

SECTION 6, CHAPTER 1

ELEMENTARY THEORY

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Introduction

1. All matter, whether solid, liquid or gas, is made up of tiny particles called **molecules**. These can be further subdivided into **atoms**. When a molecule consists of atoms of one kind only it is said to be an **element**, but if it contains atoms of different kinds it is termed a **compound**.

Atomic Structure

2. Modern theory indicates that all atoms are built up of infinitely small particles of positive electricity called **protons**, even tinier particles of negative electricity called **electrons**, and particles known as **neutrons** which are electrically neutral. An atom consists of a central nucleus made up of protons and neutrons, and around this nucleus a number of electrons revolve in orbits, in the manner of planets round a central sun (Fig. 1).

3. The number of protons, neutrons and electrons varies according to the type of atom; e.g. the hydrogen atom consists of a nucleus of one proton with one electron revolving round it, the helium atom has a nucleus of two protons and two neutrons with two orbital electrons, while the copper atom consists of a nucleus of 29 protons and 35 neutrons with 29 orbital electrons. In the normal state the number of protons equals the number of electrons, and the atom as a whole is electrically uncharged.

4. The electrons are held in their orbits by the attraction which exists between unlike electrical charges, and they travel in these orbits at extremely high speeds. Thus it becomes apparent that energy is associated with, and locked up in, every atom. A minute proportion of this energy can be released by causing an electron to move from

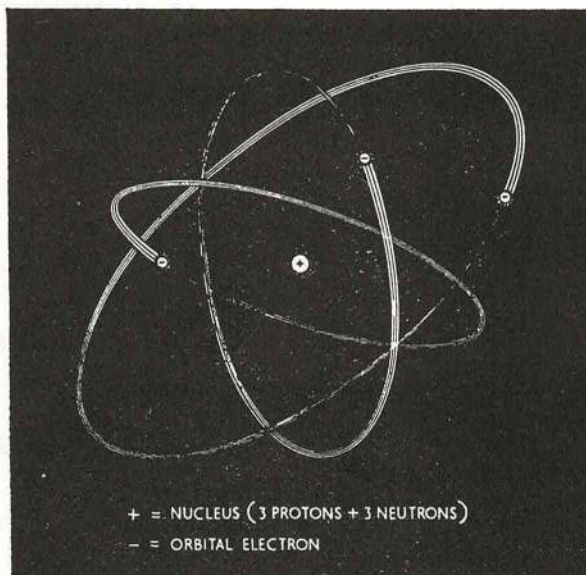


Fig. 1. LITHIUM ATOM.

its orbit in one atom to an orbit in another atom, and this is, in fact, the underlying principle of all electrical power. The statement that electricity is generated simply means that electricity, already in existence, is given motion in a specific direction. The generator does not "make" electricity any more than a pump which is impelling a stream of water through a pipe "makes" the water.

Free Electrons

5. The electron is held to its orbit by the attraction of the positively-charged protons of the nucleus. The force of the attraction will vary with the distance between electron and nucleus, and since the electron's orbit is elliptical in shape it follows that the attraction will not be constant. When the orbit of an electron is very large the attraction at two points in the orbit will be very weak indeed, and the electron is then likely to come under the influence of the nucleus of an adjoining atom. In such circumstances it will attach itself to the new nucleus, usurping the orbit of another electron, which in turn will take up an orbit around a different nucleus. Such electrons, moving in haphazard fashion from atom to atom, are termed "free" electrons. The energy released by their change of orbit is, however, self-cancelling, as the electrons are moving in all directions.

Ionization

6. When "free" electrons exist they can be removed from their own atom and attached to another by the application of force, or energy. The removal or addition of an electron does not alter the nature of the atom, but it alters its state of electrification and generally its chemical properties. An atom which has either a surplus or a deficit of electrons is termed an **ion**, and is said to be ionized.

7. A negative ion is an atom which has acquired an additional electron and has, therefore, an excess of negative over positive charges. The negative ion will repel electrons and will readily part with the excess electron it has acquired in an effort to regain its former status as a neutral atom. Similarly a positive ion, which has been deprived of an electron, will attract electrons in its effort to become once more a neutral atom.

8. A body is said to be charged when some of its atoms have been ionized. When these atoms have a deficit of electrons (i.e., when they are positive ions) the body is said to have a positive charge; a surplus of electrons in the ionized atom constitutes a negative charge. The amount of electrification of the body depends upon the number of ionized atoms. Normally no single atom gains or

loses more than one electron when a body is charged or electrified.

Potential

9. The earth is regarded as being electrically neutral, and thus forms an ideal reference point for judging the state of electrification of other bodies. The energy acquired by a charged body during electrification can be likened to the potential energy that is acquired by a weight when it is lifted and held against the force of gravity. Just as this potential energy is dissipated when the weight regains its original state by falling to the ground, so the electrical energy of the charged body is dissipated when, by being brought into contact with the earth, the state of 'electron unbalance' in the charged body is eliminated by the transfer of electrons between the body and the earth. A charged body is, in consequence, said to have **electrical potential**, more usually abbreviated to 'potential': if the body has a deficiency of electrons its potential is positive; if its electrified state arises from an excess of electrons its potential is negative.

Potential Difference

10. The charged or electrified state is essentially a state of strain, in which the acquired electrical energy is displayed as a force which tends to restore the neutral (uncharged) status of the body by attracting to it charges that are of opposite sign. This force is an **electric force**: it must exist between any two points having potentials of different magnitudes, even if both potentials are of the same sign; it tends to move electrons from the point of more negative (or less positive) potential to the point of less negative (or more positive) potential, and since it is apparent between points of different potential, it is referred to as **potential difference** (p.d.).

Electric Current

11. The "poles" of a battery are bodies which are positively and negatively charged. If these poles are connected by a wire made of a substance which is rich in free electrons (good conductor), these electrons will be attracted by the positive pole and repelled by the negative pole. An orderly "drift" or stream of electrons will thus take place through the wire from the negative to the positive poles of the battery. This stream

of electrons is an **electric current**. The current will continue to flow until either the whole of the positive ions at the positive pole have become neutral atoms once more by attracting electrons, or the negative ions have regained their neutral status by getting rid of all their surplus electrons.

Types of Current

12. There are two main forms of current: **direct current** (d.c.); and **alternating current** (a.c.). Direct current flows steadily in one direction only. Alternating current is a current the direction of flow of which alternates in direction; the time of flow in one direction is known as a half-cycle, and the duration of all half-cycles is the same.

Conductors and Insulators

13. Substances which have large numbers of "free" electrons in their composition will readily permit a flow of electrons to take place under the influence of electrical forces. Such substances are known as **conductors**. Most metals have this property, known as **conductance**, in varying degrees; copper and silver are particularly good conductors.

14. In other substances the electrons rotate around their nuclei in very small orbits, being firmly held by the attraction of their nuclei. Consequently it is very difficult to dislodge these electrons and so set up the flow of electrons which constitutes a current. These substances are termed **insulators**.

15. No rigid line can be drawn between conducting and insulating substances, since every substance will permit the flow of electrons to some extent if sufficient electrical force is applied. In practice, the term "insulator" is applied to substances in which the flow of electrons is so minute, compared with that in a good conductor in similar circumstances, that it can be considered negligible.

Current Effects

16. An electric current has been defined as an orderly stream of electrons, or negative electrical charges, proceeding in a given direction. Such a movement of electrons through a conductor will give rise to certain definite effects, all of which are closely related to the rate of flow of the current.

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These effects are as follows:—

(a) **Heating.** A conductor which is carrying current becomes heated. This property is made use of in lamps, electric fires, etc.

(b) **Magnetic.** A conductor which is carrying current is surrounded by a magnetic field. This property is the basis of all electrical machinery.

(c) **Chemical.** Effects which result from a convection current flowing through an electrolyte may be considered under two headings, viz:—

(i) A deposition effect, by which elements held in solution in the electrolyte are set free or deposited on the surfaces or **electrodes** by which the current enters and leaves the electrolyte. This is the basis of electro-deposition or plating processes.

(ii) A reconstitution effect, in which the current through an electrolyte, on reversal of direction, reverses the deposition effect to restore electrodes and electrolyte to their original condition. This effect is exploited in the electric storage or secondary cell.

Generating Devices

17. An electric current can only flow through a conductor if a p.d. exists between any two points in the conductor, and a continuous flow of current is possible only if the p.d. is maintained continuously. A p.d. between two points, as has already been shown, arises from a preponderance of positive ions (or dearth of electrons) at the high-potential point relative to the low-potential point, and if this preponderance (or dearth) is to be preserved at a constant level then one electron must be withdrawn from the high-potential point for every 'current' electron that reaches it from the conductor. This is the function of a **generating device**; by the expenditure of some non-electrical form of energy the device withdraws electrons from the point of higher potential; in detaching these electrons from their orbits it endows them with energy (i.e. the non-electrical energy supplied to the generating device is converted, either wholly or in part, into electrical energy), and it then transfers them to the low-potential point to maintain the p.d. and thus stabilize current flow.

18. Generating devices take a variety of forms, the two most widely-used forms

being the electric cell, in which chemical energy is utilized for the withdrawal and transfer of electrons from the high-potential to the low-potential point, and the mechanical generator, a term used to describe any device which uses mechanical (kinetic) energy, in association with magnetism, to perform the same function.

The Electric Circuit

19. A conductor (or group of conductors) in which one or more of the various effects of an electric current are exploited for practical purposes is generally referred to as a **load**, and the two extremities of the load, between which the p.d. must be maintained to procure a continuous flow of current, are known as the **poles** of the load, the load extremity towards which electrons move by virtue of the maintained p.d. is termed the **positive pole**, while the extremity where the electrons enter the load from the generating device is the **negative pole**.

20. In the same way the extremities of a generating device, by which electrons enter and leave it, are known as the positive and negative poles, respectively, of the device. Since it is not usually practicable to connect the poles of the load directly to the poles of the generating device, connection between these points is made by conductors whose sole purpose is to provide a pathway offering the minimum of electrical resistance to the passage of the 'current' electrons; in practice, various other devices for the control and, if desired, measurement of current and p.d. between various points may be incorporated in these 'transmission' conductors as required.

21. The complete assembly of generating device, loads and transmission network offers a closed path for the continuous circulation of energy-bearing electrons and is known as an **electric circuit**, although the same term is also used to describe any network of conductors that connect points of different electrical potential.

22. Since it is essential that all electrons emanating from the generating device should be confined to the circuit—any 'leakage' is a loss of energy—all conductors of a circuit must be effectively insulated, to prevent accidental contact with adjoining conducting material. Fig. 2 shows a complete circuit for the provision of light from mechanical (kinetic) energy; a mechanical generator

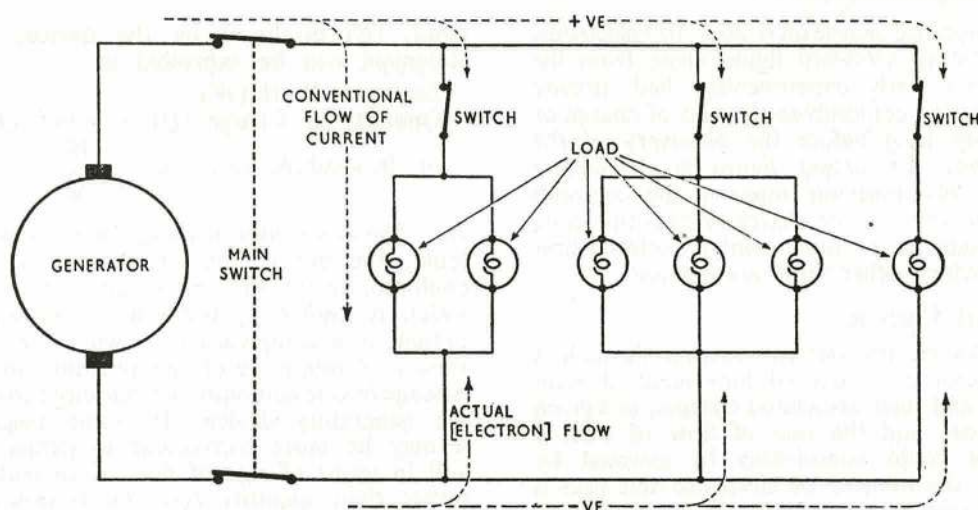


Fig. 2. ELECTRIC CIRCUIT FOR PRACTICAL CONVERSION OF KINETIC ENERGY INTO LIGHT.

converts the kinetic energy of a 'prime mover', such as an internal combustion engine, into electrical energy and thus maintains a p.d. across a group of six filament lamps, where the heating effect of an electric current is used to provide light.

Electromotive Force

23. Provided the generator of the circuit illustrated is operating and the necessary switches are closed to complete the electron-path of the circuit, two energy-conversion processes occur simultaneously—kinetic energy is converted into electrical energy by the generator, and electrical energy is converted into heat, for the most part in the conductors of the lamps where it is usefully employed in producing light, but also to a smaller extent in the other conductors of the circuit, including those of the generator itself. These two processes form the basis of two important electrical concepts: in the first process an electric force is *produced* by the generating device, while in the second an electric force is *applied* between any two points in the circuit. This latter force is already familiar as a p.d.; the force *produced* by the generating device is termed **electromotive force (e.m.f.)**.

24. Looked at in another way, the e.m.f. in a complete circuit is the force that drives (or is available to drive) current through the entire circuit, while the p.d. between two points of a current-carrying circuit—a p.d.

must always exist between the points to cause the flow of current between them—is that fraction of the e.m.f. which drives current through that particular part of the circuit. A special case occurs when, with the generating device operative, the circuit is incomplete; no current can flow in the circuit because the electron-path is not continuous, there is no p.d. between points of the circuit (exterior to the generator) that are in contact with the *same* pole of the generator, but the whole of the e.m.f. appears between any two external points that are in contact with *opposite* poles of the generator. The magnitude of the p.d. between the output terminals of a generator, cell, etc., *on open circuit* is thus the same as the e.m.f. developed by the device.

BASIC ELECTRICAL UNITS

Unit of Quantity

25. An electrified body displays an electric charge because of a surplus or deficiency of electrons, relative to protons, in the body. The charge on every electron is identical in magnitude, and the electron-charge could, in consequence, constitute a unit by which quantities of electricity could be assessed. Unfortunately, the electron charge is of extremely small magnitude and such a unit of quantity could not possibly meet practical requirements. The practical (m.k.s.) unit of charge or quantity of electricity (symbol Q) is the **coulomb**, a unit which is equivalent to

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the combined charge on 6.26×10^{18} electrons. This rather awkward figure arose from the fact that early experimenters had already evolved the coulomb as the unit of charge or quantity long before the discovery of the electron; the actual figure is of almost completely academic interest—the coulomb as a definite unit of charge or quantity in its own right, is the fundamental electrical unit from which other units are derived.

Unit of Current

26. An electric current flowing through a conductor is an overall movement of electrons, and their associated charges, in a given direction, and the rate of flow of such a current could conceivably be assessed by noting the number of electrons that pass a given point in the conductor in one second. A unit based on the individual electron is, of course, extremely small and quite unrealistic for practical purposes of current-flow measurement, just as the individual electron-charge is much too small for use as a unit of quantity. The practical m.k.s. unit of rate of flow of current (symbol I) is called the ampere (usually written as amp. or A) and has been evolved from the coulomb, the m.k.s. unit of quantity: the rate of flow of current through a conductor is one ampere if the quantity of electricity passing a given point in the conductor in one second is one coulomb. This can be shown as

$$\text{Current (I amperes)} = \frac{\text{Quantity or Charge (Q coulombs)}}{\text{Time (t seconds)}}$$

$$\text{or, more simply } I = \frac{Q}{t} \quad (1)$$

The milliamper (written as milliamp. or mA), which is 10^{-3} amp., and the microampere (microamp. or μA), which is 10^{-6} amp., are used to express the values of small currents: for example, 0.013 A is more usually shown as 13 mA, while 0.0000037 A is more conveniently represented by $3.7 \mu\text{A}$.

Units of E.M.F. and P.D.

27. In order that a current may flow steadily through a complete circuit (see para. 17) there must be an e.m.f. maintained continuously in the circuit by some form of generating device which, by doing work, converts a non-electrical form of energy into electrical energy. The quantity of energy or work (W) involved is proportional to the quantity of electricity or charge (Q) that passes through the generating device and the

e.m.f. (E) produced by the device; this statement can be expressed as

$$\begin{aligned} \text{Energy or Work (W)} &= \\ \text{Quantity or Charge (Q)} \times \text{e.m.f. (E)} \\ \text{or, in symbols only } E &= \frac{W}{Q} \quad (2) \end{aligned}$$

28. The m.k.s. unit of energy or work is the joule; the m.k.s. unit of quantity is the coulomb, hence the m.k.s. unit of e.m.f., which is known as the volt, is the e.m.f. present in a complete circuit when the conversion of one joule of energy results in the passage of one coulomb of electricity through the generating device. In some respects it may be more convenient to define the volt in terms of rate of flow of current (I) rather than quantity (Q): by transposing equation (1) it will be seen that

$$\begin{aligned} Q &= It, \\ \text{hence equation (2) can be expressed as} \\ E &= \frac{W}{It} \quad (3) \end{aligned}$$

29. The fact that a current flows through any conductor forming part of a complete circuit implies that a p.d. (V) exists between any two points in the conductor, and this p.d. is, of course, a fraction of the e.m.f. (E) that exists in the circuit as a whole. The m.k.s. unit of p.d. is, in consequence, the same as that of e.m.f., i.e. the volt: a p.d. of one volt is present between two points on a circuit if the amount of electrical energy that is converted into some non-electrical form between these points is one joule for each coulomb of electricity that passes. Alternatively if a current of one ampere flowing for one second between two points provides one joule of electrical energy for conversion into non-electrical energy, then the p.d. between the two points is one volt. These alternative statements can be expressed as

$$V = \frac{W}{Q} = \frac{W}{It} \quad (4)$$

Note. The International Volt, which has a status comparable to that of the International Ampere, is derived from the e.m.f. produced by the Standard Weston or cadmium cell—the p.d. across the terminals of a Weston cell on open circuit is, for all practical purposes, completely constant, and the International Volt is defined as 0.98169 of the e.m.f. of a Weston electric cell at 20°C .

Unit of Energy

30. Energy is the ability or capacity to do work, and the m.k.s. unit of energy and work is the joule—this holds good for all forms of energy, both electrical and non-electrical. The work (W) that is done in an electric circuit between points having a difference of potential of V volts when a charge of Q coulombs is moved between these points can be stated (by transposing equation 4) as

$$W = V \times Q \text{ (joules)} \quad (5)$$

Thus one joule of electrical energy is expended (converted into another form) when one coulomb of charge moves through a p.d. of one volt. Similarly, the work done when a current of I amperes flows between the two points for t seconds is

$$W = V \times I \times t \text{ (joules)} \quad (6)$$

hence the joule, as an electrical unit, can also be defined as the energy that is expended when a current of one ampere flows for one second between points that differ in potential by one volt.

Unit of Power

31. Power is the rate at which work is done, and the electrical power in a circuit is the rate at which electrical energy in the circuit is being converted into another form. The m.k.s. unit of electrical power, like that of any other form of power, is the watt—this is the rate of working when one joule of electrical energy is expended (converted) in one second. The power (P) in a circuit in which W joules of energy are converted in t seconds is thus

$$P = \frac{W}{t} \text{ (watts)} \quad (7)$$

and since $W = VIt$ joules (equation 6)

$$P = \frac{VIt}{t} = VI \text{ (watts)} \quad (8)$$

Unit of Resistance

32. The fact that a current heats a conductor when passing along it is due to an attribute of the conductor called its resistance—resistance can therefore be regarded as an agency that is responsible for the conversion of electrical energy conveyed by a current under the influence of a p.d. into heat. The m.k.s. unit of current is the ampere, while heat, being a form of energy, can be expressed in joules; the m.k.s. unit of

resistance which is called the ohm (symbol Ω) is the resistance of a conductor in which a constant current of one ampere generates heat at the rate of one joule per second, i.e. at the rate of one watt. One watt represents the rate of energy-conversion in a conductor through which a current of one ampere flows when a p.d. of one volt is maintained between its ends; if the whole of the energy-transformation occurring in the conductor is devoted to the production of heat, then the resistance of such a conductor is one ohm. The ohm, as defined above, is based on the True or Absolute Ampere, and is properly designated as the Absolute Ohm; the International Ohm, which ceased to be the legal standard for resistance measurement in 1948, was equivalent to 1.00048 absolute ohms; it was defined as the resistance, at 0°C., of a column of mercury, of uniform cross-section, 1.063 m. in length and of 0.0144521 kg. mass.

Multiple and Sub-Multiple Units

33. The use of prefixes to denote multiples and sub-multiples of units has already been noted, and the use of the sub-multiple prefixes has been illustrated in the treatment of the ampere. A similar procedure is adopted when dealing with very large or very small voltages, wattages and resistances. For example, 132,000 volts (the standard voltage for the National transmission grid) is more usually spoken of as 132 kilovolts (132 kV), while 1/200th of a volt (0.005 v) is generally described as 5 millivolts (5 mV); 20,000 watts is more concisely represented by 20 kilowatts (20 kW) and 20,000,000 watts by 20 megawatts (20 MW), while 1/5th of a watt (0.2 watt) is expressed as 200 milliwatts (200 mW); 20,000,000 ohms is usually referred to as 20 megohms (20 M Ω), and 1/50,000th of an ohm (0.00002 Ω) as 20 microhms (20 $\mu\Omega$).

34. The position as regards multiples of the standard units of charge or quantity and energy (the coulomb and joule respectively) is rather different, since neither of these names are used for the multiple units. One coulomb of electricity passes a given point in a circuit when a constant current of one ampere flows for one second; the coulomb can, therefore, be re-named 'ampere-second', and 3,600 coulombs can be regarded as one ampere-hour (Ah). This is the practical unit of charge or quantity when

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dealing with such processes as electro-deposition, battery-charging, etc. By the same process of reasoning, one joule (which is the amount of energy expended in a circuit in one second if the power in the circuit is one watt) can be regarded as a 'watt-second'; 3,600 watt-seconds or joules constitute a watt-hour (Wh), 1,000 watt-hours are, of course, one kilowatt-hour (kWh), and a thousand kilowatt-hours are one megawatt-hour (MWh).

Note. *Italic* characters are generally used to indicate *concepts*, such as e.m.f., energy, power, etc., while plain letters are used in the abbreviations for actual units. For example, *W* is used to indicate work and *P* to indicate power, but *W* is the recognised abbreviation for the watt, the unit of power.

RESISTANCE

Introduction

35. The flow of current (*I*) in a conductor is supported by the p.d. (*V*) between its ends and is opposed by the conductor resistance (*R*). These three factors in any conductor or network of conductors are obviously related to each other, and for practical purposes it is preferable to consider the resistance of a conductor as the ratio of the p.d. between its ends to the current it supports. Using the accepted symbols for resistance, p.d. and current, this can be expressed as

$$R \propto \frac{V}{I}$$

Ohm's Law

36 The relationships between currents flowing in metallic conductors and the voltages supporting them were first investigated by Georg Ohm, a German physicist of the early nineteenth century, who published the details of his experiments, together with the conclusions he drew from the results, in 1827. His findings represented one of the most significant steps in the advance of electrical science, and to-day they are summarized in what has become known as Ohm's Law. This states that

"The current through a fixed metallic conductor is proportional to the potential difference between its ends, provided the temperature of the conductor and other conditions remain constant".

The expression $R \propto \frac{V}{I}$, which is applicable

to conductors generally thus becomes $R = \frac{V}{I}$

for metallic conductors, and *R*, for a given conductor, is a CONSTANT so long as conditions affecting the conductor remain constant. If *V* is measured in volts and *I* in amperes, then the constant *R* is the resistance of the conductor in ohms.

Application of Ohm's Law

37. Ohm's Law defines the relationship between the current flowing through a metallic conductor and the voltage which supports the current. The law is equally applicable to the whole or any part of a circuit that consists entirely of metallic conductors, and it is advantageous to remember it as the expression.

$$\frac{V}{I} = R.$$

If *I* represents the current flowing through the conductor (in amperes) and *V* represents the voltage supporting the current (in volts), then *R* represents the resistance of the conductor in ohms. If the values of any two of the quantities *V*, *I* and *R* are known, the value of the third can be calculated from this equation. The equation can, of course, be written in three forms, and it is helpful to remember all three, thus:—

If *V* and *I* are known, to find *R* use $R = \frac{V}{I}$

If *V* and *R* are known, to find *I* use $I = \frac{V}{R}$

If *I* and *R* are known, to find *V* use $V = I \times R$

As a further help, the triangle shown in Fig. 3 (drawn with *V* at the Vertex and *R* on the Right) can be used; if the unknown quantity is hidden with the hand then the appropriate equation for ascertaining the unknown quantity is left visible.

Example 1. What is the resistance of a wire which carries a current of 22 amperes under a p.d. of 110 volts?

$$V = 110\text{V and } I = 22\text{A}$$

$$R = \frac{V}{I} = \frac{110\text{V}}{22\text{A}} = 5\ \Omega$$

Example 2. What p.d. is required to support a current of 1.5 milliamp.

through a resistor of 3 kilohms resistance?

$$I = 0.0015 \text{ A and } R = 3000 \Omega$$

$$V = I \times R = 3000 \Omega \times 0.0015 \text{ A} = 4.5 \text{ V.}$$

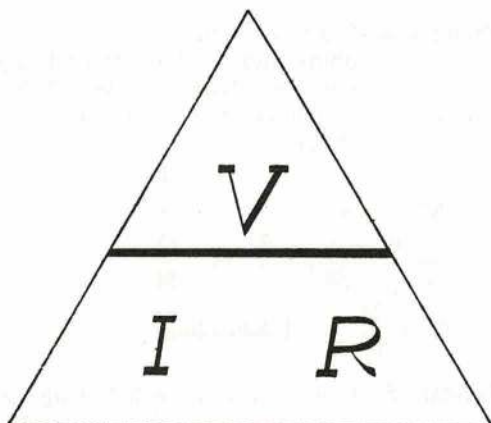


Fig. 3 OHM'S LAW TRIANGLE.

Example 3. What current flows through a wire of resistance 40 ohms when a p.d. of 200 volts is applied to its ends?

$$V = 200 \text{ V and } R = 40 \Omega$$

$$I = \frac{V}{R} = \frac{200 \text{ V}}{40 \Omega} = 5 \text{ A.}$$

Arrangement of Resistances

38. Resistors in Series. Resistors (or conductors) that are arranged 'end-to-end', so that the same current flows through each in succession, are said to be connected 'in series'. The value of the current in such a circuit is, of course, the same at all points, hence the p.d. across each individual resistor is proportional to its resistance. Suppose three resistances r_1 , r_2 and r_3 are connected in series as shown in Fig. 4, and let R be the total resistance between points A and D. Since the same current I flows through each resistance, by Ohm's Law the p.d. (V)

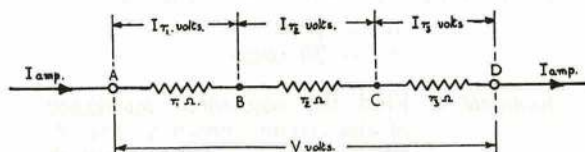


Fig. 4. RESISTANCES IN SERIES.

between A and D is equal to $I \times R$, and the p.d.s. between intermediate points are $I \times r_1$ (A - B); $I \times r_2$ (B - C); and $I \times r_3$ (C - D). From this it will be seen that

$$IR = Ir_1 + Ir_2 + Ir_3.$$

Dividing each term of the above equation by I , which is common to all, we get

$$R = r_1 + r_2 + r_3.$$

The joint resistance of resistors (or conductors) in series is thus the sum of the individual resistances: this arrangement gives maximum resistance and minimum current for a given applied p.d.

39. Resistors in Parallel. When resistors are arranged 'side-by-side' so that there is the same p.d. across each individual resistor, with each affording a separate path for a part of the total current, they are said to be connected 'in parallel'. Consider three resistors, having individual resistances of r_1 , r_2 and r_3 ohms, connected in parallel as shown in Fig. 5: the p.d. between points

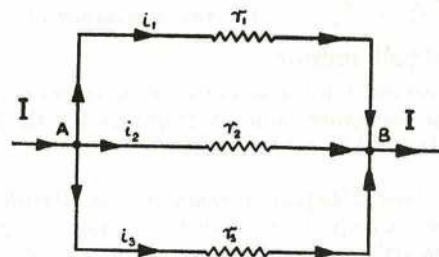


Fig. 5. RESISTANCES IN PARALLEL.

A and B is V volts, and the total current between these points is I amperes. The total current I is obviously the sum of the individual branch currents i_1 , i_2 and i_3 , i.e.

$$I = i_1 + i_2 + i_3.$$

Applying Ohm's Law to the three branches we get

$$i_1 = \frac{V}{r_1}, \quad i_2 = \frac{V}{r_2} \quad \text{and} \quad i_3 = \frac{V}{r_3}$$

If the equivalent resistance between points A and B is represented by R ohms, then

$$I = \frac{V}{R}.$$

But I is also equal to $i_1 + i_2 + i_3$, which in turn can be represented by $\frac{V}{r_1}$, $\frac{V}{r_2}$ and $\frac{V}{r_3}$ respectively, hence we get

$$\frac{V}{R} = \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3} \text{ ohms.}$$

Divide each term of this equation by V :—

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r_2} + \frac{1}{r_3} \text{ ohms.} \quad (1)$$

The reciprocal of the joint resistance of resistors (or conductors) in parallel is thus the sum of the reciprocals of the individual resistances: this arrangement gives minimum resistance and maximum current for a given p.d.—the joint resistance of a parallel group of resistors is always less than the resistance of any one of the individual resistors.

40. Equal Resistances in Parallel. A special case arises when all the individual resistors of a parallel group are of the same resistance value. Suppose three resistors, each having resistance r ohms, are connected in parallel. Equation 1 of the previous paragraph then becomes

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r}$$

and $R = \frac{r}{3}$, i.e. $1/3$ the resistance of one individual resistor.

Hence the joint resistance of n resistors of equal resistance value is equal to $1/n$ th the resistance of one of them.

41. Two Unequal Resistances in Parallel. If two unequal resistances, r_1 and r_2 , are connected in parallel, equation 1 can be developed as follows:—

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2},$$

$$\text{hence } R = \frac{r_1 r_2}{r_1 + r_2} \quad (2)$$

The joint resistance of two unequal resistances in parallel can be found by dividing the product of the two resistances by their sum

Calculation of Values in Multi-Resistor Circuits

42. Ohm's Law and the expressions for equivalent resistance in series and parallel circuits provide the essentials needed for solving most problems concerned with resistance, current and p.d. in circuits that contain two or more individual resistances. Many circuits will be found to consist of parallel groups of resistors in series with one or more other resistors: when dealing with such circuits it is always advisable to construct a circuit diagram from the data given, enter

all the known facts on the diagram, and then proceed to simplify it by determining the equivalent resistances of all parallel groups—in this way the most complex circuits can eventually be resolved into simple series or parallel circuits.

Example 4. Three resistances of 4 ohms, 6 ohms and 8 ohms respectively are connected in parallel. What is the equivalent resistance of the group?

$$\begin{aligned} \frac{1}{R} &= \frac{1}{4} + \frac{1}{6} + \frac{1}{8} \\ &= \frac{6 + 4 + 3}{24} = \frac{13}{24} \\ R &= \frac{24}{13} = 1.846 \text{ ohms.} \end{aligned}$$

Example 5. Four resistors, each having resistance of 3.2 ohms, are connected in parallel. What is the equivalent resistance of the group?

$$R = \frac{r}{n} = \frac{3.2 \text{ ohms}}{4} = 0.8 \text{ ohm.}$$

Example 6. Two resistances of 3.1 ohms and 7.2 ohms respectively are connected in parallel. What is the equivalent resistance?

$$\begin{aligned} R &= \frac{\text{Product}}{\text{Sum}} = \frac{3.1 \times 7.2}{3.1 + 7.2} = \frac{22.32}{10.3} \\ &= 2.16 \text{ ohms.} \end{aligned}$$

Example 7. Three resistors, when connected in parallel, have an equivalent resistance of 5 ohms. Two of the resistors are of 15 ohms and 10 ohms respectively. What is the resistance of the third?

Let the resistance of the third resistor = r ohms.

$$\text{Then } \frac{1}{5} = \frac{1}{15} + \frac{1}{10} + \frac{1}{r} = \frac{2r + 3r + 30}{30r}$$

Multiply each term by $30r$ to simplify:—

$$\begin{aligned} 6r &= 5r + 30 \\ r &= 30 \text{ ohms.} \end{aligned}$$

Example 8. Find the equivalent resistance of the circuit shown at Fig. 6. What current will flow through each resistance if the circuit is connected to a 24-volt supply?

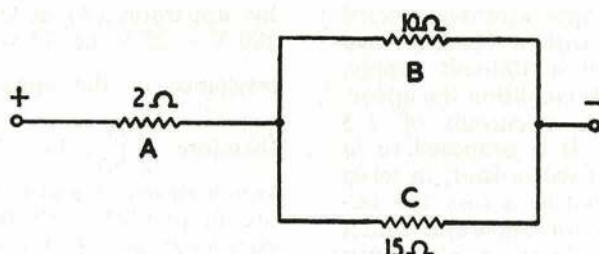


Fig. 6. CIRCUIT 'A'.

Equivalent resistance of B and C

$$= \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ ohms.}$$

Equivalent resistance of A, B and C =

$$2 + 6 \text{ ohms} = 8 \text{ ohms.}$$

Current through complete circuit ($I = \frac{V}{R}$)

$$\text{is thus } \frac{24}{8} = 3 \text{ amperes:}$$

hence current through A (carrying the full circuit current) is also **3 amperes**, the p.d. across A ($V = I \times R$) is thus $3A \times 2 \Omega$, i.e. 6 volts, and the p.d. across B and C in parallel is thus $24 V - 6 V$, i.e. 18 volts.

Current through B (applied voltage 18 V).

$$= \frac{18 V}{10 \Omega} = 1.8 \text{ amperes, and the current}$$

through C (applied voltage also 18 V) =

$$\frac{18 V}{15 \Omega} = 1.2 \text{ amperes.}$$

Example 9. Find the equivalent resistance of the circuit shown at Fig. 7. If the total current taken by this circuit is 5 amperes, what is the supply voltage?

Let the equivalent resistance of B, C and D (in parallel) = R :

$$\begin{aligned} \text{then } \frac{1}{R} &= \frac{1}{4} + \frac{1}{10} + \frac{1}{20} \\ &= \frac{5 + 2 + 1}{20} = \frac{8}{20} \\ R &= \frac{20}{8} = 2.5 \text{ ohms.} \end{aligned}$$

Equivalent resistance of E and F (in parallel)

$$= \frac{3 \times 2}{3 + 2} = \frac{6}{5} = 1.2 \text{ ohms.}$$

Equivalent resistance of B, C, D, E, F and G is thus:—

$$2.5 \text{ ohms (B, C and D)} + 1.2 \text{ ohms (E and F)} + 2.3 \text{ ohms (G)} = 6 \text{ ohms.}$$

Since resistance of A is equal to the equivalent resistance (6 ohms) of the group in parallel with it, the total equivalent resistance of the full circuit is $\frac{6 \text{ ohms}}{2} = 3 \text{ ohms.}$

To sustain a current of 5 amperes through an equivalent resistance of 3 ohms, the applied voltage ($V = I \times R$) required will be:—

$$5A \times 3 \Omega = 15 \text{ volts.}$$

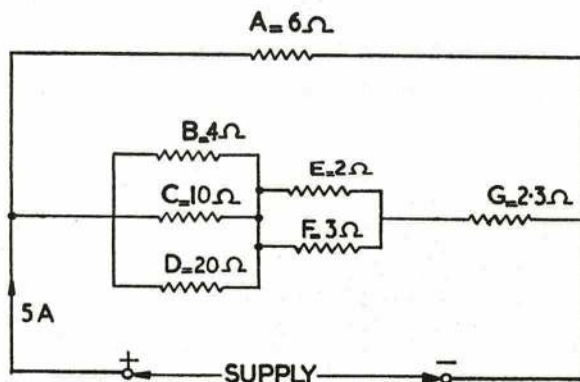


Fig. 7. CIRCUIT 'B'.

Example 10. A certain apparatus is connected in series with a 10-ohm fixed resistor to a 100-volt supply, and in this condition the apparatus takes a current of 2.5 amperes. It is proposed to fit a second fixed resistor, in series with a switch, across the terminals of the apparatus with a view to obtaining alternative currents of 2.5 amperes or 2 amperes as required. Calculate the resistance of the proposed by-pass resistor and state the current this resistor will carry when the switch is closed.

Construct a circuit diagram from the facts given, as shown at Fig. 8.

With the switch open, resistor **C** is not part of the circuit; the apparatus (**A**) and the 10-ohm resistor form a simple series circuit, in which a current of 2.5 amp. flows in all parts. The p.d. across the resistor (**B**) is thus $2.5 \text{ A} \times 10 \Omega$, i.e. 25 volts; the p.d. across

the apparatus (**A**) in this condition is thus $100 \text{ V} - 25 \text{ V}$, i.e. 75 volts, and the effective resistance of the apparatus ($R = \frac{V}{I}$) is

therefore $\frac{75 \text{ V}}{2.5 \text{ A}}$, i.e. 30 ohms. With the

switch closed, **A** and **C** (the by-pass resistor) are in parallel, with **B** in series with this parallel group. In this condition the current through **A** is required to be 2 amperes; the resistance of **A**, as has already been determined, is 30 ohms, hence the p.d. across **A** (and also across **C**) required to sustain this current of 2 amperes through **A** is thus $2 \text{ A} \times 30 \Omega$, i.e. 60 volts. The p.d. across **B** is thus $100 \text{ V} - 60 \text{ V}$, i.e. 40 volts, and the

current through **B** is, in consequence, $\frac{40 \text{ V}}{10 \Omega}$,

i.e. 4 amperes. The current through **C** is the total current less the apparatus current, i.e. $4 \text{ A} - 2 \text{ A}$, or 2 amperes, and since this current is supported by a p.d. of 60 volts, the

resistance of **C** is $\frac{60 \text{ V}}{2 \text{ A}}$, i.e. 30 ohms.

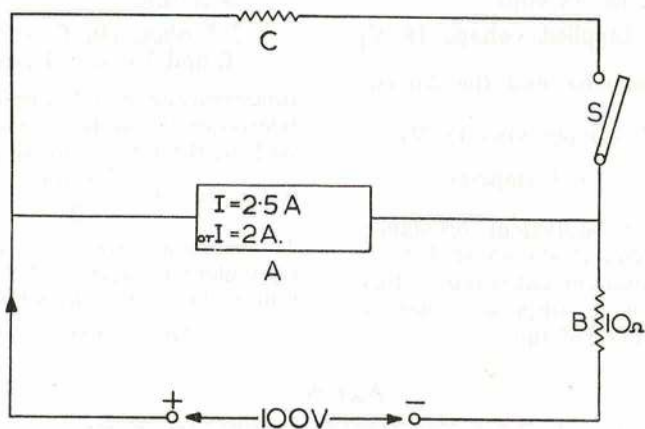


Fig. 8. CIRCUIT 'C'.

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