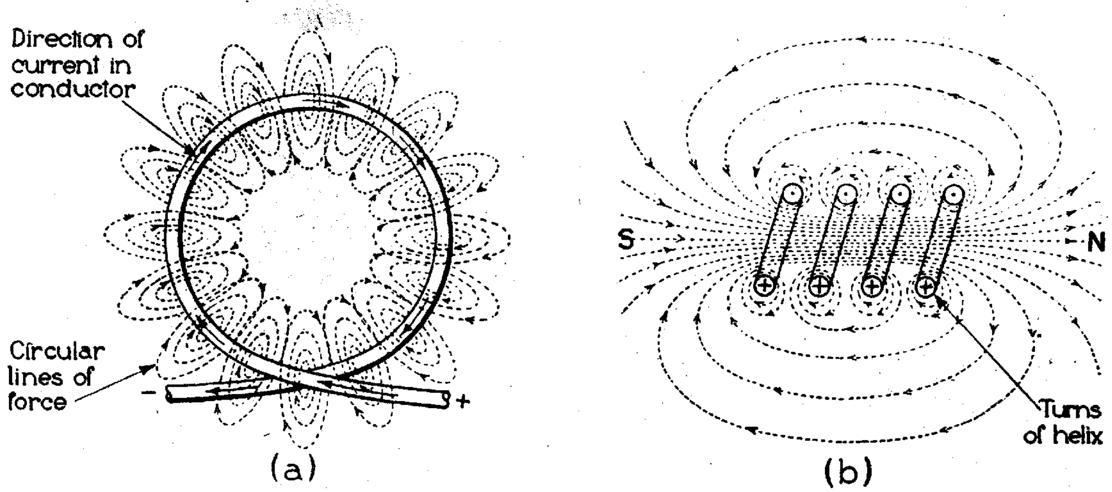


FIG. 9, CHAP. II.—Magnetic fields around coiled conductors.

8. If the current carrying conductor is bent into a single loop as in fig. 9a the relative configuration of its magnetic field will not be altered, but all the lines of force will now enter one face of the loop and emerge on the other. Several continuous loops wound in this way, forming a spiral or helix are said to constitute a solenoid. The configuration of the magnetic field of a solenoid is shown in fig. 9b, and it will be seen that the resultant field is similar to that of a bar magnet. All the phenomena associated with a bar magnet can, in fact, be reproduced equally well by the solenoid. The magnetic polarity of a solenoid depends upon the direction of current round the windings, and is generally found by the following rule. " Looking at one end of the coil, if the current flows in a clockwise direction, the end nearest the observer is a south-seeking pole. If current is in anti-clockwise direction, the nearest end is a north-seeking pole." The appropriate mnemonic for this rule is shown in fig. 10.

FIG. 10, CHAP. II.—Mnemonic for polarity of solenoid.

The galvanometer

9. The deflection of a magnetic needle by the current in a single conductor will be increased if the current is increased, or if the number of current-carrying conductors affecting the magnet is increased, provided that these conductors are so arranged that their effect is cumulative. This is easily achieved by winding a short coil of many turns of wire on a suitable " former," the magnetic needle being pivoted in the centre. Such an arrangement is called a galvanometer.

By applying Ampere's Swimming Rule, it will be observed that the current in the portions of conductor above the needle tend to deflect the latter in the same direction as those beneath the needle, so that by using a great many turns of fine wire, very small currents may be detected. The essential portions of this type of galvanometer are shown in fig. 11. Other, and more sensitive, types of current indicator will be discussed in the chapter devoted to Measuring Instruments.

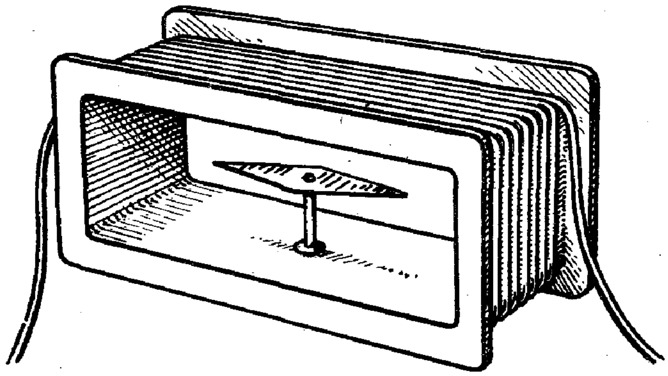


FIG. 11, CHAP. II.—Galvanometer.

THE ELECTRO-MAGNETIC SYSTEM OF UNITS (E.M.U.)

Magnetic field strength

10. If two like poles of equal strength, situated 1 cm. apart in vacuo, repel each other with a force of one dyne, they are said to be unit poles, or to possess unit pole strength.

The force exerted between such poles at any distance is inversely proportional to the square of the distance between them. This is the Second Law of Magnetism and was demonstrated by Coulomb. Suppose an isolated pole has n lines of force associated with it, and is surrounded by a spherical surface of r cm. radius, the pole being at its centre. Since the area of this surface is $4\pi r^2$ sq. cm. the number of lines of force passing through each square centimetre will be $\frac{n}{4\pi r^2}$. If another concentric spherical surface is situated at a distance of $2r$ centimetres from the pole, the number of lines passing through each square centimetre will be $\frac{n}{4\pi(2r)^2}$ or $\frac{n}{16\pi r^2}$. Thus at twice the distance from the pole, only one-fourth of the lines of force are effective in any given area.

The strength or intensity of a magnetic field at any point is measured by the force exerted on a unit pole if placed at that point, and is denoted by the symbol H. Magnetic field strength is measured in dynes per unit magnetic pole, thus a pole of strength m units sets up at a distance d cms. from it in a vacuum a magnetic field strength of $\frac{m}{d^2}$ dynes per unit magnetic pole. The name " oersted " is sometimes used for a field strength of one dyne per unit pole.

Magnetic Flux

11. The total number of lines of force flowing from the north seeking or N-pole to the south seeking or S-pole has already been referred to as the magnetic flux. The lines of force bounding any area in the field perpendicular to its direction may be considered to enclose a tube of flux. From a unit magnetic pole it is assumed that one unit tube of flux passes through each square centimetre of the surface of a sphere one centimetre in radius, having the pole as its centre. The surface area of this sphere being 4π square centimetres it follows that 4π unit tubes of flux are assumed to emanate from or terminate upon a unit pole. The unit of magnetic flux is the unit tube or Maxwell, the symbol for magnetic flux being Φ.

The moment of a magnet is the product of the pole strength and the distance between the poles. If the pole strength is m unit poles and they are l centimetres apart, the moment is ml units.

The intensity of magnetisation of a magnet is its moment per unit volume or $\frac{ml}{v}$. If the magnet is of uniform cross section A the volume v will be lA, and the intensity of magnetism which is denoted by J will be $\frac{ml}{lA}$ or $\frac{m}{A}$. Hence the important result that the intensity of magnetisation is synonomous with the pole strength per unit area.

Flux density is defined as the number of unit tubes of flux passing through unit area in the field perpendicular to its direction. In a vacuum a magnetic field of strength H oersteds sets up a magnetic flux density of B_0 tubes per square centimetre, B_0 being numerically equal to H although its unit is different. In a uniform magnetic field of area A and strength H oersteds the total magnetic flux is B_0A tubes, which is numerically equal to HA. The name gauss is given to the unit of flux density, one gauss being equal to one unit tube per square centimetre.

12. If a piece of soft iron of uniform cross section is introduced into the uniform field just mentioned, the iron will be magnetised by induction, and it will develop magnetic poles upon (or near) each surface perpendicular to the field. Suppose these poles each to be of pole strength m units, then the intensity of magnetisation upon each of these surfaces will be $J = \frac{m}{A}$. The soft iron now has its own field, the flux through the iron due to its own magnetism being 4π times the pole strength at either end, and the total flux will therefore be B_0A due to the magnetising agency and $4\pi m$ due to the induction, or, putting for the total flux Φ,

$$\Phi = 4\pi m + B_0A$$

The flux density in the iron, $B = \frac{\Phi}{A}$

$$B = \frac{4\pi m}{A} + B_0$$

$$= 4\pi J + B_0$$

$$= B_0\left(\frac{4\pi J}{B_0} + 1\right)$$

Since Bo is equal numerically to H, the field strength causing the magnetisation,

$$B = H\left(\frac{4\pi J}{H} + 1\right)$$

The factor $\left(\frac{4\pi J}{H} + 1\right)$ is called the permeability of the material in which the field is situated, and is denoted by μ. The relation between B and H is therefore written $B = \mu H$.

Permeability

13. (i) The permeability of a medium may be defined as the numerical ratio of the flux density to the magnetic field strength, or alternatively as the ratio of the flux density in the medium to the flux density in vacuo for the same value of magnetic field strength. The absolute value of the permeability is not known, but it is assumed to be unity in vacuo, and the value for other materials is then only comparative. According to the nature of their permeability, materials are divided into ferro-magnetic, dia-magnetic and para-magnetic materials. Ferro-magnetic materials are those whose permeability is large compared to unity. A characteristic of these substances is that μ itself is not constant but varies according to the flux density, the nature of this variation being shown by the slope of the magnetisation or B/H curve of the material,

as shown in fig. 12. Iron, steel, nickel and cobalt and many, but by no means all, of their compounds are ferro-magnetic, and included in this group are the manganese bronzes discovered by Heusler in 1898, which have magnetic properties somewhat resembling those of cast iron.

(ii) Para-magnetic substances are those which have a permeability which is constant for all values of flux density and is only slightly greater than unity. They behave magnetically like iron, but to a much smaller degree, while dia-magnetic substances behave in an entirely opposite manner, the permeability being constant for all values of flux density, but slightly smaller than unity. The most dia-magnetic substance known is bismuth, and a ball of bismuth is not attracted by a magnet but repelled, while a bar of bismuth placed in a magnetic field sets itself in a transverse direction instead of in line with the direction of the field as a ferro-magnetic bar would do. The separation of materials into para-magnetic and dia-magnetic classes is not yet complete, since

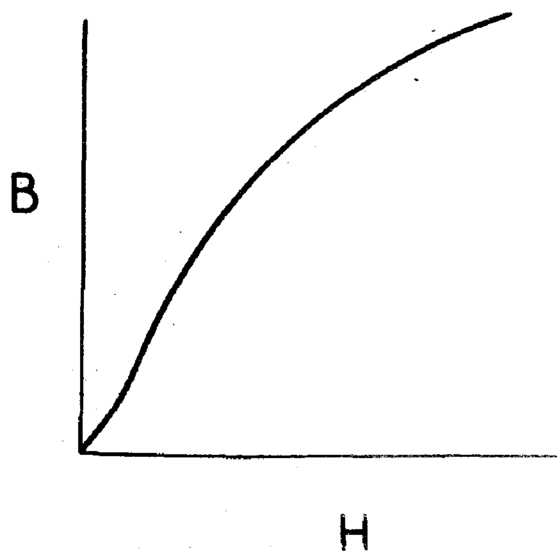


FIG. 12, CHAP. II.—B/H curve.

the measurements necessary are so delicate and usually have to be made in air (which is itself para-magnetic) in the presence of the earth's magnetic field. It is now believed that no substance is absolutely non-magnetic, and a recent tabulation of magnetic properties of various substances gives solid oxygen, manganese, iron oxide, platinum, chromium, tantalum, and aluminium as definitely para-magnetic; sodium, potassium, and wood as doubtful; copper, sulphur, glass, zinc, quartz, lead, silver, gold, mercury, and bismuth as dia-magnetic, the most para-magnetic materials being mentioned first, and the most dia-magnetic last. It must be again emphasised that in para-magnetics and dia-magnetics, the permeability only differs from unity by a few parts in a thousand, e.g. for solid oxygen, $\mu \doteqdot 1 \cdot 0053$, while for bismuth $\mu \doteqdot \cdot 99983$. In ordinary engineering practice it is therefore usual to consider that all materials other than the ferro-magnetics have a permeability of unity.

Difference of magnetic potential

14. Consider any two points in space, A and B, and let a unit north-seeking pole be transferred from A to B in the presence of magnetic forces in the region between the two points. In doing this, a definite amount of work is performed, which is independent of the shape of the path along which the unit pole is moved. The amount of work (in ergs per unit pole) is the difference of magnetic potential between the points A and B. If work is done on the charge in moving from A to B, B is at a higher potential than A, while if work is done by the magnetic forces themselves, A is at a higher potential than B, that is a north-seeking pole tends to move from a point of high to a point of low potential.

The electromagnetic unit of current

15. Since a magnetic field can be produced by an electric current, it is desirable that unit current should be capable of definition in terms of its magnetic effect. The electromagnetic unit of current is the current which, when flowing in a wire bent into an arc of a circle of 1 centimetre length and 1 centimetre radius, produces unit magnetic field strength at the centre of the arc. It may also be defined as the current which, flowing in a single circular turn of 1 centimetre radius, produces at the centre of the circle a magnetic field strength of 2π units. The E.M.U. of current is equal to 10 amperes, and since the unit of quantity is unit current × unit time, it follows that the E.M.U. of quantity is 10 coulombs. Having defined the E.M.U. of quantity the E.M. units of electric P.D. and resistance follow :—

(i) Unit P.D. is the P.D. between two points when the work done in carrying unit quantity from one point to the other is 1 erg.

$$\text{1 unit of P.D. (E.M.U.)} = \frac{\text{1 erg}}{\text{1 unit of Quantity (E.M.U.)}} = \frac{\frac{1}{10^7}\text{ joule}}{\text{10 coulomb}} = \frac{1}{10^8}\frac{\text{joule}}{\text{coulomb}} = \frac{1}{10^8}\text{ volt.}$$

$\therefore$ 1 volt $= 10^8$ electromagnetic units of P.D. or E.M.F.

(ii) Unit resistance (E.M.U.)

$$\text{1 ohm} = \frac{\text{1 volt}}{\text{1 amp.}} = \frac{10^8\text{ E.M.U. P.D.}}{\frac{1}{10}\text{ E.M.U. current}}$$

$$= 10^9 \text{ electromagnetic units of resistance.}$$

Magnetic field strength at the centre of a current-carrying loop

16. Consider a conductor carrying a current of I E.M. Units, bent into a circular loop of radius r centimetres. Then the magnetic field strength set up at the centre, by a very short element of the conductor of length δl, is δH, and $\delta H = \frac{I.\delta l}{r^2}$. The total field strength H is the sum of all such elements of field strength due to the whole length of the conductor, which is $2\pi r$ centimetres, and therefore $H = \frac{I}{r^2} \times 2\pi r = \frac{2\pi I}{r}$ dynes per unit magnetic pole. If the current is in amperes, then $H = \frac{2\pi I}{10r}$ dynes per unit magnetic pole, or oersteds.

Force exerted upon a conductor in a magnetic field

17. (i) If a unit pole is placed at the centre of a circular loop, which is carrying a current of I E.M. Units there will be a mutual force between the unit pole and the equivalent pole producing the magnetic field of strength H. Since this equivalent pole is the conductor itself, the force will be radial, and therefore perpendicular to the length of the conductor. This result is perfectly general, the force between a magnetic field and a conductor always tending to urge the conductor in a direction perpendicular to its length and also perpendicular to the lines of force. A straight conductor of length l centimetres, carrying a current of I E.M. Units, when placed in a uniform field of strength H dynes per unit magnetic pole, will experience a force of HlI dynes in a direction mutually perpendicular to both conductor and field.

(ii) The direction in which the conductor tends to move may be deduced from a consideration of the elastic properties of the imaginary lines of force. Fig. 13 shews a conductor situated in a magnetic field, and carrying a current which is assumed to flow out of the paper towards the reader. At (*a*) the magnetic field due to the current is shewn superimposed upon the original field, and it will be observed that on the left hand side of the conductor the lines of force are in the same direction, so that in this region the effect of the current is to strengthen the field, while on the right hand side of the conductor the field of the latter is in opposition to the original field and tends to reduce the field strength on this side. In reality the two fields of the magnet and conductor respectively combine to form a resultant field as indicated at (*b*). Owing to the repulsive action of parallel lines of force acting in the same direction, the distribution of the flux tends to become uniform, and if the conductor is free to move it will be displaced to the right, which is the region in which the field is weakest. The action may be attributed to the tendency of the lines composing the distorted magnetic field on the left of the conductor to

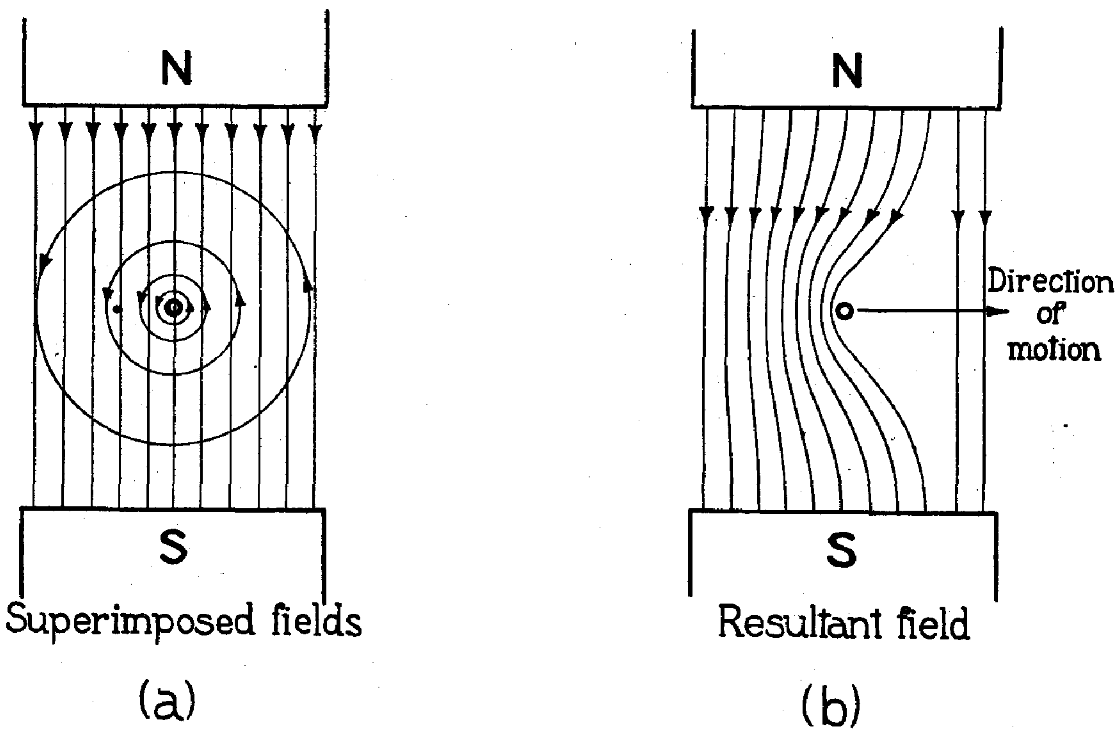


FIG. 13, CHAP. II.—**Movement of current-carrying conductor in magnetic field.**

shorten as much as possible and also to repel each other, so that the field tends to resume its original uniform distribution. The direction in which the conductor will move may be remembered by Fleming's Left Hand Rule, i.e. extend the thumb, forefinger and middle finger of the left hand in mutually perpendicular directions. Place the **F**orefinger in the direction of the **F**ield, the m**I**ddle finger in the direction of the current (**I**) and the thu**M**b then indicates the direction of **M**otion of the conductor.

Work done by change of flux

18. If a conductor of length l centimetres carrying a current of I E.M.U. is placed in a field of strength H dynes per unit magnetic pole, the force acting on it, from the above discussion, is HlI dynes. Now let the conductor be moved in opposition to this force through a distance of d centimetres, the work done being $HIld$ dyne-cms. or ergs, $= W$. Now H is numerically equal to the flux density, if the permeability of the medium is unity, ld is the area A swept by the conductor and HA is the change of flux linking with the electric circuit. The work performed

is therefore equal to the product of the current and the change of flux, and this result is found to be true for any shape of electric circuit. Algebraically, $W = I\ (\Phi_2 - \Phi_1)$, Φ_2 being the flux enclosed before the conductor is moved and Φ_1 the flux enclosed after the operation has taken place.

The average rate at which this work is done, or the power expended, is

$\frac{W}{t} = I\left(\frac{\Phi_2 - \Phi_1}{t}\right)$ ergs per second. Power being the product of E.M.F. and current, it is apparent that the expenditure of mechanical energy has resulted in the production of an E.M.F.

Since $$\frac{W}{t} = EI,\ EI = I\,\frac{(\Phi_2 - \Phi_1)}{t}.$$

$$E = \frac{\Phi_2 - \Phi_1}{t} \text{ (E.M. units of E.M.F.)}$$

$$\text{or } E = \frac{\Phi_2 - \Phi_1}{10^8 t} \text{ volts.}$$

It should be observed that the creation of this E.M.F. is due to the conversion of mechanical energy into electrical energy. There has been no expenditure of magnetic energy although the magnetic flux played an important part in the transformation.

Magneto-motive force

19. The work done in carrying a unit pole round any closed path in a magnetic field is called the magneto-motive force in that path. First of all, let us suppose that the magnetic field is produced by a single circular turn of current-carrying conductor, and that the path of the pole does not encircle the conductor. Then if work has to be done on the pole in moving it towards the conductor, an equal amount of work is done on the pole by the magnetic forces in moving it away from the conductor, and in taking the pole round the complete path the total external energy supplied is zero. That is, the M.M.F. round a closed path not encircling a conductor is zero. If however a unit pole is carried round a path encircling a conductor, which carries a current of I E.M.U., every line from the pole will link with the circuit, the total number of tubes of flux linking with the conductor, will be 4π tubes (by definition of a unit pole). The M.M.F. round this path will therefore be $4\pi\ I$ ergs, and is independent of the shape of the path. Again, if instead of a single loop we have a solenoid of N turns, and the unit pole is taken round a path linking with all the turns, the work done will be $4\ \pi IN$ ergs. The number of flux linkages is equal to the actual flux linking with the conductor multiplied by the number of turns linking with the flux.

Field strength inside a toroidal coil

20. Imagine a long thin solenoid uniformly wound with an insulated conductor, upon a cylindrical rod and carrying N turns in all; after winding the coil may be removed from the rod and the ends of the coil brought round to meet each other so that the winding forms a ring of radius r cms. Such a coil is called a toroid, or toroidal coil, and is shewn in fig. 14.

Suppose a current if I E.M. Units is flowing through the conductor. If a unit magnetic pole is taken round the mean circumference of the ring threading each turn of wire in succession the work done will be $2\ \pi\ r\ H$ ergs, H being the field strength inside the coil, which is at present unknown. But $2\ \pi r\ H$ ergs is also the M.M.F. round the path, and the latter quantity has been shewn to be $4\ \pi NI$ ergs. Equating the two expressions

$$2\pi r H = 4\pi NI$$

$$H = \frac{4\pi NI}{2\pi r} = \frac{4\pi NI}{l}$$

where l is the length of path of the unit magnetic pole, or $2\pi r$ cms.

If we imagine further that the radius r increases without limit, any portion of the length of the winding may be considered as approximately a straight solenoid, and so for a solenoid whose length is large compared to its diameter the field strength at its centre is given by the expression

$$H = \frac{4\pi NI}{l} \text{ if } I \text{ is in E.M.U.}$$

and $$H = \frac{4\pi NI}{10l} \text{ if } I \text{ is in amperes.}$$

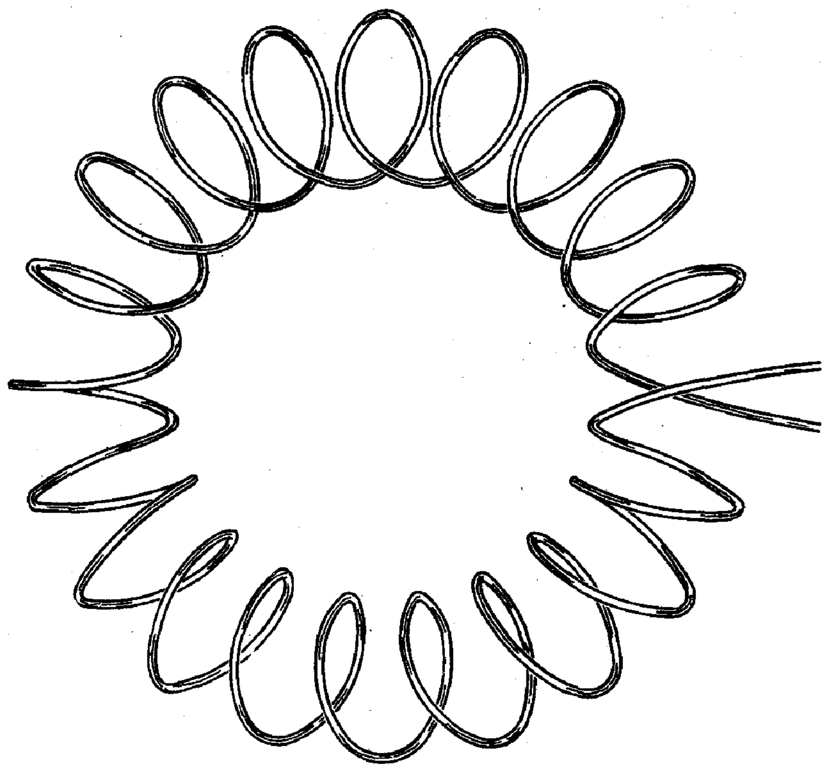


FIG. 14, CHAP. II.—Toroidal coil.

The quantity denoted by H is sometimes referred to as the magnetising force of the solenoid.

Suppose the space inside the winding to be filled with a substance of permeability μ. Then the flux density inside the material is B, or μH tubes per square centimetre, and the total flux is BA tubes.

$$\Phi = \frac{4\pi IN}{10\,l} \times \mu A \text{ maxwells.}$$

This equation is sometimes called Ohm's law for the magnetic circuit. It may be written

$$\Phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{\text{M.M.F.}}{S}$$

$$\text{where M.M.F.} = \frac{4\pi IN}{10}$$

$$S = \frac{l}{A\mu} = \text{the reluctance of the magnetic circuit.}$$

The magneto-motive force (M.M.F.) is $\frac{4\pi}{10}$ (or 1·257) times the ampere-turns, and the reluctance of the magnetic circuit is the opposition offered by the substance to the establishment of a flux

It is thus analogous with the resistance of an electric circuit, i.e. its opposition to the establishment of a current. The appropriateness of the analogy is evident from an examination of the formula for the resistance of a conductor :—

$$R = \frac{l\,\varrho}{A}$$

where l is the length of the conductor, A the area of its cross-section and ϱ a constant for the material. Similarly, the reluctance of a magnetic path is given by

$$S = \frac{l}{A\,\mu}$$

Thus the permeability of the magnetic circuit in some respects resembles the specific conductivity of the electric circuit, but whereas the specific conductivity $\left(\frac{1}{\varrho}\right)$ of a conductor is constant at uniform temperature, the permeability of a ferro-magnetic material varies with the flux density as explained later.

Example

Calculate the ampere-turns necessary to produce a flux of 10,000 lines in a closed iron ring of cross sectional area 4 sq. cm., μ being assumed constant and equal to 1000, and the mean length of a magnetic line in the iron being 20 cms.

Since
$$\Phi = \frac{1{\cdot}257\,I\,N}{S}$$

$$I\,N = \frac{S\;\Phi}{1{\cdot}257} = {\cdot}8\,S\;\Phi$$

$$S = \frac{l}{A\,\mu} = \frac{20}{4 \times 1000}$$

$$\therefore I\,N = {\cdot}8 \times \frac{20}{4000} \times 10{,}000$$

$$= \frac{8 \times 20}{4} = 40 \text{ ampere turns.}$$

B. H. CURVES

The Thomson permeameter

21. In the last example, it was assumed that the permeability of the iron core was constant, and equal to 1000. In the case of ferro-magnetic materials, the permeability depends upon the quality of the iron, the flux density, and the temperature, as well as upon the previous magnetic history of the sample concerned.

The relation between B and H may be determined experimentally. If this relationship is plotted with H as abscissa and B as ordinate, the resulting graph is called the B/H curve for the particular sample. The Thomson permeameter is an early form of apparatus employed for such a determination, although more rapid methods are now generally used. The theory of the permeameter exhibits the quantitative relationship between B and H directly, and is chosen as an illustration for this reason.

The permeameter is shewn diagrammatically in fig. 15. It consists of a massive soft iron magnetic circuit or yoke, carrying a winding of stout insulated copper wire, the number of turns and consequently the ampere-turns for a given current, being known. A " test piece " of the iron whose magnetic properties are under investigation forms the core of the electro-magnet, the magnetic circuit being completely closed. This necessitates an accurate fit of the test piece in the annular opening at C, and perfect contact of the end of the test piece on the machined surface A.

The test consists of a measurement of the force necessary exactly to neutralise the magnetic attraction between the yoke and the test piece at this surface, this attractive force being caused by a known magneto-motive force due to the winding.

The magnetic force of attraction between two surfaces with an area of contact A cm² is given by the equation :—

$$P = \frac{B^2 A}{8\pi} \text{ dynes.}$$

(P is used to denote this force, because it is, in common parlance, the " pull " of the magnet.)

If the " pull " is known, B can be obtained by simple manipulation of the formula, thus

$$B = \sqrt{\frac{8\pi P}{A}}$$

while H, the magnetising force, is equal to $\frac{4\pi IN.}{10l}$

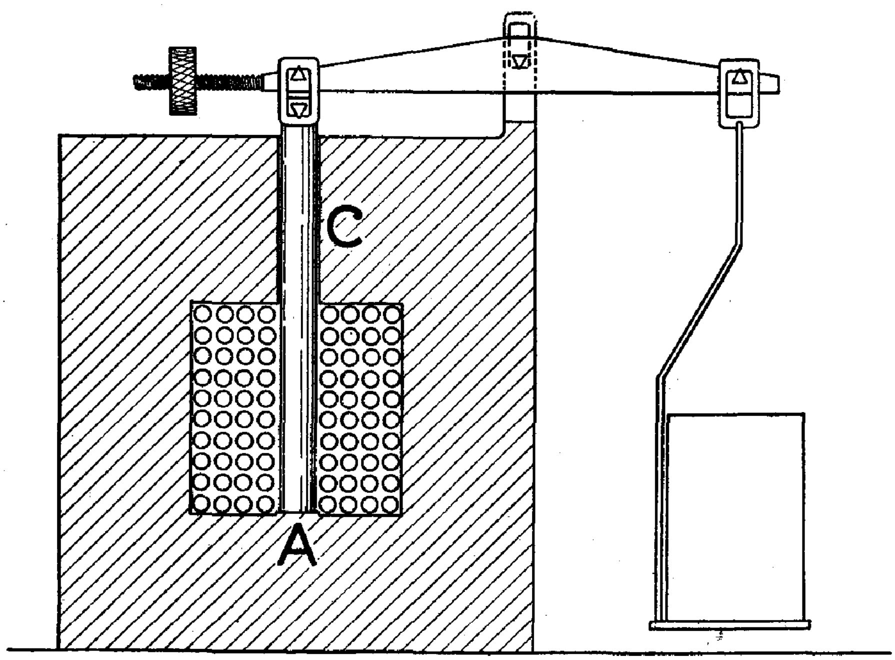


FIG. 15, CHAP. II.—Permeameter.

Example

In the permeameter shewn, the test piece is 20 cms. long and 2 cms. diameter. The magnetising winding carries 200 turns. It is found that a weight of 5,000 grams is just sufficient to overcome the attraction at A, when 1 ampere flows in the coil. Determine the magnetising force, H, the flux density, B, and the permeability of the sample at this flux density.

$$H = \frac{4\pi}{10} \times \text{ampere turns per cm.}$$

$$= \frac{4\pi}{10} \times \frac{200}{20} = 4\pi \text{ or } 12{\cdot}57 \left(\frac{\text{dynes}}{\text{cm}^2}\right)$$

This is also numerically equal to the flux density in the gap when the test piece is absent.

$$P = 5{,}000 \text{ grams or } 4{,}905{,}000 \text{ dynes.}$$

$$\text{Now } A = \pi r^2 \text{ and } r = 1 \text{ cm.}$$

$$\therefore A = \pi \text{ cm.}^2$$

$$B = \sqrt{\frac{8\,\pi P}{A}} = \sqrt{8 \times 4905000}$$

$$= \sqrt{39{,}240{,}000} = 6264 \text{ tubes/cm.}^2 \text{ or gauss.}$$

The permeability is the ratio $\frac{B}{H}$ or $\frac{6264}{12{\cdot}57}$

$$\therefore \mu = 498.$$

Note.—The reluctance of the massive yoke is neglected in comparison with that of the test piece. A correction for this could be applied if necessary.

22. Typical B/H curves for various ferromagnetic materials are shewn in fig. 16 and the variation of μ with flux density for a particular sample of mild steel, in fig. 17. The molecular

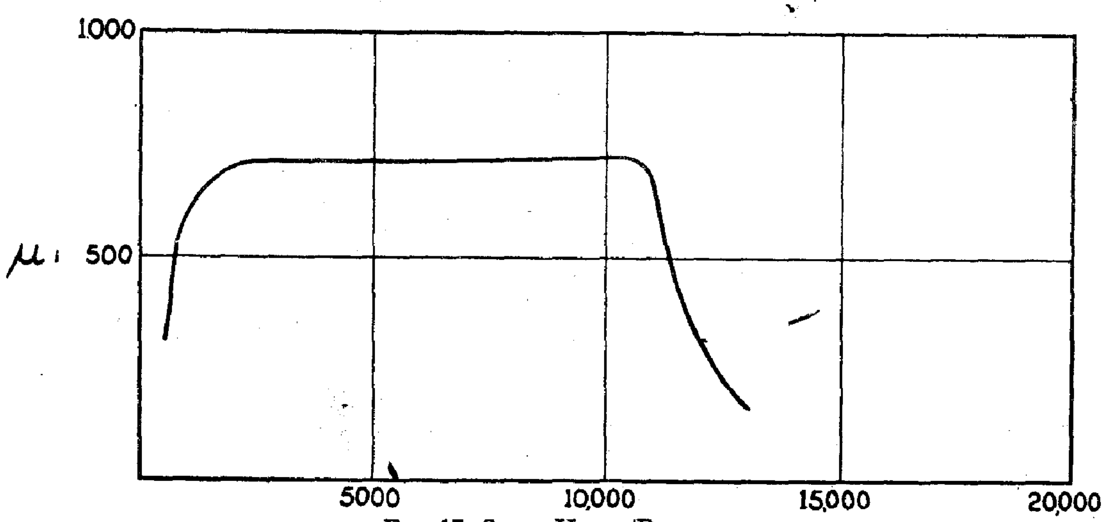
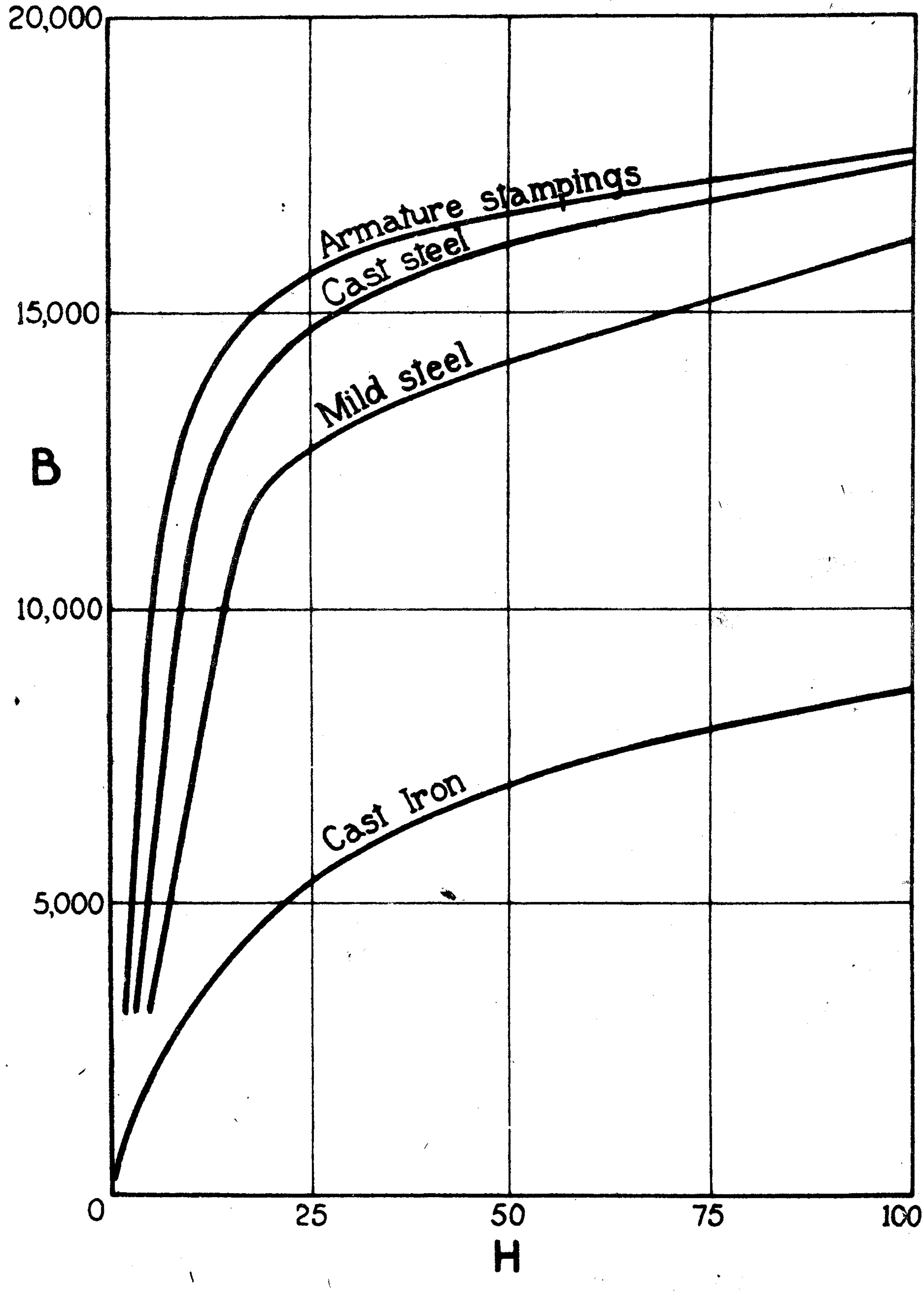


FIG. 17, CHAP. II.—μ/B curve.

theory of magnetism outlined in a preceding section was in part derived from a study of B/H curves of various materials. The variation of flux density, as the magnetising force is increased, is generally somewhat as follows. Commencing with a totally unmagnetised sample, the application of a small magnetising force will result only in the partial disturbance of some of the closed magnetic chains resident in the sample, and does not result in the production of an appreciable external field; the ratio of B to H (i.e. the permeability) is therefore low for small values of B, as shewn in fig. 17. Once the closed magnetic chains have been broken up, the increase of flux density is practically proportional to the increase in magnetising force, and over a considerable range of flux density the permeability is constant. When practically all the molecular magnets have been aligned with their axes in line with the axis of the specimen, further increase of magnetising force is only devoted to a slight improvement of this alignment, little increase in flux density resulting, hence the permeability again falls to a low value. In the latter state the sample is said to be magnetically saturated.

Hysteresis. The hysteresis loop

23. If a series of B/H measurements are taken, with increasing values of H, from zero up to some definite value, say H_1, and a new series then taken with decreasing values of H, from H_1 to zero, it will be found that the plotted results give two different B/H curves, the latter



B/H CURVES OF IRON AND STEEL

FIG. 16.
CHAP. II.

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lying above the former, so that on the descending curve when H is zero, B still has a finite value, which is called the remanence of the material. This is an illustration of the fact that any ferro-magnetic substance possesses to some extent the property of retentivity, i.e. having been magnetised by some external means, the sample retains its magnetism when the magnetising force has been withdrawn. This remanent or residual magnetism can be removed by applying a magnetising force in such a manner that the sample tends to become magnetised with opposite polarity. The magnetising force thus necessary to overcome the residual magnetism is called the coercive force. If the sample is taken through a complete cycle of magnetisation, i.e. from $H = +H_1$ through $H = 0$, to $H = -H_1$ and then back through $H = 0$ to $H = +H_1$ as shewn in fig. 18, the graphical representation of the B/H relationship is called a hysteresis loop. The magnetisation may be considered to lag behind the magnetising force, and the term hysteresis effect is used to describe the phenomenon. Hysteresis may be regarded as an expression of the work done in overcoming the friction between the molecules of the substance undergoing magnetisation; this work is converted into heat, and is therefore irrecoverable. Hysteresis losses are only of importance in the case of iron subjected to successive cycles of magnetisation.

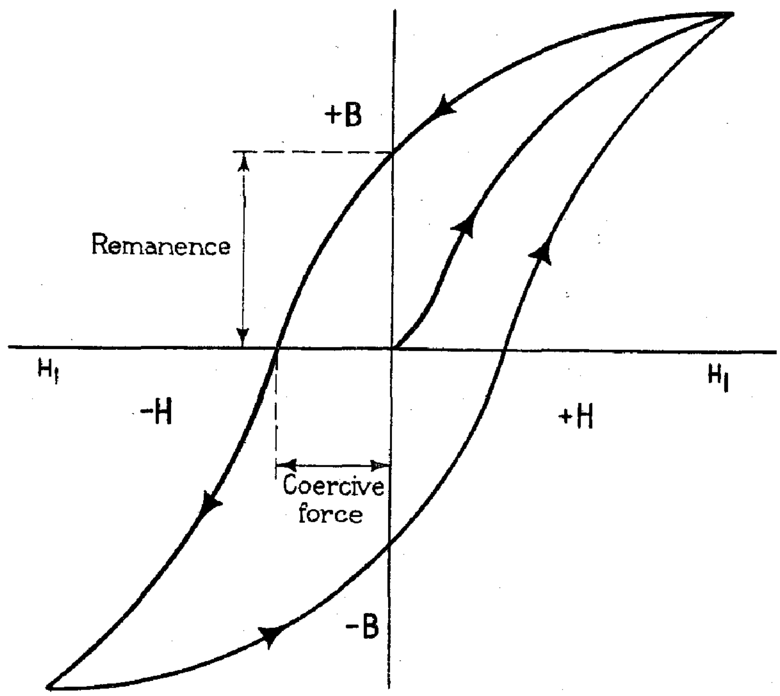


FIG. 18, CHAP. II.—Hysteresis loop.

INDUCED E.M.F.

24. We have seen that whenever there occurs a change in the amount of magnetic flux linking with an electric circuit, the energy expended in changing the flux linkage is partially converted into electrical energy and consequent production of an electromotive force. The flux may be set up by a permanent magnet, an electromagnet, an adjacent current-carrying conductor or a current in the circuit in which the E.M.F. is induced. The methods of producing an induced electromotive force may therefore be classified as follows :—

(i) Moving flux, stationary conductor, used in rotating field alternators.

(ii) Stationary flux, moving conductor, used in common forms of D.C. generators and some types of alternator.

(iii) Varying flux, stationary conductor.

(*a*) Mutual induction, a varying flux in one circuit setting up an E.M.F. in an adjacent one.

(*b*) Self induction, the variation of flux set up by a change of current in a circuit inducing an E.M.F. in the circuit itself.

A simple example of the first class is depicted in fig. 19 in which a coil of wire is connected to a sensitive galvanometer. When the magnet is stationary, no current flows and there is no deflection of the meter. On dropping the magnet into the coil, the magnetic flux links with each turn of the coil in succession and during the time the flux linkage is changing, the induced E.M.F. causes a flow of current with a resulting deflection of the needle. When the magnet comes to rest (in the position shewn by a dotted outline) the flux is again stationary with regard to the circuit and the induced E.M.F. falls to zero. On withdrawing the magnet the change of flux linkage again produces a momentary E.M.F. but in the opposite direction and this is shewn by a momentary reverse deflection of the galvanometer needle.

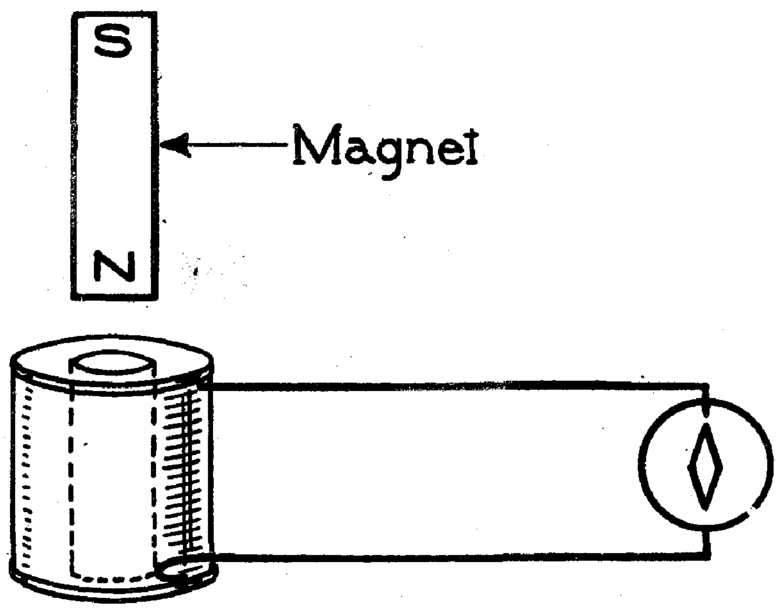


FIG. 19, CHAP. II.—Induction of E.M.F. by motion of magnetic field.

25. An interesting example of the production of electromotive force by the second method is to wind a coil of many turns on a " former " about two feet square, the ends of the winding being connected to a very sensitive galvanometer. Allow the coil to hang vertically by its connecting leads, and turn it sharply through 180° when a deflection of the galvanometer will be observed. The E.M.F. in this case is due to the change of linkage between the coil and the magnetic field of the earth and this apparatus when suitably calibrated can be used to determine the magnetic field strength of the earth at any point.

In order to meet the contingency in which only one portion of a circuit is actually situated in a given field, it is often convenient to speak of the amount of flux cut by a conductor owing to relative motion between conductor and field. As an example, consider a conductor moving across a magnetic field established between two unlike poles, as in fig. 20. The change of flux linkage with the whole circuit is evidently equal to the number of tubes of flux through which the conductor passes, and the conception of cutting is particularly useful because it lends itself to the application of the following mnemonic for finding the direction of the induced electromotive force.

Fleming's right hand rule :—

Extend the thumb, forefinger and middle finger of the right hand in three mutually perpendicular directions. Point the thu**M**b in the direction of **M**otion, the **F**orefinger in the direction of the **F**ield, then th**E** **M**iddle **F**inger gives the direction of the induced E.M.F.

The magnitude of the induced E.M.F. can be derived by application of the formula

E (average) $= \frac{N}{10^8} \frac{(\Phi_2 - \Phi_1)}{t}$. If the conductor moves across the field perpendicularly with velocity u centimetres per second and the length of the conductor is l centimetres, the area swept by

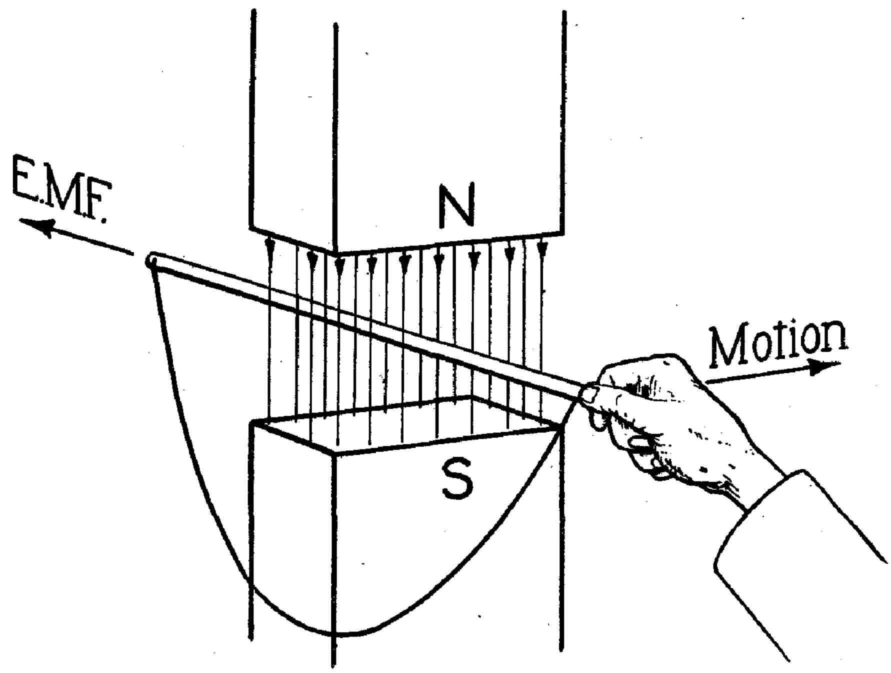


FIG. 20, CHAP. II.—Induction of E.M.F. by motion of conductor.

the conductor per second is lu square centimetres. As $\Phi = BA$, $\Phi_2 - \Phi_1 = B\ (A_2 - A_1)$ the area $A_2 - A_1$ being the area swept by the conductor, or lu square centimetres per second, hence the change of flux is Blu tubes per second, and is numerically equal to the induced electromotive force in E.M.U. The E.M.F. is thus equal to $Blu \times 10^{-8}$ volts.

If the conductor is not moving through the conductor perpendicularly, but is cutting it at an angle θ, the component of its velocity perpendicular to the flux is $u \sin \Theta$ and the induced E.M.F. will be $Blu \sin \theta \times 10^{-8}$ volts. This condition arises in practical dynamo and alternator construction, and further consideration is therefore postponed until Chapter IV.

Faraday's law

26. The phenomenon of electro-magnetic induction was first discovered by Faraday, who summarised the effects in his law of electromagnetic induction, which may now be stated :—

An induced electromotive force is established whenever a change occurs in the magnetic flux linking with an electric circuit. The magnitude of the E.M.F. is proportional to the rate of change of flux linkage.

It must not be supposed that a complete conductive circuit must exist in order that an E.M.F. may be produced. For example, if the circuit consists of a metallic conductor connected to a condenser, the E.M.F. set up by a change of flux linkage in the circuit will set up a conduction current in the metallic portion and a displacement current in the dielectric. Extending this principle still further, a change of magnetic flux through a dielectric substance sets up a displacement current in the dielectric, and consequently an electric strain in the material. The electric strain is equivalent to an electric field strength which is measured in volts per metre. This extension of Faraday's law to a dielectric material is the basis of the theory of electromagnetic radiation.

Lenz's law

27. Whenever an induced E.M.F. is set up in an electric circuit and a current thereby established, the conductor experiences a force owing to the interaction between the original flux and that produced by the current in the conductor. The direction in which this force will tend to urge the conductor can be found by applying the left hand rule and is always such as to oppose the motion causing the induced E.M.F. Lenz formulated this principle in his general law :—

Every induced E.M.F. opposes in some manner the change of conditions which produced the E.M.F.

The reader should verify this law by applying the left hand rule to the conductor shewn in fig. 20, in which an E.M.F. is induced by moving the conductor to the right. The force on the conductor due to the flow of current in the circuit will tend to move the conductor to the left.

Mutual induction

28. An electric circuit (A) consisting of a solenoid, battery and switch is shown in fig. 21. On closing the switch a conduction current will be established, and consequently a magnetic flux will thread the coil, its general configuration being shewn in the figure. A portion of the

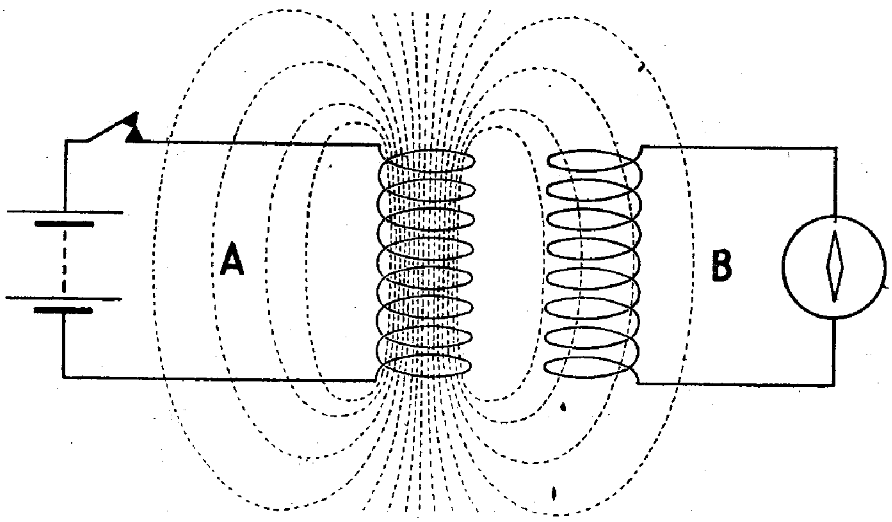


FIG. 21, CHAP. II.—Mutual induction.

flux also links with the adjacent circuit (B). The number of flux linkages evidently depends upon the current in circuit A, the size and shape of the two circuits, and their relative positions. The number of flux linkages common to both circuits is called the mutual flux linkage. It is evident that the mutual flux linkage would be unchanged if the source of E.M.F. were transferred to circuit B and the magnitude of the E.M.F. adjusted so that the current flowing was of the same value as in the original circuit.

Reverting to the arrangement shewn in fig. 21 suppose that a change of current occurs in circuit A ; there will be a change in the total flux produced and consequently a change of flux linkage with circuit B. Now a change of flux linkage, by Faraday's law, gives rise to an E.M.F. and therefore an E.M.F. will be induced in circuit B. It is termed an E.M.F. of mutual induction.

The presence of this E.M.F. can be detected by completing circuit B by means of a sensitive galvanometer, which will indicate a flow of current in circuit B whenever a change of current occurs in circuit A.

Since the flux linkage is proportional to the current, i, and to a constant depending upon the relative shapes and sizes of the circuits, we may write

$$\text{Flux linkage} = M\,i$$

where M is the constant referred to above. It is called the mutual inductance between the two circuits.

The induced E.M.F. is numerically equal to the time rate of change of flux linkage, that is

$$E = \textit{rate of change of } M\,i$$

Since M is by definition constant

$$E = M \times \textit{rate of change of } i$$

which is frequently written

$$E = M\frac{di}{dt} \qquad \ldots \qquad (a)$$

The symbol $\frac{d}{dt}$ will be used frequently in subsequent paragraphs as an abbreviation for " the rate of change with respect to time, of........". Thus $\frac{d}{dt}i$, or $\frac{di}{dt}$ must be thought of as denoting " the rate of change of current with respect to time ".

The equation given above serves to define a unit of mutual inductance. In electromagnetic units the mutual inductance of a circuit is one unit if E.M. unit voltage is induced in it when the rate of change of current is one E.M. unit per second. Similarly in practical units, the mutual inductance is one unit if one volt is induced in it when the rate of change of current is one ampere per second. This practical unit of mutual inductance is called the Henry. It is equal to 10^9 E.M. Units.

Calculation of mutual inductance

29. The mutual inductance between two circuits is not readily calculated in any but the simplest instances. A case which lends itself to calculation is that of two toroidal coils on the same former, one winding being wound over the other. It may then be assumed that when a current is set up in either coil the whole of the magnetic flux will link with both coils, or (introducing a term frequently employed to convey the same meaning) no magnetic leakage occurs. The permeability of the material will also be taken as constant. Let a varying current of intensity i amperes at any moment flow in one coil which has N_1 turns. Then the flux set up is given by the equation

$$\Phi = \frac{4\pi}{10}\,\frac{iN_1}{S},$$

S being the reluctance of the magnetic path. Now only a change of flux can produce an E.M.F. in the second circuit and since every factor of the right hand member of the equation is constant except the current, the rate of change of flux with respect to time, $\frac{d\,\Phi}{dt}$, must be

$$\frac{d\,\Phi}{dt} = \frac{4\,\pi\,N_1}{10\,S}\,\frac{di}{dt}$$

and this is also equal to the E.M.F. induced in each turn of the winding N_2. The total E.M.F. in this winding is therefore

$$e = \frac{4\pi}{10}\ \frac{N_1N_2}{S}\ \frac{di}{dt}\ \text{E.M.U.}$$

$$\text{or } E = \frac{4\pi}{10^9}\ \frac{N_1N_2}{S}\ \frac{di}{dt}\ \text{volts} \qquad (b)$$

comparison of this equation with equation (*a*) shews that both can only be true if M (in henries) is equal to $\frac{4\,\pi}{10^9}\ \frac{N_1N_2}{S}$.

Self-induction—Inductance

30. Referring to fig. 21 above, let it be supposed that the circuit B is non-existent. On closing the switch, a conduction current and consequently a magnetic flux is established just as before. Now the magnetic flux links with the circuit A, and any change of current will result in a change of flux linkages through the circuit. The flux linkages are proportional to the intensity of the current and also to a constant L, which depends upon the shape and size of the coil. It is therefore permissible to write

$$\text{Flux linkage} = L\ i$$

The constant L is called the coefficient of self-induction, or the inductance of the circuit, the latter expression being usual in radio practice. Now as the value of the induced E.M.F. in any circuit is numerically equal to the rate of change of flux linkages,

$$E = \frac{d}{dt}\ (L\ i)$$

since L is constant by definition

$$E = L\frac{di}{dt}$$

The unit of inductance is defined in exactly the same way as the unit of mutual inductance. The practical unit is the Henry, and is the inductance of a circuit in which a current change of one ampere per second sets up an induced E.M.F. of one volt.

By Lenz's law this self-induced E.M.F. must oppose the change of current which produced it, that is, it is actually proportional to $-\frac{di}{dt}$. This tendency of the induced E.M.F. may therefore be indicated by writing

$$E = -L\frac{di}{dt} \qquad (c)$$

Calculation of inductance

31. Referring to the example of calculation of mutual inductance given above, the flux set up by the instantaneous current of i amperes is

$$\Phi = \frac{4\pi}{10}\ \frac{iN}{S}$$

the rate of change of the flux is

$$\frac{d\,\Phi}{dt} = \frac{4\pi}{10}\ \frac{N_1}{S}\ \frac{di}{dt}$$

and the total change of flux linkage with the N_1 turns of the winding will therefore be $\frac{4\pi}{10}\ \frac{N_1^2}{S}\ \frac{di}{dt}$, which is equal to the induced E.M.F. in E.M.U.

In practical units

$$E = -\frac{4\pi}{10^9}\frac{N_1^2}{S}\frac{di}{dt} \quad \ldots \quad (d)$$

the minus sign being inserted in order to satisfy Lenz's Law. Comparison with equation (c) above indicates that

$$L = \frac{4\pi}{10^9}\frac{N_1^2}{S} \text{ henries}$$

This formula would be perfectly general if two conditions were always satisfied, that is, if it were certain that the whole of the flux linked with every turn of the winding, and if the reluctance S could be calculated for any shape or size of circuit. In practice, the formula is used in conjunction with a form factor which takes the " geometry " of the circuit into consideration. (The term geometry is nowadays frequently used as an omnibus term for " shape, size and disposition of parts ").

The henry is too large for general use, although coils of 1,000 henries inductance are occasionally met with, e.g. the secondary winding of a high class intervalve transformer. For use in radio practice the henry is subdivided as under

1 millihenry $= \frac{1}{1{,}000}$ henry, or 10^{-3} henry.

1 microhenry $= \frac{1}{1{,}000{,}000}$ henry, or 10^{-6} henry

1 absolute unit $= \frac{1}{1{,}000{,}000{,}000}$ henry, or 10^{-9} henry.

The absolute unit of inductance is also called the centimetre. This can be justified on theoretical grounds, and practically by the fact that the inductance depends upon the size and shape of the circuit, that is upon measurements of length.

Effect of inductance in an electrical circuit

32. When an electrical circuit possesses the property of inductance, and it must be remembered that no circuit can be entirely without it, the effect of its presence is to oppose any change in the value of the current, or alternatively any alteration in the state of the electrons or electric charges from a state either of rest or of uniform motion. Now in connection with matter, the property having the same nature is called inertia and we may therefore say the inductance is the electrical analogue of inertia. To take a concrete example, consider the circuit shewn in fig. 22, in which the battery of E.M.F. E volts is connected to the coil of inductance L henries and

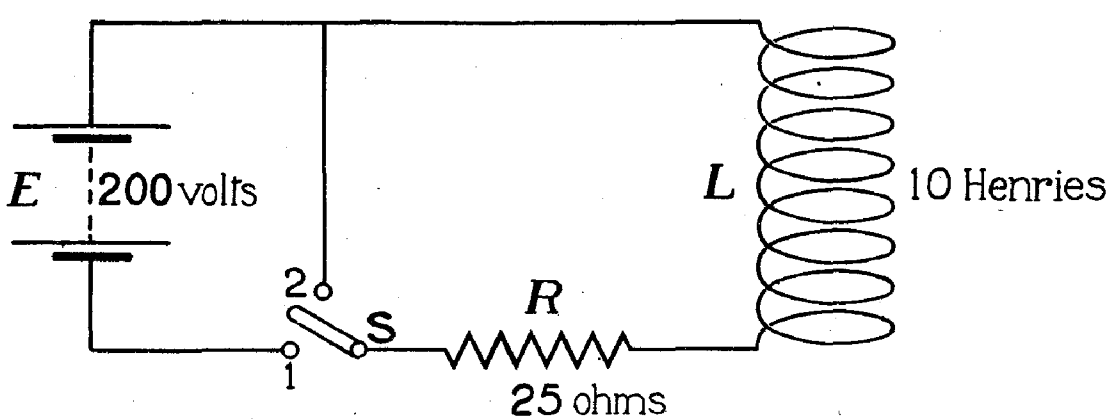


FIG. 22, CHAP. II.—Circuit possessing resistance and inductance.

R ohms resistance by means of a switch S, which is so designed as to short circuit the coil at the instant of disconnecting the battery. On placing the switch S into position 1, a current will be established. Now by Ohm's law, this current is equal to $E \div R$, without reference to time, and if the circuit were absolutely non-inductive the current would assume this value instantaneously.

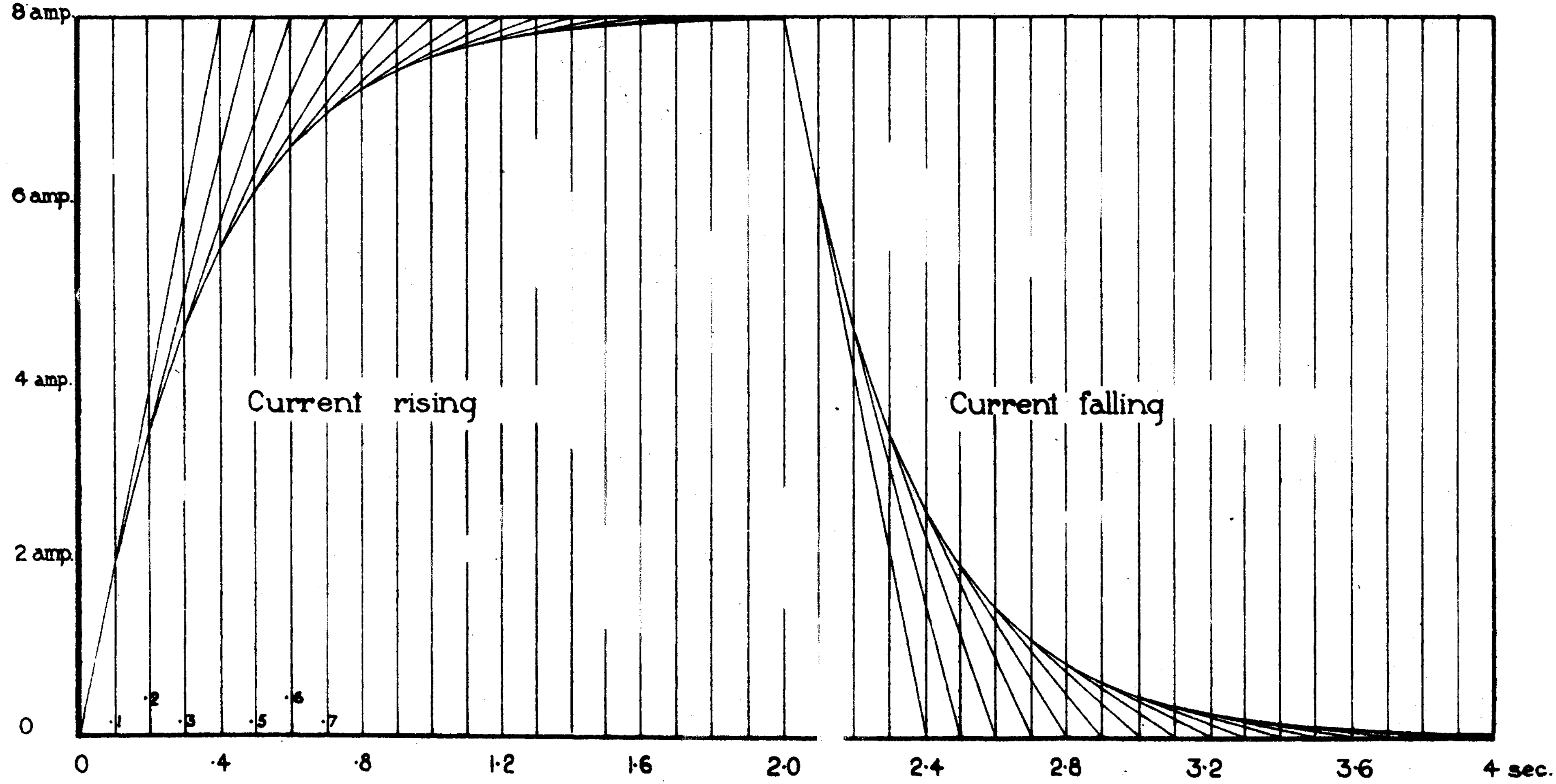
If the applied E.M.F. is 200 volts, $R = 25$ ohms and $L =$ zero, the current would be 8 amperes from the instant of closing the switch until the circuit was broken, when the current would fall to zero instantly. On the other hand, if the circuit possessed an inductance L of 10 henries, but offered absolutely no resistance, the current would commence to grow uniformly at a rate of $\frac{E}{L}$ or 20 amperes per second, and would ultimately reach an infinite value. If we assume for purely theoretical purposes that the current cannot exceed the value $\frac{E}{R}$ to which it would be limited by the presence of resistance, the current, growing at the rate of $\frac{E}{L}$ amperes per second, would reach this value in a time T which is given by the relation

$$\frac{ET}{L} = \frac{E}{R}$$

$$T = \frac{L}{R}$$

With the circuit constants assumed above, $T = \frac{10}{25} = \cdot 4$ second. The time taken by the current to reach the value $\frac{E}{R}$, assuming that its original rate of increase were maintained, is called the time constant of the circuit. It is analogous to the time constant of the circuit possessing both capacitance and resistance, the charging and discharging processes of which were explained in Chapter I. The initial rate of increase of current cannot be maintained, however, because the growth of current through the coil sets up around it a magnetic flux of increasing density, which links with the conductor and induces in it an E.M.F. (Faraday's law). In accordance with Lenz's law this E.M.F. tends to oppose the change of conditions which produced it, i.e. it opposes the growth of current, and is said to be a counter-E.M.F.

33. After a short interval of time, say $\cdot 1$ second, the current has risen to 2 amperes, the P.D. across the resistance will be $2 \times 25 = 50$ volts, and the voltage available to overcome the electrical inertia of the inductance, or to increase the magnetic flux, is only $200 - 50 = 150$ volts. The current will continue to rise, but at a rate of $\frac{150}{L}$ or 15 amperes per second instead of 20 amperes per second as originally, and if this rate were maintained the current would reach 8 amperes in a further $\cdot 4$ second. The growth of current can in fact be obtained by a graphical construction identical with that used in Chapter I for the charging of a condenser, and fig. 23 shews the current growth with the circuit constants given above. It will be observed that by this graphical construction, the current reaches $68 \cdot 5$ per cent. of its final value in the time $T = \frac{L}{R}$. If sufficiently small time intervals are taken, however, a more accurate curve results, and in reality the current would reach $63 \cdot 2$ per cent. of its final value in the time T. The graph shows that the current would not reach the value 8 amperes given by Ohm's law until about two seconds had elapsed from the time of completing the circuit. It must not be thought that this phenomenon invalidates Ohm's law; it really proves it, for if we say that $I = \frac{E}{R}$, in this particular instance, a major error is committed, because the total E.M.F. acting in the circuit



RISE AND FALL OF CURRENT IN AN INDUCTIVE CIRCUIT

FIG. 23
CHAP. II.

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is not E, but $E - L\frac{di}{dt}$, so that the true relation between current, E.M.F. and the circuit constants is

$$i = \frac{E - L\frac{di}{dt}}{R}$$

or $Ri + L\frac{di}{dt} = E$

the small letter i being used to indicate that this equation gives the instantaneous value of the current at some interval of time after closing the switch. The mathematical solution of this equation is given in a note at the end of the chapter.

34. Reverting to the circuit diagram of fig 22, suppose that after the current has been established for some time the switch is suddenly placed in position 2. The inductance and resistance are then disconnected from the supply and short-circuited upon themselves. The current through the inductance now commences to die away, and the decreasing flux, which collapses into the conductor, sets up an induced E.M.F. which opposes the change of current, and therefore tends to maintain it at its original value. If there were no resistance the current would commence to fall at a rate of $\frac{E}{L}$ amperes per second, as shown graphically in the figure. In the first 0·1 second the current would fall to 6 amperes, and the P.D. across the resistance would be 150 volts. The current then continues to fall at a rate of $\frac{150}{L}$ or 15·0 amperes per second, and its rate of decrease becomes less and less as time goes on. The " decay " curve, as it is called, is obtained by the graphical construction previously outlined.

If instead of short-circuiting the coil by means of the switch the circuit is simply broken, the effect of the voltage gradient (i.e. the electric field strength in volts per centimetre, vide Chapter I) across a minute gap of air at the instant of metallic disconnection, causes ionisation of the air in the gap which then becomes partially conductive, and the current continues to flow across the gap in the form of an electric arc. This arcing is accentuated by the presence in the circuit of coils of large inductance, and in such circuits some steps are usually taken to reduce such effects of the arc as burning of the switch contacts, one method adopted being to connect a condenser in parallel with them.

Energy stored in a magnetic field

35. Just as energy is stored in the electric field when an electrical condenser of capacity C farads is charged to a difference of potential of V volts, the energy stored being $\frac{1}{2}CV^2$ joules, so when a magnetic field is established it can be regarded as a storage of energy. While the magnetic field is being established the current grows slowly to a final steady value of $\frac{E}{R}$, E being the E.M.F. in the circuit and R its total resistance. The amount of energy stored can be found as follows :—

The current starting from zero value, reaches the final value $I = \frac{E}{R}$ in t seconds, the average rate of charge of current being therefore $\frac{I}{t}$ amperes per second, and the average current $\frac{L}{2}$ amperes. The average counter-E.M.F. by Faraday's law is $L \times$ *rate of change of current* or $\frac{LI}{t}$ volts, L being in Henries.

The average rate at which energy is stored is therefore $\frac{LI}{t} \times \frac{I}{2}$ volt-amperes, which is the power expended in creating the magnetic field. Since this power is only expended during the time the current is growing, the expenditure goes on for t seconds, and the work done is $\frac{LI}{t} \times \frac{I}{2} \times t$ or $\frac{1}{2}LI^2$ joules.

It has been said that the energy is stored, the implication being that it is restored to the source of supply at some future period. This takes place when the current is caused to fall to zero, for example by withdrawing the original source of energy from the circuit. The collapse of the magnetic field results in a change of flux linkage with the circuit and an E.M.F. is set up which by Lenz's law tends to oppose the fall of the current. As a result the current does not immediately fall to zero when the E.M.F. is withdrawn but dies away slowly and eventually becomes zero when all the stored energy has been expended.

Energy density

36. In Chapter VII it is shown that under certain conditions an electromagnetic wave may be radiated from an electric circuit, this wave consisting of an electric field such as is discussed in Chapter I, and a magnetic field of the nature just considered. In the theoretical consideration of this radiation it is convenient to refer to the amount of energy stored in magnetic form per unit volume of space, which is called the energy density. The energy density of the field inside the winding of a toroidal coil can easily be found if it is assumed that the flux density over the whole cross-section of the coil is uniform. Let the toroid carry N turns, the length of the mean magnetic line passing through all the turns be l centimetres, and the cross-section of the coil be A square centimetres. Then the inductance of the coil is $\frac{4\pi N^2 A \mu}{l}$ E.M. units, and the energy stored in the magnetic field is $\frac{1}{2} LI^2$ ergs. if both L and I are expressed in E.M.U. Hence

$$W = \frac{1}{2} \times \frac{4\pi N^2 A \mu}{l} I^2 \text{ ergs.}$$

But as the volume of the whole magnetic field inside the coil is $A\,l$ cubic centimetres, the energy stored in each cubic centimetre is

$$\frac{4\pi I^2 N^2 A \mu}{2l} \times \frac{1}{A\,l} = \frac{4\pi I^2 N^2}{2l^2} \text{ ergs}$$

and the energy density is

$$\left(\frac{4\pi I N}{l}\right)^2 \times \frac{\mu}{8\pi} \text{ ergs per cm.}^3$$

As $\frac{4\pi I N}{l}$ is the magnetic field strength inside the coil, it may be denoted by H and therefore the energy density is $\frac{\mu H^2}{8\pi}$ ergs per cubic centimetre. This result is true for any distribution of the magnetic field.

TYPES OF INDUCTANCE

37. A coil which has been deliberately produced for the purpose of introducing the property of inductance into a circuit is properly called an inductive coil or inductor, but it is more generally termed simply an inductance. Inductances may be classified as large or small, the former term indicating those having a value of the order of henries, and these are constructed by winding many turns of wire upon a closed iron core, or upon a similar core having a small air gap. Such coils are only employed when the frequency is comparatively low. Small inductances are those having an inductance of the order of microhenries, and are either without iron cores, or have cores of a special iron alloy composition. A coil wound upon a core having unit permeability is always spoken of as an " air core " inductance, irrespective of the material upon which it is actually wound, because the latter has no influence upon the value of the inductance, which depends solely upon the geometry of the winding.

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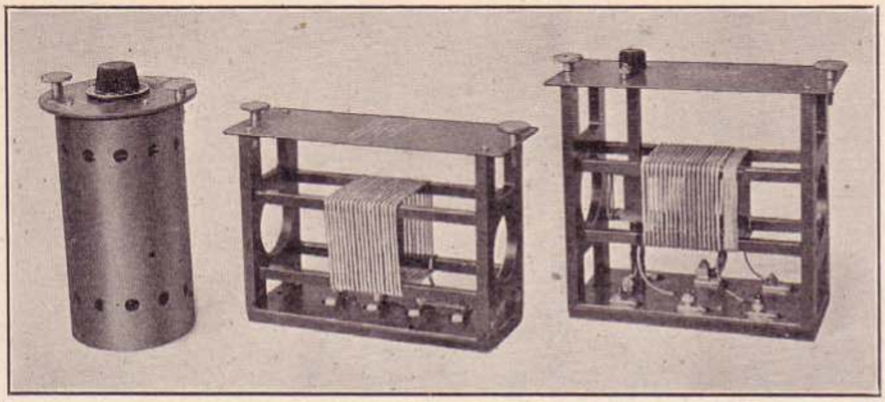

FIG. 24A, CHAP. II.—Typical air-core inductances.

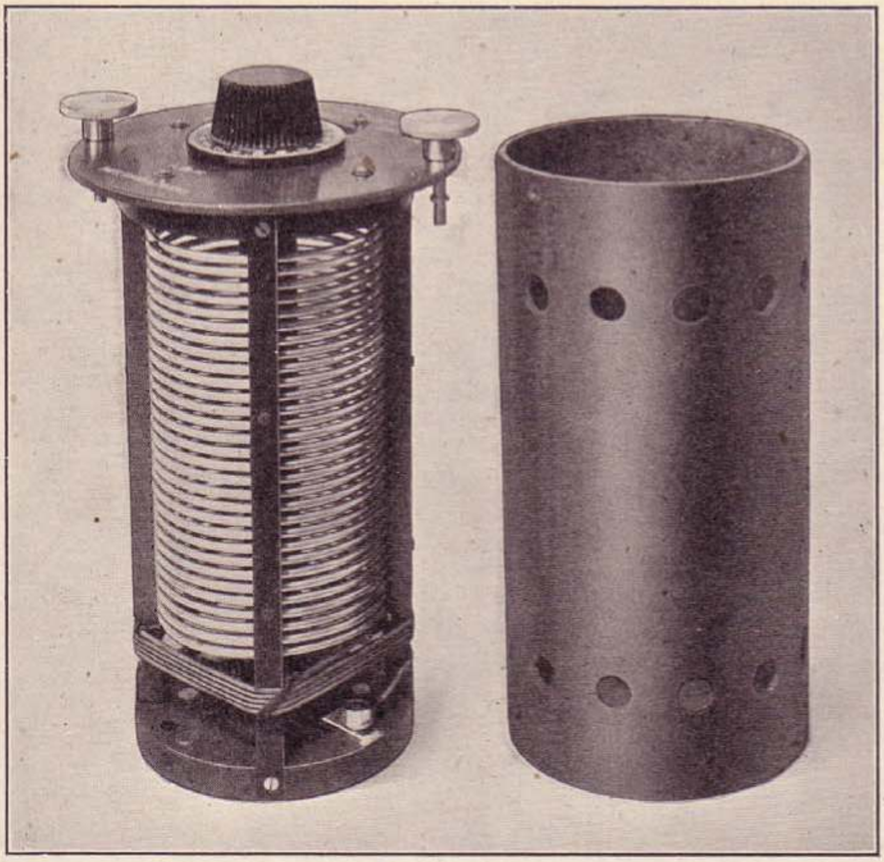

FIG. 24B, CHAP. II.—Continuously variable air-core inductance.

38. Air core inductances are met with in many different sizes and shapes as components of radio transmitters and receivers. Three typical coils are shown in fig. 24a. The construction of the one on the left is shown in fig. 24b. The coil is of copper tubing, silver plated in order to avoid oxydisation, and so reduce the surface resistance. The inductance is continuously variable within certain limits by means of a spring clip which embraces the conductor. The clip is carried on a radial arm which is mounted upon a nut working on a fixed brass screw extending along the whole length of the axis of the coil. The pitch of the thread is the same as that of the winding, and the screw is surrounded by a guide tube or sleeve of insulating material which is slotted along its length to allow the nut, and consequently the radial arm, to move axially. The insulating sleeve is rotated by means of the milled head shown, and as the screw cannot rotate the spring contact arm is constrained to travel round the turns of the coil. An indicating device is fitted in order to show the position of the contact.

The two inductances on the right (fig. 24a) are of fixed value, the winding being of stranded wire wound upon an insulating framework. That on the extreme right actually carries three distinct coils, the main winding being the stoutest conductor ; the turns are spaced by a distance approximately equal to the overall diameter of the conductor. On the right a second winding is carried on the same framework and is linked with the main winding by mutual induction. The third winding is carried on a smaller framework which is mounted upon a vertical bar. This winding is also linked with the main coil by mutual induction, the flux linkage being variable by rotating the smaller coil by means of a knob on the upper panel.

Receiving inductances are usually of solid wire although multistranded wire is sometimes used. When the inductance is to be adjustable in large steps, a series of tappings is made and the points connected to a rotary switch. When the value of inductance is to be adjustable to fine limits the variometer construction is adopted.

The variometer

39. The variometer principle depends upon the presence of mutual inductance between two circuits. If two coils of inductance L_1 and L_2 respectively are separated by such a distance that the interlinkage of their magnetic fields is negligible, the inductance of the two in series can be shown to be $L_1 + L_2$ (Chapter V). If, however, they are brought closely together so that the field of each coil interlinks with the turns of the other, mutual inductance exists between them. An applied E.M.F. E will then cause a current i to flow through both coils and the rate of increase of the current must be the same in each. If the coils and their respective inductances are designated by L_1 and L_2, and the mutual inductance between them by M, then owing to the rise of current there is a counter-E.M.F. in L_1, due to its self induction, its value being $-L_1\frac{di}{dt}$ and as the growing field of L_2 is also threading L_1, an additional counter-E.M.F. $-M\frac{di}{dt}$. In the second coil, the growth of its own flux causes a counter-E.M.F. $L_2\frac{di}{dt}$, and the growth of the flux of L_1, which also embraces L_2, causes an additional counter-E.M.F. in it which is equal to $-M\frac{di}{dt}$. Hence the total counter E.M.F. is

$$-(L_1 + L_2 + 2M)\frac{di}{dt}.$$

But as the perfectly general expression for the counter-E.M.F. in such conditions is $-L\frac{di}{dt}$, L being the total inductance of the circuit, the latter is equal to $L_1 + L_2 + 2M$. It will be noted that in the above example, the mutual flux linkage is so disposed that the counter-E.M.F. due to it is in the same direction as the counter-E.M.F. of self induction. This is not necessarily so, and in fact the counter-E.M.F. of mutual induction in both coils will be reversed if the direction of the field of one coil is reversed. This can be achieved either by actually turning the coil through 180° or by reversing the connections to its ends. The above reasoning again applied then shows that the total inductance is given by

$$L = L_1 + L_2 - 2M.$$

The fact that mutual inductance may have either positive or negative sign, while self-inductance is always positive, is of considerable importance. An immediate application is the variometer (*see* fig. 25). In this instrument two coils are mounted concentrically with each other, and connected in series. The inner coil is arranged to rotate with reference to the outer through an angle of 180°, and the mutual inductance between the two coils is therefore variable from $+ M$ to $- M$.

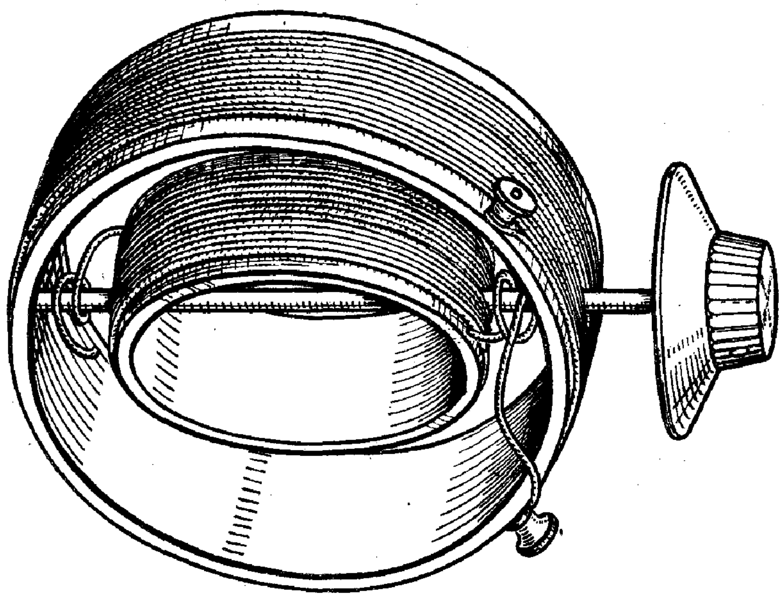

FIG. 25, CHAP. II.—Variometer.

The whole inductance of the two coils in series including this mutual inductance is from $L_1 + L_2 - 2M$ to $L_1 + L_2 + 2M$, and the variation is perfectly smooth throughout this range. Instead of being connected in series, the two coils may be connected in parallel. The value of the inductance is then rather more difficult to deduce and will be given in Chapter V. It is shown in that chapter that the inductance of two coils in parallel without mutual inductance between them is

$$L = \frac{L_1 L_2}{L_1 + L_2},$$

while if a mutual inductance M exists between them

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}.$$

It should hardly be necessary to add that the same units of inductance must be used throughout; if L_1 is given in μH, L_2 and M must also be expressed in μH, while if L_1 is given in henries the same unit must be adopted for the other values before insertion into the equation.

Where space is limited it may be found desirable to utilise the basket-weave coil. This is wound upon a disc, which has an odd number of radial slots, generally seven or nine. These slots extend from the circumference inwards to a depth of about one half of the overall radius. The winding is commenced at the inner end of one slot, passing the wire alternately to one side of the disc or the other, as shown in fig. 26. The inductance of such a coil may be calculated by formula (*b*) below. Many formulae have been developed for the calculation of the inductance of multilayer coils of various shapes, but it is frequently more practicable to measure the inductance by a practical method than to calculate it. An example of such a measurement is given in the chapter dealing with radio-frequency measurements.

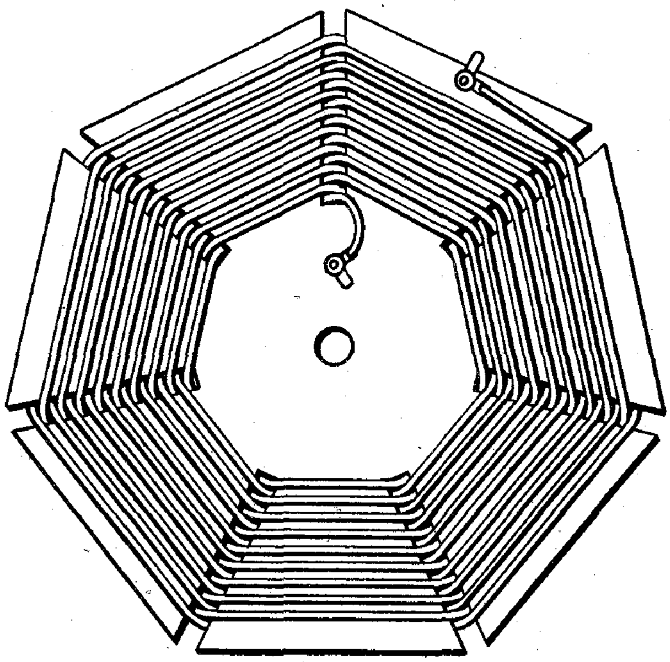

FIG. 26, CHAP. II.—Basket weave coil.

Calculation of inductance of air-core coils

40. In certain emergencies it may be necessary to construct extempore inductance coils for some special purpose, and in these circumstances it is usually convenient to adopt the single layer solenoidal method of winding. The form factor for a single layer solenoid is a complex mathematical function of the ratio of length to diameter of winding, and a formula often used is $L = \frac{d}{2} N^2 f$ where d is the diameter of the coil measured from centre to centre of the wire, N is the number of turns and f is the form factor. For values of $\frac{\text{length}}{\text{diameter}}$ lying between ·1 and 10, the latter can be represented approximately by an empirical formula $f = \frac{r}{8 \cdot 5 r + 10 l}$, and the above expression for inductance then becomes,

$$L = \frac{r^2 N^2}{8 \cdot 5\, r + 10\, l} \qquad \text{(a)}$$

where L is the inductance in microhenries, r the mean radius and l the length of the solenoid, all measurements being in inches. The former upon which the coil is wound is assumed to be of circular section, but if a hexagonal or octagonal former is used the effective radius may be taken the mean of the radii of the inscribed and circumscribed circles on the section. If the coil is of solenoidal form, but having more than one layer, a similar formula may be used, but the radius must be half the mean diameter of the winding, that is

$$r' = \frac{d_1 + d_2}{4}$$

d_1 being the diameter of the former and d_2 the diameter of the wound solenoid, while for the length, the quantity $l' = l + \frac{1}{2}(d_2 - d_1)$ must be substituted. The inductance can then be calculated from the formula

$$L = \frac{(r')^2 N^2}{8 \cdot 5\, r' + 10\, l'}$$

THE TELEPHONE RECEIVER

41. The first instrument handled by the embryo wireless operator is generally the telephone receiver, or rather the head set comprising the headband, cord and a pair of receivers. The telephone receiver is a device which converts variations of electrical current into sound waves, that is, electrical energy into mechanical energy. It consists of a permanent magnet upon which are mounted two soft iron pole pieces, each carrying a magnetising winding. The two coils are connected in series in such a manner that current in a given direction strengthens both poles, while current in the opposite direction weakens them. A circular diaphragm of soft iron or stalloy is so mounted that both pole pieces normally exert a slight pull on its centre, although the diaphragm does not quite touch the pole pieces. The general arrangement is shewn in fig. 27.

42. The action of the telephone receiver may be explained with reference to the circuit given in fig. 20 Chapter I in which it is placed in series with a carbon microphone and a suitable

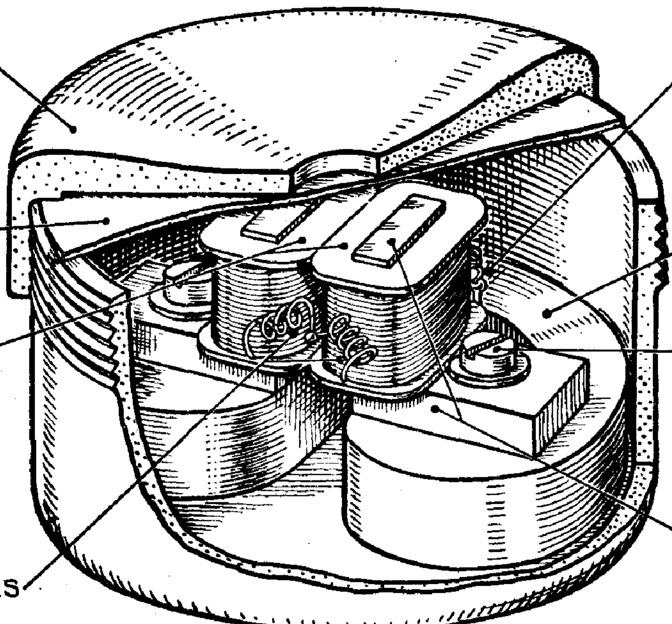

Fig. 27, Chap. II.—Telephone Receiver.

battery. When the microphone is unaffected by sound (a term frequently used to express this state being " quiescent ") a small steady current flows round the coils of the telephone receiver, which sets up a magnetic flux in addition to that provided by the permanent magnet. The direction of current should be such that the two fluxes are additive, and the diaphragm will be attracted rather more closely to the pole pieces than in the absence of this polarising current. On speaking into the microphone, however, the changes in the resistance cause changes in the value of the current round the windings, and corresponding changes in the magnetic flux. An increase of flux will cause further attraction of the diaphragm, and a decrease will result in its partial release from attraction, so that the diaphragm is set into vibration in a manner corresponding to the variation of current, and a sound is emitted by the diaphragm which resembles that originally impressed upon the microphone. The necessity for the inclusion of the permanent magnet is not obvious from the foregoing explanation, for if it were omitted, the variation of current in the winding round the soft iron pole pieces would still cause a varying pull on the diaphragm. The permanent magnet has two functions (i) it gives the receiver greatly increased sensitivity for the same current changes; (ii) in its absence the vibration of the telephone diaphragm would take place at twice the rate of vibration of the microphone diaphragm, and the sound emitted by the telephone would not closely resemble the original sound. Taking these points in order, it will first be shown that the effect of the permanent magnet is to give a greater " pull " on the diaphragm for a given variation of current in the winding than would be obtained in its absence. Let the flux density in the air gap between pole pieces and diaphragms be B. The pull on the diaphragm is given by the equation :—

$$P = \frac{B^2}{8\pi} \text{ dynes per square centimetre (para. 21).}$$

The flux density B may be separated into two components, viz., B_p caused by the permanent magnet and the steady component of current, and $\pm B_c$ caused by the variation of current. The above equation then becomes

$$P = \frac{(B_p \pm B_c)^2}{8\pi} \quad \frac{\text{dynes}}{\text{cm.}^2}$$

or omitting the divisor 8π which will not affect the argument

$$P = (B_p \pm B_c)^2$$

or

$$P = B_p^2 \pm 2B_pB_c + B_c^2.$$

The pull P_o on the diaphragm in the quiescent condition can be obtained by observing that the component B_c is then zero, hence

$$P_o = B_p^2$$

and the additional pull due to the variation of current is

$$P_c = \pm 2B_pB_c + B_c^2.$$

This is obviously the portion of the attractive force with which we are immediately concerned. If the two components of flux density B_p and B_c are acting in the same direction, the pull on the diaphragm will be found by using the positive sign in the above expression. If the two fluxes are in opposition, the negative sign will be appropriate.

Examination of this equation shows that if the flux density B_c is zero, that is if no permanent magnet (or its electrical equivalent) is fitted, the attractive force will be $P_c = B_c^2$ only. The presence of the permanent magnet increases the pull caused by a given current from B_c^2 to $\pm 2B_pB_c + B_c^2$, that is, in the ratio of 1 to $1 + 2\frac{B_p}{B_c}$. As B_p may be hundreds of times as large as B_c it is apparent that the sensitivity of the receiver is increased enormously by the presence of the permanent magnet. It was stated above that if the current circulated round the pole pieces in such a direction that the resulting flux was of the same polarity as that of the permanent magnet the two fluxes were additive. This is a desirable state of affairs, giving the equivalent of a permanent magnet of even greater pole strength than that of the actual magnet. An additional reason for so arranging the direction of current is that unless this uniformity of polarity is maintained, the two fluxes will be in opposition and the magnetising force of the current will tend to demagnetise the permanent magnet and the sensitivity of the telephones will be impaired. It is therefore desirable to check the polarity of the connections of telephone receivers which are intended for use in the above circumstances and always connect them in such a way that the direct current will set up a flux tending to strengthen the pole pieces.

In order to avoid this necessity it is usual in modern practice to connect the microphone and battery to one winding of a small transformer, forming what is called the primary circuit, while the telephones are connected to the ends of a secondary winding. Under these conditons no steady component of current circulates round the telephone windings, but any variation of current in the primary winding causes a variation of flux which embraces the secondary winding and induces in the latter an E.M.F. which at all instants varies in magnitude in accordance with the current variations. As the secondary circuit is completed by the telephone windings, a varying current will be established and will affect the diaphragm in the manner previously explained.

43. Attention may now be devoted to the second reason given above for the inclusion of a permanent magnet. Various possible conditions are illustrated in fig. 28. The first series of diagrams shews the state of affairs when the permanent magnet is absent, but with a large D.C. component of current in the windings, the latter serving the same purpose as the permanent magnet to some extent. It will be observed that the movement of the diaphragm, and therefore the sound emitted from the receiver, is a copy of the original sound wave.

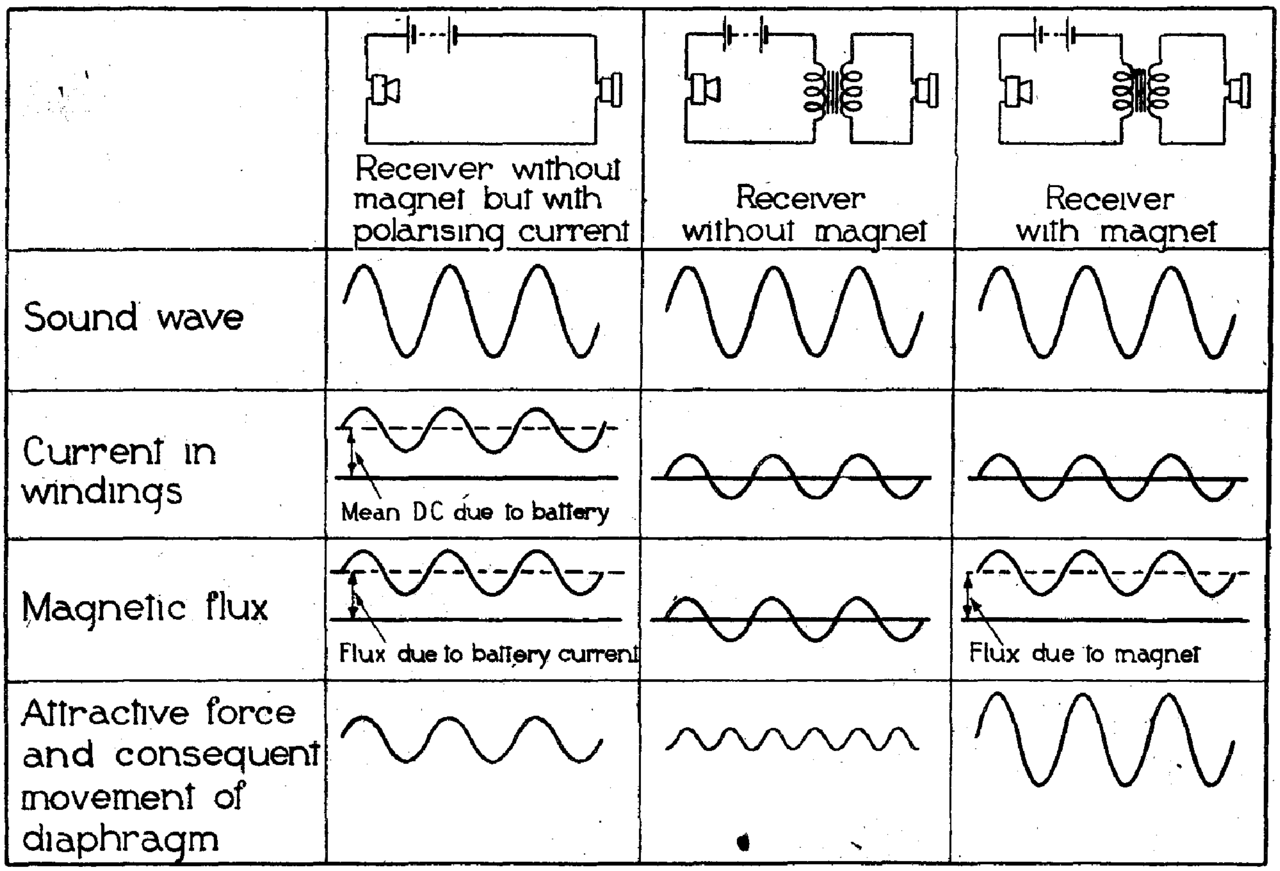

FIG. 28, CHAP. II.—Effect of polarising flux in telephone receiver.

In the second and third series of diagrams the circuit contains a transformer as dealt with above. If no permanent magnet is fitted, the diaphragm is initially under no strain whatever, but will be attracted whenever a current is established in the winding, no matter what its direction may be. The result of this is that the diaphragm is set into vibration in such a way that a single variation of current from zero to some peak value, back to zero, then to an equal peak value in the opposite direction, and finally to zero, causes two separate pulls on the diaphragm, and the latter vibrates at twice the rate at which the current variations are taking place, hence the sound emitted by the receiver is not a copy of the original sound wave. The effect of the permanent magnet is shown in the third series. Here the variation of flux caused by the current variation is superimposed upon the steady flux, and an increase of current causes an increased pull, while when the current decreases the reverse is the case, hence the diaphragm undergoes one variation of displacement for each complete change of current.

NOTE.—Solution of the equation $Ri + L\frac{di}{dt} = E$.

The equation must first be rearranged thus,

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \quad \ldots \quad 1.$$

Multiplying each term by $\varepsilon^{\frac{R}{L}t}$

$$\frac{di}{dt}\varepsilon^{\frac{R}{L}t} + \frac{R}{L}i\,\varepsilon^{\frac{R}{L}t} = \frac{E}{L}\varepsilon^{\frac{R}{L}t} \quad \ldots \quad 2.$$

The left-hand side is equal to $\frac{d}{dt}(i\,\varepsilon^{\frac{R}{L}t})$

where both i and t are variables, for

$$\frac{d}{dt}(i\,\varepsilon^{\frac{R}{L}t}) = \frac{R}{L}i\,\varepsilon^{\frac{R}{L}t} + \varepsilon^{\frac{R}{L}t}\frac{di}{dt}$$

Hence, integrating both members of the equation

$$i\,\varepsilon^{\frac{R}{L}t} = \frac{E}{L}\int \varepsilon^{\frac{R}{L}t}.\,dt$$

$$= \frac{E}{R}\varepsilon^{\frac{R}{L}t} + C \quad \ldots \quad 3.$$

where C is a constant depending upon the initial conditions.

Finally,

$$i = \frac{E}{R} + C\,\varepsilon^{-\frac{R}{L}t} \quad \ldots \quad 4.$$

To determine C, we observe that $i = 0$ when $t = 0$, and that both E and R are constant. Hence, at the instant $t = 0$,

$$0 = \frac{E}{R} + C\,\varepsilon^{\circ}$$

$$\text{or } C = -\frac{E}{R}, \text{ whence}$$

$$i = \frac{E}{R}(1 - \varepsilon^{-\frac{R}{L}t}) \quad \ldots \quad 5a.$$

t being restricted to positive values only, for a negative value of t means a time previous to the closure of the circuit, and it must be assumed that the current is zero until this instant. The equation shewing the decay of the current can be derived from the preceding by varying the initial conditions. If at the time $t = 0$, E becomes zero, equation (4) becomes

$$i = C\,\varepsilon^{-\frac{R}{L}t}.$$

and to determine C, we observe that at the time $t = 0$, i has some value which may be denoted by I_0. If the current has been flowing for a period sufficiently long, the value of I_0 is given by Ohm's law, i.e. $I_0 = \frac{E}{R}$

and $$i = \frac{E}{R} \varepsilon^{-\frac{R}{L}t} \quad \text{.. .. .. .. 5b.}$$

The equations (5*a*) and (5*b*) are frequently referred to as the equations of logarithmic growth and decay.

It will be observed that when $t = \frac{L}{R}$ these equations become

$$i = \frac{E}{R}(1 - \varepsilon^{-1}) \quad \text{.. .. .. 6a (current growing)}$$

$$i = \frac{E}{R}\varepsilon^{-1} \quad \text{.. .. .. 6b (current decaying).}$$

Now $\varepsilon = 2{\cdot}71828$, $\varepsilon^{-1} = \frac{1}{2{\cdot}71828} = {\cdot}36788$ and therefore, when $t = \frac{L}{R}$

$$i = \frac{E}{R}(1 - {\cdot}36788) \quad \text{.. .. .. .. 7a (current growing)}$$

$$i = \frac{E}{R} \times {\cdot}36788 \quad \text{.. .. .. 7b (current decaying)}$$

shewing that when growing, the current will reach the value ${\cdot}63212 \frac{E}{R}$ in the time $\frac{L}{R}$, while when decaying the current will fall to ${\cdot}36788$ of its initial value in the time $\frac{L}{R}$.


LIGHTNING MK. 1
COVER PITOT HEAD