CHAPTER VI.—THE TRANSFORMER
COUPLED RESONANT CIRCUITS
THE TRANSFORMER

1. One of the greatest advantages of an alternating current supply is the ease with which the transformation of a low to a high voltage or *vice versa* can be performed, without the aid of rotating machinery. This transformation is effected by means of a piece of apparatus called a static transformer, or more commonly, a transformer, which consists essentially of three circuits, namely, a magnetic circuit linking two electric circuits. The latter are termed the primary circuit, which contains the fundamental source of all the energy transformed or dissipated in the apparatus, and the secondary circuit respectively. The magnetic circuit is the volume of space occupied by magnetic flux, and may consist of a path of ferromagnetic material such as iron, or a non-magnetic material such as air, giving rise to what are called iron-core and air-core transformers respectively. The principles are the same in each type, but it is convenient to treat them in an entirely different manner, and in this chapter the term transformer may be regarded

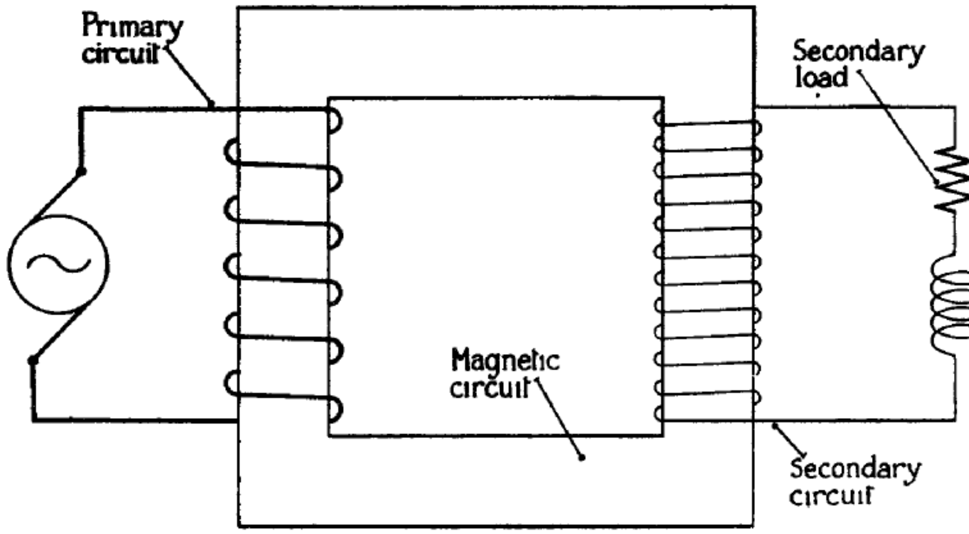


FIG. 1, CHAP. VI.—Principle of transformer.

as an abbreviation for iron-core transformer; the theory of air-core transformers is more conveniently treated in the section on "coupled circuits." The transformer is represented diagrammatically in fig. 1, and consists of a ferromagnetic core, which is built up of laminations, the best swedish soft iron, or stalloy, being usually employed in order to reduce hysteresis and eddy current losses. These laminations are insulated from each other by varnish or thin paper. Over the core are wound the two electrically conductive circuits, primary and secondary, the windings being brought to terminals. If the secondary winding is given more turns than the primary, the transformer will give a secondary E.M.F. which is higher than the primary terminal P.D. and the contrary is the case if the secondary winding is given fewer turns than the primary. The two forms are therefore known as step-up and step-down transformers respectively. The two windings are well insulated from each other by micanite and varnished cloth, and if very high voltages are to be handled, possibly wound upon insulating sleeves. The general arrangement is shewn in fig. 2, in which two principal types of construction are illustrated, namely the core type (*a*) and the shell type (*b*). It will be observed that the core type follows closely the theoretical diagram given previously, except that instead of carrying one winding on each of the two "limbs" of the core, both primary and secondary windings are distributed equally between the two. It may be said that in the core type the magnetic circuit is to a great extent covered by the coils, which are freely exposed, while in the shell type, the winding is chiefly enclosed by the magnetic circuit, which is itself exposed. It is usual to divide the high voltage winding into separately insulated sections in order to reduce the potential difference between the successive layers of wire. This leads to a decrease in the amount of insulation required by the wires themselves, the result being a net gain in cheapness and efficiency. Generally, the core is built in two or more parts so that the coils can be wound separately and afterwards placed in position, the portions of the core being afterwards either butted together or preferably interleaved forming what is called an imbricated joint. Fig. 3 shews the latter type of core.

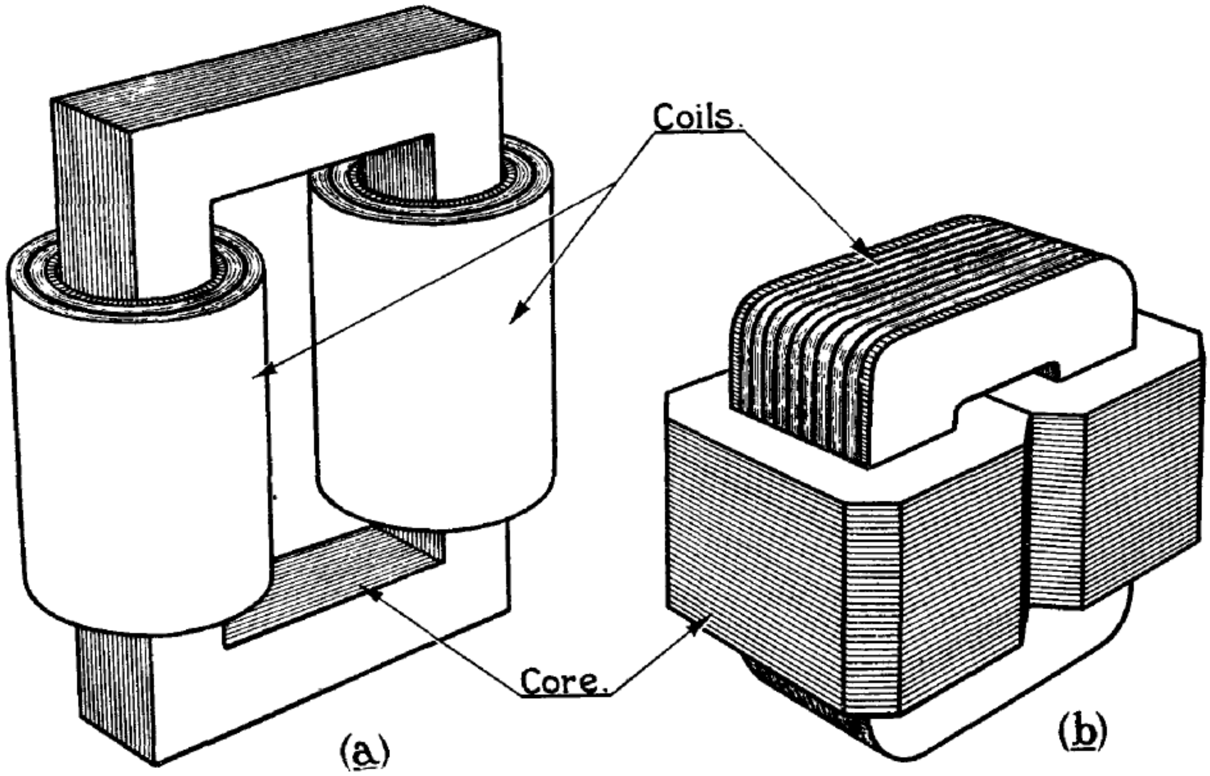


FIG. 2, CHAP. VI —Transformers, core and shell types.

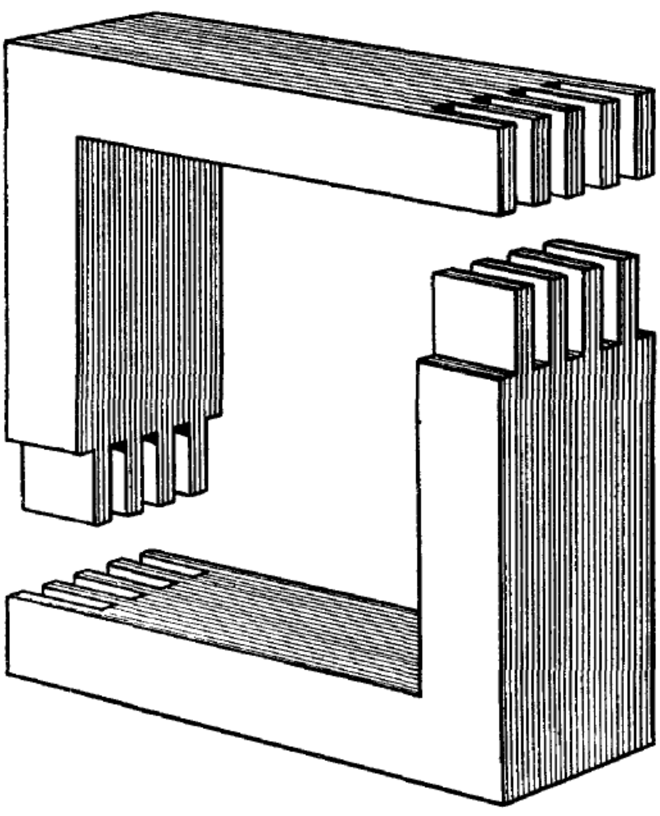


FIG. 3, CHAP. VI.—Imbricated joint in core.

Notation

2. Owing to the large number of variable quantities which must be taken into consideration when dealing with the action of the transformer, the vector diagrams are apt to appear rather complicated, but provided that the diagram is drawn step by step as detailed in the text, it will be found that the vectors themselves convey the whole of the essential theory.

The notation which is used is as follows :—

Primary terminal P.D.	V_p
Primary induced counter-E.M.F.	E_1
Component of terminal P.D. equal to latter	V_1
Secondary induced E.M.F.	E_2
Secondary terminal P.D.	V_2
Primary no-load current	I_o
Iron-loss component of I_o	I_t
Magnetising component of I_o	I_m
Primary load current	I_1
Secondary load current	I_2
Total primary current	I_p
Flux established by primary magnetising current	Φ_m
Flux due to primary load current	Φ_1
Flux due to secondary load current	Φ_2
Primary leakage flux	$_L\Phi_1$
Secondary leakage flux	$_L\Phi_2$
Counter E.M.F. due to primary leakage flux	$_LE_1$
Counter E.M.F. due to secondary leakage flux	$_LE_2$
Total fluxes and voltages allowing for magnetic leakage :—	
In primary circuit	Φ_1', E_1'
In secondary circuit	Φ_2', E_2'
Primary circuit constants :—	
Resistance of winding	R_1
Inductance of winding	L_1
Number of turns	N_1
Primary impedance, no load	Z_o
Primary inductance, no load	L_o
Effective reactance of primary, no load	X_o
Effective resistance of primary, no load	R_o
Equivalent primary resistance	R_p
Equivalent primary reactance	X_p
Secondary circuit constants :—	
Resistance of winding	R_2
Inductance of winding	L_2
Number of turns	N_2
Equivalent secondary resistance	R_s
Equivalent secondary reactance	X_s
Turns ratio, $\frac{N_2}{N_1}$	T

It will be assumed that the primary terminal P.D. is of sinusoidal waveform, and it will then only be necessary to deal with R.M.S. values of voltage, current and flux.

Primary current and voltage, secondary on open circuit

3. If an alternator is connected to the primary terminals of the transformer, the secondary winding being left on open circuit, an alternating current will flow in the primary winding, and will establish a corresponding flux in the iron core. The primary winding is then acting simply as an inductive coil and the current will be equal to

$$I_o = \frac{V_p}{Z_o}$$

In all practical transformers the inductive reactance of the primary winding is very large indeed compared with its resistance, and I_o which is called the no-load primary current, will lag on the applied voltage by nearly 90°. The vector diagram fig. 4a shews the relative phases of I_o, the counter-E.M.F. of self-induction, E_1, which is numerically equal to $\omega L_1 I_o$ and the reactive component, V_1, of the applied voltage, which is equal and opposite to E_1. A component R_oI_o of the terminal voltage is also shewn; this provides for the unavoidable losses which occur even

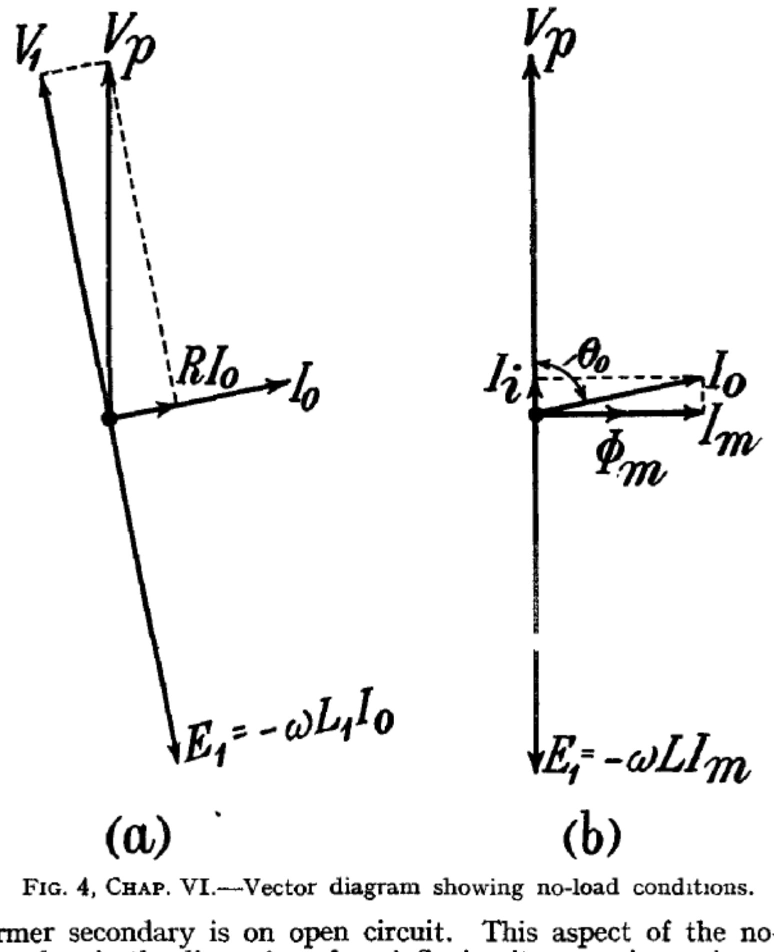


FIG. 4, CHAP. VI.—Vector diagram showing no-load conditions.

when the transformer secondary is on open circuit. This aspect of the no-load condition is identical with that taken in the discussion of an A.C. circuit possessing resistance and inductance in series (Chapter V). In dealing with the transformer, however, it is more convenient to consider that the resistance causing the no-load losses is in parallel with the primary winding. The no-load current then consists of two components in quadrature, first the loss component which is denoted by I_i and is in phase with the applied voltage V_p, and second a component I_m called the magnetising component which lags on the applied voltage by 90°, and is responsible for the establishment of the alternating flux Φ_m. The flux is in phase with the current I_m and the counter-E.M.F. of self-induction, E_1, lags on I_m by 90°. The relationship between the various components of voltage and current is shewn in fig. 4b.

4. In the type of transformer under discussion, the primary and secondary windings are arranged on the core in such a way that practically all the flux produced by the magnetising current will link with every turn of the secondary winding ; in consequence the flux will cause an induced E.M.F. not only in the primary but also in the secondary winding, the E.M.F. in every turn of winding being of the same magnitude. The total E.M.F. induced in the primary is by Faraday's law equal to N_1 times the rate of change of flux, or $N_1 \frac{d\Phi_m}{dt}$ and the secondary induced E.M.F. will be N_2 times the rate of change of flux or $N_2 \frac{d\Phi_m}{dt}$, N_1 and N_2 being the number of turns in the primary and secondary windings respectively. From this we may immediately deduce the most important law of the transformer.

$$\frac{\text{Secondary E.M.F.}}{\text{Primary counter-E.M.F.}} = \frac{\text{Turns on secondary winding}}{\text{Turns on primary winding}}$$

or, algebraically,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = T$$

This is called the transformation ratio of the transformer. In a step-up transformer T is greater than unity, and in a step-down transformer, less that unity. As the secondary E.M.F. and the primary counter-E.M.F. are both caused by the same changing flux, they must be in phase with each other, and since the primary counter-E.M.F. is (practically) 180° out of phase with the voltage applied to the primary, the secondary induced E.M.F. must also be 180° out of phase with the applied voltage. To illustrate the relation between the voltage applied to the primary and the induced secondary E.M.F. suppose a transformer to have 100 primary turns and 3,300 secondary turns the secondary being on open circuit. An applied voltage of 220 volts will cause a small current to flow, and the counter-E.M.F. induced in the primary winding will be nearly but not quite 220 volts, the difference being utilised to supply the small power losses. The counter-E.M.F. induced in each turn of primary winding will be $\frac{220}{100} = 2{\cdot}2$ volts per turn, and a similar E.M.F. will also be induced in each turn of secondary winding, hence the total secondary E.M.F. will be $2{\cdot}2 \times 3300$ or 7260 volts.

Effect of secondary load current

5. (i) If a circuit is connected to the secondary terminals, the induced E.M.F. in the latter winding will establish a secondary load current. This current in its turn will set up in the core a magnetic flux which is proportional to the ampereturns on the secondary winding. The establishment of a secondary current therefore tends to weaken the flux caused by the primary current, but actually this effect does not occur, because as soon as secondary current starts to flow, reducing the total effective flux, the primary counter-E.M.F. is also reduced, and the applied voltage is able to supply an increased primary current called the primary load current, which in its turn sets up a flux which is equal and opposite to that caused by the secondary load current. This equality must signify that the additional primary ampereturns are equal to the secondary ampereturns or Primary load current $\times N_1 =$ Secondary load current $\times N_2$

$$\frac{\text{Primary load current}}{\text{Secondary load current}} = \frac{N_2}{N_1} = T.$$

If I_1 and I_2 are the primary and secondary load currents

$$I_1 = TI_2$$

The total primary current is the vector sum of the original no-load current and the primary load current. The magnitude of the latter will increase directly with the magnitude of the secondary current, and when the secondary is supplying a current approaching the normal load for which it is designed, the primary load current is so much greater than the no-load component that the latter may be disregarded. Under these conditions, the relation between primary and secondary currents may be written

$$I_p = TI_2$$

where I_p is the total primary current.

(ii) The phase relation between the primary current and terminal P.D. depends upon the nature of the impedance of the appliance to which the secondary winding is supplying current and can be shewn by vector diagrams. In order to preserve balance in these, the following, convention is adopted. If the transformation ratio is less than unity (step-down) the vectors representing secondary voltages are drawn to a scale T times as large as the primary voltages,

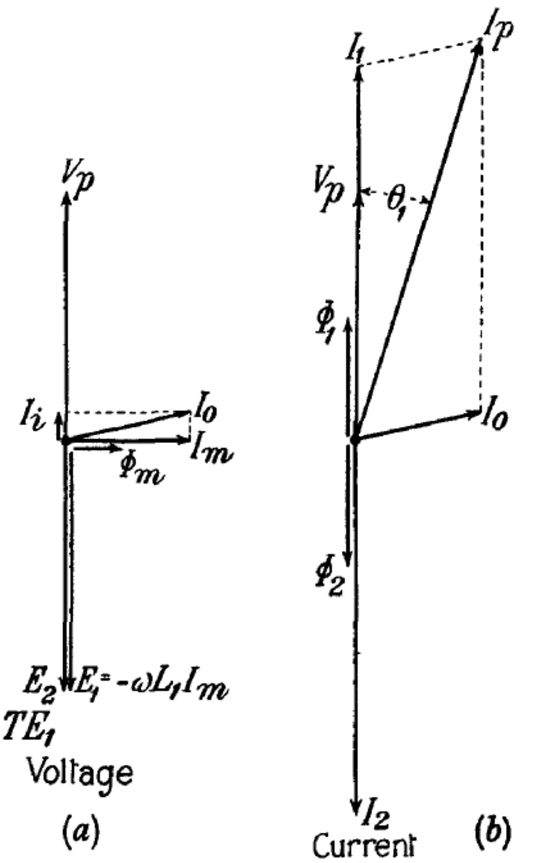


FIG. 5, CHAP. VI.—Voltage and current relations with purely resistive load.

while the primary currents are drawn on a scale T times as large as the secondary currents. If the transformation ratio is greater than unity these scales are adjusted in an opposite manner. In effect therefore the vector diagrams are drawn as for a transformer of unity turns ratio.

Resistive load on secondary

6. The effect of connecting to the secondary winding a circuit possessing resistance only is shewn by the vector diagrams, fig. 5. In fig. 5a I_m is the magnetising component of the primary no-load current I_o, I_i the loss component. The primary flux Φ_m is caused by I_m and is in phase with the latter. Lagging upon this flux by 90° are the induced voltages E_1 and E_2, the former being the primary counter-E.M.F. and the latter the secondary induced E.M.F. E_1 is to all intents and purposes equal to the applied voltage V_p and is 180° out of phase with it, while E_2 is in phase with E_1, hence the secondary E.M.F. is 180° out of phase with the applied voltage.

Now consider the effects caused by a secondary load current I_2 which is in phase with the secondary E.M.F., E_2. This current will produce a flux Φ_2 in phase with I_2 as shewn in fig. 5b, and as already stated the effect of this flux is to reduce the total flux linking with the primary winding, so that the inductance of the latter is decreased. As a result an increased primary current will flow setting up a flux Φ_1 which is equal and opposite to Φ_2. The flux Φ_1 is caused by a component of primary current which is in phase with it and is termed the primary load current I_1, and it is apparent that I_1 must be exactly opposite in phase to I_2 in order that the phase relationship between Φ_2 and Φ_1 shall be correct. The total primary current I_p is the vector sum of the primary load current I_1 and the no-load current I_o, and is obtained by the usual construction. It will be seen that I_p lags upon the applied voltage V_p by an angle θ_1 which is inversely proportional to the load current, i.e. with a small secondary current. θ_1 is nearly 90° but when the secondary winding is caused to give a large current the primary current and primary voltage are very nearly in phase. This may be expressed by stating that when on

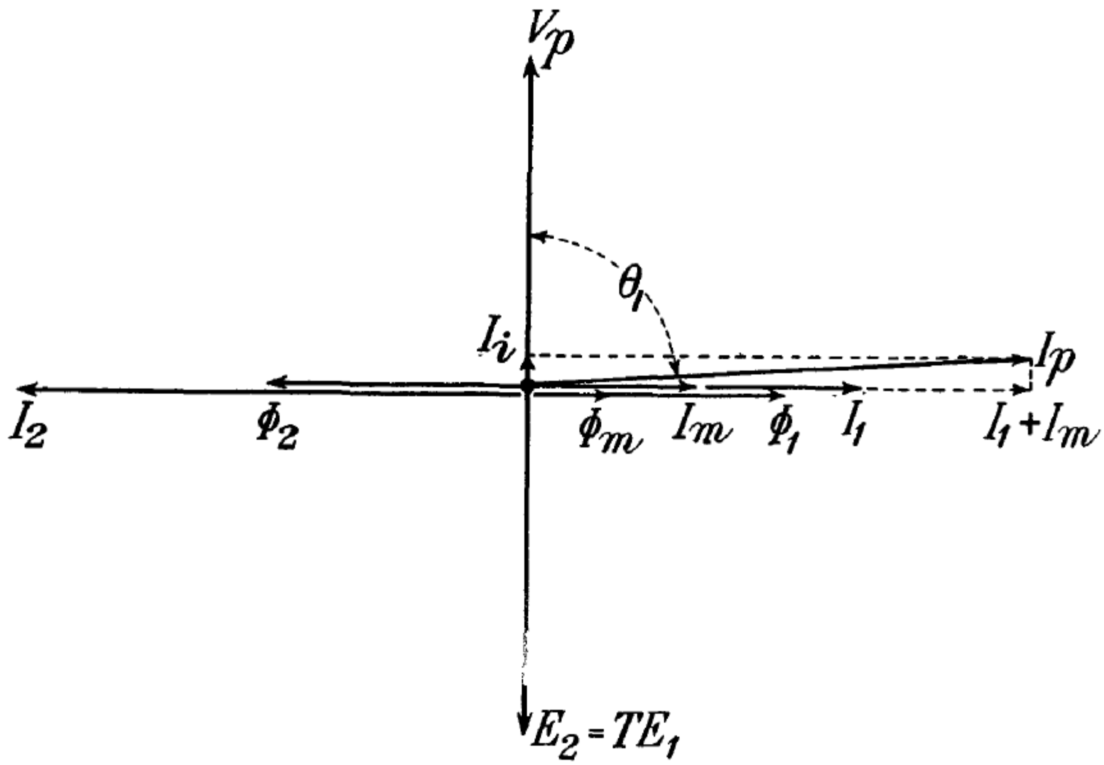


FIG. 6, CHAP. VI.—Voltage and current relations with purely inductive load.

full load with a purely resistive circuit the transformer acts as if it were practically noninductive, or alternatively, that with such conditions of loading the power factor of the primary circuit approaches unity.

Purely reactive loading

7. (i) If the secondary load is replaced by an inductance having negligible resistance the resulting conditions are shewn in the vector diagram fig. 6. Here again I_m is the magnetising current and the flux Φ_m is in phase with it. The vectors E_1 and E_2 are the primary and secondary induced voltages as before and V_p equal in magnitude to E_1, but of opposite phase, is the primary terminal voltage. It should be noted that in this and all future vector diagrams, the vector E_1 is not actually drawn, because it coincides with E_2 owing to the adjustment of relative scales of current and voltage, paragraph 5. The secondary current I_2 will lag on the secondary voltage E_2 by 90°, and will set up the flux Φ_2, but in order that the total flux may remain constant and equal to Φ_m, a primary load current I_1 will be established, its phase being such that its resultant flux

Φ_1 is equal and opposite to Φ_2. The primary load current lags on the applied voltage by 90°, and the total primary current I_p which is the vector sum of I_1, I_m and I_i, will lag on V_p by an angle θ_1 which is nearly but not quite 90°.

(ii) The effect of a purely capacitive secondary load is shewn in the vector diagram, fig. 7, the magnetising current I_m and flux Φ_m being as before, and E_2 the secondary E.M.F. lags by 90° on these. The secondary load current I_2 leads on E_2 by 90°, and the primary load current I_1 is 180° out of phase with I_2. The primary load current I_p is the vector sum of I_1 and I_o. Under certain conditions, a resonant condition may exist, and this occurs when the primary load current is exactly equal (and of opposite phase) to the magnetising current. The total primary current I_p is then only that due to the iron losses, and so far as the primary terminal

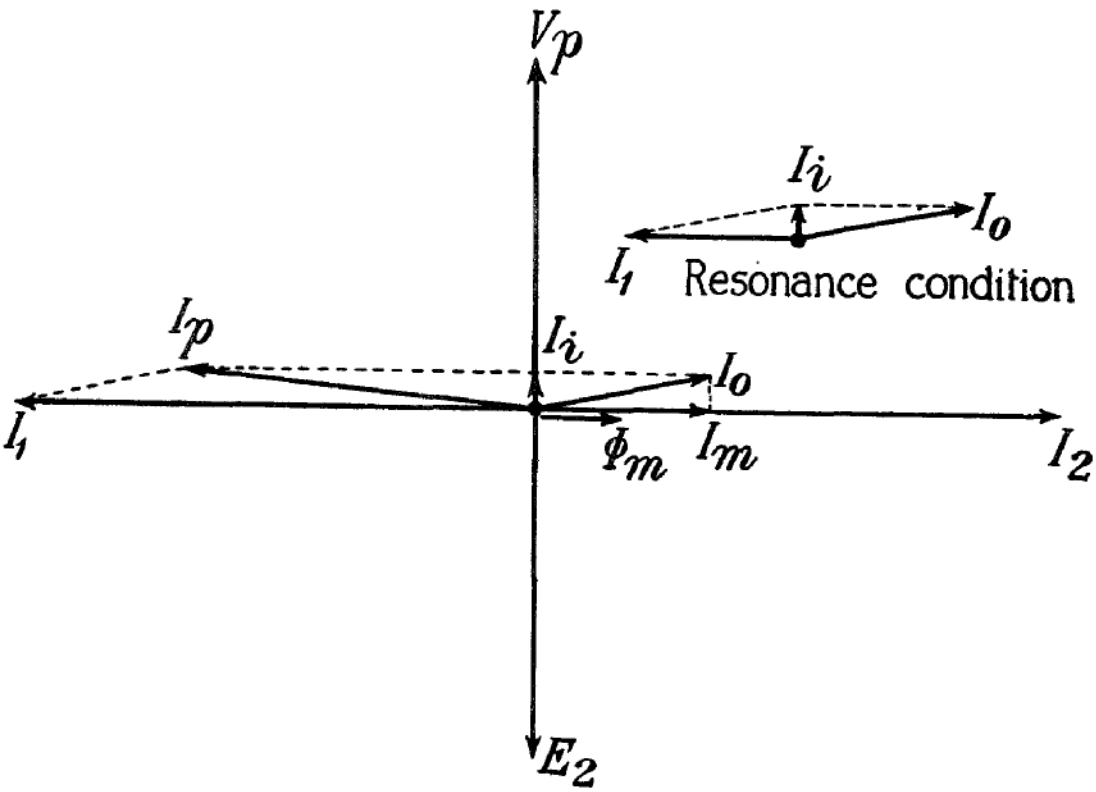


FIG. 7, CHAP. VI.—Voltage and current relations with purely capacitive load.

P.D. is concerned the transformer with its capacitance load behaves like a rejector circuit. This condition never occurs in power transformers but is of importance in the small iron-core transformer used in audio frequency amplifiers.

Secondary load both resistive and reactive

8. (i) As a rule, the appliance to which the secondary winding is connected will possess both reactance and resistance. In order to investigate the action under such conditions, consider the secondary load to consist of a resistance of r_2 ohms and an inductance of l_2 henries connected in series, and the impedance of the load will then be $\sqrt{r_2^2 + \omega^2 l^2} = Z_2$ ohms. The vector diagram is given in fig. 8, and as before, the secondary induced E.M.F. E_2 will cause a current I_2 to flow, its magnitude being $I_2 = \frac{E_2}{Z_2}$ ohms ; this current will lag on by an angle $\theta_2 = \frac{\tan^{-1} \omega l_2}{r_2}$

The primary load current I_1 is of opposite phase to I_2 in order to establish the necessary counter-balancing flux, and the total primary current is the vector sum of I_1 and I_o, lagging upon the applied voltage V_p by the angle θ_1 which is slightly greater than θ_2, the inequality becoming smaller as the secondary current increases. When the transformer is giving its full secondary current, it may be considered that the primary current lags on the applied E.M.F. by an angle equal to θ_2 which is tantamount to an assertion that the transformer itself has no effect upon the phase difference, but merely serves to alter the voltage applied to the external circuit connected to the secondary terminals.

(ii) In fig. 9 is shewn the condition which arises when the secondary load possesses both resistance and capacitive reactance. Here E_1 and E_2 lag by 90° on the magnetising current, and I_2 leads on E_2 by an angle θ_2, the primary load current I_1 being of exactly opposite phase in order to produce the necessary counter-balancing flux. The primary current I_p is the vector

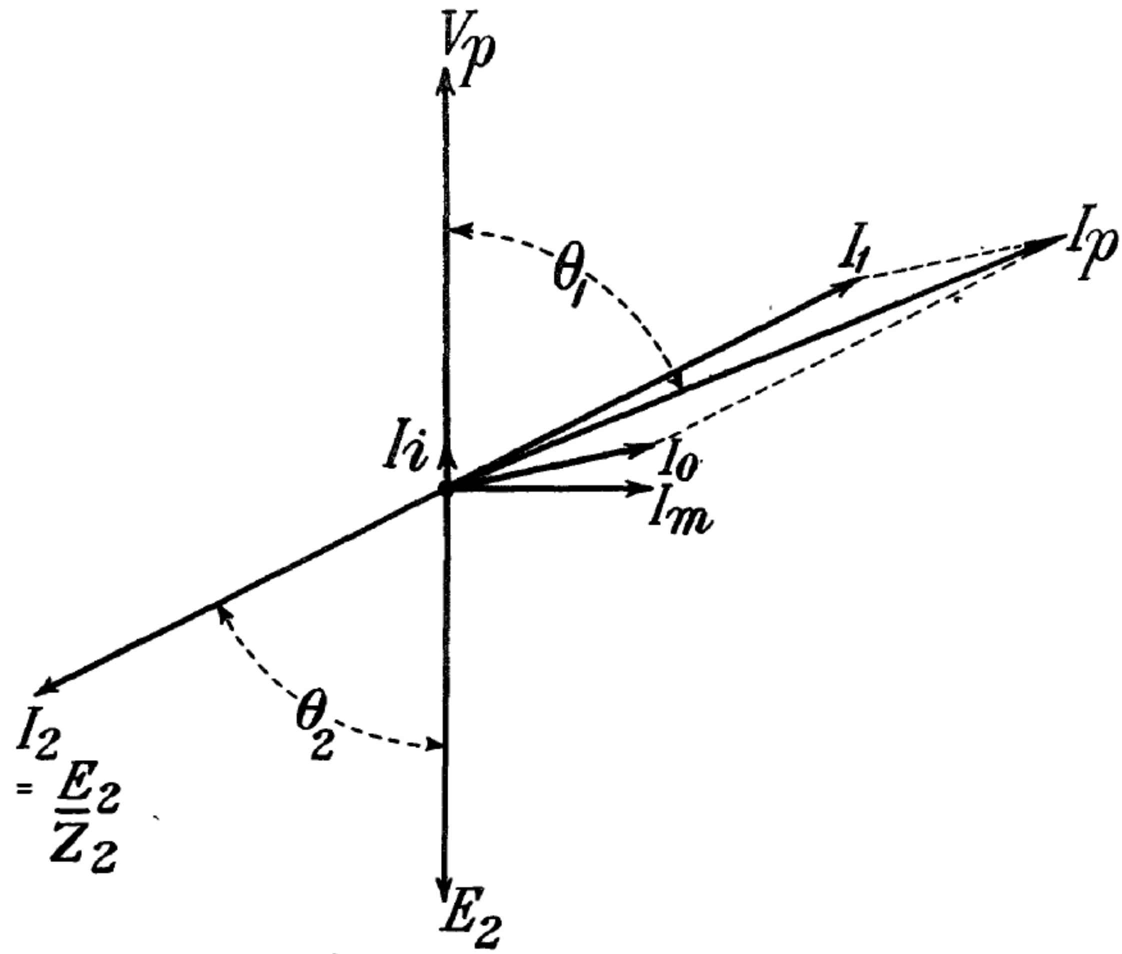


FIG. 8, CHAP. VI.—Voltage and current relations with load possessing both resistance and inductance.

sum of I_1 and I_o, and leads on the applied voltage V_p by the angle θ_1 which is slightly less than θ_2. In the inset diagram the resonance condition is depicted. In this case the vector sum of I_1 and I_o, i.e. I_p is in phase with the applied voltage V_p, and the circuit behaves as if it possesses neither inductance nor capacitance, imposing only a resistive load upon the alternator.

Losses in transformers

9. In the circuits just discussed, it will be observed that, if the no load current is neglected, the following relations hold, namely $I_p = I_2 \times T$, $E_2 = E_1 \times T = V_p \times T$. From this it follows that $I_p \times V_p = I_2 \times T \times \frac{E_2}{T}$ or $I_p V_p = I_2 E_2$ This indicates that if the losses are

neglected, the power input is equal to the power output, which is of course to be expected from the principle of conservation of energy. In practice this equality is not achieved owing to the conversion of a portion of the energy supplied into heat, and it is convenient to divide the losses according to the portion of the transformer in which they occur. This leads to the conception of (i) iron losses (ii) copper losses and (iii) losses due to magnetic leakage.

Iron losses

10. The iron losses are caused by (*a*) eddy currents in the iron, and (*b*) hysteresis.

Eddy currents.—These are due to the effects already noted in connection with the armatures of dynamo-electric machinery (Chapter IV), and are reduced but not entirely eliminated by lamination of the iron core. The degree to which the sub-division is performed depends to some

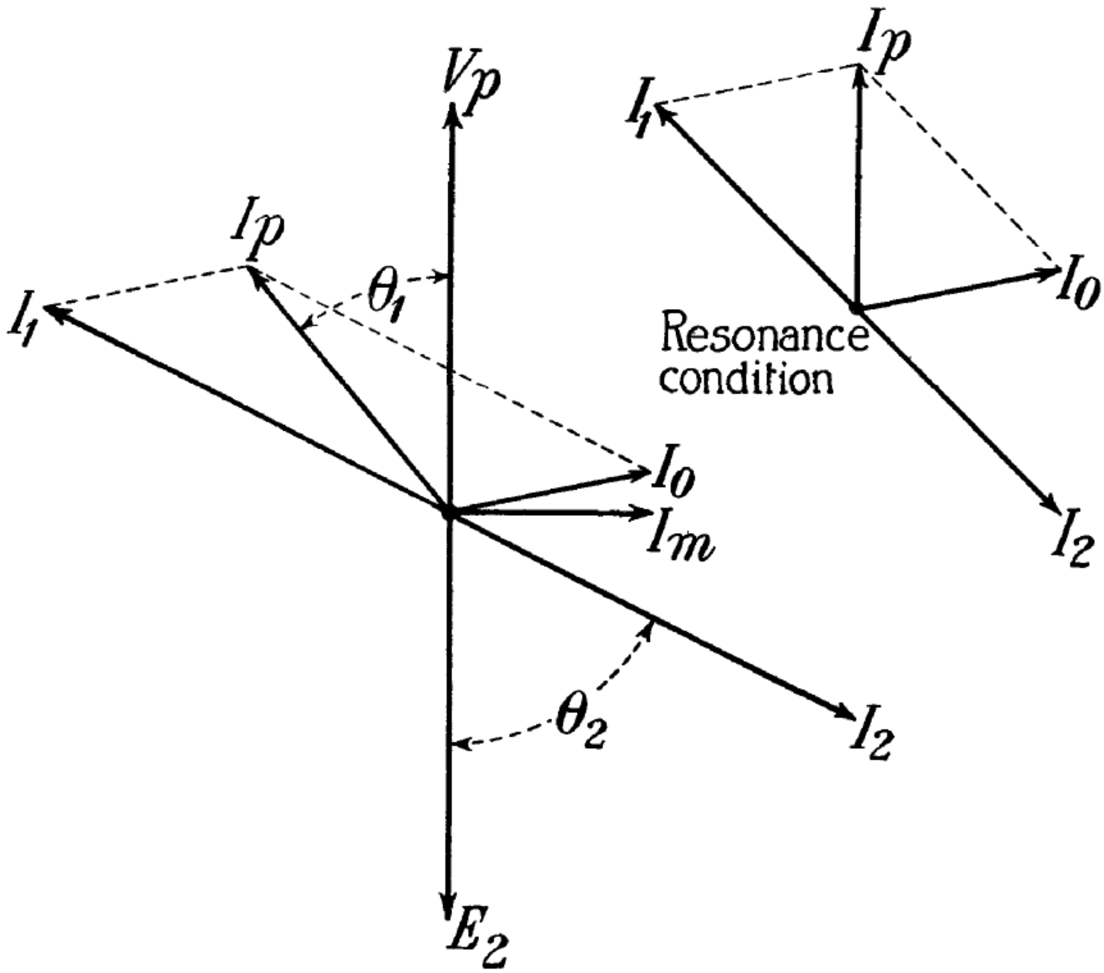


FIG 9, CHAP. VI.—Voltage and current relations with load possessing both resistance and capacitance.

extent upon the kind of iron employed, and also upon the frequency for which the transformer is designed. At the standard commercial frequency (50 cycles per second) a thickness of ·014 inch is frequently adopted if soft iron is used, but with stalloy the laminations may be as much as ·03 inch in thickness. At lower frequencies thicker laminations may be used, but at higher frequencies it becomes increasingly difficult and expensive to achieve the degree of lamination which is necessary if the iron losses are not to be increased. At frequencies of the order of hundreds or thousands per second therefore, the iron losses are much greater than at commercial frequencies.

Hysteresis losses.—During each alteration of the magnetising current, the core is carried through a complete cycle of magnetisation, and the variation of the flux density B under the influence of the magnetising force H gives rise to a B/H curve which is actually a closed loop. This signifies that the energy expended in creating the flux is not entirely restored to the electric circuit when the flux is destroyed, or that energy is expended in changing the magnetic state of the material. This phenomenon was dealt with in Chapter II, and it is only necessary to add that in practice it is usual to assume that the energy expended may be represented by an equation of the form

$$W_h = hB^{1\cdot 7}.$$

where W_h is the energy required to carry the iron through a complete magnetic cycle, in ergs per cubic centimetre, and h a constant for the particular kind of iron. For materials customarily employed in transformers h usually lies between ·001 and ·0015. As an example, consider the power P_h required to overcome the hysteresis loss in a transformer core, the cross section of which is 100 square centimetres and the length of the mean magnetic circuit is 200 centimetres. The hysteretic constant h may be taken as ·001, and the frequency 50 cycles per second, while the flux density may be as high as 10,000 C.G.S. units.

$$\text{Then } P_h = \cdot 001 \times 10000^{1\cdot 7} \times 10^{-7} \text{ joules per cm}^3 \text{ per period}$$

$$= \cdot 001 \times 10000^{1\cdot 7} \times 10^{-7} \times 100 \times 200 \times 50 \frac{\text{joules}}{\text{sec.}} \quad (10000^{1\cdot 7} = 6{,}310{,}000)$$

$$\therefore P_h = \cdot 001 \times 6\cdot 31 \times 10^6 \times 10^{-7} \times 10^2 \times 2 \times 10^2 \times 50$$

$$= 10^3 \times 6\cdot 31 \times 10^6 \times 10^{-7} \times 10^2 \times 10^2 \times 10^2 = 631 \text{ watts.}$$

The iron losses are practically the same on no load as on full load, and in the former condition are responsible for practically the whole of the losses, because the actual ohmic resistance of the primary is so small that, when only the small no-load primary current is flowing, the I^2R loss in the conductor is negligible. It will be remembered that the no load current was divided into two components, one of which, I_m, was said to establish the flux and to cause no loss of energy, while the other, I_i, was held responsible for the losses under no-load conditions. As we have now seen, these are almost entirely due to the iron and consequently I_i is referred to as the iron-loss current.

Copper losses

11. These are caused by the actual ohmic resistance of the windings, and as stated above are negligible when the secondary is unloaded. When a secondary current is established, however, the primary current increases and the copper losses in the primary circuit are equal to $I_p^2R_1$, while the copper losses in the secondary circuit become $I_2^2R_2$, hence the total copper losses are

$$P_c = I_p^2R_1 + I_2^2R_2 \text{ watts.}$$

In consequence of this waste of power the resistance is responsible for a voltage drop in each winding, and to produce a given number of magnetising ampereturns, the primary terminal voltage must be greater than that calculated by ignoring the effect of resistance, while the resistance of the secondary winding causes its terminal P.D. to be less than the induced E.M.F. E_2. These consequences are shown in a vector diagram (fig. 10). The no-load current I_o and the voltages E_1 and E_2 are as in previous diagrams. Assuming that the load possesses both inductance and resistance, the secondary current I_2 lags upon E_2 by the angle θ_2, and a component of voltage $e_2 = I_2 R_2$ in phase with I_2 represents the internal secondary P.D. due to the copper loss. The secondary terminal voltage is the vector difference between E_2 and e_2 and is shown by the vector V_2. Similar effects occur in the primary, and it is seen that I_p the primary current is the vector sum of I_o and I_1 the no-load and load currents respectively. The copper loss in this circuit is responsible for an internal P.D. equal to $I_p R_1 = e_1$ which is in phase with I_p, and the total voltage V_p which must be applied to the primary is now not V_1 equal and opposite to E_1, as in previous instances, but the vector sum of e_1 and V_1.

It has been shown that the total copper losses are equal to $I_p^2\ R_1 + I_2^2\ R_2$ watts. It is sometimes convenient to regard these as being caused by a single resistance placed either in the primary or secondary circuit. As I_2 is practically equal to $\frac{I_p}{T}$, the copper losses may be written $I_p^2 R_1 + I_p^2\ \frac{R_2}{T^2}$ or $I_p^2\left(R_1 + \frac{R_2}{T^2}\right)$ watts and if no copper losses existed in the transformer itself a resistance $R_1 + \frac{R_2}{T^2}$ placed in series with the primary circuit would have the same effect as the

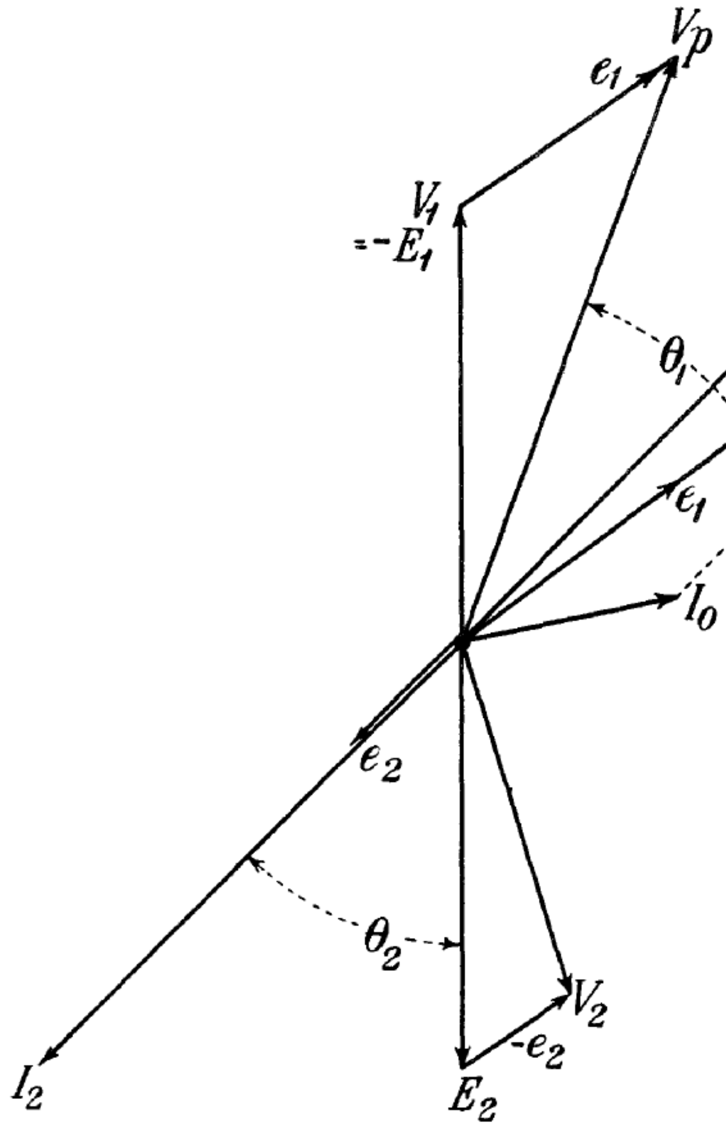


FIG. 10, CHAP. VI.—Effect of internal resistance of windings.

actual primary and secondary resistances, both with respect to the losses and to the P.D. at the secondary terminals. The expression $\left(R_1 + \frac{R_2}{T^2}\right)$ is therefore called the equivalent resistance of the transformer referred to the primary circuit, or more briefly as the equivalent primary resistance, and is generally denoted by R_p. Alternatively, it may be assumed that the whole of the copper losses are caused by a resistance equal to $R_2 + T^2 R_1$ in the secondary circuit, the primary circuit then being assumed to have no copper losses; the expression $(R_2 + T^2 R_1)$ is called the equivalent secondary resistance, and is denoted by R_s. It may be observed that if the windings are so designed that the current density is the same in both primary and secondary windings, that is, if each winding carries the same current per square millimetre of cross-sectional area, the cross section of each winding is inversely proportional to the number of turns, while the length of each winding is proportional to the number of turns, provided that the primary and secondary are intermingled and not wound one over the other. In consequence, $T^2 R_1$ is always approximately equal to R_2, and the equivalent resistance approximately equal to double the true resistance, i.e. $R_p \doteqdot 2R_1$, or $R_s \doteqdot 2R_2$.

Example.—A 25 K.V.A. transformer has a step up of 10 to 1, and is designed for a primary voltage of 250 volts. The primary winding has a resistance of ·02 ohms and the secondary 1·8 ohms. Find the equivalent secondary resistance, the fall of terminal P.D. at full load, and the copper losses at full load.

The equivalent secondary resistance $= R_2 + T^2 R_1$,
$= 1{\cdot}8 + 10^2 \times {\cdot}02.$
$= 3{\cdot}8$ ohms.
N.B.—$2R_2 = 3{\cdot}6$ ohms.

Full load primary current (neglecting no-load component)

$$= \frac{25000 \text{ volt-amperes}}{250 \text{ volts}} = 100 \text{ amperes.}$$

The voltage drop due to primary resistance $= 100 \times {\cdot}02$
$= 2$ volts.

Hence the secondary voltage, instead of being 2500 volts will be 2480 volts.

Full load secondary current $= \dfrac{I_p}{T} = \dfrac{100}{10} = 10$ amperes.

Voltage drop due to secondary resistance $= 10 \times 1{\cdot}8 = 18$ volts. Hence the secondary terminal P.D. $= 2480 - 18$.
$= 2462$ volts.

Copper loss at full load $= I_p^2 R_1 + I_2^2 R_2$
$= 100^2 \times {\cdot}02 + 10^2 \times 1{\cdot}8$
$= 200 + 180$
$= 380$ watts.

Using the equivalent secondary resistance :—

Total resistance drop $= I_2 (R_2 + T^2 R_1) = I_2 R_s = 10 \times 3{\cdot}8 = 38$ volts.
Secondary terminal P.D. $= 2500 - 38 = 2462$ volts.
Copper losses $= I^2{}_2 (R_2 + T^2 R_1)$
$= 100 \times 3{\cdot}8 = 380$ watts.

The use of the "equivalent resistance" method obviously reduces the amount of arithmetic required.

Losses due to magnetic leakage

12. Up to the present it has been assumed that the whole of the flux set up by the magnetising current passes through every turn on both windings of the transformer, but this is not entirely true. Considering a transformer with the general design shown in fig. 11; the current in the primary winding sets up a flux, a portion of which may leave the core and pass through the air, and these tubes of flux do not link with the secondary winding. As they do not assist in producing a secondary E.M.F. they are known collectively as the primary leakage flux. Similarly,

the current flowing in the secondary will produce some tubes of flux which do not link with the primary winding, and these constitute the secondary leakage flux. The leakage flux due to each winding is produced by the current in that winding and is independent of the other, and each portion is therefore in phase with and proportional to the current responsible for its existence, both leakage fluxes increasing with an increase of load current. This is in contrast with the main flux linking with both windings, which being proportional to the magnetising current I_m remains constant at all loads. Since the primary ampereturns and secondary ampereturns are nearly equal the leakage fluxes will also approach equality, because the reluctance of their

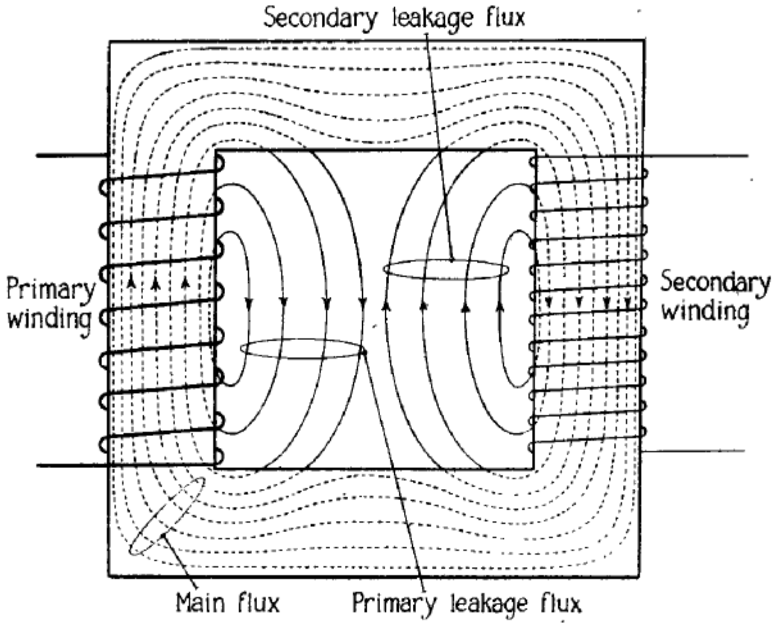


FIG. 11, CHAP. VI.—Primary and secondary leakage flux.

respective paths cannot be very different. The effects of magnetic leakage are shewn in the vector diagram fig. 12, in which the common flux is shewn by Φ_m, and lagging on this by 90° are the induced voltages E_1 and E_2 in the primary and secondary windings respectively. E_1 is counterbalanced by a component of the applied voltage which is represented by V_1. The secondary voltage E_2 gives rise to the secondary current I_2, and I_2 in turn produces the flux Φ_2 of which a portion ${}_L\Phi_2$ represents the leakage flux. The actual secondary flux is the vector sum of the common flux Φ_m and the secondary leakage flux ${}_L\Phi_2$, and is indicated by the vector Φ_2'. The effect of the secondary leakage flux is to cause a reduction in the total induced secondary voltage, which may be considered to be due to a component of the secondary E.M.F. which lags by 90° on the leakage flux, as shewn by the vector ${}_LE_2$. The effective E.M.F.acting in the secondary winding is the sum of the vectors E_2 and ${}_LE_2$ and is given by E_2'. It will be observed that this vector lags by 90° on the effective secondary flux Φ_2'. The primary load current I_1 produces the flux Φ_1 counterbalancing the flux Φ_2, and the primary current I_P is the vector sum of I_1 and the no-load current I_o. The primary leakage flux ${}_L\Phi_1$ is in phase with I_P, and the total primary flux is therefore Φ_1'. The result of the leakage flux may be represented by considering that an additional counter E.M.F. ${}_LE_1$ is induced in the primary winding, and this must be overcome by an additional applied voltage, hence the terminal P.D. is not V_1, 180° out of phase with E_2, but V_P which leads upon the resultant primary flux Φ_1' by 90°.

Effects of magnetic leakage

13. (i) It will be observed that owing to magnetic leakage, the applied E.M.F. has to be increased in value in order to obtain a given secondary current, and this applied E.M.F. leads on the primary current by a greater angle than would be the case in the absence of leakage. The latter therefore appears to add an effective inductance in series with the primary winding, and

this is called the leakage inductance of the primary. Again in the secondary circuit the terminal voltage is reduced by the effect of leakage from E_2 lagging by 90° on the common flux, to E_2' lagging by an angle greater than 90°. The secondary current I_2 lags on the actual secondary E.M.F. E_2' by the angle θ_2, whereas in the absence of the secondary leakage flux it would lag by the same angle upon the voltage E_2. The effect of the magnetic leakage is therefore to increase the angle of lag between the secondary current and the common flux, and is equivalent to the

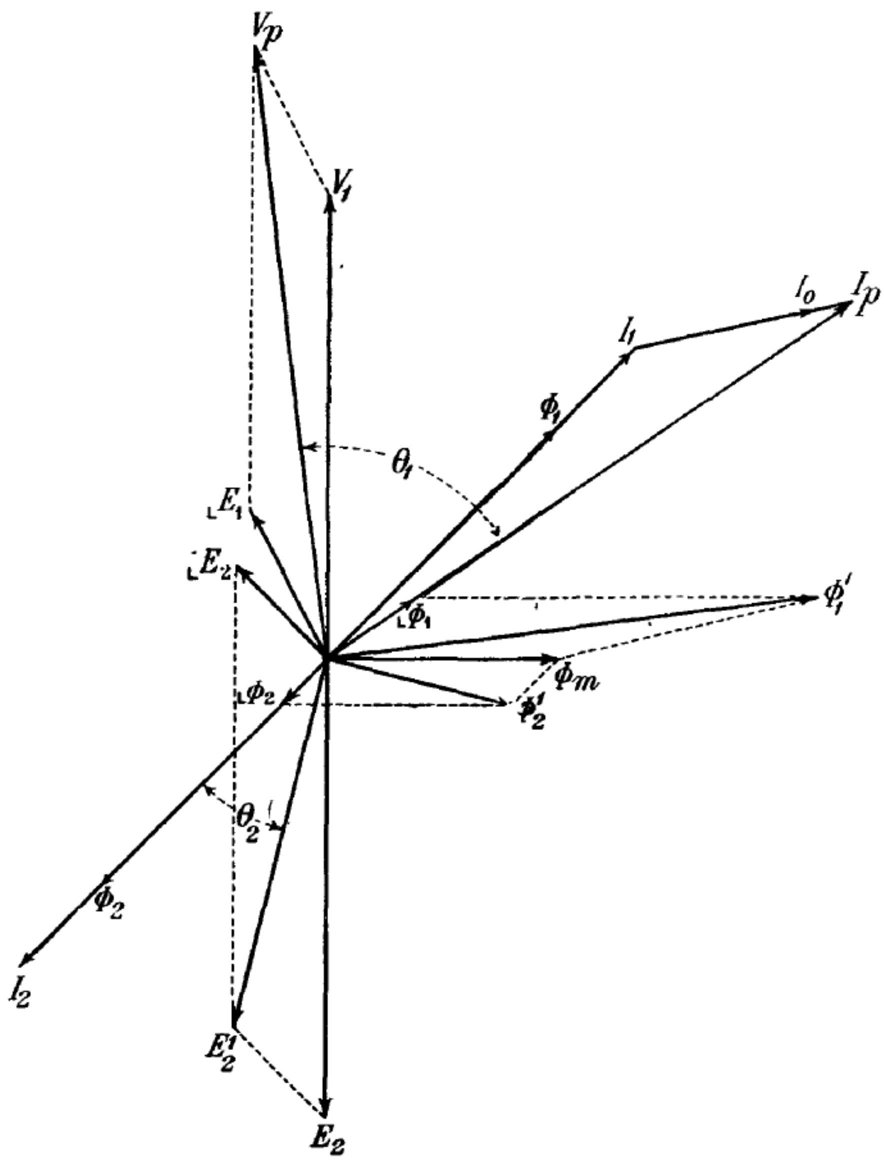


FIG. 12, CHAP. VI.—Effect of leakage flux.

introduction of an inductive reactance into the secondary circuit. Both leakage inductances cause a reduction in the secondary terminal P.D. for a given applied voltage, and in practice certain steps are taken to reduce this reactive voltage drop. One expedient, applicable chiefly to transformers of the core type, is to use concentric windings, the secondary being wound in sections over the primary. In the shell type of transformer, the windings may be sectionally wound and the primary and secondary sections intermingled upon the centre limb, with the same object. The arrangement of one winding on each limb of the core, as in fig. 1, is hardly ever adopted.

(ii) The effects of both copper losses and magnetic leakage are illustrated by the vector diagram given in fig. 13, which shews the conditions when the secondary load is both resistive and inductive. The secondary current I_2 lags on the secondary induced E.M.F., and in phase with I_2 is the internal voltage drop I_2R_2, while the reactive drop I_2X_2 due to magnetic leakage leads on I_2 by 90°. The total internal voltage drop is the vector sum of I_2R_2 and I_2X_2 and is given by I_2Z_2, the secondary terminal P.D. being therefore V_2, which is the result of subtracting

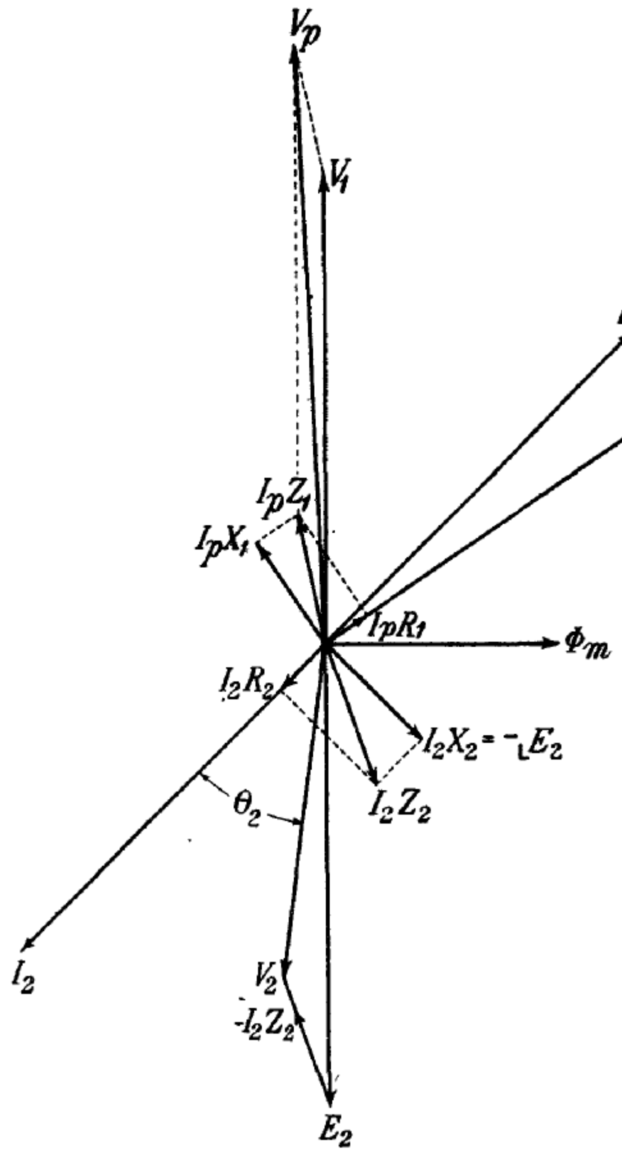


FIG. 13, CHAP. VI.—Effect of leakage flux and internal resistance.

the internal volts drop I_2Z_2 from the secondary E.M.F. E_2. I_2 is seen to lag on V_2 by an angle θ_2. The primary current I_p is the sum of the primary load current I_1 and the no-load current I_o, and the internal resistive volts drop I_pR_1 is in phase with I_p while the reactive volts drop I_pX_1 leads on I_p by 90°. The total internal primary volts drop is the vector sum of I_pR_1 and I_pX_1, and is shewn in the diagram by I_pZ_1. The primary applied voltage V_p is the vector sum of the internal drop I_pZ_1 and the component V_1 which is equal and opposite to the primary counter-E.M.F.

Equivalent circuit of the transformer

14. The behaviour of a power transformer may be represented with a high degree of accuracy by an equivalent circuit, in which the transformer itself is considered to be an ideal one, giving a secondary E.M.F. equal to the primary terminal voltage multiplied by the turns ratio, and having no internal losses whatever. The primary resistance R_1 and leakage reactance X_1 are considered to exist outside the actual transformer, and the same applies to the secondary resistance R_2 and leakage reactance X_2. As it must be assumed that no current will flow through the primary winding of this ideal transformer unless the secondary is on load, it is necessary to add an imaginary circuit which will account theoretically for the no-load current, and this is done by inserting a circuit consisting of a resistance R_o and inductive reactance X_o in parallel

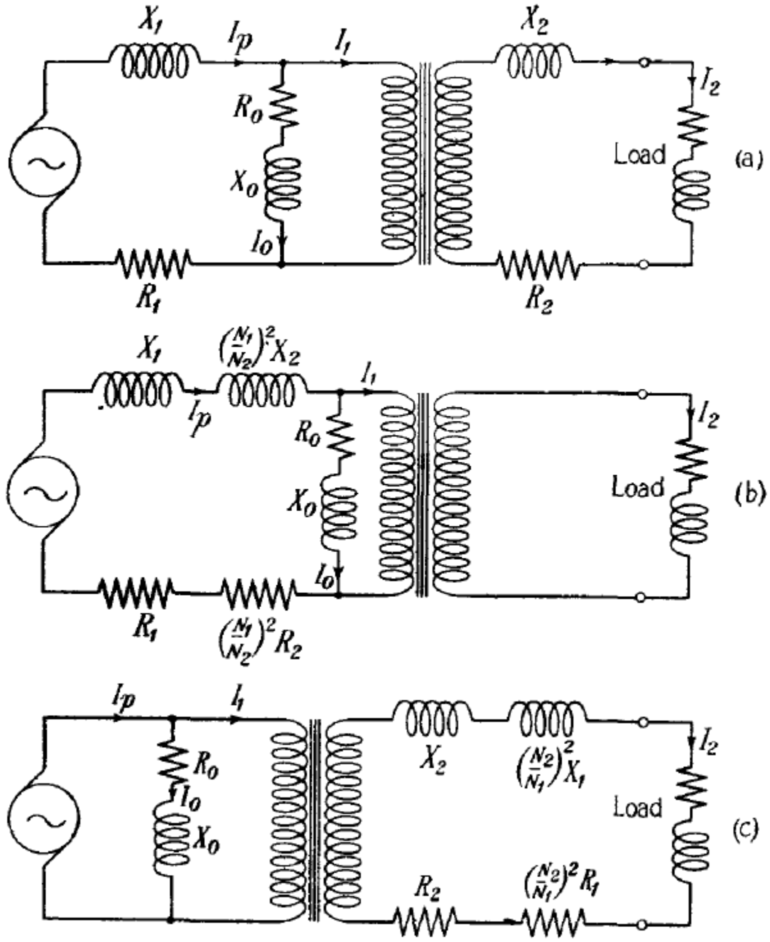


FIG. 14, CHAP. VI.—Equivalent circuits of transformer.

with the primary winding (fig. 14a). A further simplification may be made by using the equivalent primary resistance $R_p = R_1 + \frac{R_2}{T^2}$ and the equivalent leakage reactance $X_p = X_1 + \frac{X_2}{T^2}$ as in fig. 14b, or the equivalent secondary resistance $R_s = R_2 + T^2R_1$ and equivalent secondary leakage reactance $X_s = X_2 + T^2X_1$, fig. 14c. The value to be assigned to the quantities R_o and X_o can be determined by what is called the open circuit test upon the actual transformer. This is performed by connecting the transformer to supply mains of the voltage and frequency for which it was designed, a voltmeter (V_1), ammeter (A_1) and wattmeter (W) being included, as in fig. 15. The voltmeter (V_2) is inserted merely as a check on the transformation ratio and may be omitted if the turns ratio is known. In any event it should be of high resistance or of the electrostatic type. The wattmeter may be considered to give only the iron losses, because the copper losses are utterly insignificant under no-load conditions, while the ammeter (A_1) gives the no-load current, which may be divided into its two components, namely the iron-loss component I_i and true magnetising component I_m, by the following method. Referring to the vector diagram fig. 4 which shows the applied primary voltage V_p, the no-load current I_o and its components, it is seen that $I_i = I_o \cos \theta_o$ and $I_m = I_o \sin \theta_o$. Taking the 25 K.V.A. transformer previously referred to as an example, suppose the no-load current to be 2·5 amperes, and the wattmeter reading to be $P_i = 250$ watts. The power P_i is equal to $V_p I_o \cos \theta_o$, and therefore the iron loss component I_i of the no-load current is given by the equation $I_i = \frac{P_i}{V_p}$.

In this particular instance $I_i = \frac{250}{250} = 1$ ampere and the true magnetising component I_m is equal to $\sqrt{I_o^2 - I_i^2}$ or $\sqrt{2 \cdot 5^2 - 1} = 2 \cdot 265$ amperes. Hence $\tan \theta_o = \frac{I_m}{I_i} = 2 \cdot 265$, and $\theta_o =$ 66° approximately. It must not be supposed that these figures are representative of an efficient transformer.

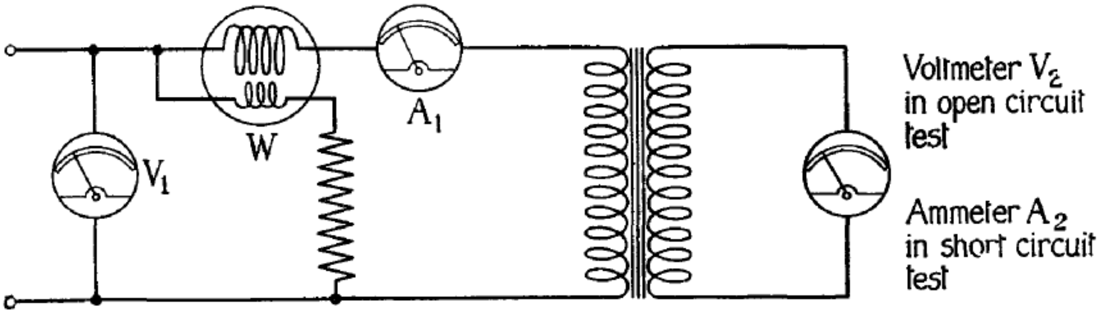


FIG. 15, CHAP. VI.—Arrangements for open circuit and short circuit tests of transformer.

The open circuit impedance Z_o of the primary is equal to $\frac{V_p}{I_o}$ or $\frac{250}{2 \cdot 5} = 100$ ohms, and the resistance component of this is found by dividing the wattmeter reading by the square of the no-load current giving

$$R_o = \frac{250}{2 \cdot 5^2} = \frac{250}{6 \cdot 25} = 40 \text{ ohms.}$$

and the reactive component is now easily found because $Z_o = \sqrt{R_o^2 + X_o^2}$, whence

$$X_o = \sqrt{Z_o^2 - R_o^2} \text{ or}$$
$$X_o = \sqrt{100^2 - 40^2}$$
$$= \underline{\underline{91 \cdot 5 \text{ ohms.}}}$$

15. The equivalent secondary resistance R_s and leakage reactance X_s are determined by means of the short circuit test. To perform this the secondary winding is short circuited by an ammeter (A_2), fig. 15, of very low resistance, and the primary winding is supplied at normal frequency, but with only sufficient terminal P.D. to cause full load secondary current to flow. The P.D. current and power supplied to the primary are then observed, by the instruments used in the previous test.

As the primary P.D. is low, only a comparatively weak flux is established and the iron losses are negligible compared with the copper losses. The wattmeter may therefore be considered to give the latter losses only.

Example.—The secondary of the transformer previously considered is connected to an ammeter of negligible resistance, and is found to give 10 amperes when the primary terminal

P.D. is 20 volts, the primary current 102·5 amperes and the input power 380 watts. Find the resistance and reactance of the transformer, assuming these are concentrated in the secondary winding.

Equivalent secondary resistance $R_s = \frac{380}{10^2} = 3{\cdot}8$ ohms.

The primary angle of lag is given by

$$\cos\theta_1 = \frac{\text{Power}}{\text{volt-amperes}} \text{ in primary}$$
$$= \frac{380}{102{\cdot}5 \times 20} = {\cdot}185$$
$$\therefore \theta_1 = 79° \text{ (approx.).}$$

The primary and secondary angles of lag are practically equal, because the secondary winding is giving its full load current.

Hence $\tan\theta_2 = \tan\theta_1 = 5{\cdot}3$

Now $\tan\theta_2 = \frac{X_s}{R_s}$

$$\therefore X_s = 5{\cdot}3R_s$$
$$= 5{\cdot}3 \times 3{\cdot}8$$
$$= 20 \text{ ohms.}$$

As the transformer has a 10 to 1 step-up, these values are 100 times as great as the equivalent primary resistance R_p and reactance X_p which become

$$R_p = {\cdot}038 \text{ ohms.}$$
$$X_p = {\cdot}2 \text{ ohms.}$$

Transformer efficiency

16. The efficiency of a transformer is measured by the ratio

$$\frac{\text{Power supplied to transformer—power wasted}}{\text{Power supplied to transformer}}$$

The power wasted is the sum of the copper and iron losses, the former increasing with the square of the secondary current and the latter being practically constant at all loads. If the copper losses at full load are known, e.g. as the result of a short circuit test, they are easily calculated for any other load, being $\frac{1}{4}$ of the maximum when the secondary current is half the full load, $\frac{1}{16}$ when the secondary current is one quarter full load and so on. In the 25 K.V.A. transformer which has been used in previous illustrations, the full load losses are, copper loss 380 watts, iron loss 250 watts, or a total of 630 watts, and the output is 25 K.V.A. If the secondary load is nonreactive, therefore, the input power for full load will be 25630 watts and the efficiency $\frac{25000}{25630} \times 100 = 97{\cdot}5$ per cent.

At three-quarters full load the copper losses will be $(\frac{3}{4})^2 \times 380 = 213{\cdot}75$ watts, the iron loss 250 watts, and the total losses $213{\cdot}75 + 250 = 463{\cdot}75$ watts. The output will be $\frac{3}{4} \times 25000 =$ 18750 watts, and the efficiency $\frac{18750}{19194} = 97{\cdot}6$ per cent.

At one-half full load the copper losses will be only 95 watts, and the total losses $95 + 250 =$ 345 watts, while the output will be 125000 watts, hence

$$\eta = \frac{12500}{12845} = 97{\cdot}2 \text{ per cent.}$$

At one-quarter full load, again

copper losses = 24 watts (approx.)
iron losses = 250 watts

Total losses 274 watts.

and $\eta = \frac{6250}{6524} = 95 \cdot 7$ per cent.; at only 1/10 of full load the total losses are 254 watts, the output 2500 watts and $\eta = \frac{2500}{2754} = 90 \cdot 7$ per cent.

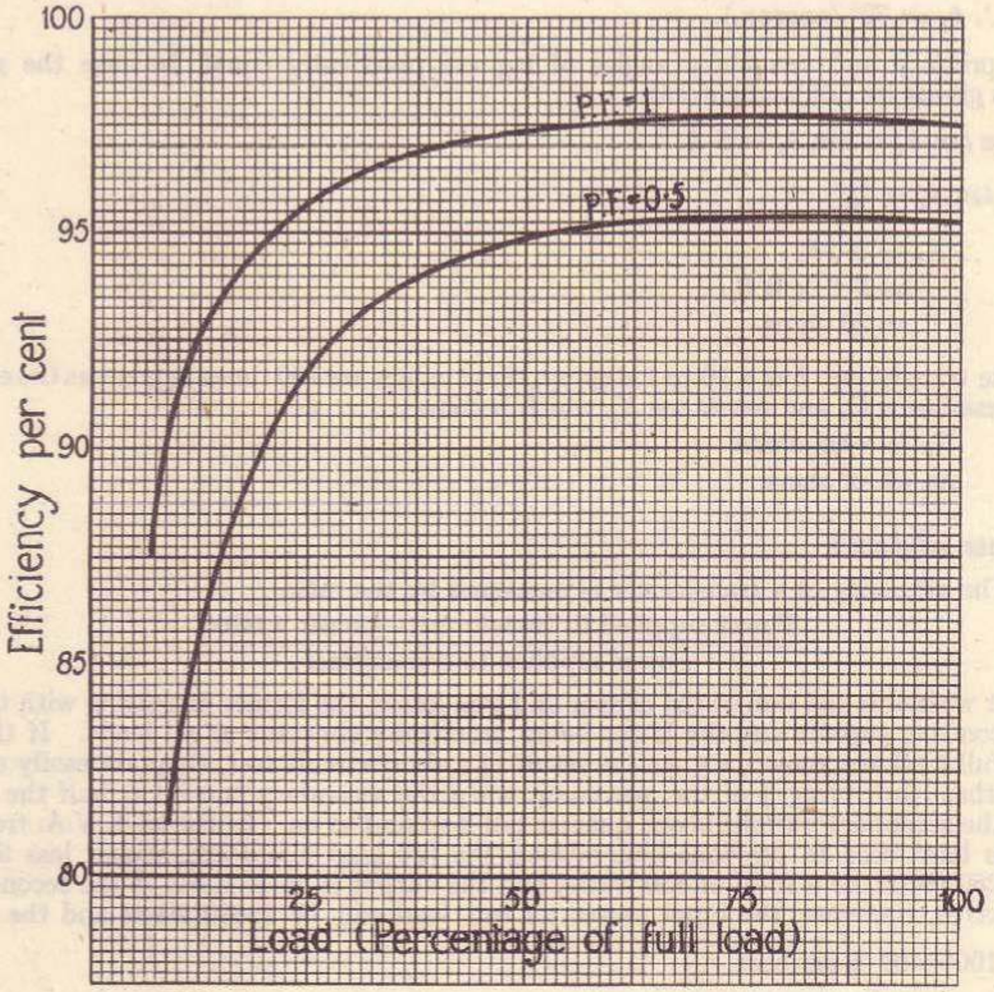


FIG. 16, CHAP. VI.—Variation of efficiency with load and power factor.

When the secondary load is reactive, the output in K.V.A. must be multiplied by the power factor of the load in order to obtain the output power; for example if the above calculations are repeated with a power factor of ·5, the output watts are halved in each case, but the losses for each condition of loading are unchanged. At full load, therefore, the output is 25000 × ·5 or 12500 watts, the input 12500 + 630 watts and $\eta = \frac{12500}{13130} = 95 \cdot 1$ per cent.

At half full load, the total losses are 345 watts, and the output 6250 watts.

$$\eta = \frac{6250}{6595} = 94{\cdot}8 \text{ per cent.}$$

and so on. The efficiency at various degrees of loading and for the power factors 1·0 and 0·5 respectively are shown graphically in fig. 16.

It will be seen from the diagram that maximum efficiency does not occur at full load but at about 80 per cent. of full load. This is because maximum efficiency is achieved under loading conditions at which the copper and the iron losses are equal. If P_i is the iron loss and P_c the full load copper loss, the fraction of full load at which the efficiency is a maximum is $\sqrt{\frac{P_i}{P_c}}$ e.g. in the transformer dealt with in the above example, the full load copper loss is 380 watts, the iron loss 250 watts, and the efficiency is a maximum at $\sqrt{\frac{250}{380}} = {\cdot}81$ of full load, the output being then 20,300 watts, the input 20,800 watts, and the efficiency approximately 97·6 per cent.

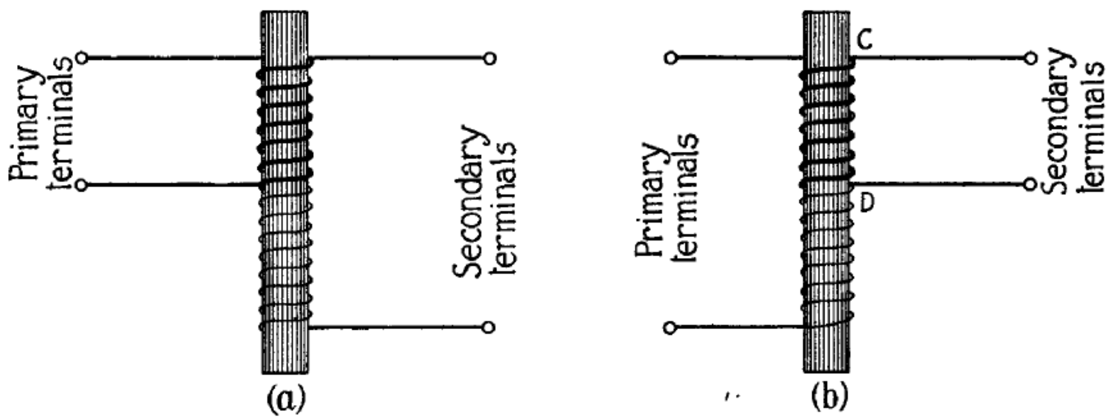


FIG. 17 CHAP. VI.—Auto-transformers.

The Auto-transformer

17. This is a transformer in which a portion of the winding is common to both primary and secondary circuits, and is only used where a low transformation ratio is required. Figs. 17a and 17b show the connections of the windings in the step-up and step-down types respectively, the portion of winding forming the primary being shown by a heavy line in the first instance, and the secondary winding by a heavy line in the second; thus the heavy line represents the conductor carrying the greatest current. The voltage applied to the primary winding causes a small magnetising current to flow, and a counter-E.M.F. of self-induction is set up in these turns which practically counterbalances the applied voltage. This current establishes in the core a magnetic flux which links with the whole of the secondary turns, and consequently induces in this winding an E.M.F. which is equal to $\frac{N_2}{N_1}$ or T times the applied voltage. When the secondary is on load, a secondary current will flow, tending to establish an additional flux in the core, which as in the ordinary transformer is neutralised by an equal and opposite flux set up by an increase of primary current, the latter being in antiphase with the secondary current which also flows in the common portion of the windings, and these turns therefore carry only the difference between the primary and secondary currents. This means that when only a small transformation ratio is desired the turns common to both windings need only be of a cross section sufficient to carry the difference between these two currents at full load. The copper losses are thus less than in a two-coil transformer designed for the same output and the same iron losses. The first cost of an

auto-transformer is also less than that of an ordinary transformer of the same output, owing to the saving of copper. These advantages are obviously only obtained when the primary and secondary currents are not very different, that is at low transformation ratios. The principal disadvantage of this type lies in the fact that there is direct connection between primary and secondary circuits. Thus in fig. 17b if a break occurs in the winding between C and D, the full primary voltage is applied to the device forming the secondary load, and the insulation of the latter will be damaged unless designed to stand this increased voltage.

Auto-transformers are used in the service to vary the voltage applied to the primary winding of the H.T. transformer of a rectifying system. Thus in the service Panel, Rectifying, Type A. which is designed for operation from a 230 volt supply, an auto-transformer having four secondary tapping points is provided, giving approximate step-down ratios of 2/1, 1·5/1, 1·2/1, and finally a 1/1 ratio, the corresponding voltages applied to the primary winding of the rectifier system being 110 volts, 150 volts, 190 volts and 230 volts.

Regulation

18. The regulation of a transformer is defined as the difference in terminal secondary voltage under full-load and no-load conditions, when the primary terminal P.D. is maintained at a constant value. That is

$$\text{Regulation} = E_2 - V_2$$

The percentage regulation or " pressure rise " is the regulation expressed as a percentage of the secondary P.D. at full load or

$$\text{Percentage regulation} = \frac{E_2 - V_2}{V_2} \times 100.$$

It should be noted that if the secondary load current is a leading one owing to the secondary external circuit possessing capacitive reactance, the secondary terminal P.D. on load may be higher than under no-load conditions.

Cooling

19. In order to radiate the heat generated in the core and windings either air or oil cooling is adopted. Air cooling is achieved by mounting the transformer in a protective casing of expanded metal, so that a free circulation of air may take place round the core and windings, and this circulation is assisted or increased if necessary by electric fans. Oil cooling consists of suspending the transformer in a tank containing insulating oil, which penetrates the windings and conducts the heat generated to the tank itself. The latter may be fitted with radiating fins in order that the heat may be rapidly radiated. In very large transformers the oil itself is cooled by water which circulates in pipes carried inside the tank. Such steps are not necessary in the comparatively small transformers used in the service, and plain oil-cooling is usually sufficient.

The purity of the oil used for cooling is of the utmost importance. It must contain neither dust particles, fluff, etc., nor water. Before filling a transformer tank it is advisable to stand the cans of oil in a warm place with the screw stopper removed for several hours and to strain the oil before filling the tank.

Extempore transformer design

20. (i) Although under ordinary conditions it is not necessary for transformer design to be undertaken by service personnel, circumstances may arise, particularly during hostilities, under which it is desirable to adapt an existing transformer for a voltage or frequency differing from its original rating, or even to construct a small transformer for temporary service pending the delivery of a correctly designed article. In the former event, a possible method of procedure is

outlined below, a concrete example being taken. It is required, then, to calculate the windings for a stalloy core having the dimensions given in fig. 18, to operate on 200 volt, 100 cycle mains, and to supply the following output :—

H.T. secondary, two windings each giving 100 milliamperes at 1000 volts.

L.T. secondary, two windings each giving 10 amperes at 15 volts.

The total output is therefore

H.T. :—	$1000 \times \cdot 1 \times 2 =$	200 volt-amperes.
L.T. :—	$15 \times 10 \times 2 =$	300 volt-amperes.
Total		500 volt-amperes.

For simplicity the power factor of the load will be assumed to be unity and the power output 500 watts.

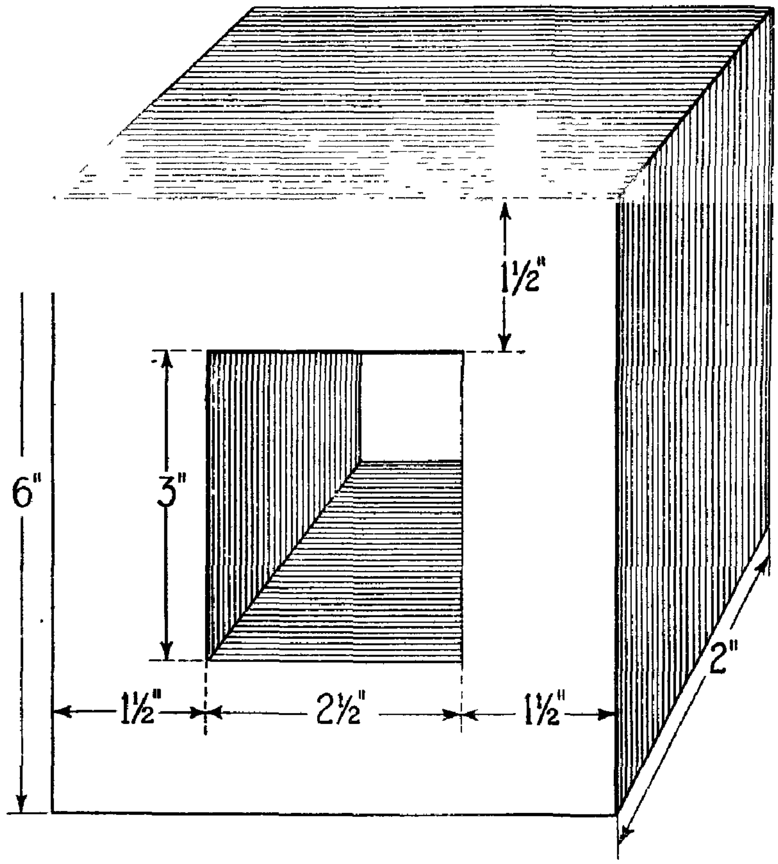


FIG. 18, CHAP. VI.—Dimensions of core.

(ii) The first step is to decide upon a tentative value for the peak flux density $\mathscr{B}$. This may be found as follows. Assume an efficiency which is reasonably capable of fulfilment, say 94 per cent. and calculate the total losses on full load. If P_0 is the output, P_I the input, $P_I - P_0$ the losses and η the efficiency, $P_I = \frac{P_0}{\eta}$ in the particular example $P_I = 500 \div 0{\cdot}94 = 532$ watts, and the losses will be 32 watts ; if the copper and iron losses are equal the iron losses will be 16 watts. Weigh the core, excluding clamping plates and bolts, or calculate its weight by finding the total volume of the iron in cubic inches and multiplying by the weight of one cubic inch of iron, 0·28 lb. The cross section of the core is $1\frac{1}{2}$ in. $\times$ 2 in. but of this a proportion is insulating material between stampings; allowing ten per cent. for this the iron cross-section (A) is $0{\cdot}9 \times 3 = 2{\cdot}7$ sq. in. and the volume 51 cubic in. of which 0·9 is iron. The approximate amount of the iron is therefore $46 \times {\cdot}28 = 13$ lb. and the iron loss will be $\frac{16}{13} = 1{\cdot}23$ watts per lb. If the thickness of the stampings is ·03 inch reference to fig. 22 at the end of this section shews that the peak flux density should not exceed 6,700 gauss.

(iii) Next, the voltage induced in each turn of the windings must be found. Since $e = \frac{Nd\Phi}{10^8 dt}$, and the flux is sinusoidal, $\frac{\mathscr{E}}{N} = \frac{\omega\Phi_{max}}{10^8}$, $\mathscr{E}$ being the peak voltage. What is actually required is the R.M.S. voltage per turn or $\frac{E}{N}$, and $E = \frac{\mathscr{E}}{\sqrt{2}}$ hence

$$\frac{E}{N} = \frac{{\cdot}707\,\omega\, A\,\mathscr{B}}{10^8}$$
$$= \frac{4{\cdot}44 f A\,\mathscr{B}}{10^8}$$

This is the fundamental formula in transformer design. In this formula A must be expressed in square centimetres. In the present example, $A = 2{\cdot}7$ sq. in. or $2{\cdot}7 \times 6{\cdot}45$ sq. cm. and

$$\frac{E}{N} = 4{\cdot}44 \times 6700 \times 2{\cdot}7 \times 6{\cdot}45 \times 100 \times 10^{-8}$$
$$= {\cdot}520 \text{ volts per turn.}$$

The number of turns required for each winding now follows :—

Primary voltage 200

Primary turns $= \frac{200}{{\cdot}52} = 390.$

H.T. secondary voltage = 1000
H.T. secondary turns $= 390 \times 5 = 1950$ } per winding.

L.T. secondary voltage = 15
L.T. secondary turns $= \frac{15}{200} \times 390 = 30$ } per winding.

N.B.—The number of turns on each winding should be a whole number.

(iv) The wire gauges may now be decided upon. Commencing with the primary, the full-load current will be $\frac{532 \text{ watts}}{200 \text{ volts}} = 2{\cdot}66$ amperes, and the current density in the copper should not exceed 1,000 amperes per square inch. Referring to Table I Appendix A it is seen that at this density, No. 17 s.w.g. is hardly capable of carrying 2·66 amperes while No. 16 s.w.g. will be operated at less than the above density and will therefore develop less heat. The even number wire gauges are in more general use than the odd numbers and 16 s.w.g. is more likely to be available than No. 17. If the number of volts per turn is less than four, single cotton covered wire may be used, although double cotton covered is preferable for the primary and silk-covered wire for the H.T. secondary. The space required for the actual winding is found from Table XI, Appendix A. It is there stated that No. 16, s.c.c., gives 198 turns per square inch of winding space. The 390 primary turns therefore require 1·97 sq. in. Each L.T. secondary is to carry 10 amperes and from the abovementioned tables it is found that No. 10 s.w.g. is suitable and single cotton covered wire of this gauge takes 54 turns per square inch. Each winding then occupies $\frac{30}{54} = {\cdot}556$ sq. in. It will be convenient to divide the primary winding into two equal portions, one on each limb, and to arrange one-half of the primary, one L.T. secondary and one H.T. secondary on each limb of the core. The space occupied so far is 1·97 sq. in. for the primary and 1·112 sq. in. for the two L.T. secondaries, a total of 3·082 sq. in. As the total window space is $2\frac{1}{2}$ in. $\times$ 3 in. $= 7{\cdot}5$ sq. in. this leaves say 4·4 sq. in. for the H.T. secondary and for insulation.

The H.T. secondary must carry 0·1 ampere and a suitable wire is No. 30 s.w.g. (d.s.c.). This gives 4,500 turns per sq. in. and each winding will occupy $\frac{1950}{4500}$ = ·432 sq. in. or a total of ·864 sq. in. There is therefore ample space for insulation between windings. It is of interest to recalculate the space occupied by the windings, assuming that only d.c.c. wire is available. It is then found that the space available for insulation is rather small but that the number of turns of given gauges could be accommodated, provided great care is taken to form the coils correctly and to use the minimum of tape binding on them. The suggested arrangement is shewn diagrammatically in figs. 19 and 20. In fig. 19, the additional insulation at the angles of the core should

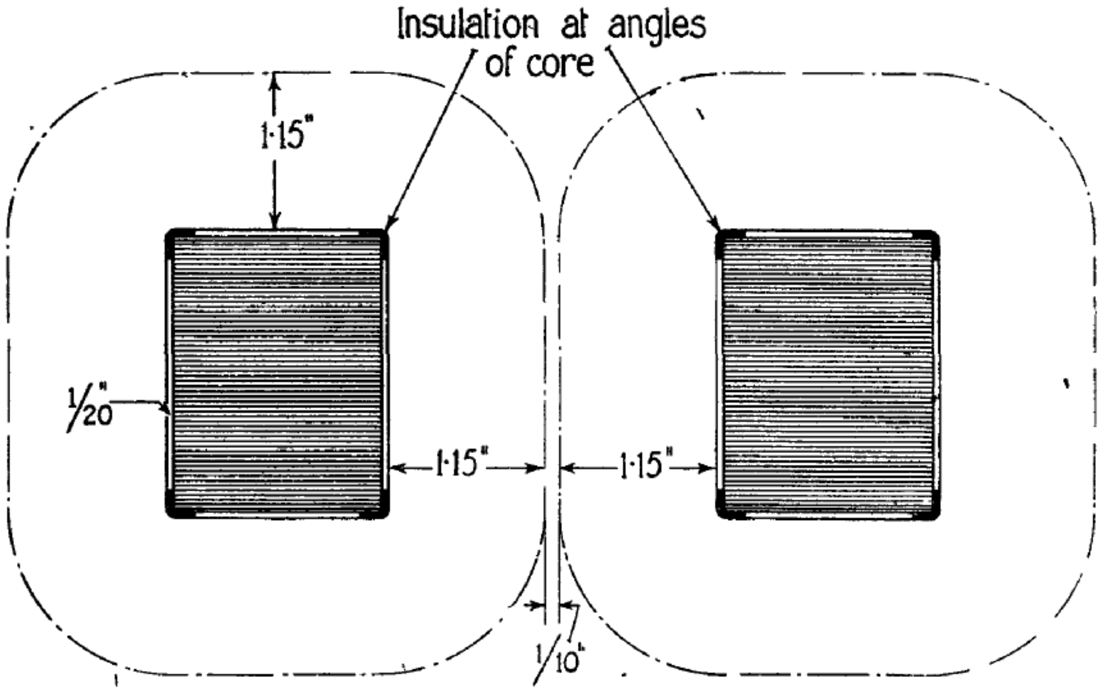


FIG. 19, CHAP. VI.—Sectional plan of core and windings.

be noted ; it is at these points that breakdown of the insulation beteeen the H.T. secondary and the core is most liable to occur. In fig. 20, it will be observed that the windings have been sectionalised to a greater extent than suggested above, each H.T. secondary being divided into six sections so that the peak P.D. between the ends of each section is less than 240 volts. Each half of the primary is also divided into two sections and intermingled with the secondaries in order to reduce magnetic leakage as much as possible.

21. (i) Before proceeding further it is advisable to calculate the temperature rise which will take place if 32 watts are dissipated. For this purpose the following empirical formula may be used :—

$$\theta = \frac{250\,(P_I - P_o)}{\text{Total cooling surface in sq. in.}}$$

where θ is the temperature rise in degrees centigrade. In calculating the cooling surface, the superficial area of the windings and the iron are added, but parallel surfaces within 0·5 inch of each other should be omitted as experience shews that such surfaces contribute little or nothing to the cooling. On this basis, the cooling area, calculated from figs. 19 and 20, will be about 160 sq. in. and the temperature rise

$$\theta = \frac{250 \times 32}{160} = 50^\circ\ \text{C.}$$

Cotton covering begins to char at 85° C. and in commercial design the final temperature is not allowed to exceed 80° C. If the normal temperature is 15° C., the final temperature reached by the transformer under discussion will be 65° C. which is perfectly satisfactory.

(ii) Now calculate the copper losses at 65° C. The temperature coefficient of copper is ·004 and the resistance of all windings will be 20 per cent. greater than at 15° C. The length of the "mean turn" is about one foot, and the following results are easily obtained.

Primary	390 turns =	130 yds. =	·97 ohm at 15° C. =	1·17 ohms at 65° C.
H.T. secondary	3900 ,, =	1300 ,, =	254 ,, ,, ,, =	305 ,, ,, ,,
L.T. ,,	60 ,, =	20 ,, =	·037 ,, ,, ,, =	·045 ,, ,, ,,

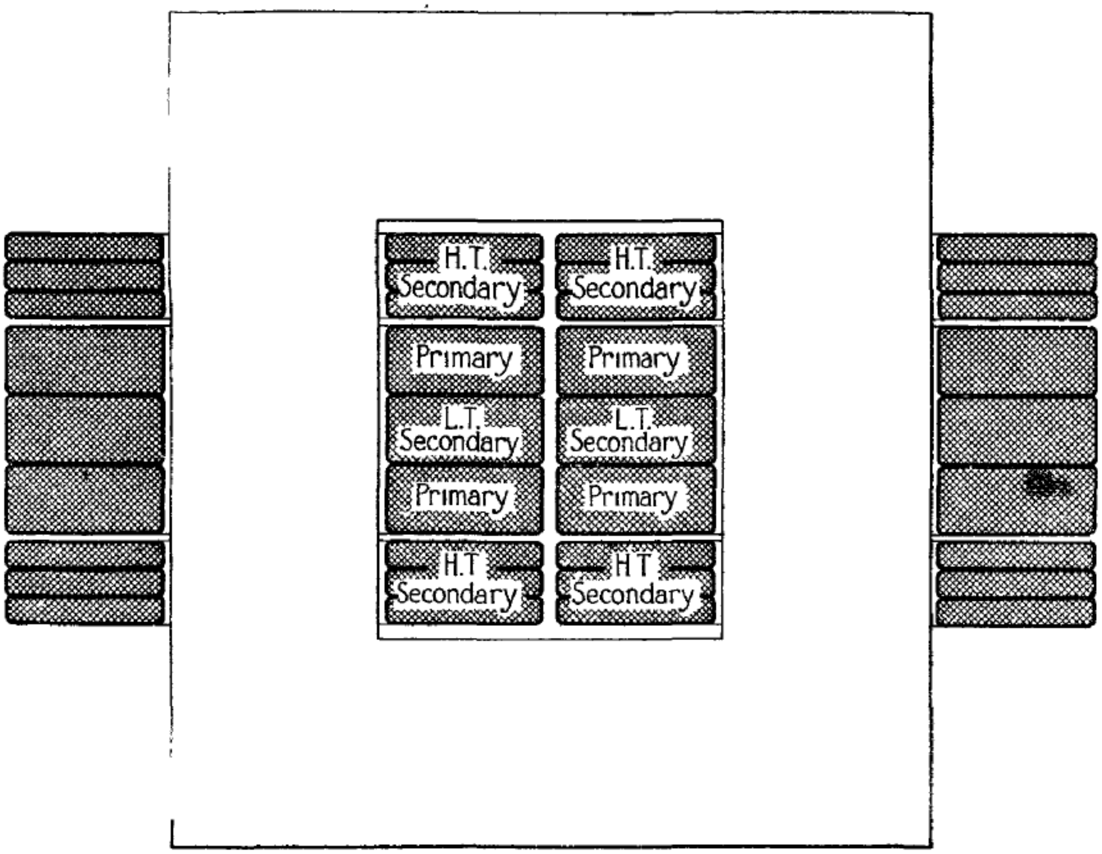
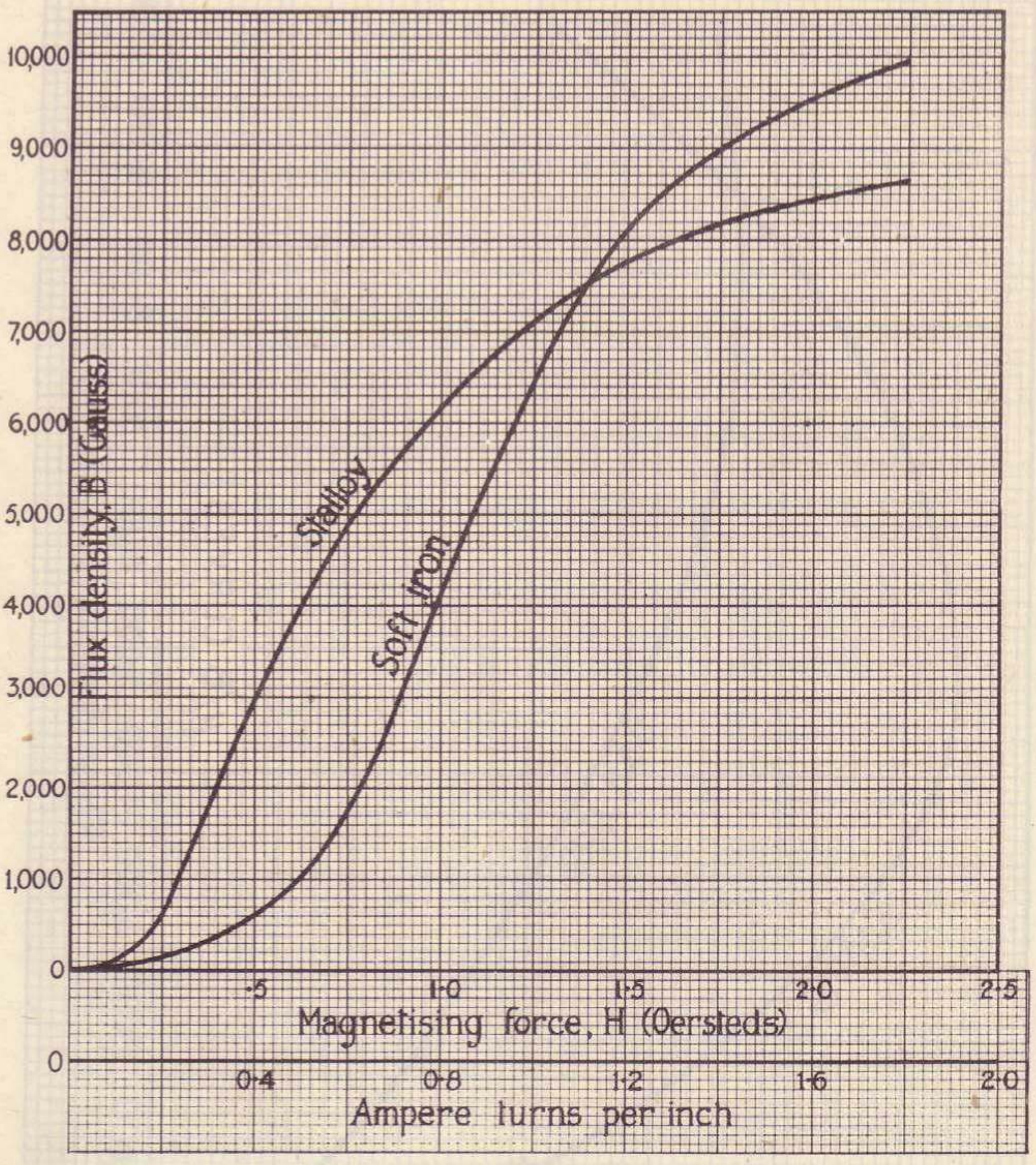


FIG. 20, CHAP. VI.—Sectional elevation of core and windings.

Copper losses.

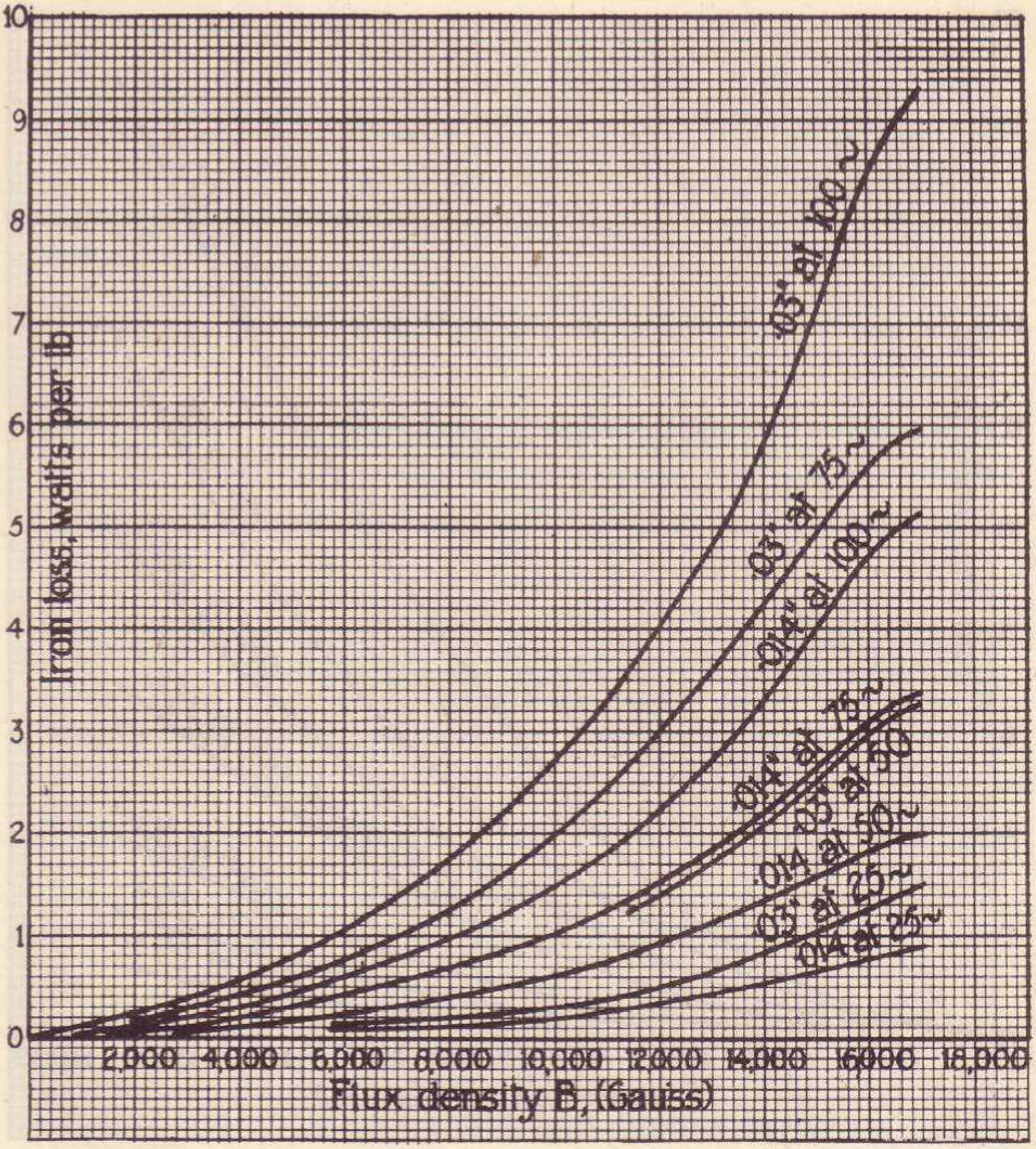
Primary	$2{\cdot}7^2 \times 1{\cdot}17 =$	8·5 watts.
H.T. secondary	$\cdot 1^2 \times 305 =$	3·05 ,,
L.T. ,,	$10^2 \times \cdot 045 =$	4·5 ,,
		16·05 watts

The primary current is taken as 2·7 amperes instead of 2·66 in order to allow for the magnetising current. If the B/H curve of the core material is given the magnetising current may be found thus. For stalloy at a flux density of 6,700 gauss, fig. 24 shews that a M.M.F. of ·92 ampere-turns per inch is required. The length of the mean tube of flux in the iron is 17 inches,



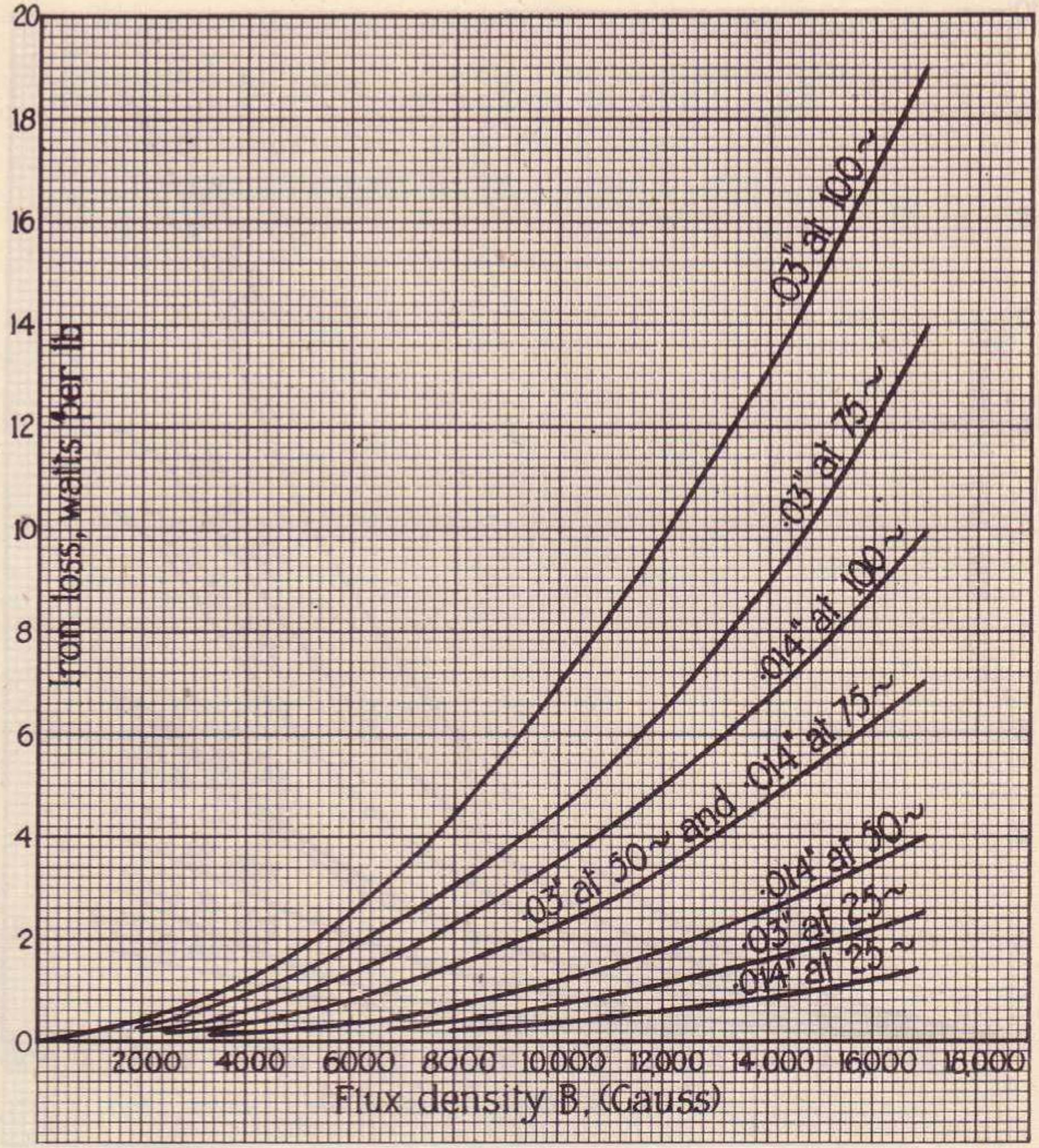
B/H CURVES

FIG. 23
CHAP. VI.



VARIATION OF IRON LOSS WITH FLUX DENSITY
(STALLOY STAMPINGS)

FIG. 22
CHAP. VI



VARIATION OF IRON LOSS WITH FLUX DENSITY
(SOFT IRON STAMPINGS)

FIG. 21
CHAP. VI

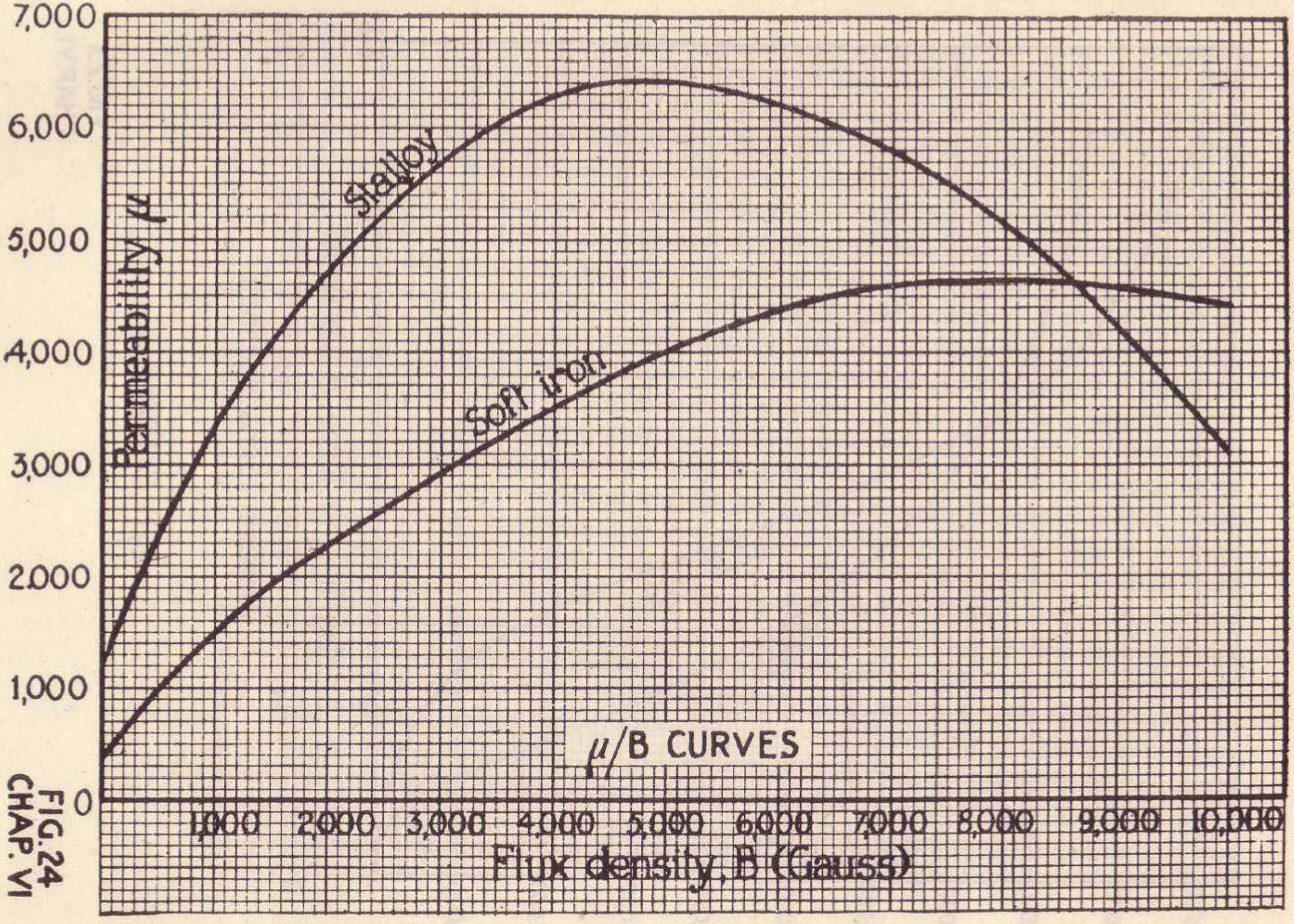


FIG. 24
CHAP. VI

and the total M.M.F. $17 \times \cdot 92 = 15\cdot 7$ ampere-turns (peak value). The R.M.S. magnetising current is therefore $\dfrac{\cdot 707 \times 15\cdot 7}{290} = \cdot 283$ amperes. The total primary current is

$$I_p = \sqrt{2\cdot 66^2 + \cdot 283^2}$$
$$= 2\cdot 69 \text{ amperes}$$

agreeing with the above estimate of 2·7 amperes.

(iii) The regulation cannot be calculated accurately because the primary and secondary leakage inductances are not known, but assuming that the leakage is negligible the volts drop in the primary winding will be $2\cdot 7 \times 1\cdot 2 = 3\cdot 24$ volts, in the H.T. secondary $\cdot 1 \times 305 = 30\cdot 5$ volts and in the L.T. secondary $10 \times \cdot 045 = \cdot 45$ volts. The terminal P.D.'s on full load are then easily found.

Primary terminal P.D. $= 200 - 3\cdot 24 = 196\cdot 76$ volts.
H.T. secondary E.M.F. $= 196\cdot 76 \times 10$
$= 1967\cdot 6$ volts.
H.T. secondary, terminal P.D. $= 1967\cdot 6 - 30\cdot 5$
$= 1937\cdot 1$ volts.
L.T. secondary E.M.F. $= \dfrac{196\cdot 75 \times 15}{200} = 14\cdot 75$ volts.
L.T. secondary, terminal P.D. $= 14\cdot 75 - \cdot 45 = 14\cdot 30$ volts.

22. When a small transformer has to be improvised, the only core material usually available is ordinary soft iron sheet or tinplate. Provided it is carefully annealed from a suitably high temperature, and operated at low flux density, such material is fairly satisfactory. The simplest procedure is to choose dimensions which appear to be reasonable and then calculate the performance. An assumption of core cross-section and flux density will determine the volts per turn and hence the number of turns on each winding, the size of wire and winding space. The core itself may be built up from long strips which are threaded through the windings and bent round to form a closed magnetic circuit.

Example.—A transformer is required to supply 10 amperes at 20 volts from 200 volt 50 cycle mains. Soft iron of ·03 in. thickness is available.

Assume an efficiency of 91 per cent. The input is then $\dfrac{200}{\cdot 91} = 222$ watts.

Losses = 22 watts.

Assume, e.g., iron loss = 12 watts.
copper loss = 10 watts.

From fig. 22 a suitable flux density will be 6,000 gauss, giving an iron loss of ·8 watt per lb., and $\dfrac{12}{\cdot 8} = 15$ lb. of iron is required, the volume being $15 \div \cdot 28 = 53\cdot 5$ or say 54 cu. in. A gross core cross section of 2 in. × 2 in. or 4 sq. in. and a mean flux line of 15 in. giving a gross volume of 60 cu. in., will give a window of $1\frac{1}{2}$ in. × 2 in. The iron cross-section will be only $\cdot 9 \times 4 = 3\cdot 6$ sq. in. and the iron volume 54 cu. in., as already stated.

The volts per turn will be

$$\frac{E}{N} = \frac{4 \times 1\cdot 11 \times 50 \times 6000 \times 3\cdot 6 \times 6\cdot 45}{10^8}$$
$$= \frac{2 \times 1\cdot 11 \times 6 \times 3\cdot 6 \times 6\cdot 45}{1000}$$
$$= \cdot 31 \text{ volts/turn}$$

Primary turns $\frac{200}{\cdot 31} = 645$.

Secondary turns $\frac{20}{200} \times 645 = 64\cdot 5$.

Primary current ..	..	1 ampere	.. ..	20 s.w.g., 567 turns per sq. in.
Secondary current	..	10 amperes	.. ..	10 s.w.g., 54 turns per sq. in.

Primary winding space .. $\frac{645}{567} = 1\cdot 14$ sq. in.
Secondary winding space $\frac{64\cdot 5}{54} = 1\cdot 2$ sq. in.
} Total, $2\cdot 34$ sq. in.

Single silk covered wire being used in both windings.

Thus ample room for insulation is available in the window space as the highest voltage is less than 300. The mean turn will be say 11 inches in length. Roughing out the dimensions it is estimated that the cooling surface will be about 125 sq. in. and the temperature rise

$$\frac{250}{125} \times 22 = 44° \text{ C.}$$

Taking the mean turn as one foot the total length of primary conductor is 645 feet or 215 yds. and its resistance $\cdot 215 \times 23\cdot 54 \times 1\cdot 8 = 9\cdot 2$ ohms. The length of the secondary winding will be about $21\cdot 5$ yds., and its resistance $\cdot 0215 \times 1\cdot 862 \times 1\cdot 8 = \cdot 0723$ ohms.

The copper losses in the primary will be $1^2 \times 9\cdot 2 = 9\cdot 2$ watts and in the secondary, $10^2 \times \cdot 0723 = 7\cdot 23$ watts, hence the total copper loss will be $13\cdot 43$ watts.

This is sufficiently near the required performance to justify the adoption of the suggested core dimensions, and the arrangements of core and windings may now be drawn with accuracy, the length of mean turn, the cooling surface, and the copper losses recomputed. Finally the magnetising current may be estimated as in the previous instance.

COUPLED CIRCUITS

23. When two circuits are so arranged that electrical energy can be transferred from one circuit to another owing to the existence of some form of impedance which is common to both circuits, the latter are said to be coupled together and are briefly designated as coupled circuits. Coupled circuits are usually divided into three classes, according to the nature of the common impedance, giving rise to (i) magnetic or inductive coupling (ii) electric or capacitive coupling (iii) resistive coupling. The most familiar case of magnetic coupling is that of the power transformer already considered, but coupled circuits are also of considerable importance in radio-frequency practice, both in transmission and reception. The individual circuits are referred to as the primary and secondary respectively, and usually possess both inductance and capacitance in addition to inherent resistance ; the primary and secondary circuits are usually tuned to the same frequency.

Mutual inductive coupling

24. (i) This is shewn diagrammatically in fig. 25a. The primary circuit consists of the inductance L_p, the resistance R_p and the capacitance C_p, and energy is supplied by an alternator having an R.M.S. voltage of E volts. The secondary circuit consists of the inductance L_s, capacitance C_s and resistance R_s, and the common impedance is that due to the mutual flux-linkage between L_p and L_s, its magnitude at any frequency $\frac{\omega}{2\pi}$ being ωM. When an E.M.F. is applied to the primary circuit an alternating current I_1 will be established and will set up an alternating flux round the coil I_1. This varying flux will embrace the inductance L_2 setting up and alternating E.M.F. in the coil and consequently a current in the secondary circuit. By Lenz's law, this secondary flux must act upon the primary circuit in such a manner as to oppose its original

cause, hence the secondary current induces in the primary circuit a counter-E.M.F. The presence of the secondary circuit is therefore responsible for the following phenomena. First, the current in the primary circuit is not the same as it would be in the absence of the secondary current, for since the primary E.M.F. causes a current in the secondary circuit, additional energy must be supplied by the alternator, and consequently the effective impedance of the two coupled circuits must be different from that of the primary alone. Second, the resonant frequency of the whole circuit is modified by the presence of the secondary. The manner in which the latter effect arises may be shewn as follows. Let the primary E.M.F. be E, and the frequency of the alternator $\frac{\omega}{2\pi}$. Then assuming an alternating current I_1 to be established in the primary circuit, the secondary induced voltage will be $E_2 = \omega M I_1$. If the reactances of the primary and secondary circuits are X_p and X_s respectively, and the resistances R_p, R_s are negligible, the secondary current will

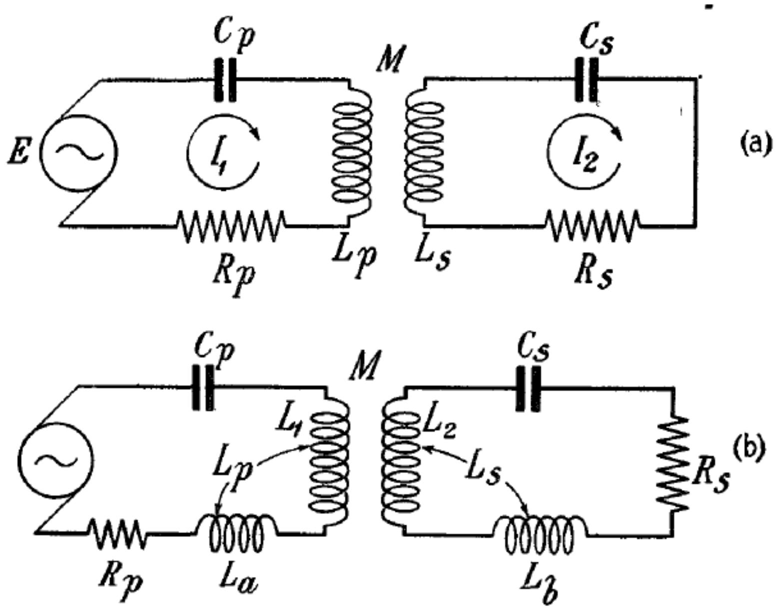


FIG. 25, CHAP. VI.—Mutual inductive coupling.

be $\frac{E_2}{X_2}$ and $I_2 = \frac{\omega M}{X_s} I_1$; this secondary current in its turn induces an E.M.F. in the inductance L_p, the value of which is $\omega M I_2$ or $\frac{\omega^2 M^2 I_1}{X_s}$ so that the total E.M.F. which is acting in the primary circuit is not E but $E + \frac{\omega^2 M^2}{X_s} I_1$.

(ii) The magnitude of the primary current can now be found, for it is equal to the total E.M.F. divided by the opposition of the circuit or

$$I_1 = \frac{E + \frac{\omega^2 M^2}{X_s} I_1}{X_p}$$

which simplifies to

$$X_p I_1 = E + \frac{\omega^2 M^2}{X_s} I_1$$

$$I_1 \left(X_p - \frac{\omega^2 M^2}{X_s} \right) = E$$

or finally

$$I_1 = \frac{E}{X_p - \frac{\omega^2 M^2}{X_s}}$$

The denominator $X_p - \dfrac{\omega^2 M^2}{X_s}$ is the ratio of voltage to current in the primary circuit, that is, its apparent reactance, and this has been modified from its original value X_p by the presence of the secondary circuit. The resonant frequency of the whole circuit can now be established, for it is that frequency which makes the reactance zero, that is

$$X_p - \frac{\omega^2 M^2}{X_s} = 0$$

i.e.

$$X_p\, X_s - \omega^2 M^2 = 0.$$

Now for simplicity assume that the two circuits are identical so that $L_p = L_s$, $C_p = C_s$, $X_p = X_s = \omega L_p - \dfrac{1}{\omega C_p}$, $X_p\, X_s = \left(\omega L_p - \dfrac{1}{\omega C_p}\right)^2$, and

$$\left(\omega L_p - \frac{1}{\omega C_p}\right)^2 - \omega^2 M^2 = 0$$

$$\omega^2 L_p^2 - \frac{2L_p}{C_p} + \frac{1}{\omega^2 C_p^2} - \omega^2 M^2 = 0$$

or

$$(L_p^2 C_p^2 - M^2 C_p^2)\, \omega^4 - 2\, L_p C_p\, \omega^2 + 1 = 0$$

which may be treated as a quadratic equation. Solving it to find ω^2,

$$\omega^2 = \frac{2\, L_p C_p \pm \sqrt{(2\, L_p C_p)^2 - 4\, (L_p^2 - M^2)\, C_p^2}}{2\, (L_p^2 - M^2)\, C_p^2}$$

$$= \frac{2\, L_p C_p \pm 2\, M C_p}{2\, (L_p^2 - M^2)\, C_p^2}$$

$$= \frac{L_p \pm M}{(L_p^2 - M^2)\, C_p}.$$

Now $L_p^2 - M^2 = (L_p + M)\,(L_p - M)$ and therefore the two values of ω^2 given by this equation are

$$\omega_1^2 = \frac{1}{(L_p + M)\, C_p}$$

$$\omega_2^2 = \frac{1}{(L_p - M)\, C_p}.$$

25. (i) In order that the values of ω_1 and ω_2 may be conveniently expressed without introducing the absolute value of the mutual inductance M, it is now desirable to introduce the "coefficient of coupling," or "coupling factor." The latter is defined algebraically as the ratio $\dfrac{M}{\sqrt{L_p L_s}}$ and is denoted by the symbol k, hence in the particular instance where $L_p = L_s$, $k = \dfrac{M}{L_p}$,

and
$$\omega_1^2 = \frac{1}{L_pC_p(1+k)}$$
$$\omega_2^2 = \frac{1}{L_pC_p\,(1-k)}$$

It will be found that the same result is obtained even if L_p is not equal to L_s, provided that $L_pC_p = L_sC_s$. If the latter relation is not satisfied, the circuits as a whole still have two resonant frequencies, but the condition is of little practical importance. The resonant frequencies of the circuits are directly derived from the values of ω_1^2 and ω_2^2, and are

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi\sqrt{L_pC_p\,(1+k)}} = \frac{f_r}{\sqrt{1+k}}$$
$$f_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi\,\sqrt{L_pC_p\,(1-k)}} = \frac{f_r}{\sqrt{1-k}}$$

where f_r is the resonant frequency of each individual circuit, $\frac{1}{2\pi\sqrt{L_pC_p}}$ or $\frac{1}{2\pi\sqrt{L_sC_s}}$.

(ii) The value of the coefficient of coupling depends upon the ratio of the flux linking with both circuits to that linking with the individual circuits. It has been shown in Chapter II that if the whole of the flux embraces every portion of both circuits $M = KN_pN_s$, $L_p = KN_p^2$ $L_s = KN_s^2$, N_p and N_s being the number of turns in the primary and secondary inductances and K a constant depending upon the geometry of the circuits. In these circumstances

$$k = \frac{M}{\sqrt{L_pL_s}} = \frac{KN_pN_s}{\sqrt{KN_p^2 \;\times\; KN_s^2}} = 1$$

i.e. the coefficient of coupling is unity. As it is impossible to achieve the totality of mutual flux-linkage specified above, the constant k can never reach the value unity in any practical circuit, although it is approached closely if the inductances L_p and L_s are intermingled on a common iron core as in the power transformer. The coefficient of coupling can be reduced by any method which reduces the amount of mutual flux linkage, e.g. by separating the circuits in space, using a core of non-magnetic material, and by turning the coils into such a relative position that the flux due to the primary current does not link with any portion of the secondary circuit. When calculating the coefficient of coupling between two circuits, some care is necessary, for instance in circuits arranged as in fig. 25b. The total primary inductance in this arrangement is $L_p = L_a + L_1$ and the secondary inductance $L_s = L_b + L_2$. Let us suppose that the mutual inductance between L_1 and L_2 has been measured, and its value known to be M. Then the coupling factor for the coils L_1, L_2 alone is $k = \frac{M}{\sqrt{L_1\,L_2}}$ but for the circuits as a whole is $\frac{M}{\sqrt{L_p\,L_s}}$ which may be very much less. An example of this is a type of air-core transformer in which the secondary winding L_s possesses a large number of turns, while the primary winding L_p consists of a few turns wound closely over one end of the secondary coil. The mutual inductance between L_p and the few secondary turns immediately underneath may be very nearly equal to L_p, i.e. the coupling factor between L_p and an equal inductance forming part of L_s may be unity. The coupling factor for the whole circuit will then be $\frac{M}{\sqrt{L_pL_s}} = \frac{L_s}{\sqrt{L_pL_s}} = \sqrt{\frac{L_p}{L_s}}$, and when two coils are arranged in this way, the coupling factor decreases with an increase of secondary inductance, i.e. with the number of turns wound on the secondary, and consequently an increase of turns ratio in order to achieve a larger step-up of voltage may be stultified by the reduction of the coupling factor.

Example.—In fig. 25b, the coefficient of coupling between the coils L_1 and L_2 is ·8. If $L_1 = 50\mu H$, $L_a = 100\ \mu H$, $L_2 = 70\ \mu H$, $L_b = 200\ \mu H$, find the coefficient of coupling between the two circuits.

$$\frac{M}{\sqrt{L_1 L_2}} = \cdot 8$$

$$\therefore\ M = \cdot 8\sqrt{L_1\, L_2}$$

$$= \cdot 8\sqrt{50 \times 70}\ \mu H$$

$$k = \frac{M}{\sqrt{L_p\ L_s}}$$

$$= \frac{\cdot 8\sqrt{50 \times 70}}{\sqrt{(100 + 50)\ (200 + 70)}}$$

$$= \frac{\cdot 8\sqrt{3500}}{\sqrt{40500}}$$

$$= \cdot 236.$$

Resonance curves of coupled circuits

26. (i) When two circuits (individually tuned to the same frequency) are coupled together by mutual induction and an E.M.F. of variable frequency is applied, the graphical representation of the variation of current with frequency, over a band extending below and above the resonant frequency, is called the resonance curve of the coupled circuits. It is perhaps obvious that separate curves may be drawn shewing the variation of current in the primary and secondary circuits respectively, but the secondary current is of principal interest in practice, and only this current will be dealt with. When the coupling factor is very low, e.g. of the order of ·002, the peak value secondary current at the resonant frequency is very small, and the ratio $\frac{I_r}{I_n}$ is large (I_r being the current at resonant and I_n the current at any other frequency) but as the coefficient of coupling is increased, a condition is reached in which the mutual reactance ωM is equal to the geometric mean resistance of the circuits, $\sqrt{R_p R_s}$. This is termed the critical coupling, because for this value of the mutual inductance M the secondary current reaches maximum value, being then given by the equation

$$I_2\,(max.) = \frac{\omega M\ E}{\omega^2 M^2 + R_p R_s}$$

Since

$$\omega M = \sqrt{R_p R_s}$$

$$I_2\,(max.) = \frac{E}{2\sqrt{R_p\, R_s}}$$

This is the greatest current which can be obtained in the secondary circuit. An increase in the value of M does not give a further increase in secondary current, but results in the formation of two peaks in the resonance curve, the frequencies at which these peaks appear being given by the formulæ $f_1 = \frac{f_r}{\sqrt{1 + k}}$, $f_2 = \frac{f_r}{\sqrt{1 - k}}$ previously obtained. The frequencies f_1 and f_2 become more widely separated as k is increased, fig. 26 shewing the resonance curves of two circuits, the constants being the same as in the single circuit used to illustrate the simple resonance curve of the acceptor circuit (Chapter V) namely $L_p = L_s = 150\ \mu H$ $C_p = C_s = \cdot 000169\ \mu F$, $R_p = R_s = 9\cdot 45$ ohms. approx., $f_r = 1000$ k.c/s. A separate curve has been shewn for each of several values of k, and it will be observed that an increase of the coupling coefficient, from $k = \cdot 002$ upwards results in an increase of secondary current until k reaches a value $\cdot 01$. Further increase in the coupling simply has the effect of broadening the resonance curve without increasing the value of the current, the curve becoming double peaked as above stated. The maximum transfer of energy at the resonant frequency f_r will occur when the coefficient of coupling has the critical

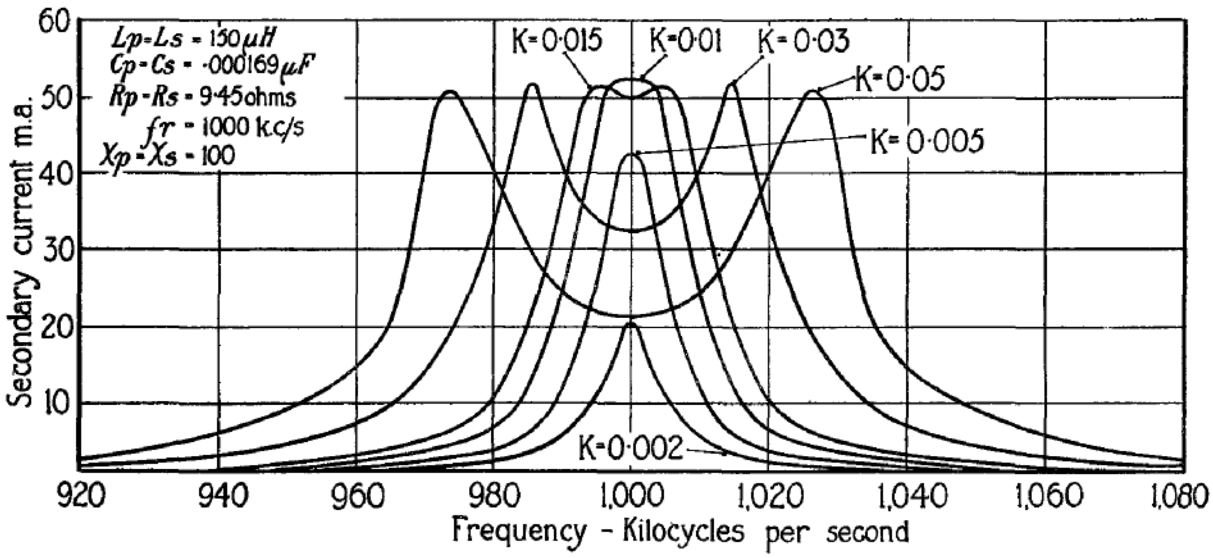


FIG. 26, CHAP. VI.—Resonance curves of coupled circuits.

value given by the relation $\omega^2 M^2 = R_p R_s$. As the value of M is not usually known it is preferable to derive an expression for the critical value of the coupling factor k, as follows :—

$$\omega^2 M^2 = R_p\, R_s$$

$$M^2 = \frac{R_p}{\omega}\ \frac{R_s}{\omega}$$

$$\frac{M^2}{L_p L_s} = \frac{R_p}{\omega L_p}\ \frac{R_s}{\omega L_s}$$

Since

$$\frac{M^2}{L_p L_s} = k^2,\ \frac{R_p}{\omega L_p} = \frac{1}{\chi_p},\ \frac{R_2}{\omega L_s} = \frac{1}{\chi_s},$$

$$k = \sqrt{\frac{1}{\chi_p} \times \frac{1}{\chi_s}}.$$

This is the value of coupling coefficient for which the transfer of energy at the resonant frequency is a maximum, and as in practical radio frequency circuits χ_p and χ_s may be of the order of 100 or more, it is apparent that very loose coupling is generally sufficient to secure this optimum transference.

(ii) Referring again to the equations connecting the coupling factor k, the resonant frequency f_r and the frequencies f_1 and f_2 at which peaks occur in the resonance curve, it will be observed that if the curve is obtained experimentally, the coefficient of coupling can be calculated from the frequencies f_1 and f_2. Thus, since

$$f_1 = \frac{f_r}{\sqrt{1+k}}, f_2 = \frac{f_r}{\sqrt{1-k}}, \quad f_1^2 + f_1^2 k = f_r, \quad f_2^2 - f_2^2 k = f_r,$$

and $$f_1^2 + f_1^2 k = f_2^2 - f_2^2 k.$$

Hence $$f_2^2 - f_1^2 = (f_1^2 + f_2^2)k$$

and $$k = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2}.$$

In fig. 26, the frequencies at which peaks occur, on the curve showing the highest degree of coupling, are approximately $f_1 = 975$ kilocycles per second, $f_2 = 1{,}025$ kilocycles per second. Hence

$$k = \frac{1025^2 - 975^2}{1025^2 + 975^2} = \frac{105{\cdot}0625 - 94{\cdot}9625}{105{\cdot}0625 + 94{\cdot}9625} = \frac{10{\cdot}1}{200{\cdot}025}$$

$$\therefore k = {\cdot}05005$$

It will also be observed that the frequencies f_1 and f_2 differ from the resonant frequency of the individual circuits by a nearly equal amount, namely 25 kilocycles per second below and above the resonant frequency; this signifies that when the coupling is of the order usually employed, that is if k is less than about $\cdot 1$, the above formula may be replaced by a simpler one with negligible error, namely

$$k = \frac{f_2 - f_1}{f_r}.$$

Using this approximation in the above example

$$k = \frac{1025 - 975}{1000}$$

$$= {\cdot}05$$

Auto-inductive coupling

27. This is shewn in fig. 27, the notation being similar to that used in previous figures.

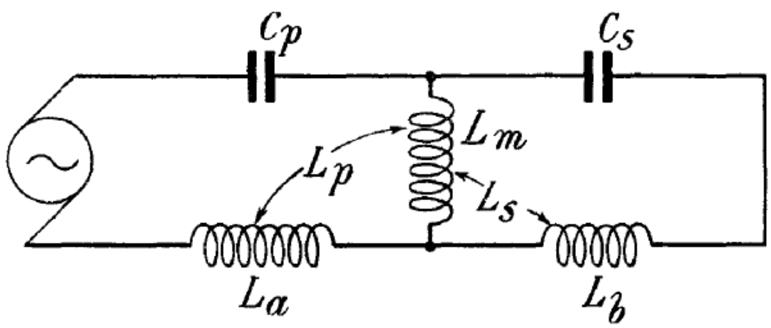


FIG. 27, CHAP. VI.—Auto-inductive coupling.

The reactance which is common to both primary and secondary circuits is that of the inductance L_m. The primary inductance is $L_p = L_a + L_m$ and the secondary inductance $L_s = L_b + L_m$.

If the circuits are so adjusted that $L_p C_p = L_s C_s$, the circuit as a whole has two resonant frequencies, which are given by the expressions

$$f_1 = \frac{1}{2\pi\sqrt{L_p C_p (1 + k)}}$$

$$f_2 = \frac{1}{2\pi\sqrt{L_p C_p (1 - k)}}$$

and
$$k = \frac{L_m}{\sqrt{L_p L_s}}$$

The degree of coupling may therefore be varied by variation in the value of the inductance L_m, but, except in the special case when $L_a = L_b$, $C_p = C_s$, such a variation will throw the circuits out of resonance with each other. If, however, the circuits are so designed that $L_b = FL_a$ and $C_a = FC_b$ (where F is any numeric whatever) the two condensers being mounted in such a way that they are varied simultaneously by a single knob, then for all settings of the condenser the following relation is satisfied, viz. $L_a C_p = L_b C_s$. The circuit as a whole still possesses two resonant frequencies, but instead of being above and below the resonant frequency of the individual circuits, they are

$$f_1 = \frac{1}{2\pi\sqrt{L_a C_p \left(1 + \frac{L_m}{L_a} + \frac{L_m}{L_b}\right)}}$$

$$f_2 = \frac{1}{2\pi\sqrt{L_a C_p}}$$

If the value of the inductance L_m is varied while the rest of the circuit constants remain at given values, the resonant frequency f_2 remains constant but the frequency f_1 decreases with increase of L_m, and the resonance curve is said to have a stationary peak at f_2 and a moving peak at the frequency f_1 which depends upon the degree of coupling. If the coefficient of coupling is defined by the equation

$$k = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2} = \frac{\omega_2^2 - \omega_1^2}{\omega_2^2 + \omega_1^2},$$

and the above expressions for ω_1^2 and ω_2^2 are inserted,

$$k = \frac{\dfrac{1}{L_a C_p} - \dfrac{1}{L_a C_p\left(1 + \frac{L_m}{L_a} + \frac{L_m}{L_b}\right)}}{\dfrac{1}{L_a C_p} + \dfrac{1}{L_a C_p\left(1 + \frac{L_m}{L_a} + \frac{L_m}{L_b}\right)}}$$

$$= \frac{L_m\left(\frac{1}{L_a} + \frac{1}{L_b}\right)}{2 + L_m\left(\frac{1}{L_a} + \frac{1}{L_b}\right)}$$

$$= \frac{L_m}{2\,\frac{L_a L_b}{L_a + L_b} + L_m}.$$

Example.—(i) In a certain circuit $L_a = 140\ \mu H$, $C_p = \cdot 001\ \mu F$, $L_m = 20\ \mu H$, $L_b = 300\ \mu H$, $C_s = \cdot 0005\ \mu F$. Find the two resonant frequencies, and the coefficient of coupling.

Since $L_p = L_a + L_m = 140 + 20 = 160\ \mu H$, $L_s = L_b + L_m = 300 + 20 = 320\ \mu H$, $L_p C = 160 \times \cdot 001$, $L_s C_s = 320 \times \cdot 0005$, $L_p C_p = L_s C_s$ and the coefficient of coupling is

$$k = \frac{L_m}{\sqrt{L_p L_s}} = \frac{20}{\sqrt{160 \times 320}}$$

$$= \cdot 0883.$$

The resonant frequency of each individual circuit is

$$f_r = \frac{1}{2\pi\sqrt{L_p C_p}} = \frac{10^6}{2\pi\sqrt{160 \times \cdot 001}}$$

$$= \frac{10^6}{2\pi \times \cdot 4} = 398000 \text{ cycles per second.}$$

$$f_1 = \frac{f_r}{\sqrt{1+k}} = \frac{398000}{\sqrt{1\cdot 0883}} = 381000 \text{ cycles per second.}$$

$$f_2 = \frac{fr}{\sqrt{1-k}} = \frac{398000}{\sqrt{\cdot 9117}} = 417000 \text{ cycles per second.}$$

(ii) If the secondary inductance is reduced to 280 μH the remainder of the constants being unchanged, find the resonant frequencies and coefficient of coupling.

$L_a C_p$ is now equal to $L_b C_s$ and the system has a stationary and a variable resonant frequency.

$$f_1 = \frac{10^6}{2\pi\sqrt{140 \times \cdot 001\left(1 + \frac{20}{140} + \frac{20}{280}\right)}}$$

$$= \frac{10^6}{2\pi\sqrt{\cdot 14\left(1 + \frac{3}{14}\right)}}$$

$$= 385000 \text{ cycles per second.}$$

$$f_2 = \frac{10^6}{2\pi\sqrt{\cdot 14}}$$

$$= 424000 \text{ cycles per second.}$$

$$k = \frac{L_m}{2\frac{L_a\ L_b}{L_a + L_b} + L_m}$$

$$= \frac{20}{2\frac{140 \times 280}{420} + 20}$$

$$= \cdot 0967.$$

Alternatively, the approximation $k = 2\,\frac{f_2 - f_1}{f_2 + f_1}$ gives

$$k = 2\,\frac{424 - 385}{809}$$

$$= \cdot 0964.$$

Auto-capacitive coupling

28. This is shewn in fig. 28 and is analogous to that just discussed. The primary capacitance

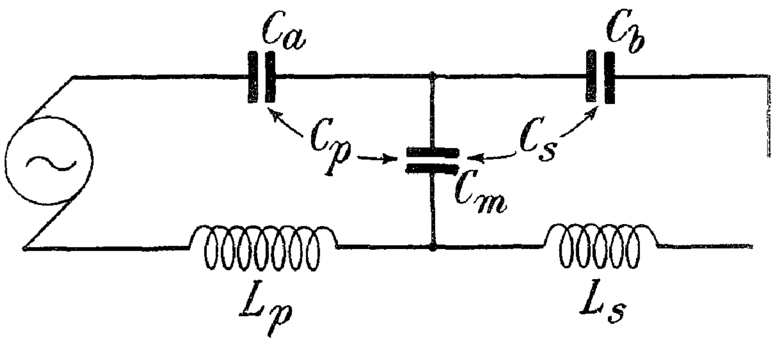


FIG. 28, CHAP. VI.—Auto-capacitive coupling.

is $C_p = \frac{C_a C_m}{C_a + C_m}$ and the secondary capacitance $C_s = \frac{C_b C_m}{C_b + C_m}$. If the circuits are so arranged that $L_p C_p = L_s C_s$, the circuit possesses two resonant frequencies, viz.:

$$f_1 = \frac{\sqrt{1-k}}{2\pi\sqrt{L_p C_p}}$$

$$f_2 = \frac{\sqrt{1+k}}{2\pi\sqrt{L_p C_p}}$$

where
$$k = \sqrt{\frac{C_a C_b}{(C_a + C_m)\ (C_b + C_m)}}$$

This condition is not normally maintained because unless $C_a = C_b$, an alteration in the coupling by adjustment of the capacitance C_m will throw the circuits out of resonance with each other. The more usual arrangement is to make $L_p C_a = L_s C_b$. The system then has one stationary resonant frequency.

$$f_1 = \frac{1}{2\pi\sqrt{L_p C_a}}$$

and a resonant frequency varying with the degree of coupling.

$$f_2 = \frac{1}{2\pi\sqrt{L_p C_a\left(1 - \frac{C_a + C_b}{C_a + C_b + C_m}\right)}}$$

Under these circumstances, if the coefficient of coupling is defined by the equation

$$k = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2}$$

$$k = \frac{C_a + C_b}{C_a + C_b + 2C_m}$$

Example.—(i) If $L_p = 160\ \mu H$, $C_a = \cdot 00111\ \mu F$, $L_s = 320\ \mu H$, $C_b = \cdot 000526\ \mu F$, $C_m = \cdot 01\ \mu F$, find the resonant frequencies and coefficient of coupling.

$$C_p = \cdot 001\ \mu F,\ C_s = \cdot 0005\ \mu F,\ L_pC_p = L_sC_s$$

$$k = \sqrt{\frac{1\cdot 111 \times 10^{-3} \times 5\cdot 26 \times 10^{-4}}{1\cdot 111 \times 10^{-2} \times 1\cdot 0526 \times 10^{-2}}}$$

$$= \sqrt{\frac{5\cdot 84 \times 10^{-7}}{1\cdot 168 \times 10^{-4}}}$$

$$= \cdot 0707.$$

$$f_1 = \frac{\cdot 9293 \times 10^6}{2\pi\sqrt{\cdot 16}} = 369000 \text{ cycles per second.}$$

$$f_2 = \frac{1\cdot 0707 \times 10^6}{2\pi\sqrt{\cdot 16}} = 426000 \text{ cycles per second.}$$

(ii) If C_b is increased to $\cdot 000555$, so that $L_pC_a = L_sC_b$, find the resonant frequencies and coefficient of coupling.

$$f_1 = \frac{1}{2\pi\sqrt{L_p C_a}}$$

$$= \frac{10^6}{2\pi\sqrt{160 \times \cdot 00111}} = 378000 \text{ cycles per second.}$$

$$\frac{1}{2\pi\sqrt{L_pC_a\left(1 - \frac{C_a + C_b}{C_a + C_b + C_m}\right)}}$$

$$C_a + C_b = \cdot 001666$$

$$C_a + C_b + C_m = \cdot 011666$$

$$1 - \frac{C_a + C_b}{C_a + C_b + C_m} = 1 - \cdot 1428 = \cdot 8572$$

$$f_2 = \frac{10^6}{2\pi\sqrt{160 \times \cdot 00111 \times \cdot 8572}}$$

$$= 410000 \text{ cycles per second.}$$

Direct inductive coupling

29. This is shewn in fig. 29, but has only been mentioned for sake of completeness, as it is rarely used in practice, for two reasons. First, in order to attain the low degree of coupling generally required, the inductance L_m must be very much larger than that of the coils L_p, L_s, and must also be adjustable to within fairly fine limits, requirements which are incompatible

with each other. Second, such a large inductance must of necessity possess considerable self-capacitance, and the circuit as a whole does not function in the manner calculated on the assumption that the self-capacitance is zero.

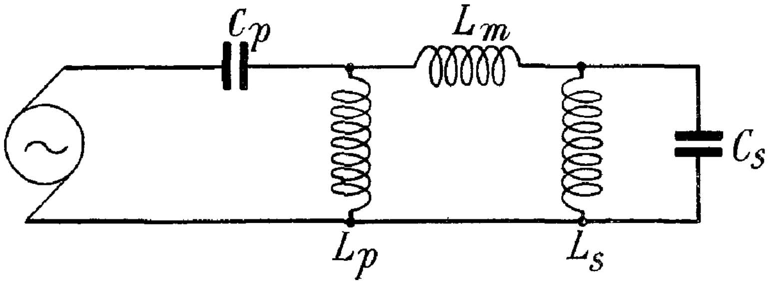


FIG. 29, CHAP. VI.—Direct inductive coupling.

Direct capacitive coupling

30. This type of circuit is analogous to that mentioned above, but a small condenser C_m constitutes the coupling device. This is one of the most convenient types of coupling and is probably in more general use than any other with the exception of mutual inductive coupling. The circuits are invariably so adjusted that $L_pC_p = L_sC_s$ and the circuit possesses a stationary frequency

$$f_2 = \frac{1}{2\pi\sqrt{L_pC_p}}$$

and a variable frequency

$$f_1 = \frac{1}{2\pi\sqrt{L_pC_p\left(1 + \frac{C_m}{C_p} + \frac{C_m}{C_s}\right)}}$$

which depends upon the degree of coupling.

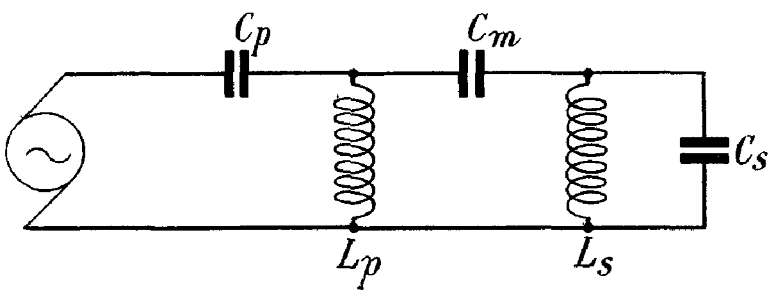


FIG. 30, CHAP. VI.—Direct capacitive coupling.

The coupling factor is found as before :—

$$k = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2}$$

$$= \frac{C_m}{2\frac{C_pC_s}{C_p + C_s} + C_m}$$

If $C_p = C_s$ this reduces to $\frac{C_m}{C_p + C_m}$. In practice the capacitance of the coupling condenser is always very small, e.g. if $C_p = C_s =$ ·0005 μF, and it is desired to maintain a coefficient of coupling of ·01 between the two circuits, the appropriate value of C_m is only ·000005 μF (approximately).

Resistive coupling

31. In fig. 31 are shewn two oscillatory circuits having a resistance R_m common to both, and by analogy with the circuits previously discussed, this arrangement may be referred to as auto-resistive coupling. It is rarely adopted deliberately for the purpose of energy transference,

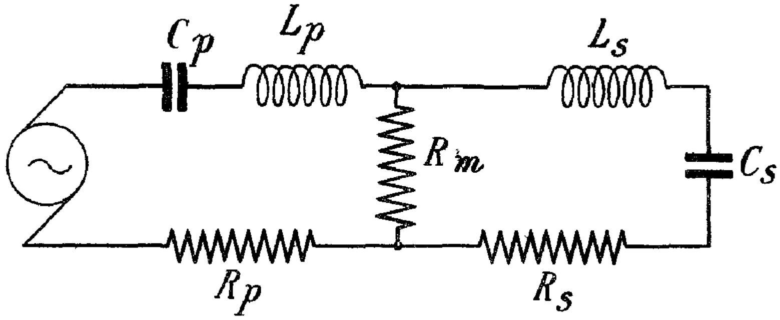


FIG. 31, CHAP. VI.—Auto-resistive coupling.

for the introduction of additional resistance must of necessity increase the damping and reduce the selectivity of the individual circuits, but such coupling may be found to exist fortuitously, for instance where earth connections are made to different points between which the resistance is appreciable. If both circuits are tuned to the same frequency and an E.M.F. of this frequency is applied, the reactance of each circuit is zero and the primary and secondary currents may be found in exactly the same manner as in D.C. practice, giving

$$I_1 = \frac{R_s + R_m}{R_pR_s + R_m(R_s + R_p)} E$$

$$I_2 = \frac{R_m}{R_pR_s + R_m(R_s + R_p)} E$$

It is of interest to calculate the relative magnitude of the primary and secondary currents in circumstances which might be found in practice. Suppose $R_p = R_s = 10$ ohms, $R_m = \cdot 1$ ohm, and the applied E.M.F. to be 1 volt. Application of the above formulae then gives

$$I_1 = \frac{10 \cdot 1}{102} \doteqdot \cdot 1 \text{ ampere, } I_2 = \frac{\cdot 1}{102} \doteqdot \cdot 001 \text{ ampere.}$$

Hence the presence of the resistance R_m gives rise to a secondary current equal in amplitude to one per cent. of that in the primary circuit, and the circuits are sometimes said to have a coupling factor of ·01. It must be appreciated, however, that as the presence of the coupling resistance has no effect upon the resonant frequency, the resonance curve of the combination possesses only a single peak, and in the true sense of the term the circuits do not possess a coupling factor. Fig. 32 shews a second form of resistive coupling which may be found to exist; it may be referred to as direct resistive coupling. In this instance the apparent coupling factor increases with decrease of the value of the coupling resistance R_m. The arrangement is rarely adopted for the reasons stated with regard to the alternative form of resistive coupling.

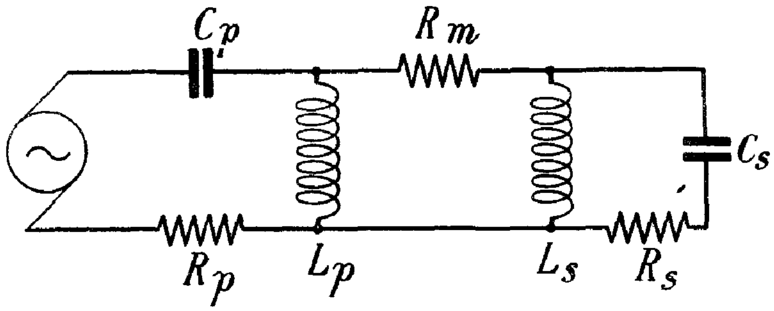


FIG. 32, CHAP. VI —Direct resistive coupling.