Chapter 15

NOISE

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CHAPTER 15

NOISE

1. INTRODUCTION

The general term Noise covers all undesirable electrical disturbances, either externally or internally generated, which tend to mask the clear reception of signals. In communications noise is often referred at as "mush", and in radar, by virtue of the appearance of the trace it produces on the screen of a CRT, as "grass".

Noise may first be divided into two general classes:-

(i) Avoidable noise

Examples of noise sources within this category are:-

(a) "Man-made" static arising from generators, ignition systems, etc.

(b) Faulty contacts in receivers.

(c) Poor quality resistors in receivers.

Avoidable noise is not considered further, since it is obvious that no theory or general prediction can be made in regard to such noise. These effects can be minimised by improved manufacture, use of high-quality components, systematic servicing, satisfactory weather protection, and care in the siting of equipments.

(ii) Unavoidable noise

This noise may be divided into two sub-classes:-

(a) External noise

Atmospheric noise, chiefly due to thunderstorms, signals from whose discharges may be propagated over very great distances. Radiation of cosmic origin.

(b) Internal noise

This class of noise covers all noise generated within the receiver, after exclusion of all avoidable internal noise due to bad contacts, etc; and in addition includes thermal noise generated in the aerial.

The subject matter of this chapter is mainly concerned with internal noise.

2. SIGNAL-TO-NOISE RATIO

Noise, from whatever source, after passing through the detector stage of a receiver presents itself on the time-base of a CRT as a ragged but very characteristic signal deflection, extending across the screen with more or less uniform amplitude (Fig. 642). On many types of display (e.g. meter), noise appears as a random variation of the position of the indicating device (pointer, electron beam, etc.) whose instantaneous position at any given moment of time may vary between wide limits, but which has a more or less clearly defined mean amplitude over any appreciable time interval.

In any element of a receiver the noise in the absence of signals has a certain mean power, and the signal in the absence of noise presents an average power for its duration. Calling these powers Noise Power and Signal Power respectively, then the ratio of signal power to noise power is called the Signal-to-Noise Ratio at that point in the receiver.

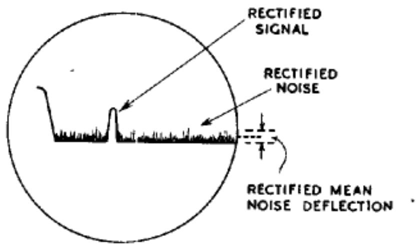

Fig. 642 - Signal and noise at CRT (A-type display).

Fig. 642 shows a typical A-type Display. As a useful guide it may be noted that a pulse signal is just distinguished on such a display by its amplitude rising above the noise level when the signal-to-noise ratio just prior to the detector of the receiver is about unity. If the ratio could be examined at previous points in the receiver (and in particular at the input to the first stage), it would be found that the ratio would increase as the input stage was approached. In the case of a centimetre equipment a ratio of signal-to-noise of between 25:1 and 100:1 must be present at the input for unity ratio just prior to the detector. In other words, to achieve a just distinguishable indication of signal on an average A-type display it is necessary to inject a signal power of 25 to 100 times the noise power existing at the input to the first stage.

3. INTERNAL NOISE

The sources of internal noise at present known to occur in normal circuits may be broadly divided into the following classes:-

(i) Thermal, Circuit or Johnson noise.

(ii) Valve noise:-

(a) Shot or Shrot noise.
(b) Partition or Division noise.
(c) Induced Grid noise.

There is good reason in radar for confining attention to this internal noise since at frequencies above about 70 Mc/s man-made static and atmospheric disturbances are both negligible in comparison with unavoidable internal receiver noise. From this point the term noise implies this class of receiver noise, bearing in mind that the use of the word unavoidable does not mean that there can be no improvement, but that with a given design using first class components, ideal contacts, etc., there is a theoretical minimum noise which can be computed for the circuit.

4. THERMAL NOISE

In an unenergised conductor the "free" or conduction electrons move about in a random manner, although over any reasonably long time-interval their mass centre remains fixed. At any instant, however, a preponderance of electrons is moving in one direction along the conductor, constituting a minute current whose mean value is zero. This current fluctuation must be associated with a corresponding minute voltage fluctuation across the ends of the conductor, whose mean value is also zero. To provide a measure of such fluctuations in all cases it is usual to compute the mean square value of the fluctuations about the average value (if any).

It can be shown that the mean square noise current in a circuit consisting of a short-circuited resistor of R ohms is given by:-

$$\overline{i_n^2} = \frac{4}{R} k.T. \Delta f \quad \text{.......................... (1)}$$

where k is Boltzmann's constant = 1·37 . 10^{-23} joules/°K *

T is the absolute temperature of the resistor in °K,
Δf is the frequency range considered, in cycles/sec, and
$\overline{i_n^2}$ is in $(\text{amps})^2$.

From the above result it follows that the mean square open-circuit noise voltage generated by a resistor of R ohms and measured by an instrument of bandwidth Δf is given by:-

$$\overline{v_n^2} = 4R.k.T\ \Delta f \quad \text{.......................... (2)}$$

and if the same units apply, $\overline{v_n^2}$ is in $(\text{volts})^2$.

As a general result it can be shown that, for any circuit consisting of ohmic resistors, coils and condensers, if the resistance (dependent on frequency) measured across any two terminals is R(f), then the total noise voltage generated across these terminals is given by:-

$$\overline{v_n^2} = \int_0^\infty 4.\ R(f).k.T.df \quad \text{.............. (3)}$$

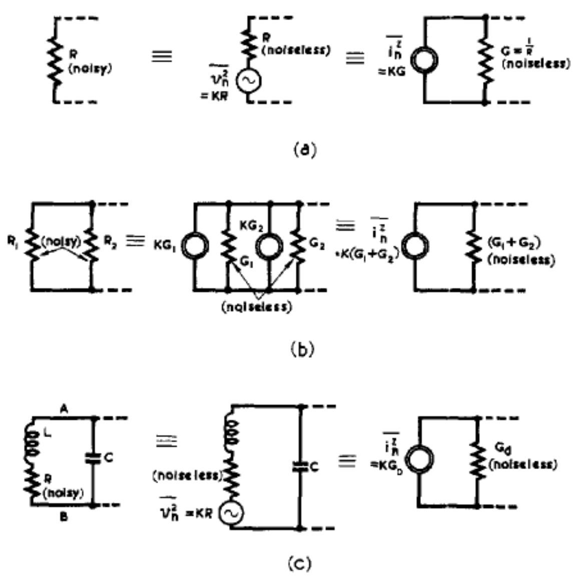

Fig. 643 - Equivalent circuit representations for thermal noise sources.

* °K indicates °Kelvin, signifying the use of the Kelvin or Absolute temperature scale, with zero at -273°C.

If the bandwidth accepted by the measuring instrument is limited to Δf, and the resistance can be considered as constant at a value R over this range of frequencies, then:-

$$\overline{v_n^2} = 4RkT\Delta f$$

as obtained above. The circuit representation, in this case, is shown in Fig. 643(a). For convenience we denote $4kT\Delta f$ by K.

When the current-generator equivalent circuit is used, it is sometimes more convenient to work with the conductance G rather than the resistance R of the resistor.

If the circuit consists of two resistors of resistances R_1 and R_2 connected in parallel, the effective short-circuit noise current as measured by an instrument of bandwidth Δf, is given by:-

$$\overline{i_n^2} = K\,(G_1 + G_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The equivalent circuit representation is shown in Fig. 643(b).

Consider the case of a high-Q tuned circuit (Fig. 643(c)), for a small bandwidth Δf near resonance. The effective open-circuit noise voltage appearing across AB, due to the resistance R in series with L is given by:-

$$\overline{v^2_{AB}} \doteqdot Q^2 \,.\, \overline{v_n^2} \text{ (where } \overline{v_n^2} = KR)$$

$$\therefore \overline{v^2_{AB}} \doteqdot Q^2.K.R.$$

But R_d = Dynamic resistance of tuned circuit = $Q^2.R = \frac{1}{G_d}$

$$\therefore \overline{v^2_{AB}} \doteqdot K/G_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Thus, near resonance, the dynamic resistance of the tuned circuit can be regarded as generating thermal noise. (This is a particular case of the general result quoted above (equation 3)).

It is described in Chap. 17 Sec. 11 how an effective radiation resistance can be attributed to an aerial. This radiation resistance is quite apart from the small, but inevitable, resistance due to the imperfect conductivity of the conductors which constitute the aerial. In calculating the performance of a receiver for amplifying signals from an aerial, the question arises as to what value of noise should be attributed to the radiation resistance of the aerial. It may be shown that if an aerial of radiation resistance R_r is confined in a uniform temperature enclosure, the aerial behaves, for noise considerations, as a resistor of value R_r producing thermal noise fluctuations. That an actual aerial is not in this state is self-evident. However, for lack of a convenient alternative in evaluating the noise in a circuit, it is usual to assign to the aerial a mean square noise voltage of KR_r (where K is normally evaluated at room temperature), in series with the radiation resistance of the aerial. This formula yields results which are sufficiently accurate to be used in practical applications.

SHOT NOISE

5. General

In any valve the emission from the cathode, because it results from the irregular departure of a large number of discrete units (electrons), has a random nature. Consequently, there are, superimposed on the mean current through the valve, fluctuations which constitute noise. This type of noise is called Shot Noise.

6. Diode Operating Under Temperature-Limited Conditions

The simplest case for consideration, in connection with shot noise, is the temperature-limited diode. The general concept of a valve working under temperature-limited conditions implies that all the electrons after emission proceed immediately to the anode independently of one another. Analysis shows that, with a diode operating under such conditions, the mean square noise current due to the shot effect when anode and cathode are effectively short-circuited is given by:-

$$\overline{i_n^2} = 2\, e.\, I_a\, \Delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where e is the magnitude of the electronic charge = $1{\cdot}59 \,.\, 10^{-19}$ coulombs;

I_a is the mean current through the valve in amperes;

Δf is the band-width under consideration,

and $\overline{i_n^2}$ is in (amps)^2.

7. Diode Operating Under Space-Charge Limitation Conditions

In practice valves are seldom used in their temperature-limited condition. The operating voltages are such that the electrons are not drawn off to the anode immediately after emission from the cathode, but a space charge is built up in the inter-electrode region in the form of an electron cloud.

Under temperature-limitation conditions, and assuming that

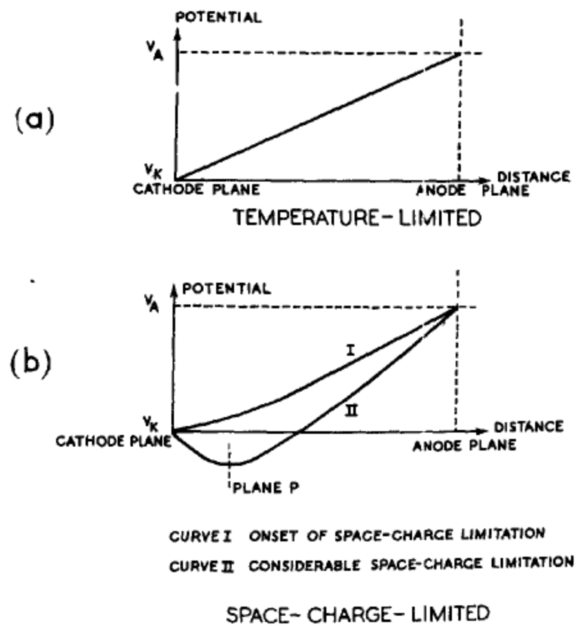

Fig. 644 - Potential distribution in anode-cathode space for a diode (a) temperature limited (b) space-charge limited.

the electrodes are plane and parallel, the potential distribution between the anode and cathode of a diode has the form shown in Fig. 644(a). Under a state of space-charge limitation there is electron interaction in the valve and it may be shown that the potential distribution has a minimum value in the inter-electrode space (Fig. 644(b)). Referring to curve II, near the cathode there is a region in which the field is directed towards the cathode, and between the cathode and anode there is a plane at which the potential is a minimum. Between the potential minimum and the anode, the electric field is directed towards the anode. It follows that, of the electrons emitted from the cathode, only those which have a component of velocity normal to the cathode sufficiently great to carry them past the plane P can ever reach the anode. The remaining electrons do not reach the anode, and thus the total anode current may be considerably less than the emission current.

Consider briefly how space-charge operation affects the shot noise in the current flowing in the anode circuit of the valve. The emission of surplus charge will momentarily increase the space charge and therefore depress the potential minimum. Consequently, certain electrons which would otherwise have had sufficient energy to overcome the potential barrier are now unable to do so. Hence the emission of electrons from the cathode, in any small time-interval δt, at a rate greater than the mean rate of emission, decreases the probability that electrons in immediately succeeding intervals will reach the anode. Thus the electron stream is to some extent ordered by the influence of the space charge, and we should therefore expect the mean square value of the fluctuating component in the anode current to be lower in the case of a valve operating under conditions of space-charge limitation than in one operating under conditions of temperature limitation. The reduction is expressed by a dimensionless factor Γ^2 (or less commonly by F^2 or A) where Γ^2 is known as the Space-Charge Reduction Factor. Under conditions of space-charge limitation, the mean square short-circuit noise current of a diode is given by:-

$$\overline{i_n^2} = 2.e.I_a\,\Gamma^2.\Delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

A typical value of Γ^2, for modern valves working under normal conditions, is of the order of $\frac{1}{20}$.

The value of Γ^2 for a diode depends on the values of the mean valve current I_a, the cathode temperature T_k, which controls the emission current, and the anode slope conductance of the valve ($G_a = {}^1/R_a$) which gives the ratio of a change of anode current with respect to a change of anode-cathode voltage. A comprehensive analysis gives a relation between Γ^2 and the factors I_a, T_k and G_a which is illustrated by the graph of Fig. 645. In this graph Γ^2 is plotted against:-

$$\frac{I_a.e}{G_a.k.T_k}$$

where e is the magnitude of the electronic charge ($1{\cdot}59 \times 10^{-19}$ coulombs),

k is Boltzmann's constant ($1{\cdot}37 \times 10^{-23}$ Joules/$^\circ$K),

and T_k, G_a and I_a are expressed respectively in $^\circ$K, mhos and amps. This parameter reduces to:-

$$\frac{11{\cdot}6\ .\ I_a\ .\ 1000}{G_a\ .\ T_k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The analysis and the relation illustrated in the figure are valid only for a relatively high degree of space-charge limitation. However, the condition of a high degree of space-charge is generally realised with modern valves, in particular those with oxide-coated cathodes under normal operating conditions.

An examination of the figure shows that the curve for Γ^2 bears a considerable resemblance to a rectangular hyperbola over the greater part of the range. Assuming that the curve is of this character, it follows that:-

$$\Gamma^2 . \frac{I_a . e}{G_a.k.T_k} = \text{constant.}$$

If we write $\theta = \dfrac{2\Gamma^2.I_a . e}{4G_a.k.T_k}$

we obtain $\Gamma^2 = \dfrac{4G_a.k.T_k.\theta}{2I_a . e}$ .... (9)

θ has been plotted for various values of $\dfrac{I_a . e}{G_a.k.T_k}$ and Γ^2 in Fig. 645, and it can be seen that over the useful range (i.e. except for very small currents) it is almost ideally constant, with an average value of about 0·65.

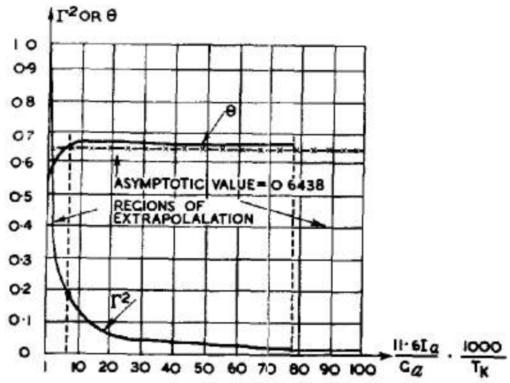

Fig. 645 - Graph of Γ^2 and θ plotted against $\dfrac{11.6I_a}{G_a} . \dfrac{1000}{T_k}$ for a diode operating under conditions of space-charge limitation.

The analysis gives Γ^2 and θ for the region between the dotted boundaries of Fig. 645. For the region on the extreme left analysis is complex whilst for the region on the extreme right the condition on which the analysis is based, i.e. a high degree of space-charge limitation, does not hold.

It has previously been stated that the mean-square noise current in a diode working under space-charge limitation conditions is given by:-

$$\overline{i_n^2} = 2e.I_a.\Gamma^2.\Delta f$$

Replacing Γ^2 by its value obtained from equation (9) we have:-

$$\overline{i_n^2} = 4k\ (\theta.T_k)\ G_a.\Delta f \quad .... (10)$$

This last equation is identical in form with a thermal noise formula, and therefore the shot noise in a diode working under space-charge limitation conditions can be considered formally as thermal noise generated by the diode slope conductance G_a at a modified temperature $\theta\ T_k$.

8. Negative-Grid Triode Under Space-Charge Limitation Conditions

The transition from a diode to a triode may be considered as a process of inserting grid wires at the desired distance from the cathode. A new space-charge distribution will result which, for sufficiently close grid spacing, will be sensibly uniform over the grid-plane. If, therefore, relatively high-mu valves (close grid spacing) are considered, the analysis for the diode may be extended to triodes. If the grid is sufficiently negative, all electrons which surmount the potential barrier in the cathode-grid space pass through the grid structure to the anode. Thus the current through the grid structure is identical with that reaching the anode. A fictitious collector plate, at some potential, could be placed in the plane of the grid such that it would collect the same space-current as previously passed through the grid structure. If it is feasible to neglect the effect on the space-charge barrier of electrons in the grid-anode space of the triode, then the diode suggested above would, for noise considerations, be a truly equivalent diode. For a finely-wound grid structure, this hypothesis is admissible, since not only are the electrons in the grid-anode space further from the space-charge barrier than those in the cathode-grid space, but also the grid acts as an electrostatic screen between the electrons in the grid-anode space and the space-charge barrier.

The problem resolves itself into a determination of the relationship between the conductance of this equivalent diode, and some measurable quantity relating to the actual triode. It is to be expected that the slope conductance of the equivalent diode G_a is related to the mutual conductance of the triode G_m and simple analysis shows in fact that the following relationship holds.

$$\frac{G_m}{\sigma} = G_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (11)$$

where σ is a function of amplification factor, transit-time factors and electrode spacings of the triode. If the amplification factor is very large σ is almost unity. For all conventional valves σ lies between 0·5 and unity, and is nearly unity for a pentode.

Using the results expressed in equations (10) and (11), we see that the mean square noise current in a negative-grid triode, operating under space-charge limitation conditions, is given by:-

$$\overline{i_n^2} = \frac{\theta}{\sigma} \cdot 4 \cdot k \cdot T_k \cdot G_m \cdot \Delta f \quad \ldots\ldots\ (12)$$

From equations (9) and (11)

$$\Gamma^2 = \frac{2 \cdot k \cdot T_k \cdot \theta \cdot G_m}{I_a \cdot e \cdot \sigma}$$

or including the numerical values of k, e and θ

$$\Gamma^2 = \frac{0{\cdot}11}{\sigma} \cdot \frac{G_m}{I_a} \cdot \frac{T_k}{1000} \quad \ldots\ (13)$$

Fig. 646 shows the variation of I_a, G_m and Γ^2 for an Acorn Triode (Mullard AT4). The anode voltage is 200 volts and σ has been taken as unity.

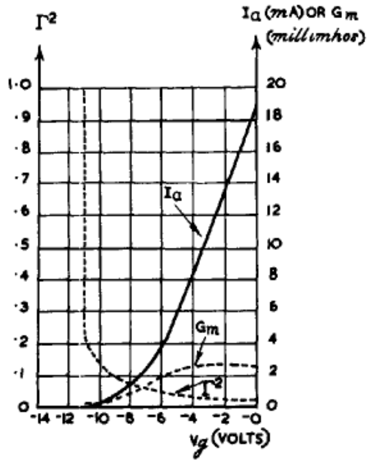

Fig. 646 - Graph showing values of I_a, G_m and Γ^2 for a Mullard AT4 for different values of grid-cathode voltage.

9. PARTITION NOISE

Since the current in a valve is composed of discrete entities emitted from the cathode with random direction and velocity the precise distribution of the electron stream at any instant is unpredictable. In the case of a diode or negative-grid triode, it can be said that all electrons which surmount the space-charge barrier will arrive at the anode. In the case, however, of a valve with more than one collecting electrode no more can be said than that, as the result of the transit of a large number of electrons, the mean currents to the anode and the other collecting electrode are both known. The final destination of each individual electron will be a matter of pure chance. This form of randomness relating to the valve current will constitute a fresh source of noise, which is known as Partition Noise.

It is to be expected, on the basis of random division of the electron stream of a valve between two collectors, the anode and screen-grid, that the partition noise in the anode circuit is equal to that in the screen circuit. Further, the partition noise should depend only on the total valve current and the relative division of anode and screen currents. At first sight it might seem a formidable problem to confirm these suppositions experimentally or even to show the existance of partition noise independently of shot noise. However, a relatively simple experiment achieves the object. In the circuit shown in Fig. 647, if the cathode resistor R is made sufficiently large the equivalent mutual conductance G_m can be reduced to a negligibly small value while the mean conduction current I_a is maintained by suitable electrode potentials. It has been shown (equation (13)) that the shot noise reduction factor Γ^2 is proportional to

$$\frac{G_m}{I_a};$$

hence a sufficient reduction of G_m reduces the anode shot noise ($\overline{i_n^2} = 2eI_a\Gamma^2\,\Delta f$) to negligible proportions. The partition noise, on the other hand, providing it depends only on I_a and the relative division of anode and screen current, should remain unchanged, and in fact does so.

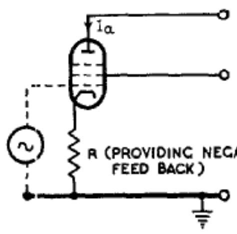

Fig. 647 - Circuit for reducing shot noise without affecting partition noise.

In the entire absence of shot noise (a purely theoretical condition) the mean-square short-circuit partition noise current in the anode (or screen) circuit is given by:-

$$\overline{i_n^2} = 2e\,.\,\frac{I_a\,.\,I_s}{I_a + I_s}\,.\,\Delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

where $\overline{i_n^2}$ is measured in (amps)^2,

and I_a and I_s are respectively the mean anode and screen currents in amps.

It is evident that the partition noise in the anode circuit of a pentode decreases to zero as the mean current flowing to the screen grid is reduced to zero. Hence design which results in the reduction of the mean screen current leads to less noisy pentodes. The screen current can be reduced in one of two ways:-

(i) The pitch of the screen grid can be increased and the diameter of its wire reduced. This, however, reduces the screening between anode and grid, and decreases the control of the screen grid on the electron stream.

(ii) The method of beaming can be used. This can be achieved by arranging the meshes of the various grids in such a way that the electron stream passes between the wires of the screen grid, i.e. the electron stream is focused.

In general shot and partition noise are present together. An analysis which considers the contributions to noise in the anode lead by electrons which:-

(i) Cross the potential barrier, which is due to space charge, and finally reach the anode circuit,

(ii) Cross the potential barrier and finally reach the screen grid circuit,

(iii) Do not have sufficient emission energy to cross the potential barrier, but which contribute to anode and screen circuit noise through resultant fluctuations of the potential barrier,

leads to the general formula:-

$$\overline{i_n^2} = 2eI_a \cdot \left(\frac{I_s + \Gamma^2 I_a}{I_a + I_s}\right) \cdot \Delta f \quad \ldots\ldots\ldots\ldots (15)$$

In the formula (15) $\overline{i_n^2}$ represents the total mean square noise current in the anode circuit, and Γ^2 is computed on the basis of the total current I in the valve. It is clear that, if $\Gamma^2 = 0$ (i.e. theoretical absence of shot noise), expression (15) reduces to expression (16).

xxx <u>Examples of the value of the mean square noise current in the anode circuit of a valve</u>

We shall evaluate the mean square noise currents in the anode circuit of a valve, which is operated firstly as a pentode and secondly as a triode.

The following data applies to an Acorn Pentode (Mullard AP4) operating as a pentode:-

$G_m = 2{\cdot}25$ mA/volt : $I_a = 6$ mA : $I_s = 1{\cdot}8$ mA : $I = I_a + I_s = 7{\cdot}8$ mA.

The temperature T_k of the cathode of the valve can be taken as 1,100°K.

The mean square noise current in the anode circuit of the pentode is given by:-

$$\overline{{}_p i_n^2} = 2e.I_a \left(\frac{I_s + \Gamma^2.I_a}{I_a + I_s}\right) \Delta f \text{ (from expression (15)).}$$

$$\text{where } \Gamma^2 = \frac{0{\cdot}11}{\sigma} \cdot \frac{G'_m}{I} \cdot \frac{T_k}{1000} \cdot$$

As an approximate relation

$$\frac{G_m}{G'_m} = \frac{I_a}{I} \text{ or } \frac{G'_m}{I} = \frac{G_m}{I_a}$$

where G'_m is the total conductance of the pentode, $\dfrac{\partial I}{\partial V_g}$.

Assuming $\sigma = 1$

$$\Gamma^2 = 0{\cdot}11 \; . \; \frac{2{\cdot}25}{6} \; . \; \frac{1100}{1000}$$

$$= 0{\cdot}045 \; .$$

Consequently

$$\overline{{}_p i_n^2} = 2e.\Delta f. \; 6{\cdot}10^{-3} \; . \; \left(\frac{1{\cdot}8 + 0{\cdot}045 \; . \; 6}{7{\cdot}8}\right)$$

$$= 2e.\Delta f. \; 1{\cdot}60 \; . \; 10^{-3} \; .$$

The mean square noise current in the anode circuit of the valve used as a triode is given by:-

$$\overline{{}_t i_n^2} = 2e \; I \; \Gamma^2.\Delta f \; .$$

Assuming that the total space current is the same as that when the valve is used as a pentode

$$\overline{{}_t i_n^2} = 2e.\Delta f \; (7{\cdot}8 \; . \; 0{\cdot}045) \; . \; 10^{-3}$$

$$= 2e.\Delta f \; . \; 0{\cdot}35 \; . \; 10^{-3} \; .$$

It follows that the valve used as a triode is less than one quarter as noisy as it is when used as a pentode.

10. INDUCED GRID NOISE

The primary noise sources (thermal, shot and partition) discussed in the previous sections, have a common characteristic; the mean square noise current is proportional to the total bandwidth involved, but is otherwise independent (at least over a wide range of frequencies) of frequency.

Another type of noise known as Induced Grid Noise (sometimes as High Frequency Noise) results from the movement of charges in the grid circuit which are induced electrically as a result of variations in the space current. If an external impedance is present between grid and cathode a noise voltage is generated which will, in turn, react upon the electron stream in the valve to produce additional valve noise.

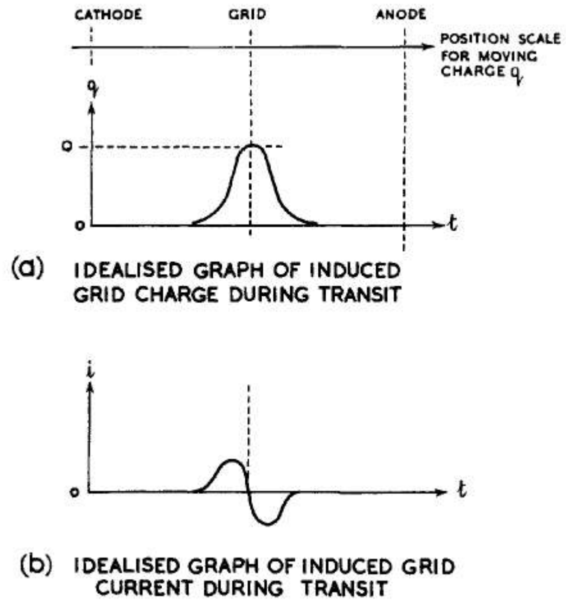

Fig. 648 - Induced charge and current at grid.

The mean square noise current due to induced grid noise is dependent on frequency and is in fact of importance only above the frequency limit where transit-time effects become significant.

The process leading to the production of induced grid noise can be explained as follows:-

If there is a uniform flow of electrons through a valve any induced current in an earthed grid, brought about by the approach of electrons to the grid, is counteracted by an opposite induced current produced by electrons receding from the grid. However, if a concentration of charge -Q, surplus to the average flow of charge in the anode-cathode space, passes across the valve and through a closely-wound grid mesh, then the variation with time of the charge q induced on the grid is of the form shown in Fig. 648(a). Consequently, since the induced current is the rate of change of the induced charge with time, the current pulse in the external grid circuit takes the form shown at (b).

If a Fourier analysis is performed with respect to a pulse of this form, it may be shown that no appreciable contribution of energy is made by frequency components small compared with $\frac{1}{\tau}$ where τ is the time of transit of electrons through the valve. Thus high frequency noise is evident only at frequencies for which there is a significant component of input conductance due to transit time (Chap. 7 Sec. 25).

Analysis gives the following expression for the mean square short-circuit noise current in the grid circuit:-

$$\overline{{}_g i_n^2} = 4k.T.\ 4{\cdot}8\ G_t\ .\ \Delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where T is the room temperature (°K).

k is Boltzmann's constant

G_t is the input conductance (mhos) due to transit-time loading only

and $\overline{{}_g i_n^2}$ is in (amps)^2.

These formulae are valid for only the initial part of the frequency range in which transit time is important.

The equivalent circuit for computation of the mean square grid fluctuation voltage $\overline{{}_g v_n^2}$ is shown in Fig. 649. The component of mean square short-circuit noise current in the anode circuit is then given by:-

$$\overline{{}_a i_n^2} = G_m^2\ .\ \overline{{}_g v_n^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Induced grid noise, which is noise produced in the circuit of an electrode not a collector of electrons, is of considerable importance in UHF in such valves as the klystron (see Chap. 16 Sec.1).

Example

A valve operating at 200 Mc/s has an input conductance G_t due to transit time of 0·0005 mhos. The external circuit conductance G (including leakance) is 0·0005 mhos. The mutual conductance G_m of the valve is 0·003 mhos. The bandwidth Δf of the receiver is 2 Mc/s and room temperature T is 300°K. Find the contribution of induced grid noise to the short-circuit noise current in the anode circuit.

$\overline{{}_g i_n^2} = 4.k.T.\ 4{\cdot}8\ .\ G_t.\Delta f$

from expression (16).

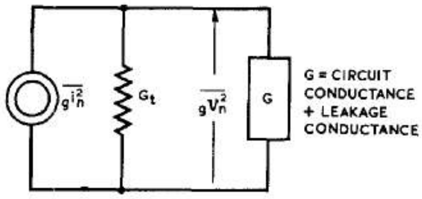

Fig. 649 - Equivalent circuit for computation of mean square voltage in grid circuit due to induced grid noise.

Therefore

$\overline{{}_g i_n^2} = 8.10^{-17}\ (\text{amps})^2$

$\overline{{}_g v_n^2} = \dfrac{\overline{{}_g i_n^2}}{(G + G_t)^2}$ (See Fig.8)

$= \dfrac{8.10^{-17}}{(2\ .\ 0{\cdot}0005)^2} = 8.10^{-11}\ (\text{volts})^2.$

$\overline{{}_a i_n^2} = G_m^2\ .\ \overline{{}_g v_n^2}$ from (17)

$= \dfrac{9}{10^6}\ .\ 8\ .\ 10^{-11}$

$= 72\ .\ 10^{-17}\ (\text{amps})^2$

Thus $\sqrt{\overline{{}_a i_n^2}} = 2{\cdot}68\ .\ 10^{-8}$ RMS amps

$= 0{\cdot}0268$ RMS microamps.

If the total effective output resistance is 10,000 ohms, the noise voltage developed in the output is given by:-

$\overline{{}_a v_n^2} = 0{\cdot}0268\ .\ 10{,}000 = 268$ RMS microvolts.

11. FURTHER SOURCES OF NOISE

Apart from the shot, partition and induced grid noise in valves, thermal noise in the resistors and noises generated in the aerial, which have already been described, there are various other sources of noise which may arise in circuits. These are briefly mentioned here, for completeness, although they have very little practical importance in radar circuits.

(i) Noise arising from Flicker Effect

This is a type of noise which sometimes occurs in audio-frequency amplifiers, particularly if valves with oxide-coated cathodes are used. It is probably due to random changes of the emitting properties of the various parts of the cathode. The duration of these changes is of the order of a millisecond. Owing to this relatively long duration of the fluctuations, the noise voltage produced across an anode load is large only if the load is of high impedance for frequencies below about 1 Kc/s. Consequently Flicker effect is of small importance at radio-frequencies.

(ii) Noise due to Ionisation Effects

If a valve is imperfectly evacuated, and ionisation takes place, there is a further source of noise. This is of little practical significance since a high vacuum normally obtains in modern valves.

(iii) Secondary Emission Effects

For some operating conditions of a valve, appreciable secondary emission of electrons may take place from an electrode bombarded by electrons from the cathode. This secondary emission results in an increase in the noise of the valve.

(iv) Noise arising from certain types of resistors

Certain non-metallic resistors (carbon resistors) may give rise to fluctuation voltages of a random nature when a direct current passes through them. This noise, which is in excess of thermal agitation, is proportional to current and also increases with the resistance.

THE EQUIVALENT NOISE RESISTANCE OF A VALVE

12. General

In the computation of total noise in a receiver and in particular of the noise resulting from a single amplifier stage it is necessary to combine the effect of thermal noise in the grid circuit with that due to shot and partition noise arising in the valve. It is desirable to replace noise sources in various parts of the circuit by a single equivalent noise generator at some suitable point; this is called "referring" the noise to this point. To simplify calculation and provide rapid comparison of the merits of different valves from the aspect of signal-to-noise performance it is most convenient to refer valve noise to the grid circuit.

In the grid circuit is placed a fictitious noise generator which would produce in an ideal noiseless valve a quantity of noise equal to that actually produced in the valve under examination. To facilitate immediate combination with true thermal noise generated in the grid circuit by ohmic resistance present, it is standard practice to represent the valve noise generator as a fictitious noisy resistance in series with the external grid circuit. This resistance "generates" the requisite amount of thermal noise to account for the valve noise present. Thereafter the valve itself may be regarded, for the purpose of calculation, as an ideal noiseless amplifier.

13. Triode Valves

In Fig.650, R_e is the fictitious noise resistance to be computed. The mean square short-circuit shot noise current in the anode circuit of a triode is given by:- .

$$\overline{i_n^2} = \frac{\theta}{\sigma} \cdot 4 \cdot k \cdot T_k \cdot G_m \cdot \Delta f.$$

(see expression (12)).

The mean square noise voltage generated by R_e is given by:-

$$\overline{{}_g v_n^2} = 4 \cdot k \cdot T \cdot R_e \cdot \Delta f = K . R_e$$

(see expression (2))

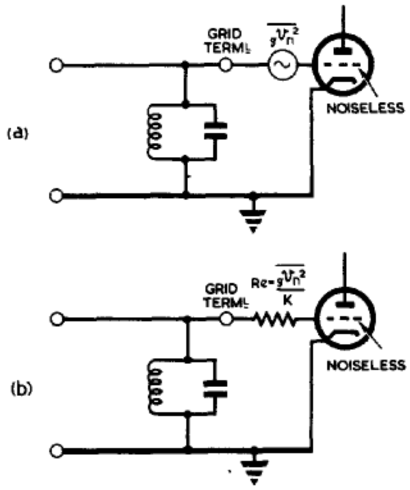

Fig. 650 - Circuit illustrating equivalent shot noise resistance of a valve.

and the resulting mean square

short-circuit noise current in the anode circuit is:-

$$\overline{i_n^2} = G_m^2 \cdot \overline{v_n^2}$$

$$= G_m^2 \cdot 4 \cdot k \cdot T \cdot R_e \cdot \Delta f.$$

Hence it follows that:-

$$G_m^2 \cdot 4 \cdot k \cdot T \cdot R_e \cdot \Delta f = \frac{\theta}{\sigma} \cdot 4 \cdot k \cdot T_k \cdot G_m \cdot \Delta f$$

or $$R_e = \frac{T_k}{T} \cdot \frac{\theta}{\sigma} \cdot \frac{1}{G_m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

It will be noted that R_e is independent of the bandwidth considered and this is an important advantage of this method of estimating valve noise. In the case of valves with oxide-coated cathodes, assuming

$$T_k = 1{,}100^\circ K,\ T = 300^\circ K \text{ and } \theta = 0 \cdot 65:-$$

$$R_e \approx \frac{2 \cdot 4}{\sigma} \cdot \frac{1}{G_m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

A commonly quoted figure for R_e is $\frac{2 \cdot 5}{G_m}$ and this formula always gives a reasonable estimate of the equivalent noise resistance referred to the grid circuit.

14. Pentode Valves

For a pentode valve it is usual to combine shot and partition noise in the formula for the equivalent noise resistance. This resistance is then given by:-

$$R_e \doteqdot \left(1 + 8 \cdot 7\, \sigma \frac{I_s}{G_m} \frac{1000}{T_k}\right)\left(\frac{2 \cdot 5}{\sigma} \cdot \frac{I_a}{I_a + I_s} \cdot \frac{1}{G_m}\right)$$

G_m being the anode-current, grid-voltage mutual conductance, $\frac{\partial I_a}{\partial v_g}$.

The following table gives the noise resistances of some common valves operating under specified conditions.

Valve Type		Valve Class	Operating Conditions			
			I_a(mA)	I_s(mA)	G_m(mA/volt)	R_e
AP4		Pentode (Acorn)	6	1·8	2·25	6,120
6SK7	(CV1981)	Pentode	9·2	2·4	2	10,500
6SJ7	(CV591)	Pentode	3	0·8	1·6	5,800
6AC7	(CV660)	Pentode	10	2·5	9	720
RCA956	(CV649)	Pentode	5·5	1·8	1·8	9,400
RCA955	(CV1059)	Triode	4·5	-	2·0	1,250

It should be noted that the expressions quoted above for the pentode valve are based on two plausible assumptions concerning the behaviour of electrons emitted from the cathode:-

(i) that the "fate" of every electron emitted (i.e. whether it terminates on screen or anode) is a matter of pure chance independent of its point of emission from the cathode.

(ii) that the influence of an individual electron may be regarded as "spread out" over the whole space-charge. In other words, the depression of the space-charge minimum as a result of "surplus" electrons (see Sec.7) is assumed to act in every case on the anode and screen currents in the same manner, in proportion to their mean values I_a, I_{sg}.

These assumptions may not be fully justified in modern valves and therefore in some cases a small correction factor may be found desirable in practice to allow, for example, for the effect of "grid alignment", which would tend to alter assumption (i) above. Again, very close electrode spacing without reduction of the cross-sectional area of the cathode might result in violation of assumption (ii) calling for suitable modification.

15. VALVE NOISE UNDER FEEDBACK CONDITIONS

If negative feedback is present in a valve amplifier the noise current generated in the output circuit is modified since the noise currents flowing in the feedback circuit will introduce compensating voltages between grid and cathode which will reduce the noise. This is illustrated in Fig.651 for the important case of current feedback (see Chap.16 Sec.12). Fundamentally, the effect of the feedback on the noise current may be considered as the result of a limiting process: each component of noise current produces a corresponding feedback voltage at the input which in turn causes a further (diminished) noise current to flow in the valve and so on. The result of adding the various terms of the series thus formed, for the circuit of Fig.651, in which the anode of the valve is effectively short-circuited to earth, is to reduce the noise current which flows through the feedback impedance according to the formula

$$(\overline{i_n^2})_{\text{with feedback}} = \frac{(\overline{i_n^2})_{\text{without feedback}}}{(1 + R_f\, G_m)^2}$$ (compare Chap. 7 Sec.16).

If now we consider the formula (15) for noise in a pentode viz,

$$\overline{i_n^2} = 2e\, I_a \frac{(I_s + \Gamma^2\, I_a)\ \Delta f}{I_s + I_a}$$

then it is the first term

$\frac{2e\, I_a\, I_s\, \Delta f}{I_a + I_s}$, depending only on the ratio of anode and screen currents, which is unaffected by feedback, whereas the second term, corresponding to the proportion of shot noise developed in the output circuit, is affected by the feedback as indicated above. The presence of the Γ^2 factor in this term,

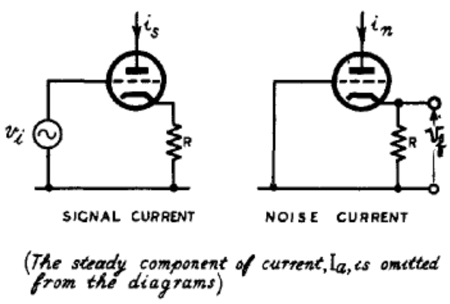

Fig. 651 - Noise under feedback conditions.

$$\frac{2e\, I_a^2 \Gamma^2\, \Delta f}{I_a + I_s}$$

indicates the susceptibility to feedback, since it expresses the dependence of the noise current on the grid-cathode voltage and space-charge. On the other hand, the product $I_a I_s$ is, to a first approximation, independent of variations in the cathode field and is therefore independent of the feedback.

16. SUMMARY

It has now been shown in the preceding sections that the physical explanation for the requirement of a comparatively high signal-to-noise ratio at the input stage of a receiver lies in the fact that while there are no new primary sources of signal power in the receiver between input and display there are many new sources of noise of different kinds, and the signal-to-noise ratio must inevitably deteriorate (if the same bandwidth is maintained) from aerial to output.

In the PPI type of display the signal and noise appear as variations in the brightness of the trace. Under such circumstances it is correspondingly more difficult to distinguish a weak signal from the noise background than with an A-Type display, and therefore the signal-to-noise ratio just prior to the detector of the receiver must be greater than unity (perhaps greater than 4) if the signal indication is to be distinguished from the noise.

On the other hand when the signal is used to measure angles of elevation or azimuth it is possible to use the integrated effect of the gated signals from successive recurrence periods so that the bandwidth of the signal fed to the output indicator or automatic following unit is very much less than the bandwidth at the detector stage immediately following the video frequency amplifier. This reduction in bandwidth may result in a considerable improvement in the resultant signal-to-noise ratio at the output, since the required signal power is concentrated in a relatively narrow band and is not reduced in the same ratio as the noise power, which is distributed uniformly throughout the band. For this reason it may be possible for a radar set to track successfully by auto-following a target which does not give a signal-to-noise ratio of even 1:1 at the detector stage.

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