Chapter 19

CONTROL SYSTEM

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CHAPTER 19

CONTROL SYSTEMS

1. INTRODUCTION

A control system, in general terms, consists of an arrangement of elements (amplifiers, converters, human operators, etc.) interconnected in such a way that the operation of each depends on the results of the operation of one or more other elements, and the purpose of which is to control some process or machine.

The whole system commences with the input element, the operation of which should be independent of the control system, and the output element is the one which affects directly the process or machine to be controlled. There may be several input or output elements. In most cases in radar the output element controls the position of the load, which may consist of a needle indicator, a shaft leading to some other control system, an aerial array or cabin which must be rotated to follow a target, or some other device.

A common example of a control system is the accelerator of an automobile engine; the position of the pedal, or input element, controls the applied engine torque which, together with external conditions depending on the resistance to motion, inertia, etc., determines the speed of the car. The road wheels may be regarded as the output elements. Such a system could be described as a position-velocity system, or more exactly, a displacement-controlling-velocity system, since the position or displacement of the input element ultimately controls the velocity of the output elements. It should be noted that the relation is not necessarily a linear one, and that it will be affected by differing external conditions.

The above is an example of an automatic, power-amplifying control system. In the case of a recording barometer, the variation in air pressure acting on a diaphragm usually provides the power to operate the recording device directly, so that the system is not power-amplifying. The supply of air to a pipe organ, on the other hand, requires power to be supplied by a motor or by a human operator; in either case power amplification is provided, a pressure gauge indicating the quantity of air required to maintain an adequate bellows pressure, and the motor or organ blower responding accordingly.

The mechanism of an automatic control system may be hydraulic, mechanical or electrical. In any case the essential links are indicated in Fig. 896. Fig. 896(a) shows a "straight through" arrangement in which input controls output directly, as with a Bowden cable (non-power - amplifying) or as in the motor-car throttle control (power-amplifying).

The first part of this chapter is devoted to a consideration of electrically operated data transmission systems, which may be represented schematically by the arrangement of Fig. 896(a), particular emphasis being given to remote position-indicating devices. The second part deals with Servo Systems.

A servo system is a control system which is Error-Actuated. As indicated at (b), at some stage of the control sequence the output quantity, or some function of it, is compared with a similar function of the input quantity, so that the whole system operates on the principle of reducing to zero the error, or difference between input and output. The process of comparing output with input is called Resetting. A servo is an automatic power-amplifying reset control system.

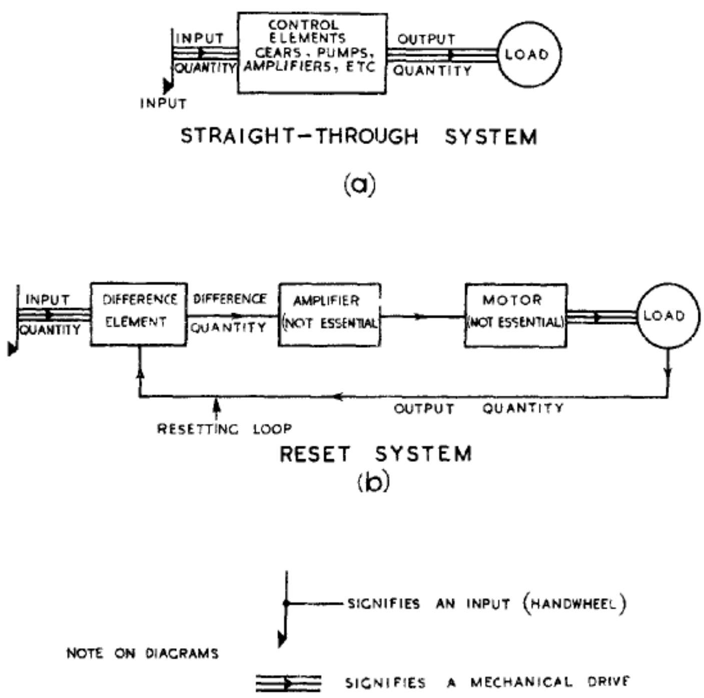


Fig. 896 - Fundamental arrangements of elementary control systems.

According to this definition , certain electronic circuits are servos whereas others are not. A simple valve amplifier is not a servo, since there is no reset. Some feedback amplifiers, on the other hand, employ the resetting principle and thus come into the servo category. For example, in a cathode follower the output quantity (cathode potential) is compared with the input quantity (grid potential) and the power amplifying properties of the valve are utilised to minimise variations in the difference between the two (error voltage).

DATA TRANSMISSION SYSTEMS (NON-RESETTING)

2. General

The function of a Data Transmission System is to convey to some remote receiver data which is represented by conditions at a transmitter. Usually this data is represented both at the transmitter and at the receiver by a mechanical movement, but this is not necessary. Such systems may be variable either smoothly or step-by-step; the transition from the one type to the other is gradual and ill-defined. For example, a wire-wound potentiometer is essentially a step-by-step device, since the division of the total resistance into its two parts changes with movement of the potentiometer arm by finite steps, corresponding to the resistance of a single turn of the wire. In effect, however, the magnitude of these steps can be so reduced as to make the error negligible, so the device is assumed to be smoothly variable.

The commonest types of data requiring transmission in radar are those giving numerical indication of the position of the transmitter; e.g. the Bearing or Elevation of an aerial system. Similarly, measured Range may be indicated by the angular movement of a calibrated transmitting shaft, although it may be transmitted as a variable voltage, the indication at the receiver being by means of a calibrated voltmeter.

A data transmission system is Synchronous if there is negligible delay between the setting of the transmitter and the adaptation of the receiver in conformity with the conditions at the transmitter; i.e. the follow-up time may be taken as zero. Systems which embody servo action usually exhibit a time-delay in operation which may or may not be sensibly constant for all inputs. They can seldom be considered as synchronous.

A system is linear if equal movements at the transmitter give rise to equal changes in indication at the receiver. Where duplicate systems are used, transmitting coarse and fine data, it is usually essential that the systems should be linear, otherwise one revolution of the fine transmitter would not correspond to the same incremental movement of the coarse one. The alternative, a non-linear gearing system, is not normally practicable. The use of such coarse-and-fine systems makes great accuracy possible without complicated or finely machined construction. For example, in a rotational system a maximum error of one part in 3,600 means, with a single, or coarse, indicator, an accuracy of $\pm 0{\cdot}05^{\circ}$ in 360°. With a coarse-and-fine system an error of $\pm 3^{\circ}$ can be permitted in both indicators without any deterioration in accuracy. This allows greatly increased manufacturing tolerances and often increases the ease of operation.

Various non-electrical synchronous transmission systems are in everyday use, but only electrical methods are dealt with in the following sections. Mechanical devices, such as flexible-cable rotary drives or push-pull controls can be used, but are inconvenient over distances of more than a few yards and in any case are prone to fatigue. Hydraulic and pneumatic devices have not been extensively applied to Service radar transmission problems. Electrical methods require only a multi-core cable between transmitter and receiver, and great accuracies may be maintained over long distances.

3. Performance of Transmission Systems

The criteria by which a transmission system is judged depend largely upon the use to which it is put. If it is required to operate a light pointer only, power considerations generally, and efficiency in particular, are not usually important. If it is required to drive a heavy mechanical load these considerations may predominate over all others. If the system is not power-amplifying, the power to drive the load at the receiver must be provided by the source operating the transmitter. Systems which are power-amplifying are frequently of the servo type, the transmission being carried out at a low power level and the signal power being amplified at the receiver by a servo.

The criteria usually applied to transmission system are as follows:

(i) Stiffness: this is defined as the magnitude of the disturbing torque that has to be applied to the load to displace it by a unit angular amount from its position corresponding to a fixed input.

(ii) Maximum static errors with or without load.

(iii) Dynamic errors; velocity and acceleration lags.

(iv) Frequency response.

(v) General reliability, size, weight, etc.

Some of these criteria are dealt with more fully in Secs.15 and 17-19.

4. Step-by-Step Systems for Transmitting Position Data

(i) Lamp system. In this system a lamp lights at the receiver to indicate the position of the moving arm at the transmitter. The very simplicity of this system, as indicated by the circuit of Fig. 897, makes it particularly useful where reliability is of paramount importance. The method can be adapted to any degree of accuracy by the use of coarse and fine dials with lamps at regular intervals on the circumference. Even so, a large number of cable cores is required. Since this is an On-Off system no errors are introduced by line losses, provided the lamps always light when (and only when) they are switched on. By the use of gas-filled lamps the current can be kept very small. Whilst the method is adaptable to pointer-matching receivers it is purely an indicating system and is not readily adapted for power drives. Either AC or DC supplies may be used.

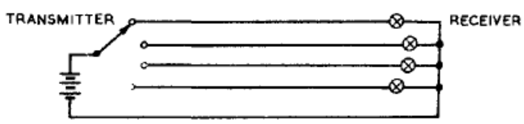


Fig. 897 - Lamp system.

(ii) M-Type System. The M-Type Transmission System, or M-Motor as it is sometimes called, is a relatively high-power analogue of the lamp system, the receiver current being used to provide a driving torque instead of energising a lamp. The method is illustrated in schematic form in Fig. 898. As the brush at the transmitter rotates and "makes" on each segment in turn of the transmitter switch, the corresponding armature winding at the receiver is energised, and the magnetic field set up produces a torque on the soft-iron rotor, which is magnetically asymmetrical, tending to pull its magnetic axis into line with the field. If the brush makes on two segments at once, both of the corresponding windings are energised and the rotor tends to take up a position intermediate between the field

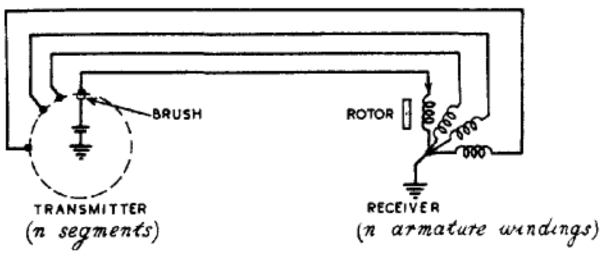


Fig. 898 - Schematic arrangement of M-type transmission system.

axes due to the individual windings. In practice this overlap interval is often brief and then does not greatly reduce the inherent errors of the system. If there are n segments the maximum off-load error (i.e. assuming there is no load torque) is $\frac{360^\circ}{2n}$. This possibility of error is accompanied by a 180° ambiguity if an unpolarised rotor is used, since in this case a reversal of the rotor does not change the magnetic stability of the system. If the rotor is polarised, two positions of equilibrium remain for each position of the transmitter brush, but one of these is unstable and of little practical importance. As the transmitter arm is rotated steadily the receiver rotor follows in jumps of $\frac{360^\circ}{n}$. The relative dispositions of brush and armature windings should normally be adjusted so that the error is zero when the brush is in the centre of each segment.

A variant of the elementary circuit is shown in Fig. 899. By the use of three fixed brushes and dead segments on the rotary switch the field at the receiver is made to rotate in steps of 30°, corresponding to a 12-segment switch of the elementary type shown in Fig. 898.

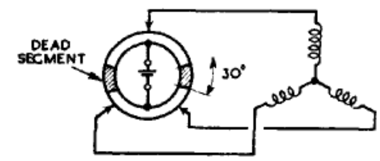


Fig. 899 - M-type transmission.

The receiver is shown wound for a two-pole field. The use of a four pole winding in the receiver reduces the step to 15°, but at the expense of introducing 180° ambiguity, even with a polarised rotor. Such a four-pole winding is illustrated in Fig. 900. Only one complete winding is shown, for the sake of clarity. The windings for the sets "2" and "3" are the same as for "1", each starting at the appropriately numbered point and ending at an earthed point. If the transmitter of Fig. 899 is used with such a receiver there is a 1:2 ratio between the movement of the rotor and that of the transmitter switch.

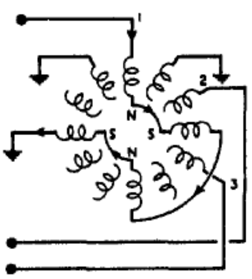


Fig. 900 - 4-pole receiver.

Fig. 901 shows an alternative form of transmitter switch providing a 1:1 ratio when used with a four-pole receiver. Neglecting the brief overlap period when a brush bridges two segments, the receiver field advances in 30° steps, i.e. 12 per revolution. There is a 180° ambiguity even with a polarised rotor.

A transmitter switch which provides 24 steps/rev. is shown in Fig. 902. Segments similarly numbered are connected together. The effect if this is used with the receiver of Fig. 898 is the same as if a 1:2 (step-up) gear were introduced between transmitter and receiver. The brushes may be duplicated, if desired, as shown (unshaded). If an 8-pole receiver is used, the 1:2 ratio is removed, but there are four stable positions of equilibrium with a polarised and eight with an unpolarised rotor.

The principal advantage of the M-type system is that it combines power amplification with relatively high efficiency. It may be used with an alternating supply provided the rotor, if polarised, is fed from an inphase source. Its chief disadvantage lies in the large number of ambiguities which must be permitted if small incremental steps are to be obtained.

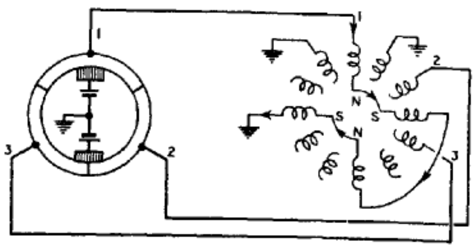


Fig. 901 - Alternative form of M-type system.

In its practical form the M-motor is not Self-Aligning; i.e., once the receiver is out-of-step with the transmitter it requires some external cause to bring it into step again, since any one of the ambiguous positions of the receiver rotor is as stable as any other.

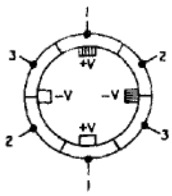


Fig. 902 - Transmitter providing 24 steps/rev.

5. Potentiometer and Voltmeter Systems

The basis of many transmission techniques is the potentiometer and voltmeter arrangement of Fig. 903. The voltmeter, in the simple circuit, is calibrated to indicate the displacement of the potentiometer slider from the zero end. Theoretically, if a wire-wound potentiometer is used, the indications are stepped, but in practice the steps may be smaller than the static error of the meter. Dynamic accuracy depends on the stiffness/inertia and damping ratios of the meter; (see Sec. 19). The accuracy is affected by anything causing variation in load current, such as line resistance or fluctuations in battery voltage. Changes of temperature may affect the accuracy of the voltmeter through mechanical changes or through variation of resistance.

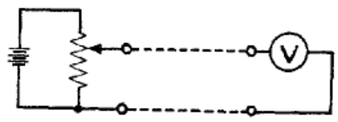


Fig. 903 - Potentiometer and voltmeter system.

All of the systems employing the basic principles of the circuit of Fig. 903, some of which are described in Sec. 6, are essentially power-amplifying. That is to say, the power which provides the output torque to bring the needle or rotor into alignment may be much greater than the power necessary to move the input potentiometer arm. On the other hand, these systems are not suitable for high powered drives, because of their very low efficiency. In general the load current must be small compared with the current flowing in the various branches of the transmitter potentiometer, so that if high powers were required at the receiver prohibitive dissipation would be necessitated at the transmitter. In this sense these systems compare unfavourably with the non-power-amplifying selsyns, described below, or the power-amplifying M-meter, (sec. 4).

The simple circuit of Fig. 903 may be modified by the substitution of an AC supply and meter for the DC arrangement. This introduces further sources of inaccuracy, of which variations in waveform and frequency of the supply are the most prominent.

One form of voltmeter commonly used is indicated in Fig. 904. This requires a 3-core cable, but is independent of the supply voltage. Such an arrangement is fundamental in all systems which are independent of supply fluctuations, and, in some cases, of line resistance (provided this is the same for all the lines).

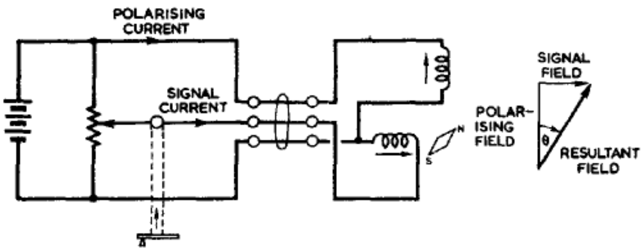


Fig. 904 - Electrodynamic system with DC polarized voltmeter.

The direction of the resultant field at the receiver depends on the position of the slider at the transmitter, and is indicated by a magnetised rotor. This rotor aligns itself with the resultant flux, which is inclined at an angle θ_o to the flux axis of the polarising winding. The system is non-linear, the movement of the input slider being proportional to tan θ_o. For DC operation a permanently magnetised needle may be used. For AC working the needle must be magnetised from a source in synchronism with the potentiometer supply.

6. Ring-Potentiometer Systems

The resolvers described in Chap. 3 Secs. 18-19, may be adapted as transmitters or receivers for use in data transmission systems, as indicated in Fig. 905. In this case sine-graded resistive potentiometers are used to form the transmitter, fed from a DC source, and a pair of inductive resolvers with a single magnetic rotor, which aligns itself with the resultant flux, form the receiver. The transmitter potentiometers are so wound that

$$V_x \propto \sin \theta_i$$

and $V_y \propto \cos \theta_i$, where θ_i is the angular rotation of the mutually perpendicular potentiometer arms from some reference position. Provided each resolver is wound so as to produce a uniform flux of magnitude proportional to the input voltage, the inclination θ_o of the resultant field at the receiver, is given by

$$\tan \theta_o = \frac{V_x}{V_y} = \tan \theta_i$$

Hence $\theta_o = \theta_i + n.180°$

This system is therefore linear. The ambiguity (n = 0 or 1) may be resolved by the use of a polarised rotor, in which case one of the equilibrium positions is unstable. If an unpolarised rotor is used the receiver must have identical scales, each occupying 180°, if ambiguity is to be avoided; (Fig. 906).

The system may be used with an alternating supply. In this case inductive potentiometers may replace the resistive ones. The rotor must be polarised from an AC source in synchronism with that which feeds the potentiometers.

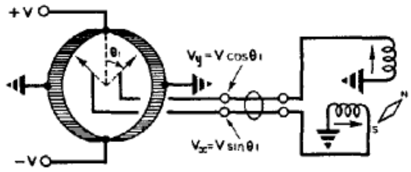


Fig. 905 - Two-coil receiver fed from sine-graded potentiometer.

An alternative arrangement of the sine-graded ring potentiometer system is shown in Fig. 907. This is typified by the term Inverted, indicating that the supply is fed to the potentiometers via the rotatable potentiometer arms, the outputs to the receiver being taken from fixed points. Separate ring potentiometers are necessary for the x and y windings of the receiver.

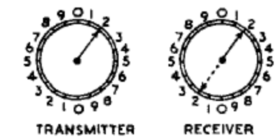


Fig. 906 - Resolution of ambiguity due to unpolarised rotor.

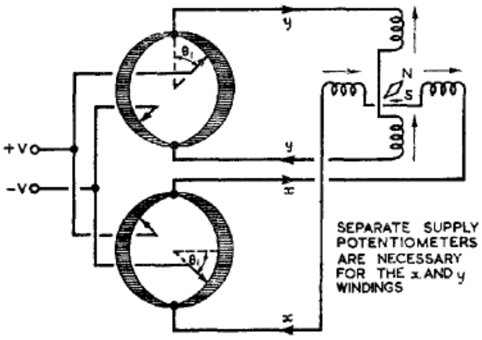


Fig. 907 - Inverted sine-graded transmitter.

Although the use of sine-graded potentiometers gives a theoretically linear relation between θ_i and θ_o, whereas non-sinusoidal potentiometers introduce non-linearity, the latter are commonly used in practice because of their comparative simplicity of construction. By the adoption of coarse-and-fine methods the errors introduced by the employment of uniformly-wound potentiometers may be made of negligible importance. Such a system is indicated in Fig. 908(a) and is analogous to the phase-shifting network described in Chap. 3 Sec. 23. The errors introduced by the potentiometer linearity are the same as for the phase-shifting network and the cause is illustrated in Fig. 908(b). This is a vector diagram indicating the direction O'P' of the resultant flux in the receiver. Using the symbols as indicated, to represent component fluxes and voltages, we have $B_y \propto V_y$ and $B_x \propto V_x$, the changes in each quantity being proportional to the angular movement θ_i of the potentiometer arm. As P moves from A to B (Fig. 908(a)) P' moves from A' to B' (Fig. 908(b)), the distance A'P' being proportional to the change in voltage from A to P, i.e. proportional to θ_i. If θ_i, is measured in degrees, and $\phi = \dfrac{\theta_i}{90}$,

then $B_x \propto \phi$

and $B_y \propto 1 - \phi$

so that $\tan\theta_o = \dfrac{B_x}{B_y} = \dfrac{\phi}{1 - \phi}$

The output alternately lags and leads the input. The error $\theta_o - \theta_i$ is zero when $\theta_i = 0^o, 45^o, 90^o$ etc. Maximum error occurs when $\theta_i = \pm 21{\cdot}5^o, 90^o \pm 21{\cdot}5^o$, etc; its value is $\pm 4{\cdot}1^o$.

Since the potentiometer is uniform it may be used either as shown in Fig. 908(a) or inverted, as in Fig. 909. This latter circuit also uses a full-wave or balanced arrangement of feed-points and receiver coils. An advantage of this arrangement is that the accuracy is not affected by voltage drops at the potentiometer output connections due to load current, provided the load is balanced and symmetrical; i.e., the input resistance of all the stator coils, measured between each input terminal and earth, is the same, and the four cable connections have equal resistance. The off-load accuracy (i.e. ignoring the effect of current drain on the transmitter) is the same for this arrangement as for that of Fig. 908(a).

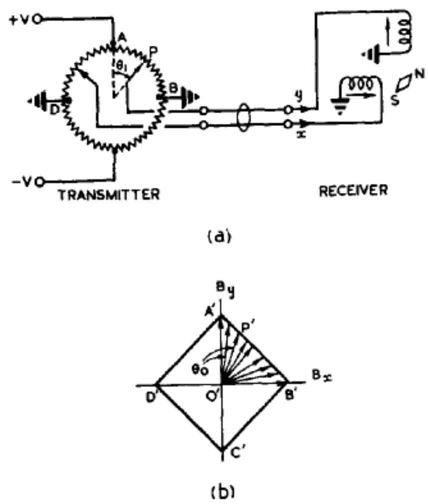


Fig. 908 - Uniform ring-potentiometer system.

A modification of the method of Fig. 909 is shown in Fig. 910 where a three-coil receiver is fed from an inverted linear ring-potentiometer. This results in an improvement in off-load accuracy, the maximum error being reduced to $1{\cdot}1^o$. The relative voltages carried by the three lines are indicated in the figure in terms of ϕ, where $\phi = \frac{\theta_i}{90^o}$.

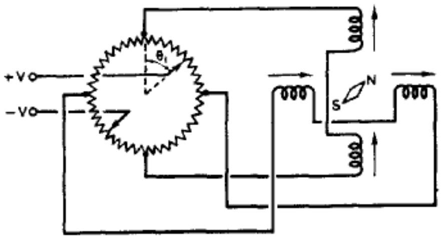


Fig. 909 - Inverted full-wave arrangement.

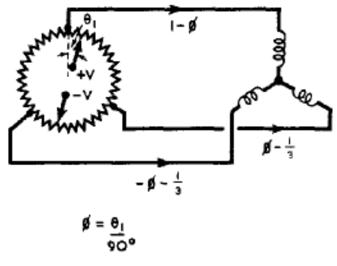


Fig. 910 - Three-coil receiver fed from uniformly wound ring-potentiometer

It may be deduced that, with the same symbols as for Fig. 908,

$$B_y \propto \frac{4}{3} - \phi$$

and $B_x \propto \sqrt{3}\,\phi$;

so that $\tan\theta_o = \frac{B_x}{B_y} = \frac{\sqrt{3}\phi}{\frac{4}{3} - \phi} = \frac{\sqrt{3}}{\frac{120}{\theta_i} - 1}$.

Provided a polarised rotor is used, no error is introduced at $\theta_i = 0^\circ$, 30°, 60° etc. Maximum error occurs at $\theta_i = \pm 13{\cdot}2^\circ$, $60^\circ \pm 13{\cdot}2^\circ$, etc.

7. Single-Phase AC Systems

Any of the systems described in Secs. 1 - 6 may be operated from single phase supplies, although generally speaking the accuracy is not as good as with DC. The systems dealt with in the following sections cannot be operated from DC supplies since, in all cases, the transmitter EMF's are magnetically induced, and, in some cases, the receivers operate correctly only if the frequency of the alternating supply is maintained at a suitable value.

The term Selsyn (from Self-Synchronous) is used for either a transmitter or a receiver designed on the principle of the inductive resolver or variable-ratio transformer. (Chap. 3 Sec. 19). The

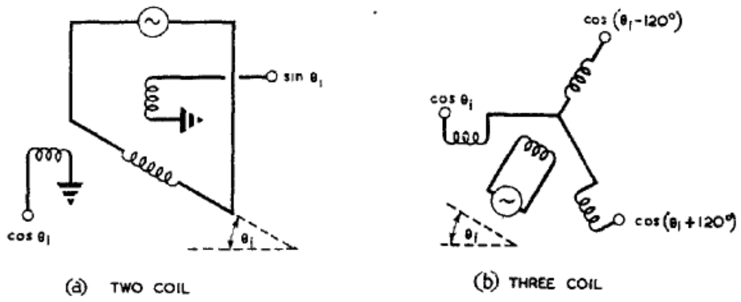


Fig. 911 - Selsyn transmitters.

transmitter fulfils the function of the sine-graded potentiometers in the circuit of Fig. 905, feeding to each receiver coil a voltage proportional to the cosine of the angle between the transmitter rotor coil and the appropriate stator coil. Fig. 911(a) shows schematically a two-coil stator and Fig. 911(b) a three-coil stator selsyn transmitter. The currents in the receiver coils have the same function as in the DC circuit, to produce a resultant flux with which the polarised rotor aligns itself. In the selsyn receiver, as in the selsyn transmitter, the soft-iron core of the rotor is polarised from an AC supply fed to the rotor winding via slip-rings.

In the Magslip (Magnetic Slip-Ring) receiver a shaped soft-iron rotor is polarised by magnetic induction from a stationary winding so that the use of slip-rings is avoided. This results in a very considerable loss in efficiency, the output torque available from a magslip receiver being sufficient to drive only a light pointer.

(Note: The term Magslip, which originally had only the meaning assigned to it above, is now employed to describe many other transmission components, some of which do not employ slip-rings, and is applied to some devices in which no movement at all occurs. The transmitters used with magslip receivers are called magslip transmitters or transmitter magslips, although they are designed on the selsyn principle, using slip-rings).

The advantages afforded by the use of selsyn-type transmitters instead of resistive potentiometers are :-

(i) they are more compact for a given output power;

(ii) they are easier to make to a given degree of accuracy;

(iii) they have a lower output impedance for a given input power, so that there is less reaction due to loading by the receiver. Also, a slightly unbalanced load does not seriously unbalance the output voltages of the transmitter.

A disadvantage is that selsyn systems are not power-amplifying. Because of its high efficiency a "power" selsyn (i.e., a large one) may be employed to drive a load which requires a large driving torque, but this torque must be provided by whatever source of power is used for driving the rotor of the transmitter. In this sense selsyns are inferior to M-motors, but compared with the latter selsyns have the advantages of :-

(i) smoothly variable output, instead of step-by-step;

(ii) absence of ambiguity; (i.e., they are self-aligning);

(iii) they require a cable with only five cores, whereas an M-motor may need many more.

8. Selsyn Systems

Fig. 912 shows a simple two-circuit selsyn system. Such a system is sometimes described as a "Two-Phase" system. The term is not used here, since "phase" is reserved for time-variations. Many of the windings used in single-phase AC transmission systems are similar to those found in polyphase motors or generators, where a rotating flux is generated. In the systems here described single phase supplies are used throughout. A flux can be made to rotate only by rotating the

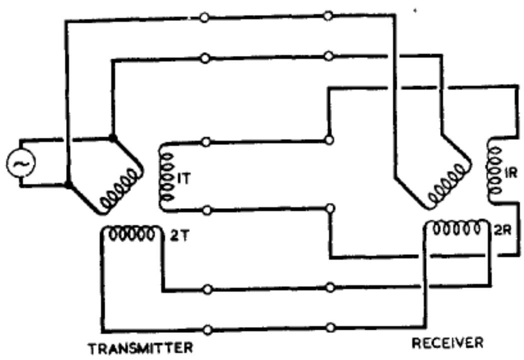


Fig. 912 - Simple two-circuit selsyn system.

appropriate field windings.

In the arrangement of Fig. 912 the rotor of the receiver is polarised from the same source as feeds the transmitter. Transmitter and receiver may be identical units, or the receiver can be of lighter construction. In the former case, when the receiver is in alignment with the transmitter the line current is zero, since the induced EMF's cancel in each of the secondary circuits. Each of the primary windings (the two rotors) draws from the supply sufficient magnetising current to set up a flux which gives a primary back EMF equal to the excess of the supply voltage over the series resistance drop. The arrows in the diagram show the flux directions for a chosen half-cycle of the supply current. Fig. 913 shows a three-circuit system. A three-circuit selsyn is preferable to a two-circuit one because :-

(i) The problem of winding the secondary circuits is simplified.

(ii) Even if the variation of mutual inductance with rotor angle is not truly sinusoidal, the error introduced at the receiver is zero every 30°, compared with every 45° for the two-circuit system, and the maximum error is less.

The three-circuit selsyn system does not require any more cable cores than the two-circuit system.

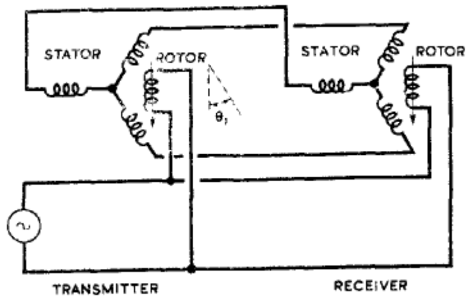


Fig. 913 - Three-circuit selsyn system.

In general several receivers may be driven from a single transmitter. Where receivers and transmitters are of identical construction interaction between receivers may be prohibitive. If one receiver is misaligned currents flow from the other receivers providing a correcting torque, and these currents cause the other receivers to be misaligned also. This interaction may be reduced by making the receivers of high input impedance compared with the output impedance of the transmitter, so that misalignment current from any one receiver is small and has little effect on the output voltages of the transmitter. Where this is done the stiffness of the receiver is small, and it is suitable for operating only a light pointer. Precautions must be taken to minimise frictional torques and errors due to mechanical unbalance of the receiver rotor.

9. Construction of Selsyn Transmitters

The principal object in the construction of a selsyn transmitter is to obtain sinusoidal variations of the mutual inductance between the primary winding and each secondary winding, as the rotor is turned. Common practice is to use three secondary windings, spaced 120° apart. Iron cores are normally used, to keep the magnetising current, at the usual power frequencies, reasonably low. Frequencies up to 1 kc/s have been used.

Usually the primary winding is put on the rotor, as this requires only two slip-rings and avoids having variable contact-resistances in the secondary circuits, the impedances of which must be accurately balanced at all times. In addition, former-wound coils can then be used for the secondary windings. However, most of the heat losses occur in the primary, and hence some makers put the primary on the stator.

The problem of making the flux linkages between the primary and each secondary winding vary sinusoidally with the angular rotation of the rotor is the same as that of securing a sinusoidal output voltage waveform from an alternator, and the methods of construction adopted in the two cases are similar.

10. Selsyn Receivers

The receiver of a selsyn-type transmission system should resemble the transmitter in having the secondary coils so wound that the mutual inductance between each secondary winding and the primary, or polarising, winding varies sinusoidally as the rotor is turned. It is an advantage if the rotor is magnetically symmetrical.

Consider a two-circuit receiver. If the rotor is unpolarised but magnetically asymmetrical, a current flowing in either stator coil tends to align the rotor with its axis of least reluctance in the direction of the magnetic axis of the coil; i.e., the rotor tends to set in the position giving maximum inductance of the coil. If the current in the coil is I_1 (RMS value) and its inductance is L_1, the torque on the rotor satisfies the relation

$T_1 \propto \frac{I_1^2}{2} \cdot \frac{dL_1}{d\theta_o}$, θ_o being the angular position of the rotor. A similar torque $T_2 \propto \frac{I_2^2}{2} \cdot \frac{dL_2}{d\theta_o}$ is developed between the second coil and the rotor, so that for equilibrium,

$$T_1 + T_2 = 0 .$$

This gives

$$\frac{dL_1}{d\theta_o} \Big/ \frac{dL_2}{d\theta_o} = -\left\{\frac{I_2}{I_1}\right\}^2 .$$

This does not lead to a simple expression for θ_o. In general the torques due to rotor asymmetry are not likely to tend to produce the same equilibrium position for the rotor as those due to mutual inductance between rotor and stator coils. Hence it is usually desirable for the rotor to be symmetrical so that $\frac{dL_1}{d\theta_o}$ and $\frac{dL_2}{d\theta_o}$ are zero. In this case the same design considerations apply to the receiver as apply to the selsyn transmitter; (Sec. 9).

11. The Magslip receiver

Simplified plan and elevation drawings of a magslip receiver are shown in Fig. 914. The L-shaped soft-iron rotor is balanced so that its equilibrium position is independent of gravity. It is polarised from a fixed cylindrical polarising winding as shown. The stator has three sets of coils arranged with their magnetic axes spaced 120° apart. The mutual inductance between any stator winding and the polarising coil should vary sinusoidally with the rotation of the rotor; i.e., the flux entering the rotor due to any stator current should vary sinusoidally with the angular position of the rotor. This desired variation of the rotor flux can be achieved if the magnetic potential U of the

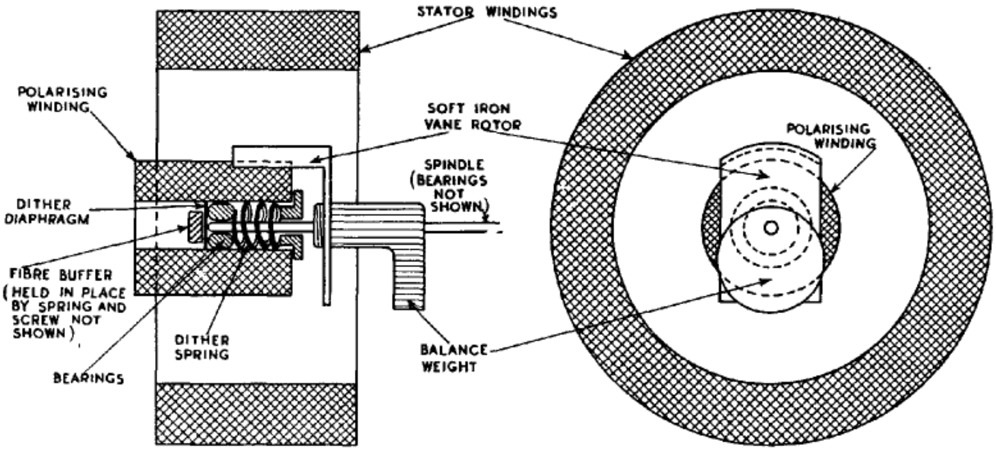


Fig. 914 - Magslip receiver.

stator surface varies sinusoidally from point to point. Using a slotted stator, this condition can be approximated to by an appropriate distribution of ampere turns. The same current flows in all the conductors of any one winding, but the number of turns in each slot is so chosen that the resulting stepped graph of magnetic potential is approximately sinusoidal; (Fig. 915). The integrating effect due to the rotor overlapping more than one slot smooths out the flux-linkages so that the effective magnetic potential is less irregular than that of Fig. 915(b).

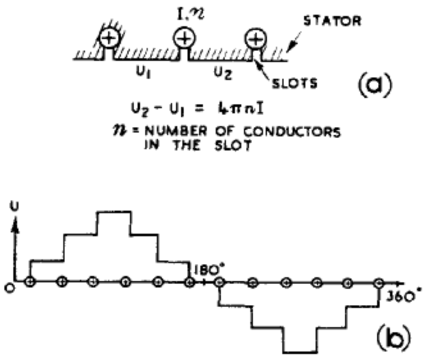


Fig. 915 - Distribution of magnetic potential for 14-slot stator.

When each of the stator windings is energised the resultant potential distribution at the stator surface is still approximately sinusoidal, as may be readily deduced by vectorial or analytical methods. There is in practice a cyclic error of magnitude approximately

6' and of period equal to the slot pitch (15° in the magslip receiver).

The effects of friction (see Sec. 15) are greatly reduced by the endwise dither of the rotor which is caused by the pulsating magnetic pull of the polarising coil.

The resonant frequency of a magslip receiver is in the neighbourhood of 2·5 c/s. The velocity lag is indicated in Fig. 916; it is about 1° at 20°/sec. The damping is low. Various methods are being investigated of increasing the damping and reducing the pronounced resonance which has deleterious effects when used with some computing devices.

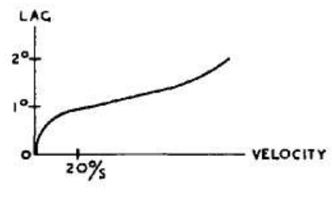


Fig. 916 - Magslip velocity lag.

12. Coincidence Indicating Systems

The object of a Coincidence Indicating System is to indicate when one shaft at the receiving end of the system is aligned with another at the transmitting end. Usually the magnitude and direction of any misalignment must be shown over a certain misalignment range.

A ring-potentiometer coincidence system is shown in Fig. 917(a). The voltmeter shows a reading whenever the angle between the two brush sets is other than 90°. The action is illustrated in Fig. 917(b). θ denotes angular rotation from the diameter AC on either ring-potentiometer, θ_i being the rotation of the feedpoints of voltage, + V and -V, and θ_o the rotation of PQ from AC. Neglecting the current drain from the first network due to the second, the variation of voltage with θ_o is shown in Fig. 917(b) for the chosen value of θ_i. Provided $\theta_o = \theta_i + 90°$ the voltage difference between P and Q is zero.

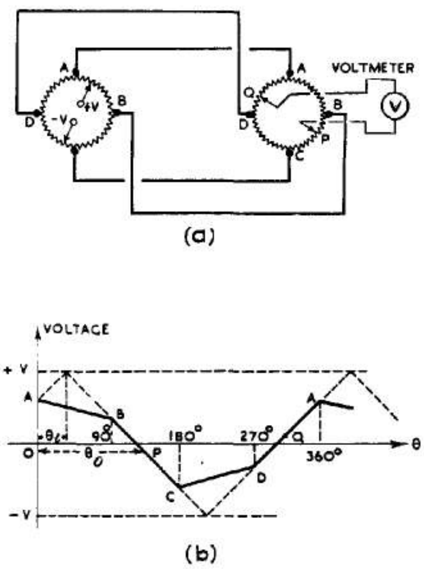


Fig. 917 - Ring-potentiometer coincidence system with curve of voltage against angular rotation.

The voltmeter may be at the same end as the power supply. It must be polarised if the sense of the misalignment is to be indicated. The system will operate with an alternating supply provided the voltmeter is polarised from an in-phase supply.

Fig. 918 shows a selsyn coincidence indicator. The flux-distribution due to rotor 1 is reproduced by stator 2. Unless rotor 2 is perpendicular to this flux, and therefore to rotor 1, a resultant EMF will be induced in rotor 2 and will be indicated by the voltmeter. The meter must be polarised, as indicated, from an in-phase supply. Although a dynamometer meter is shown, an AC-polarised moving iron meter could also be used, or a polarised DC voltmeter in conjunction with a phase discriminating rectifier; (Sec. 14).

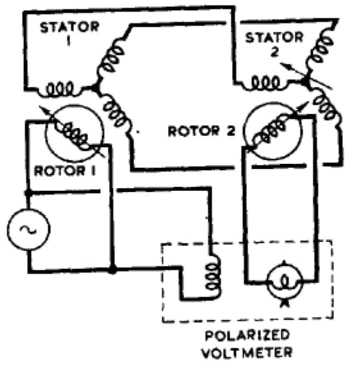


Fig. 918 - Selsyn-type coincidence system.

A selsyn system using a differential receiver is shown in Fig. 919. A differential selsyn receiver consists of a stator and a rotor, each with three symmetrical windings. The flux in the stator of the receiver is parallel to the magnetic axis of rotor 1, at an angle θ_i from the reference axis. The rotor of the receiver, carrying the rotor coils, aligns itself so that the stator flux and rotor flux axes are coincident. The rotor flux is inclined at an angle θ_o to the reference axis on the rotor (indicated by the needle attached to the rotor). Hence the inclination of the rotor needle to the reference axis of stator 1 indicates the error $(\theta_i - \theta_o)$.

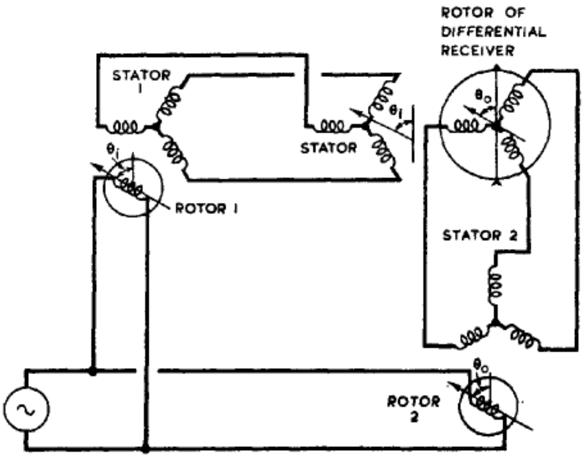


Fig. 919 - System using differential receiver.

In this system the differential receiver may be remote from either transmitter.

Coincidence indicating systems may be used to give visual indication of misaligment, or may be used, if suitably adapted, as error indicating devices in servo mechanisms.

SERVO SYSTEMS

13. General

Although attention is confined, in the subsequent analysis and description, to servos which drive a mechanical load, the principles are the same if the output is of a different kind, and the same methods of analysis may be employed.

The arrangements of the principal elements in a servo are shown schematically in Fig. 920. Sometimes the functions of two or more elements may be performed by a single co-ordinating element. It will be assumed, unless otherwise stated, that there is no inherent feedback from output to input in any individual element.

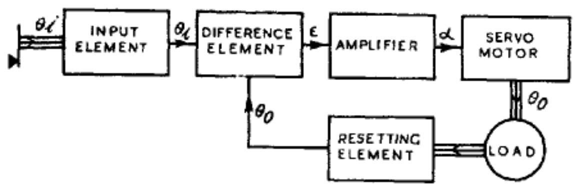


Fig. 920 - Arrangement of the principal elements of a servo (see Sec.18 for explanation of symbols).

14. Computing Devices (Converting and Calculating Elements)

In many servos it is necessary to convert operative quantities from one kind to another. For example, either the input or the error quantity may have to be converted from a mechanical movement to an electrical potential, or vice versa. It may also be required to add or subtract, multiply or divide, integrate or differentiate, various operative quantities. Any of these processes may be performed in various ways, either mechanical or electrical solutions being common.

To convert a mechanical movement to an electrical potential difference a simple linear potentiometer arrangement will suffice, as illustrated in Fig. 921. A constant voltage is applied at the input terminals A and B and the output voltage between the slider C and B is proportional to the product of the constant voltage and the slider movement. If the input voltage is not constant, but is proportional, say, to x, whilst the slider movement is proportional to y, the output is proportional to the product xy. By winding the potentiometer according to a non-linear law so that R=f(y), where R is the variable resistance and y the slider movement, the output can be made proportional to xf(y).

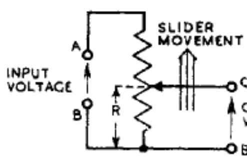


Fig. 921 - Conversion of a mechanical movement to an electric potential.

In converting from an electrical potential to a mechanical movement, a self-contained servo is usually required, since a voltmeter type of arrangement, which would otherwise suffice, does not normally provide sufficient output torque.

The fundamental arrangement is illustrated schematically in Fig. 922. The output from the resetting potentiometer is subtracted from the input in a difference amplifier. The latter controls the servo motor which drives the resetter until the input to the difference amplifier is zero. Thus the output of the potentiometer is equal to the input, and the resultant movement of the potentiometer arm is proportional to the input voltage.

Voltages may be added by using the network of Fig. 923. The result

$$V = \frac{\sum \frac{V_1}{R_1}}{\sum \frac{1}{R_1}} \quad \text{(see Chap. 1 Sec. 12)}$$

becomes, if all the resistances are equal,

$$V = \frac{\sum V_1}{n}$$ where n is the number of shunt networks.

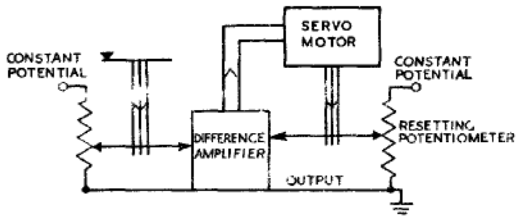


Fig. 922 - Conversion of an electric potential to a mechanical movement.

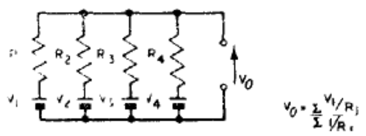


Fig. 923 - Adding network (ladder).

For direct voltages, a reversal of polarity of one of the sources changes the addition to a subtraction. Fig. 924 shows how the potentiometer input and output elements may employ this principle to form the combined resetting link and difference element of a servo. In this example the input element is a resistive potentiometer fed from + 300V, the slider being rotated by the input operator. The resetting element consists of another potentiometer fed from -300V, its slider being driven by the output shaft of the servo motor. The difference voltage $\mathcal{E}$ is taken from the junction of the two equal resistances and may be positive or negative, according as $\theta_1 \gtrless \theta_0$. If it is important that $\mathcal{E}$ should accurately represent $\theta_1 - \theta_0$, the output impedance of the potentiometer should be $\ll$ R. In the case illustrated this is usually unimportant, since strict proportionality is not necessary in a difference

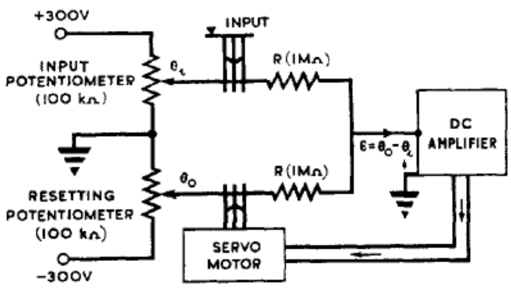


Fig. 924 - Use of potentiometers and resistance network to form combined resetting and difference elements of a servo.

element, although any non-linearity which may be introduced complicates analysis. In other cases an amplifier inserted between each potentiometer and the adding network will ensure that the output impedance is constant to a sufficient degree of approximation.

For alternating voltages the same system may be used, subtraction being achieved by a reversal of phase of one of the input signals. Alternatively, transformers may be employed, making addition and subtraction extremely simple in theory, as indicated in Fig. 925. In practice, slight changes in phase may be difficult to avoid, and can seriously affect operation. With AC, potentiometers may be of the inductive type, in which coils, instead of resistance-wire, wound on high permeability cores, take the place of the tapped resistance. One advantage of this arrangement is that with resistive loads the load current is in quadrature with the potentiometer current and does not appreciably affect the magnitude of the output voltage. An amplifier following the potentiometer is thus often avoidable.

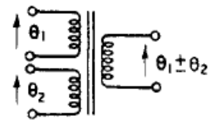


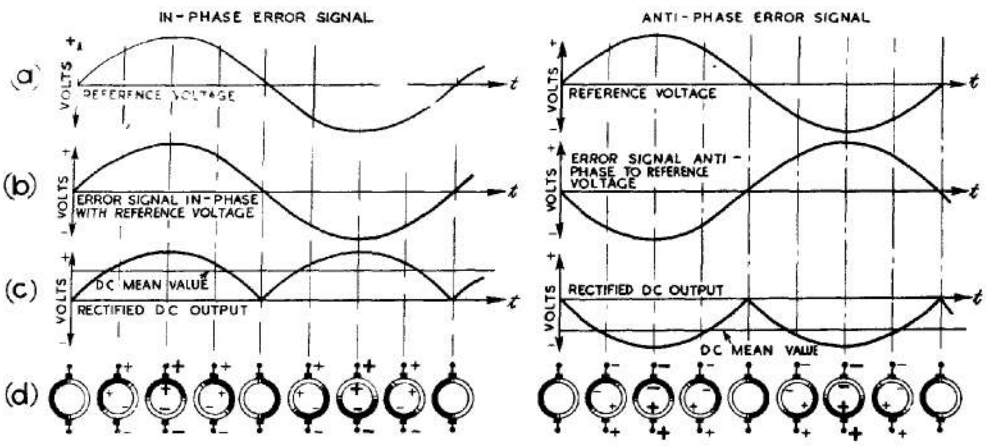
Fig. 925 - Addition or subtraction of sinusoidal voltages.

Electrical integrating and differentiating circuits are described in Chap. 2 Sec. 17. Their mechanical equivalents, in the form of ball-and-disc gear, are not often used in servos, although the "governor" type of speed control, which is a form of rate-measurer, may be employed. Integrating circuits in hydraulic servos are common, and one type consists of a cylinder with some form of valve which allows fluid to enter at a rate proportional to the size of the aperture; the output depends on the movement of a piston which is proportional to the quantity of fluid in the chamber. The piston movement is thereby made proportional to the time-integral of the valve movement.

It may be necessary in electrical servos to convert from AC to DC or vice versa. In some cases the error voltage is sinusoidal, whereas the servo motor requires a steady voltage or direct current feed, its sign determining the direction of rotation of the armature. The sign of the error depends on the phase of the alternating voltage, and it is therefore necessary to convert the sinusoidal input to a DC output by means of a phase discriminating circuit.

This may be accomplished by a synchronous rotary converter, the action of which is illustrated in Fig. 926. The commutator, which acts simply as a reversing switch, is driven at synchronous speed with reference to the frequency of the sinusoidal error voltage. At (a) is shown a fictitious reference voltage corresponding to the commutator rotation, and (b) shows the error voltage which is fed to the commutator via slip-rings. A reversal of the error voltage reverses the polarity of the brushes (d) and thus the sign of the output voltage after being filtered from the commutator.

An electronic circuit which accomplishes a similar result is illustrated in Fig. 927(a). The standard reference voltage, with which the error voltage is either in phase or antiphase, is applied to the anodes of the two valves in pushpull, so that one is conducting when the current of the other is cut off. The error voltage is applied to the control grids of both valves, so that anode current is



(a) STANDARD REFERENCE VOLTAGE
(b) ERROR SIGNAL APPLIED TO COMMUTATOR VIA SLIPRINGS
(c) OUTPUT FROM BRUSHES
(d) POSITION OF COMMUTATOR W.R.T. BRUSHES

Fig. 926 - Phase discrimination by means of a synchronous rotary rectifier.

caused to flow in one valve or the other for a half cycle, as shown in Fig. 927(b). The direction of the field in the anode coil is thus reversed with the change in phase of the error voltage and, provided the linear portions of the valve characteristics are used, the magnitude of the current will be proportional to the magnitude of the error signal. In the case illustrated a direct current is caused to flow in one or other of the differential field windings of a servo motor, according to whether the error voltage is in-phase or anti-phase with the reference voltage.

The reciprocal problem of converting a DC input of a certain polarity into an output of corresponding amplitude and phase may be solved by means of saturable reactors as well as by electronic means. Fig. 928(a) shows an iron-cored reactor fed with both direct and alternating voltages. The direct current partially saturates the core, so that the impedance presented to the AC input terminals is reduced as the direct current is increased.

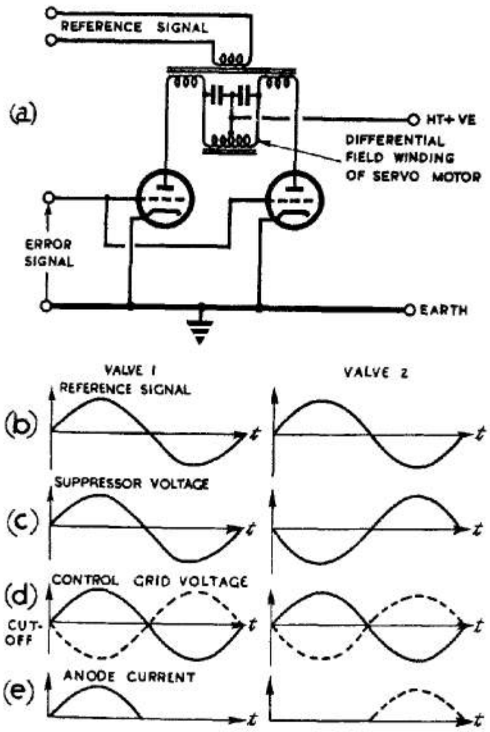


Fig. 927 - Operation of electronic AC-DC error converter (Phase-discriminating rectifier).

Four such reactors are used in the network shown in Fig. 928(b). DC polarising windings, one on each reactor, are fed from the same source. Control windings, fed from the DC error voltage source, are so connected that the DC fields in A and C are increased by the error voltage when those in B and D are decreased, and vice versa. When the error voltage is zero and the system is properly balanced, the impedances presented by the four reactors at the individual AC input terminals are the same. When the error voltage is not zero the DC fields in two opposite arms of the bridge are increased, reducing the AC impedance, and those in the other pair of arms are reduced, increasing the AC impedance. Thus a net voltage is produced at the output terminals equal to

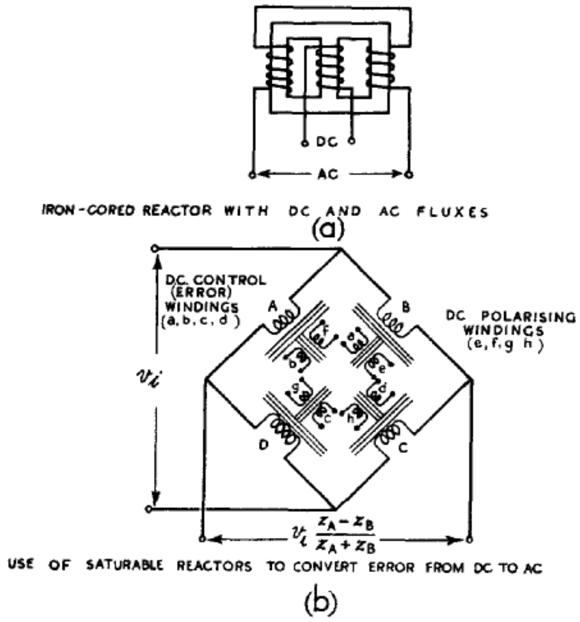


Fig. 928 - Iron-cored reactor with DC and AC fluxes, and use of saturable reactor to convert error from DC to AC.

$$\frac{v_i \quad (z_A - z_B)}{z_A \; + \; z_B}$$

and is thus in-phase or anti-phase with v_i according to the polarity of the DC error signal. Within limits the relation is linear.

15. Characteristics of the Load

Where the servo is required to provide mechanical movement the dynamic characteristics of the load, motor armature and gearing are of primary importance in their effect on the behaviour of the system as a whole.

The principal effects to be considered are inertia and friction. The former is the chief limiting factor with respect to acceleration or retardation, and the latter dissipates energy and limits the speed

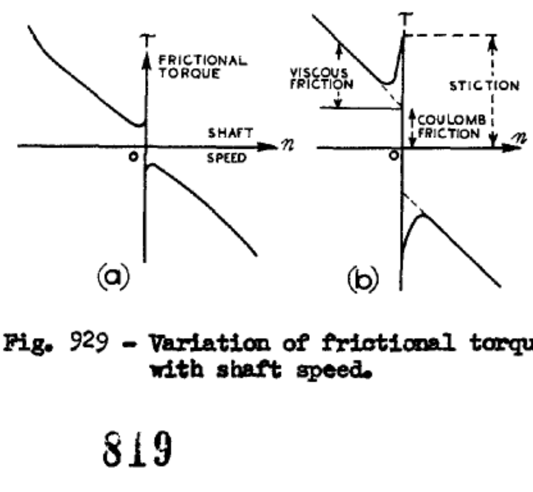


Fig. 929 - Variation of frictional torque with shaft speed.

of rotation. Fig. 929 shows the variation of frictional resistance or torque with shaft speed. There are three chief components of the friction force, stiction (or static friction), coulomb friction and viscous friction. These are indicated at Fig. 929(b). In most analyses of servos the first two types are ignored, since they are non-linear and difficult to allow for. The co-efficient of viscous friction is sometimes called Viscosity. In practice, stiction is particularly important, its presence leading to jerky motion (since the stiction torque must be overcome before motion can ensue, and this involves an error in the input to the servo motor before sufficient torque can be built up).

Various methods are used to combat stiction, one of the commonest being to dither the output shaft either by using an auxiliary motor, or by introducing a deliberate dither error into the servo mechanism. This keeps the shaft moving at a rapid rate about the equilibrium position, and provided the dither frequency is high enough (of the order of 10-1000 c/s, depending on the natural frequency of the system) the effective resistance to be overcome to cause initial motion is reduced from its stiction value almost to zero. The dither should be in a direction perpendicular to the plane of motion: e.g. a magslip receiver pointer (see Sec. 11) is caused to dither in the direction of the axis of rotation. A rod which moves in a longitudinal bearing might be given a rotational dither motion. Another method of countering stiction is that of variable duration impulsing. This is similar to the principle of the piledriver. Stiction may be sufficient to prevent motion if only a steady relatively small force is applied. If a much greater force is applied for short periods, the duration of each impulse being variable, no matter how small the mean force applied the peak force may be maintained sufficiently large to make stiction unimportant. The use of variable duration impulsing is illustrated in Fig. 930. The momentum imparted to the load is proportional to the excess of the area below the impulse curve over the corresponding area below the friction curve. Once motion has begun it is the mean frictional torque of Fig. 930 that matters, not the stiction value.

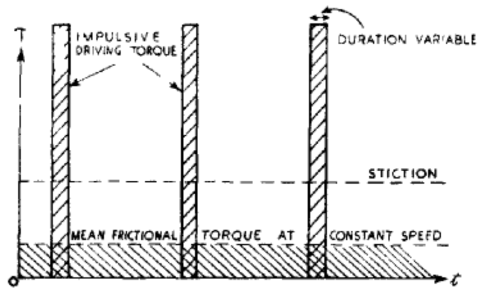


Fig. 930 - Use of variable duration impulsing to combat stiction.

16. Servo Motors

Many forms of DC or AC machines may be used in servo systems, besides other types of motors in common use, and a full description of them is impossible in this work. A few of these will be mentioned with special reference to their servo applications.

In general the motor is required to remain stationary (for a displacement-controlling-displacement servo) when some controlling current or voltage is zero, and to accelerate as this error quantity is increased, in a direction depending on the polarity (DC) or phase (AC) of the error. The motor may also possess additional features, such as automatic damping or braking devices, other than frictional resistance at the rotor bearings.

The simplest of servo motors are of the On-Off type, there being no variation of motor speed or torque with the amplitude of the error voltage. Such motors are prone to excessive Hunting; i.e., the output shaft over-runs the equilibrium or zero-error position, the drive to the motor is reversed, and the output shaft again over-runs in the opposite sense, the output thus being of an irregular, oscillatory nature. Alternatively there is likely to exist a substantial Dead Zone; i.e., the error must exceed a certain value, either positive or negative, before the motor is switched on. For accurate displacement-control servos such on-off motors are seldom desirable, and a smoothly variable type of control is necessary. In the analysis of such systems, it is generally assumed that either the output torque or the shaft speed of the motor is approximately proportional to the magnitude of the input voltage or current. Whilst it is true that many types of motor do behave in one of these ways, in general the relation between output and input is much more complicated than this. It will, however, be shown how a DC motor, by means of a separate feedback circuit, can be made to conform very approximately to the Velocity Control requirement that speed of the output shaft is proportional to the control voltage, thereby simplifying design. Where it is the output torque, rather than the output speed, which is dependent on the magnitude of the error voltage or current, the term Torque Control is applied.

DC Motors

There are two methods of controlling a DC motor with a separately excited field winding. The control voltage may be applied either to the armature or to the field winding. The advantage of the former is that when there is no output required i.e., when the motor is at rest, no power is dissipated in the armature circuit since no current flows there. The disadvantage lies in providing a supply for the armature current with a sufficiently low output impedance. For high-powered motors gas-filled control valves of the thyratron type are frequently used, and these have other drawbacks (see Chap. 6) apart from their comparative fragility. Other types of current amplifiers of a non-electronic nature are sometimes used. One of these, used in the Ward-Leonard system, is described below.

One method of controlling a DC motor by means of the field winding employs a reactor of large inductance in series with a bridge rectifier circuit feeding the armature winding. While the armature is stationary no armature reaction EMF is generated, and the AC armature supply circuit is almost purely reactive. When an error voltage develops a field, the armature accelerates in the direction corresponding to the polarity of the error, and power is drawn from the armature supply due to the back EMF, which makes the armature circuit partly resistive.

In general, series or shunt field arrangements are not suitable for use in servo motors, since a reversal of the armature voltage does not result in a reversal of the output torque. A differential-field series motor may be used, with opposing field-windings switched either mechanically or electronically by the error signal.

Two-Phase Induction Motor

Many types of this motor (using cup-shaped metal rotors, or squirrel-cage or wound armatures) are in common use. The method of control normally used is to apply a constant alternating voltage to one stator field-winding and the error voltage, in quadrature with the constant voltage, to the other winding. The motor will then accelerate in one direction or the other according to whether the error voltage leads or lags in quadrature with the constant voltage. Also, since the torque is proportional to the currents induced in the armature, which in turn depend on the magnitude of the rotating flux, the larger the error voltage the greater is the output torque. A special point in design is that when the error voltage is reduced to zero the motor will hunt if allowed to run as a single phase motor. This may be prevented by a suitable choice of reactance/resistance ratio for the armature winding. When properly designed, the armature acts as an eddy current brake when the error voltage is zero, and this feature makes the two-phase motor particularly adaptable as a servo motor.

Induction motors used in this way are not economical except at low powers, and it is in small servos that they are usually to be found.

Ward-Leonard System

This high-powered type of control gear consists of a three-element chain of prime-mover, DC generator and variable speed motor. The arrangement is illustrated in Fig. 931. The first two elements act as a current amplifier, since the power supplied to the variable speed motor depends on the current in the exciting field-winding. Various modifications exist which affect the behaviour of the system, particularly in regard to the field winding of the generator.

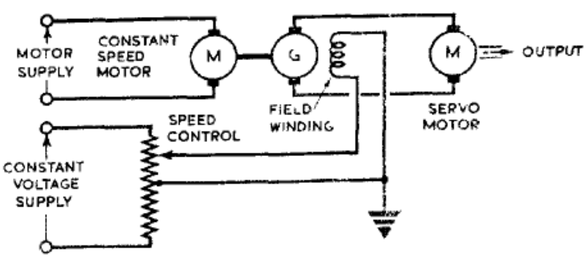


Fig. 931 - Ward-Leonard system.

Metadyne System

The metadyne is a DC generator used as in the Ward-Leonard system for driving a DC servo motor. The characteristics of the whole system depend largely on the amplification of the generator and its rapidity of response to changes in the controlling field-current. By suitable design the metadyne can be made to produce very considerable power amplification, of the order 30,000 : 1, but the same elements of design which provide high amplification limit the rapidity of response. When the metadyne is designed to give maximum power amplification it is known as an Amplidyne Generator.

One of the characteristics of a Ward Leonard system using a metadyne is that a braking torque may be automatically developed when the speed of the output shaft of the motor load exceeds the speed demanded by the magnitude of the error. This is a useful anti-hunt feature, since the brake is applied in this case before the output overshoots, i.e. before the error reaches zero.

Speed-Control System

Fig. 932 illustrates a method whereby a DC motor can be made to conform approximately to the speed-control type; i.e. its output shaft speed is proportional to the magnitude of the error voltage irrespective of the output torque, within the limits of the linearity of the motor characteristics.

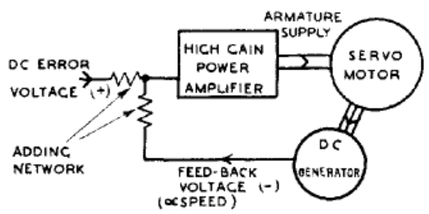


Fig. 932 - Speed control system.

A small DC generator is attached, or geared, to the output shaft. The output voltage from this generator is proportional to shaft speed within a close degree of approximation. This voltage is subtracted from the error voltage in the amplifier input circuit and the difference is used to control the DC motor. The change in torque thereby produced at the output shaft is very much more rapid than the corresponding change in error voltage, and, provided the motor does not overload, the difference between the error and feedback voltages is kept very small. Hence the error, as well as the feedback, is very nearly proportional to the speed of the output shaft.

17. Performance of Servo Systems

No standard technique has yet been adopted for measuring or estimating the complete performance of a servo system, but various tests which can be applied will be dealt with in this section. Possibly the simplest complete test which can be devised is the frequency response. A simple harmonic input is applied, and the amplitude and phase of the output are determined. The phase difference and relative amplitude for different frequencies may then be plotted on a harmonic response diagram as shown in Fig. 933. Because some elements of the preceding or succeeding stage of radar control systems may exhibit pronounced resonant properties it is frequently important not only to avoid resonances, but to introduce deliberately a small response in the servo at the resonant frequencies of the external circuits. It may be sufficient if the response to frequencies higher than one or two c/s falls off very rapidly with rising frequency, but this also tends to make for sluggish response to sudden changes of input. Where the input quantity to a servo is a received radar signal, which is prone to certain types of unwanted fluctuations, a special frequency response may be required.

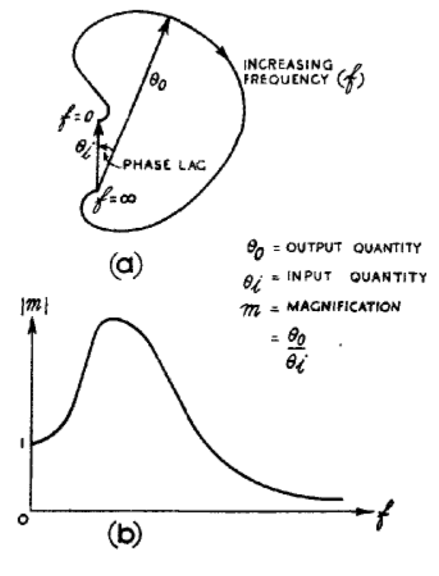


Fig. 933 - Typical harmonic response diagrams.

A second test which may be applied to a servo is its response to a sudden change of input quantity; (Unit Function). If the input quantity is altered suddenly by unit amount from the equilibrium position the manner in which the output approaches its new value is called the Unit Function Response. Complete information as to the performance of the servo to any given input can be derived from this test; (provided the servo is a linear system). In particular the rapidity of pull-in and whether it is oscillatory or non-oscillatory, and of what degree, are of considerable importance in most radar applications.

Other tests aim at subjecting the servo to such input conditions as are more directly related to the actual conditions likely to be met with in use. There is usually a maximum input velocity which can be expected and at this velocity of input there must not be introduced a velocity lag greater than the permissible error. Similarly, the acceleration and higher order lags must not exceed stipulated values, and tests may be applied to confirm this. There is liable to be a standstill error if the system is insufficiently stiff. If the system is too resilient, i.e., not stiff enough, extraneous torques due to windage, stiction etc. may introduce excessive errors.

18. Definitions and Assumptions (Applying to a displacement-displacement servo).

The input, output and error quantities are denoted by Θ_i, Θ_o, and E respectively, so that $E = \Theta_i - \Theta_o$. These are converted by gearing into proportionate quantities of the same kind - i.e., an angular rotation - so that the new quantities, θ_i, θ_o and ε are connected with the old by the relations

$$\frac{\theta_i}{\Theta_i} = \frac{\theta_o}{\Theta_o} = \frac{\varepsilon}{E} = c \text{ where c is the gear ratio.}$$

(See Fig. 934).

The input and resetting elements convert Θ_i and Θ_o into voltages or currents which are combined in the difference element, the difference being amplified and applied, either directly or through some modifying circuit, to the servo motor. The input to the motor is denoted by $\alpha = f(\theta_i, \theta_o)$ and it will be assumed that the output torque of the motor is T_o where $T_o = k_o\alpha$

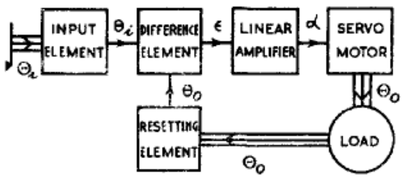


Fig. 934 - Simple error control.

The gear ratio c is often sufficiently high to ensure that the mechanical properties of the load may be neglected in comparison with those of the gearing and motor shaft. If this is not so the moments of inertia and coefficients of friction for the load may be referred to the output shaft of the motor according to the gear ratio and added to the mechanical properties of the motor.

The resultant coefficient of viscous friction will be denoted by k and the moment of inertia by J. Coulomb friction and stiction, time-lags and backlash will be neglected. For some purposes we shall assume that the output shaft is subjected to an extraneous torque T_x, due, for example to windage or stiction.

19. Simple Error Control

In this case, illustrated by the circuit of Fig. 934, the input current or voltage applied to the motor is proportional to the error E.

i.e. $\alpha = \gamma\varepsilon$, where γ is a constant, and

$$T_o = k_o\alpha = k_o\gamma\varepsilon = k_o\gamma c E .$$

k_o is a constant for a given motor, but γ depends on the amplification provided between the difference element and the motor.

The net torque produced at the rotor shaft is therefore $k_o \gamma \mathcal{E} + T_x - \frac{k d\theta_o}{dt}$, and this is equal to the rate of change of angular momentum, $J \frac{d^2\theta_o}{dt^2}$

i.e., $k_o \gamma \mathcal{E} + T_x - k \frac{d\theta_o}{dt} = J \frac{d^2\theta_o}{dt^2}$

i.e., $k_o \gamma (\theta_i - \theta_o) + T_x = J \frac{d^2\theta_o}{dt^2} + k \frac{d\theta_o}{dt}$.

Hence $J \frac{d^2\theta_o}{dt^2} + k \frac{d\theta_o}{dt} + k_o \gamma \theta_o = k_o \gamma \theta_i + T_x$ .... (1)

Putting $\theta_o = \theta_i - \mathcal{E}$, and rearranging the terms, we obtain

$$J \frac{d^2 \mathcal{E}}{dt^2} + k \frac{d\mathcal{E}}{dt} + k_o \gamma \mathcal{E} = J \frac{d^2\theta_i}{dt^2} + k \frac{d\theta_i}{dt} - T_x \text{ ....(2)}$$

These two linear differential equations form the basis of the analysis of the system.

Steady State Errors

We shall assume for the moment that a steady state can exist in each of the forms which we are about to consider. This assumption presupposes the stability of the system, which will be discussed later. By steady state we mean here a condition in which the input obeys a simple law, such as a constant velocity, or constant acceleration, while the output follows a related law with an angular lag corresponding to the magnitude of the input. We shall consider the three conditions, θ_i = constant, $\frac{d\theta_i}{dt}$ = constant, and $\frac{d^2\theta_i}{dt^2}$ = constant.

Stiffness

If θ_i and θ_o are both constant, equation (2) gives

$$\mathcal{E} = \frac{-T_x}{k_o \gamma} \qquad \text{or } k_o \gamma = -\frac{T_x}{\mathcal{E}} .$$

$k_o \gamma$ is called the Stiffness Coefficient of the servo, since it determines the magnitude of the extraneous torque which must be applied to the motor shaft to produce a given error $\mathcal{E}$. The corresponding coefficient for the load, as distinct from the motor shaft, is $c^2 k_o \gamma$, since the torque required is increased, and the angular error decreased, in the ratio c:1.

Velocity Lag

If the input velocity is constant, in the steady state

$$\frac{d\theta_i}{dt} = \frac{d\theta_o}{dt} = A, \quad \text{say;}$$

then $k_o \gamma \varepsilon = kA - T_x$, from (2);

i.e. $\varepsilon = \frac{kA}{k_o \gamma} - \frac{T_x}{k_o \gamma}$.

$\frac{k}{k_o \gamma}$ is called the velocity lag coefficient; (i.e., it is the lag per unit constant velocity).

Acceleration Lag

If $\frac{d^2\theta_i}{dt^2} = B$

$\frac{d\theta_i}{dt} = Bt + A = \Omega$, say.

It can be shown from equation (2) that

$\varepsilon = \left(\frac{J}{k_o \gamma} - \frac{k^2}{k_o^2 \gamma^2}\right) B + \frac{k}{k_o \gamma}\Omega - \frac{T_x}{k_o \gamma}$ in the steady state.

$\frac{J}{k_o \gamma} - \frac{k^2}{k_o^2 \gamma^2}$ is called the acceleration lag coefficient.

(It is immaterial whether velocity or acceleration lag coefficients are referred to the motor output shaft or to the load itself, since both error and velocity or acceleration are similarly changed by the gear ratio. Also the same figure is obtained whether the lag is quoted in radians per radian-per-second or in degrees per degree-per-second).

Using the notation of a subsequent paragraph, we may write the acceleration lag coefficient as $\frac{J}{\gamma k_o}(1 - 4\zeta^2)$, where $\zeta = \frac{k}{2\sqrt{k_o \gamma J}}$.

In a manner similar to the above, lag coefficients of higher orders may be obtained, if required. In practical applications however, it is usually the lower order lags which predominate, and if these are kept within the required minimum the total errors remain sufficiently small.

Response to Unit Function Input ($T_x = 0$)

If θ_i follows the variation (a) shown in Fig. 935 the response of θ_o may be oscillatory or non-oscillatory as shown at (b) according to the ratios of the coefficients of the characteristic equation obtained by writing D for $\frac{d}{dt}$ in equation (1) and equating to zero the coefficient of θ_o.

This gives $JD^2 + kD + k_o \gamma = 0$.

This may be written $D^2 + 2\zeta\omega_n D - \omega_n^2 = 0$,

where $\omega_n = \sqrt{\frac{k_o\gamma}{J}}$, and is called the undamped natural angular frequency;

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_o\gamma}{J}}$$ is the undamped natural frequency;

$$\zeta = \frac{k}{2J\omega_n} = \frac{k}{2\sqrt{k_o J\gamma}}$$ and is called the Damping Ratio.

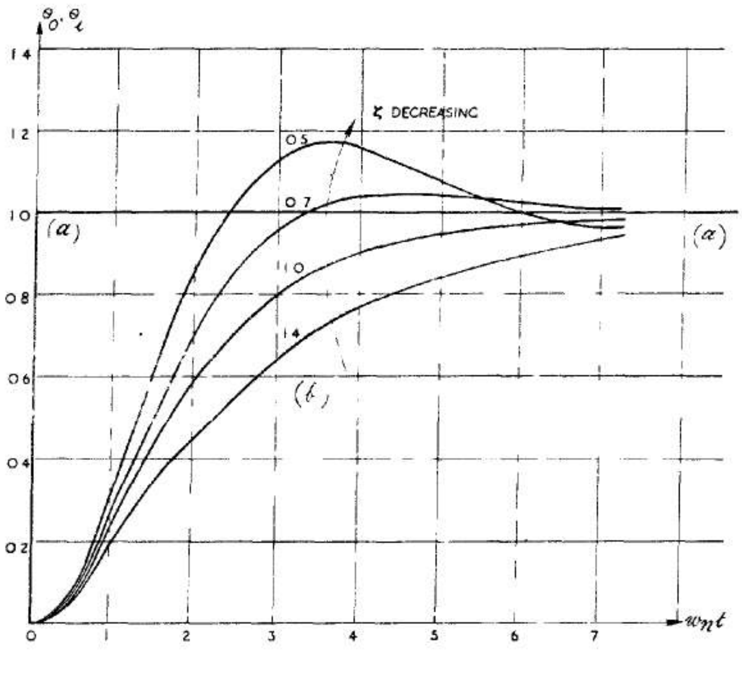


Fig. 935 - Response of simple error control servo to unit-function input.

Alternatively, $\zeta\omega_n$ may be used as a parameter, called the Damping Factor. The system is undamped if $\zeta = 0$, so that continuous oscillations are present. If $\zeta < 0$ the system is unstable (impossible in this case since neither k nor J can be negative). If $\zeta = 1$ the response is critically damped, whereas for $\zeta > 1$ it is non-oscillatory. (Compare Chap. 2 Sec.10).

Normally it is important that a markedly oscillatory response should be avoided, and in practice a value for ζ of about 0·6 to 0·8 is common. This gives a slightly oscillatory response that pulls in more rapidly than when $\zeta = 1$.

For a given value of ζ, the rapidity of pull-in depends on ω_n. The higher the natural frequency the more rapid is the response.

On the other hand there may be good reasons for keeping ω_n low, particularly because, as already stated, of undesirable resonances in other parts of the control system.

<u>Harmonic Response</u> ($T_x = 0$)

If θ_i is a simple harmonic variation of frequency $f = \frac{\omega}{2\pi}$, we may obtain the response of θ_o from equation (1) using the relations

$$\frac{d\theta_i}{dt} = j\omega\theta_i \text{ and } \frac{d^2\theta_i}{dt^2} = -\omega^2\theta_i.$$

$$\text{Hence } m = \frac{\theta_o}{\theta_i} = \frac{k_o\gamma}{k_o\gamma + jk\omega - J\omega^2} = \frac{1}{1 - \frac{J}{k_o\gamma}\omega^2 + j\frac{k}{k_o\gamma}\omega}$$

$$= \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + 2j\varsigma\frac{\omega}{\omega_n}}$$

The amplitude of the response is given by

$$|m| = \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\varsigma\left(\frac{\omega}{\omega_n}\right)^2}} \quad \dots\dots\dots\dots\dots (3)$$

Curves are plotted in Fig. 936 for different values of ς, showing the variation of $|m|$ with x, where $x = \frac{\omega}{\omega_n}$.

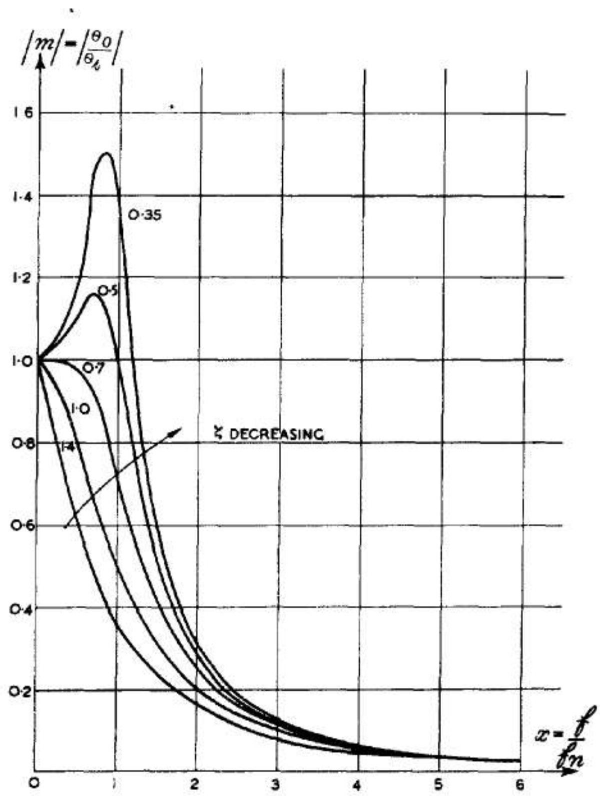


Fig. 936 - Harmonic response of simple error control servo.

20. Limitations of Simple Error Control

In this form of control system there are not normally sufficient variables to ensure that all the requirements are fulfilled. For a given motor and load k, k_o and J are fixed (apart from minor changes which depend on the gearing) so that c and γ are the only variable parameters. It is normal to choose c so that the motor can handle the maximum speed of rotation (slewing speed) without overloading during periods of high acceleration. The other requirements all involve γ and are frequently mutually conflicting. How this works out in practice is described in the following example, taken from a radar servo providing auto-follow in bearing.

The servo motor is a $\frac{1}{2}$ HP motor with a maximum speed of 2500 r.p.m.

The maximum speed required is 10°/sec. at the load. Hence c is made equal to

$$\frac{2\pi \cdot 2500}{60} \div \frac{10.2\pi}{360} = 1500.$$

At this speed the output torque is given by

$$T_o = \frac{1}{2} \cdot 550 \div \frac{2500.\ 2\pi}{60} = \frac{33}{10\pi} \text{ lb.ft.}$$

Hence k, the coefficient of viscous friction must not be greater than

$$\frac{33}{10\pi} \div \frac{2500.\ 2\pi}{60} = \frac{99}{2500\,\pi^2}$$

$$\doteqdot 0\cdot004 \text{ lbs.ft/rad/sec.}$$

The inertia of the rotor is 0·003 slug ft. (1 slug $\doteqdot$ 32 lbs. mass).

The maximum permissible error is $\frac{1}{3}$ of a degree at the maximum speed, so that the velocity lag coefficient must not be greater than

$$\frac{1}{3} \div 10 \quad {}^\circ/{}^\circ/\text{sec.}$$

Hence, using the formula of Sec. 19, we obtain

$$\gamma k_o > 30k\ .$$

For optimum damping take $\zeta = 0\cdot6$, so that

$$\frac{k}{2} = 0\cdot6\sqrt{k_o\,\gamma\,J}\ .$$

Hence $\gamma k_o = \dfrac{k^2}{1\cdot44\ J}$.

Combining these last two results, we have

$$\frac{k^2}{1\cdot44J} > 30k$$

i.e. $k > 30.\ 1\cdot44J$.

Putting J = 0·003, this becomes

$$k > 0\cdot1296.$$

This conclusion contradicts the earlier one that k must be less than 0·004.

The damping requirement could not be satisfied unless the $\frac{1}{2}$ HP motor were replaced by a much larger one, about 14 HP being required.

Further limitations arise since the velocity lag requirements conflict with the desirability of keeping f_n small.

Since $\gamma k_o > 30k$, it follows that $f_n > \frac{1}{2\pi}\sqrt{\frac{30k}{J}}$

and taking k as 0·004 this makes $f_n > 0\cdot9$. (Taking k as 0·1296 makes matters far worse).

21. The Use of Derivative and Integrated Error Control

To overcome the limitations of simple error control additional terms proportional to various derivatives or time-integrated functions of the error or of the input and output quantities are included in the quantity controlling the motor. These functions may be obtained before or after the difference element; for instance, instead of θ_i being fed to the difference element directly, it may pass through various devices so that the difference element input is

$$\theta_i + A\frac{d\theta_i}{dt} + B\int \theta_i \, dt,$$

or some similar function. We shall use the notation $f(D)$ to indicate these additional terms, where $D = \frac{d}{dt}$; $f(D)$ may include both differential and integral forms.

The inputs to the difference element then become $\theta_i + f_i(D)\theta_i$ and $\theta_o + f_o(D)\theta_o$, so that $T_o \propto \left\{\theta_i + f(D)\theta_i - \theta_o - f_o(D)\theta_o\right\}$.

Fig. 937 illustrates the method.

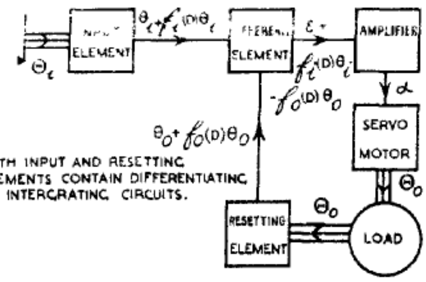


Fig. 937 - Use of derivative or integrated error control.

(The output from the difference element may also be subjected to further differentiation or integration, so that

$$T_o \propto \left\{ 1 + f_\varepsilon D \right\} \left\{ \theta_i + f_i(D)\, \theta_i - \theta_o - f_o(D)\theta_o \right\}$$

but for simplicity we have assumed that $f_\varepsilon(D) \equiv 0$.)

The differential equation of the system then becomes

$$k_o\gamma\left\{ \theta_i - \theta_o + f_i(D)\,\theta_i - f_o(D)\theta_o\right\} + T_x = kD\,\theta_o + J\,D^2\,\theta_o \equiv f_\ell(D)\theta_o$$

if we apply the same notation to the load function.

Hence we obtain,

$$\left\{ k_o\,\gamma(1 + f_o(D)) + f_\ell(D)\right\}\theta_o = k_o\gamma\left\{ 1 + f_i(D)\right\}\theta_i + T_x \quad \ldots\ldots\ldots (4)$$

and

$$\left\{ k_o\,\gamma(1 + f_o(D)) + f_\ell(D)\right\}\varepsilon = k_o\,\gamma\left\{ f_o(D) - f_i(D) + f_\ell(D)\right\}\theta_i - T_x \quad \ldots (5)$$

By a suitable choice of f_o (D) the characteristic equation, obtained by equating to zero the coefficient of θ_o in equation (4), may be adjusted in any desired manner without affecting the mechanical constants k and J. When this has been done the function $f_i(D)$ may be chosen to bring velocity and acceleration lags down to the required minima.

It may be useful to regard the coefficient of D in $k_o \gamma f_o(D)$ as constituting "artificial viscosity", since it becomes added to k in the coefficient of θ_o. This coefficient of D may be either positive or negative, so that the affect of "Negative viscosity" can be obtained. Similarly the coefficient of D^2 in $k_o \gamma f_o(D)$ may be regarded as "artificial inertia". By the introduction of these and similar quantities the various requirements which were found to be mutually incompatible in the single system of Sec. 19 may frequently be satisfied.

There are practical reasons for avoiding the use of differentiating circuits in the input to the serve. Particularly in radar auto-follow systems the input signals are liable to be jerky, so that differentiation accentuates the irregularities. Either differentiation of the output or integration is generally to be preferred. The latter inevitably involves a time lag but in practice this can often be made negligible.

In the case of Integrated Error Control $f_o(D) \equiv f_i(D) = \frac{a}{D}$, say, where a is a constant.

We obtain from (5)

$$\left\{ k_o \gamma \left(1 + \frac{a}{D}\right) + f_\ell(D) \right\} \mathcal{E} = f_\ell(D)\, \theta_i - T_x .$$

This may be written

$$\left\{ a + k_o \gamma D + D f_\ell(D) \right\} \mathcal{E} = D f_\ell(D)\, \theta - D T_x$$

$$= J \frac{d^3\theta_i}{dt^3} + k \frac{d^2\theta_i}{dt^2} - \frac{dT_x}{dt} .$$

If θ_i and T_x are both constant and the servo is stable, $\mathcal{E}$ is zero in the steady state. This also follows when $\frac{d}{dt}(\theta_i)$ is constant; i.e., the velocity lag is reduced to zero. With this form of control no matter how small the error, the output torque will eventually build up (within the limits of the motor's output power) until the extraneous torque is exceeded and the error reduced.

These effects can be produced by suitable arrangement of electrical integrating or differentiating circuits such as those described in Chap. 2 Sec. 17.

Although such techniques as these allow for great flexibility of servo design, they operate under the assumption that the amplifiers, motors etc., employed in the servo remain linear under the conditions imposed by the choice of circuit components. This assumption is justified provided output torques and speeds remain well below the maximum for the motor, and provided amplifiers are not overloaded. In practice, these assumptions frequently do not apply, particularly when the system is used for slewing (turning at high speed).

22. Speed Control (First Order Servo)

If in equation (3) of Sec. 21 we substitute f_e (D) $\equiv$ aD and f_i(D) $\equiv$ 0, we have

$$k_e \gamma \theta_o + a\, k_e \gamma \frac{d\theta_o}{dt} + k \frac{d\theta_o}{dt} + J \frac{d^2\theta_o}{dt^2} = k_e \gamma \theta_i + T_x.$$

Ignoring T_x and choosing a sufficiently large so that

$$a\, k_e \gamma \frac{d\theta_o}{dt} \gg \frac{k d\theta_o}{dt} + J \frac{d^2\theta_o}{dt^2}, \text{ we have}$$

$$\theta_o + \frac{a d\theta_o}{dt} \doteq \theta_i \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

Substituting $\theta_o = \theta_i - \mathcal{E}$ we have

$$\mathcal{E} + a\frac{d\mathcal{E}}{dt} \doteq a\frac{d\theta_i}{dt} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

This approximation will be justified in the majority of cases, exceptions being for inputs of the unit-function type, which are normally met in initial conditions only.

For such a servo the velocity lag coefficient is a, obtained by putting $\frac{d\mathcal{E}}{dt}$ = 0 in equation (7).

The harmonic response is obtained by putting

$$\frac{d\theta_o}{dt} = j\omega\theta_o \text{ so that } \theta_o = \frac{\theta_i}{1 + aj\omega}.$$

$$\text{Then } |m| = \left|\frac{\theta_o}{\theta_i}\right| = \frac{1}{\sqrt{1 + a^2\omega^2}} \quad \dots\dots\dots\dots (8)$$

It is clear that a rapid falling off in response with increasing frequency is obtained by making a large, i.e. at the expense of velocity lag.

The above equations are the same as those obtained if simple error control is employed with a motor whose speed, independent of output torque, is proportional to the error;

for in this case,

$$\frac{d\,\theta_o}{dt} \propto (\theta_i - \theta_o) = \frac{1}{a}\,(\theta_i - \theta_o), \text{ say,}$$

giving the same result as that of equation (6).

23. Second Order Servo with Zero Velocity Lag

If f_i and f_e in equation (4) are suitably chosen the equation (if T_x is ignored) reduces to :-

$$\frac{d^2\mathcal{E}}{dt^2} + 2\zeta\omega_n \frac{d\mathcal{E}}{dt} + \omega_n^2\mathcal{E} = \frac{d^2\theta_i}{dt^2} \quad .... (9)$$

where ζ and ω_n may be chosen without restriction. Comparing this equation with (2), we see that the term in $\frac{d\theta_i}{dt}$ has been eliminated; this reduces the velocity lag to zero. Other modifications to the response of the servo due to the disappearance of this term may be seen by comparing the response to unit-function input (Fig. 938) and the harmonic response (Fig. 939) with those of the simple error control servo (Fig. 935 and 936). The pull-in is much more rapid and for damping greater than or equal to critical ($\zeta \geq 1$) there is a

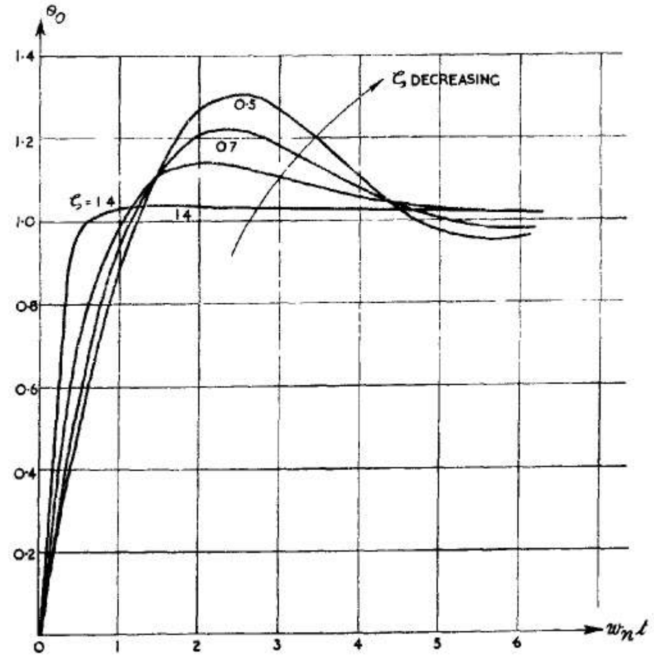


Fig. 938 - Response of second order servo (zero velocity lag) to unit-function input.

single overshoot instead of a gradual rise as in the former case. However, for the same values of ω_n and ζ the rate at which the harmonic response falls off with rising frequency is greatly reduced. The acceleration lag coefficient is given by $\frac{1}{\omega_n^2}$ compared with $\frac{1}{\omega_n^2} \cdot (1-4\zeta^2)$ for the simple error control servo. We shall compare the performance of this second order servo having zero velocity lag with that of a first order servo under the following conditions :-

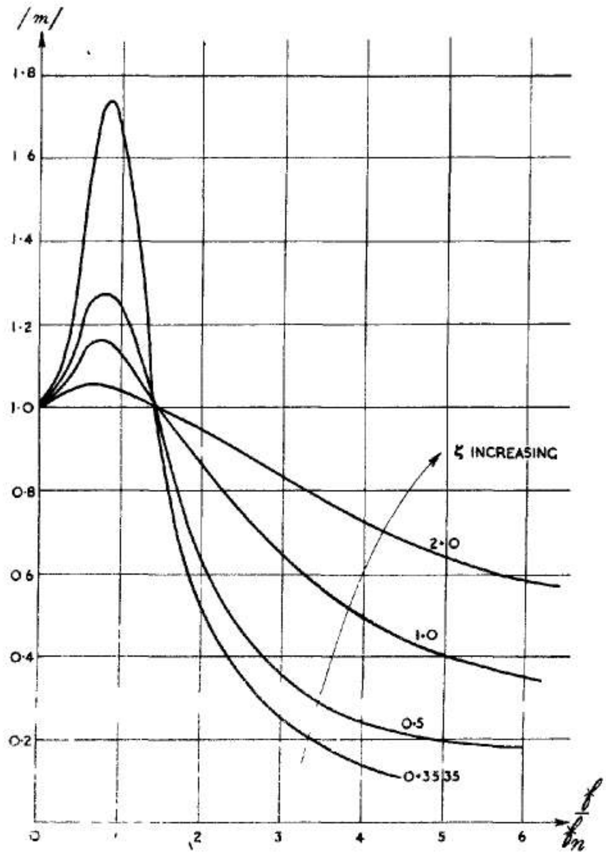


Fig. 939 - Harmonic response of second order servp (zero velocity lag).

(i) Maximum angular velocity of the input = 10°/sec.

(ii) Maximum angular acceleration of the input = $1^{\circ}/\text{sec.}^2$

(iii) At 2 c/s the response must fall to 0·4 of its value at f = 0.

For the first order servo, putting $\omega = 2\pi f = 4\pi$ when $f = 2$ c/s, and $|m| = 0{\cdot}4$ in equation (8), we obtain

$$a = \frac{1}{4\pi}\sqrt{\frac{21}{4}} \doteqdot \frac{1}{12} .$$

Since the velocity lag coefficient is a, the maximum velocity lag is

$$10 \cdot \frac{1}{12} \doteqdot 0{\cdot}8^{\circ}.$$

For this servo acceleration lag may be neglected.

The acceleration lag coefficient for the second order servo is $\frac{1}{\omega_n^2}$; hence, with the same permissible error, we obtain

$$1 \cdot \frac{1}{\omega_n^2} = \frac{10}{12}$$

so that $\omega_n = \sqrt{1 \cdot 2} \doteqdot 1 \cdot 1$.

Hence, at 2 c/s, $\frac{f}{f_n} = \frac{\omega}{\omega_n} = \frac{2 \cdot 2\pi}{1 \cdot 1}$

$\doteqdot 12$.

Fig.939 does not permit of extrapolation to this extent, but calculation gives the result that for $\frac{f}{f_n} \doteqdot 12$ and $|m| = 0 \cdot 4$,

$\zeta \doteqdot 2 \cdot 6$.

Both servos then satisfy the given conditions. They may now be compared for pull-in time in response to unit-function input.

For the first order servo the time which elapses before the error is reduced to 10% of the input is given by

$$t = \frac{a}{0 \cdot 4343}$$

$$= \frac{1}{12.0 \cdot 4343}$$

$\doteqdot 0 \cdot 2$ secs.

For the second order servo the time which elapses before the error is zero (there is a subsequent overshoot) may be shown to be given by

$$\varepsilon^{\left(2\sqrt{\zeta^2 - 1}\right)t} = 2\zeta^2 - 1 + 2\zeta\sqrt{\zeta^2 - 1} \; ; \quad (\text{provided } \zeta > 1).$$

Putting $\zeta = 2 \cdot 6$, we may deduce that

$$\varepsilon^{2.2 \cdot 4t} = 25$$

or $t = \frac{1}{4 \cdot 8} \cdot \log_\varepsilon 25$

$\doteqdot 0 \cdot 67$ secs.

It is possible to reduce the time to the first zero by choosing a smaller value of ζ , and this permits of a larger value of ω_n. If we make $\zeta = 1$ condition (iii) is satisfied provided $\frac{f}{f_n} \geqq 5$, so that $\omega_n \leqq \frac{4\pi}{5}$.

Taking the maximum value of ω_n we substitute this value in the abscissa of the appropriate curve in Fig. 938 at the point where $\theta_e = 1$,

i.e. $\omega_n t = 1$

so that $t = \frac{5}{4\pi} \doteqdot 0{\cdot}4$ secs.

Any further decrease in ζ leads to excessive overshoots so that the time to the first zero error is not an adequate criterion.

In general the second order servo is to be preferred because of the smaller lags which may be obtained for a given frequency band-width, although where rapidity of pull-in is of primary importance the first order servo may be better. However, it is not possible to satisfy condition (iii) with the first order servo and at the same time reduce the angular lag to much less than one degree (in the instance considered), and this limitation may be prohibitive.