CHAPTER VI.—THE STRENGTH OF SPARS

1. Introductory.—This chapter describes the methods usually adopted for estimating the strength of metal spars. Special consideration will be given to cases where the applicability of these methods is doubtful.

In this chapter the assumption is made (which is in general approximately true) that the minor principal axis of the spar section lies in the plane of the drag bracing system. Where for convenience of strut fitting the spar section is "skewed" or twisted over (generally to conform to the angle of stagger), the consequent interaction of vertical and horizontal loads and deflections necessitates considerable modifications. Where the angle of "skew" is considerable, special methods of calculation may have to be employed. Such cases should be referred to the Airworthiness Department.

In general a spar will be loaded both laterally and axially, the unit lateral load w being obtained as described in chapter V and the unit end load P from the stress diagram for the truss considered. It is required to find the factor N such that under loads Nw and NP the stress at the weakest point on the spar is equal to the maximum allowable stress.

2. Maximum allowable stress.—The maximum allowable stress should be obtained from an *ad hoc* strength test on a specimen representing a portion of spar concerned. Usually the test specimen is chosen to represent a portion of one of the spar bays of length equal to the length between the points of contraflexure, determined from the spar bending moment diagram. When there are no points of contraflexure the length of the specimen required will not normally exceed fifteen times the maximum depth of the spar. The spar is mounted* as illustrated in fig. 1.

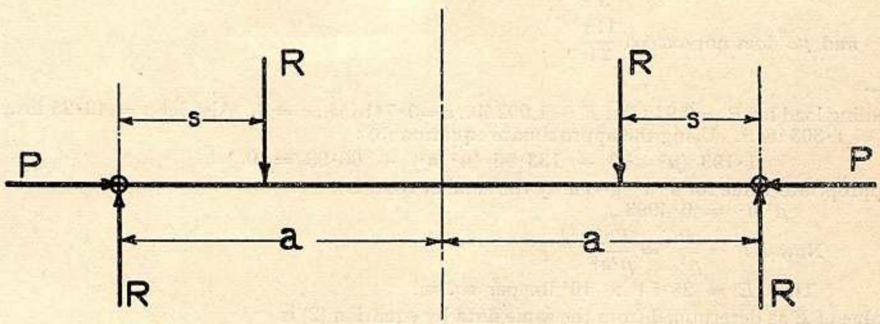

FIG. 1.—CHAP. VI.

A standard form of roller bearing pin end fitting is employed to give zero bending moment at the spar extremities and loading boxes are required at the points of application of the lateral load. The spar is supported in the drag plane at points of rib attachment, one such support being omitted to represent conditions arising when one rib fails or is shot away (chapter III, para. 21). The magnitudes of the couple Rs and the end load P are arranged to give approximately the same ratio of bending moment stress to direct stress as obtains at some critical portion of the spar when built into the aeroplane. The length of the bending arm s is chosen so that the load R gives a shear about equal to the greatest shear over that portion of the spar in the aeroplane represented by the test specimen. If the shear at any other point along the spar is considerably greater than this, and if the spar design is such that its strength in shear is in

* Other methods of tests are acceptable. If it is desired to use any method not hitherto approved prior concurrence should be obtained.

doubt, a second test specimen is sometimes necessary for a special shear test. On test R and P are increased until the spar breaks, the ratio R/P being kept constant.* Then, if R and P are the greatest loads that the spar will carry, the maximum allowable stress is

$$\frac{P}{A} + \frac{R \sin \mu s \sec \mu a}{\mu Z} \quad \ldots \quad (1)$$

where $\mu^2 = P/EI$, and A and Z are the cross section area and modulus of section obtained from the measured dimensions of the test specimen. The value of E used is such that the calculated and measured deflections of the test specimen at mid span relative to the ends of the specimen coincide at three-quarters of the failing load, *i.e.* if d be this measured deflection, then E is chosen to satisfy the equation

$$d = \frac{R \sin \mu s \sec \mu a}{\mu P} - \frac{Rs}{P} \quad \ldots \quad (2)$$

when the values of R and P are three-quarters of the maximum values, and d is positive downwards for the loading system of fig. 1.

If the P's are applied eccentrically at a distance e (measured positive upwards) from the neutral axis then $\frac{Pe\sec\mu a}{Z}$ should be added to expression (1) and e $(\sec\mu a - 1)$ to the right-hand side of equation (2).

A convenient approximation to equation (2) for finding μ^2 when the other quantities are known is

$$\frac{s^3}{6a^2}(\mu^2 a^2)^2 - \left[\left\{5s + (5e + 4d)\,p\right\} - \frac{\pi^2 s^3}{6a^2}\right]\mu^2 a^2 + pd\,\pi^2 = 0 \quad \ldots \quad (3)$$

where $p = \frac{P}{R}$ and the eccentricity e is taken into account.

This approximation is sufficiently accurate if

μs does not exceed $\frac{\pi}{3}$

and μa does not exceed $\frac{11\pi}{24}$

Example :—

At $\frac{3}{4}$ failing load let $P = 9{,}940$ lb., $R = 1{,}092$ lb., $d = 0\cdot741$ ins., $e = 0$. Also let $a = 43\cdot25$ ins., $s = 23\cdot75$ ins., and $I = 1\cdot305$ ins.[4]. Using the approximate equation (3) :

$$1\cdot193\ (\mu^2 a^2)^2 - 133\cdot96\ (\mu^2 a^2) + 66\cdot98 = 0.$$

The appropriate value for $\mu^2 a^2$ (given by the smaller root) is

$$\mu^2 a^2 = 0\cdot4993$$

Now $EI = \frac{P}{\mu^2} = \frac{Pa^2}{\mu^2 a^2}$

Thus $E = 28\cdot54 \times 10^6$ lb. per sq. in.

The value of E as determined from the same data by equation (2) is

$$E = 28\cdot50 \times 10^6 \text{ lb. per sq. in.}$$

The moment of inertia, I, is to correspond to the measured dimensions of the test specimen.

A convenient method of calculating I is as follows. Draw out the section to a conveniently large scale, and mark off the flanges and webs into small elements of equal length (say $0\cdot1$ in. long) as shown in fig. 2.

Let t_1 = thickness of flange.
t_2 = thickness of web.
d_1 = distance of any one element of the flange from the neutral axis of the spar.
d_2 = distance of any one element of the web from the neutral axis of the spar.
n_1 = number of elements in flange.
n_2 = number of elements in web.

Then area $= 0\cdot1(n_1 t_1 + n_2 t_2)$.

$I = 0\cdot1 t_1 \Sigma (d_1^2) + 0\cdot1 t_2 \Sigma (d_2^2)$.

It is advisable to tabulate the calculation.

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* A convenient method of plotting the test results is given in R. & M. 1537.

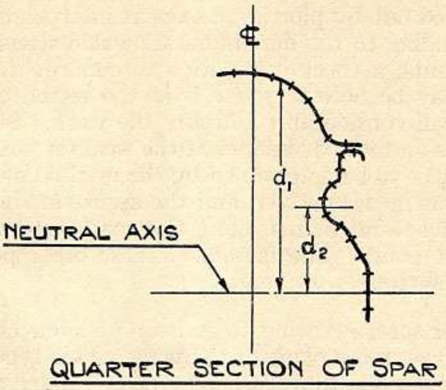

FIG. 2.—CHAP. VI.

Allowance for variation from minimum specification values in the material properties of the test specimen is made as follows. The 0·1 per cent. proof stress is obtained from a control tension test on a specimen of the material of which the spar is made,* and if this is greater than the minimum specification value the spar stress at failure estimated as above is reduced in the ratio

$$\frac{\text{Minimum specification } 0\cdot 1 \text{ per cent. proof stress}}{0\cdot 1 \text{ per cent. proof stress given by control test}}$$

If, however, the 0·1 per cent. proof stress given by the control test is *less* than the minimum specification value, the above correction is not applied. The reason for this is that this method of correction over-estimates the effect of variations in 0·1 per cent. proof stress, and hence would lead to an unsafe estimate of spar failing stress if applied to correct upwards as well as downwards. The question of correcting upwards should not arise, however, as it implies that the material of the test spar should not have been accepted. If the material specification quotes the 0·2 per cent. proof stress instead of the 0·1 per cent. proof stress, the 0·2 per cent. proof stress is to be used in making the correction described above. If ultimate stress only is quoted in the specification the scheduled 0·1 per cent. proof stress, obtained from chapter VIII, section VI, is to be taken. The maximum allowable stress, the value of E determined as above, and the moment of inertia and area corresponding to the minimum scantlings allowed by the drawings, are to be used in the calculations which follow.

3. Calculation of the stresses produced by given applied loads.—The generalised three moment theorem summarised in para. 6 is to be used for calculating the spar bending moments. The minimum realized factor N is ascertained by replacing w and P in equation (1) of para. 6, section A by Nw and NP, or, more generally, by multiplying all terms representing externally applied loads by N, and determining what value of N corresponds to a maximum spar stress equal to the maximum allowable spar stress. A direct solution for N is not readily obtainable. A convenient procedure is to calculate the maximum bending moment, and hence the maximum stress, corresponding to three trial values of N, these values being the specified ultimate factor and factors 10 per cent. greater and 10 per cent. less than this. The correct value of N which produces the maximum allowable stress at the weakest point in the spar can then be obtained

* This control specimen is to be cut from the same flat untreated strip from which the spar specimen was made, and not from the spar itself, (i) when the spar is formed by cold working with no subsequent heat-treatment, and (ii) when any heat-treatment during or subsequent to forming is not in accordance with the conditions for heat-treatment (if any) laid down in the specification. When the heat-treatment during or subsequent to forming is exactly in conformity with that laid down in the material specification, the control specimen is to be cut from the spar itself.

by interpolation, best carried out by plotting, as linear interpolation may be seriously in error. If the value of N corresponding to the maximum allowable stress falls appreciably outside the assumed $\pm$ 10 per cent. limits, a check calculation to confirm the value is desirable. Smaller limits than 10 per cent. may be necessary if α is in the region of $\pi/2$. A tabular method of arranging the calculation will considerably simplify the work. Strictly speaking, if factors are to be quoted at other points on the spar besides at the weakest point, the value of N appropriate to each such point would have to be determined by the method of trial and error just described. The work involved is not justified, however, and the approximate procedure usually adopted is to calculate the stresses at several points along the spar corresponding to the value of N as determined for the weakest point. The factors at these other points are then assumed to be inversely proportional to the stresses so found.

4. Secondary failure of spars.—Owing to end compression both spars of a plane may fail by buckling together sideways in the plane of the wing. This type of failure is called secondary failure and the strength of the spars to resist it may be calculated as follows :—

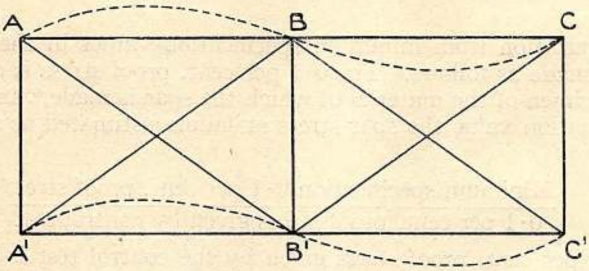

FIG. 3.—CHAP. VI.

Let fig. 3 represent the structure of a main plane where AC, $A'C'$ are the spars and AA', BB', CC' the drag struts. Then the failure in question will occur in the plane $ACC'A'$. Owing to the presence of the ribs (which act as ties between the two spars, and also, owing to the stabilising effect of the fabric, as struts), spars AB, $A'B'$ must deflect together in the same direction and by the same amounts if the lengths of the ribs between the spars are supposed invariable. The same applies to BC and $B'C'$. The only case which need be considered is that in which AB and BC deflect in opposite directions as shown in fig. 3. Hence we may regard AB as a strut pin-jointed at A and B and similarly for BC. It may be assumed that the ribs between the spars introduce no fixing moments. Referring to fig. 4, let

l = length of spar FF' = length of spar RR'.
A_F = cross sectional area of spar FF'.
I_F = moment of inertia of spar FF' for failure in the plane under consideration.
P_F = compressive end load in spar FF' when the wings are carrying unit load.
Q_F = Euler failing load for spar $FF' = \frac{\pi^2 EI_F}{l^2}$.
y_F = deflection at any point on spar FF' distant x from F.
x_L = distance of any rib from FR.
δ_F = equivalent eccentricity of spar FF'.
h_F = distance from neutral axis of spar FF' of most highly stressed fibre.
A_R, I_R, etc. = similar quantities for spar RR'.
L = load in any rib.
p_2 = 0·2 per cent. proof stress.
N = realized factor when failure occurs in one or both spars.

The spars are constrained to deflect equally by the ribs, so

$$y_F = y_R = y \text{ say} \quad \ldots \quad (1)$$

For the spar FF', when deflected under the factored loads

$$EI_F \frac{d^2y}{dx^2} = -NP_F y - NP_F \delta_F - \sum_0^x NL\,(x - x_L) \quad \ldots \quad (2)$$

For spar RR'

$$EI_R \frac{d^2y}{dx^2} = - NP_R y - NP_R \delta_R + \sum_0^x NL (x - x_L) \quad \ldots \quad (3)$$

Adding (2) and (3)

$$E (I_F + I_R) \frac{d^2y}{dx^2} = - N (P_F + P_R) y - NP_F\delta_F - NP_R \delta_R \quad \ldots \quad (4)$$

Generally

$$\delta_F = \delta_R = \delta \text{ say} \quad \ldots \quad (5)$$

If δ_F, δ_R, are unequal then the assumption is made that each is equal to δ where δ is the greater of the two

$$\therefore E (I_F + I_R) \frac{d^2y}{dx^2} = - N (P_F + P_R) y - N (P_F + P_R) \delta \quad \ldots \quad (6)$$

The signs of δ_F, δ_R are ignored so that the case considered will be the most severe for any particular numerical values of δ_F, δ_R.

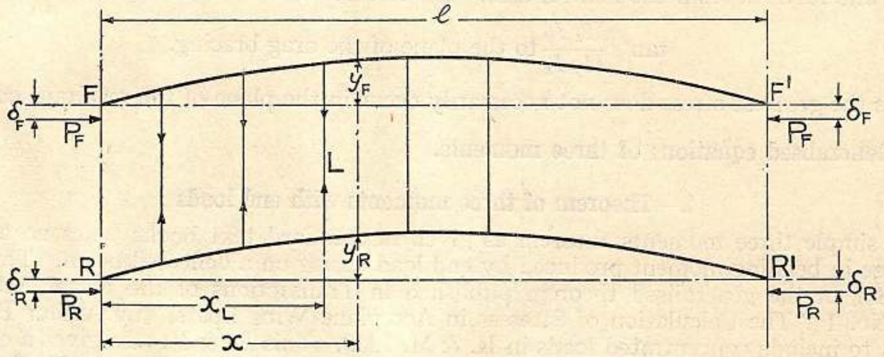

FIG. 4.—CHAP. VI.

Now (6) is the differential equation which would be obtained for a strut with moment of inertia $(I_F + I_R)$ and equivalent eccentricity δ, having a compressive end load $N (P_F + P_R)$.

The solution of the differential equation, when we put $I = I_F + I_R$ and $P = P_F + P_R$ is

$$y = G \sin \mu x + H \cos \mu x - \delta \quad \ldots \quad (7)$$

where G and H are constants to be determined and

$$\mu^2 = \frac{NP}{EI}$$

when $\quad x = 0,\ y = 0$ and so $H = \delta \quad \ldots \quad (8)$

when $\quad x = l,\ y = 0$ and so $0 = G \sin \mu l + \delta \cos \mu l - \delta$

Therefore $\quad G = \delta \dfrac{(1 - \cos \mu l)}{\sin \mu l} \quad \ldots \quad (9)$

By differentiation of (6) above it is clear that $EI \dfrac{d^2y}{dx^2}$ is maximum when $\dfrac{dy}{dx} = 0$.

By symmetry, this is when $x = \dfrac{l}{2}$.

Therefore maximum moment is

$$M_{max.} = - NP\delta \left\{\frac{(1 - \cos \mu l)}{\sin \mu l} \sin \frac{\mu l}{2} + \cos \frac{\mu l}{2}\right\} = - NP\delta \sec \frac{\mu l}{2}$$

$$= - NP \delta \sec \frac{\pi}{2} \sqrt{\frac{NP}{Q_F + Q_R}} \quad \ldots \quad (10)$$

It follows that for the spar FF' the equation

$$p_2 = \frac{NP_F}{A_F} + \frac{N (P_F + P_R)}{I_F + I_R} \delta h_F \sec \frac{\pi}{2} \sqrt{\frac{N (P_F + P_R)}{Q_F + Q_R}} \quad \ldots \quad (11)$$

gives the realized factor N at failure.

If the spar RR' fails first, the appropriate equation is

$$p_2 = \frac{NP_R}{A_R} + \frac{N\ (P_F + P_R)}{I_F + I_R}\ \delta\ h_R\ \sec \frac{\pi}{2}\sqrt{\frac{N\ (P_F + P_R)}{Q_F + Q_R}} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

In practice, all the terms in equations (11) and (12) are known except N. Both equations should be solved by trial and error and the lower of the two values of N obtained is the appropriate realized factor for the particular case of loading under consideration. It is difficult to specify any general value for δ applicable to all types of spars. Unless there are particular reasons for adopting a value greater than $l/600$ this value should be taken. It is clear that this type of failure is resisted to a certain extent by the leading edge to which the ribs are attached and to a still greater extent by the fabric when it is attached to the spars. For this type of failure, therefore, the minimum factor calculated as described above is to be multiplied by an arbitrary correcting factor which may be taken as 1·5 for a fabric covered wing and 3 for a wing with 3-ply or metal covering forward of and firmly attached to the front spar.

5. Unsymmetrical bending.—When it is necessary to consider the stresses due both to a lift bending moment M_y and to a drag moment M_z the greatest stresses will in general occur at the points furthest from the neutral axis. The neutral axis is at an angle :—

$$\tan^{-1}\frac{M_z\,I_y}{M_y\,I_z} \text{ to the plane of the drag bracing.}$$

Thus the greatest stress does not necessarily occur in the plane of the resultant couple.

6. Generalised equations of three moments.

i.—Theorem of three moments with end loads

The simple three moments theorem as given in standard text books takes no account of the change in bending moment produced by end load acting on a deflected beam. This effect is allowed for in the generalised theorem published in Transactions of the Royal Aeronautical Society No. 1 " The Calculation of Stresses in Aeroplane Wing Spars," by Arthur Berry, and extended to include concentrated loads in R. & M. 1233. This R. & M. also gives a convenient polar diagram method of drawing the bending moment diagram. *See* also R. & M. 1507, Appendix III, and the Journal of the Royal Aeronautical Society, June, 1934. The above publications should be referred to for the derivation of the formulae given below.

(i) *Notation and three moment equation.*—Referring to fig. 5 the points of support are denoted by the letters A, B, C, in order, beginning with the support nearest the tip ; the bending moments at these points by M_A, M_B, M_C . . . which are counted positive when they tend to bend the beam concave upwards, thus ⌣ ; the length of the members AB, BC . . . by $2a_1$, $2a_2$ . . . the loadings per inch run (taken positive upwards) by w_1, w_2 . . . that of the overhang being w_0 ; and the compressions (or tensions) by P_1, P_2 . . .

These end loads P_1 and P_2 are obtained by solving the truss shown in fig. 5 after applying the simple three moments theorem without end loads to the top and bottom spars A, B, C,

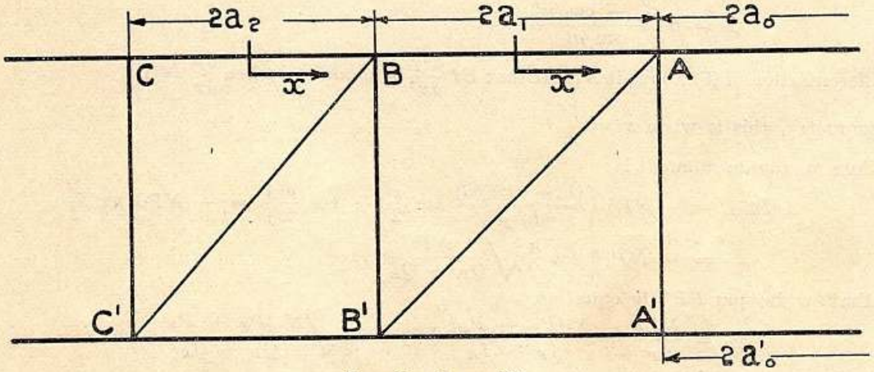

FIG. 5.—CHAP. VI.

and A', B', C' and adding the loads from the drag bracing. This is a first approximation to the end loads in the bays AB, BC and, usually, when there are no offsets, these end loads are sufficiently accurate for a single application only of equation (1) below to be necessary. When the lift wires are offset, however, more than one application of equation (6) below and hence more than one solution of the truss may be necessary for an accurate estimate of the end loads.

The deflection at any point is denoted by y, counted positive when upwards ; the distance of any point in a member from the middle point of the member is denoted by x counted positively in the direction from B to A, so that for the member AB, $x = a_1$ at A, $x = -a_1$ at B.

Then the generalised theorem of three moments for a continuous beam with compressive end load is

$$\frac{a_1}{I_1} M_A f(\alpha_1) + \frac{a_2}{I_2} M_C f(\alpha_2) + 2 M_B \left\{ \frac{a_1}{I_1} \phi(\alpha_1) + \frac{a_2}{I_2} \phi(\alpha_2) \right\}$$
$$= \frac{w_1 a_1^3}{I_1} \psi(\alpha_1) + \frac{w_2 a_2^3}{I_2} \psi(\alpha_2) \quad \ldots \quad (1)$$

where

$$\mu_1 = \sqrt{\frac{P_1}{EI_1}}, \quad \mu_2 = \sqrt{\frac{P_2}{EI_2}},$$

$$\alpha_1 = a_1 \mu_1, \ \alpha_2 = a_2 \mu_2,$$

$$f(\alpha) = \frac{3}{2} \left[\frac{2\alpha \operatorname{cosec} 2\alpha - 1}{\alpha^2} \right] = 2\phi(\alpha) - \phi\left(\frac{\alpha}{2}\right)$$

$$\phi(\alpha) = \frac{3}{4} \left[\frac{1 - 2\alpha \cot 2\alpha}{\alpha^2} \right]$$

$$\psi(\alpha) = 3 \left[\frac{\tan \alpha - \alpha}{\alpha^3} \right] = \frac{1}{3} \phi\left(\frac{\alpha}{2}\right) \left\{ 4\phi(\alpha) - \phi\left(\frac{\alpha}{2}\right) \right\}$$

If the supports are deflected by known amounts δ_A, δ_B and δ_C, measured positive upwards, then equation (1) becomes

$$\frac{a_1}{I_1} M_A f(\alpha_1) + \frac{a_2}{I_2} M_C f(\alpha_2) + 2 M_B \left\{ \frac{a_1}{I_1} \phi(\alpha_1) + \frac{a_2}{I_2} \phi(\alpha_2) \right\} + \frac{3E}{2a_1} (\delta_B - \delta_A)$$
$$+ \frac{3E}{2a_2} (\delta_B - \delta_C) = \frac{w_1 a_1^3}{I_1} \psi(\alpha_1) + \frac{w_2 a_2^3}{I_2} \psi(\alpha_2) \quad \ldots \quad (2)$$

If both members are alike, so that $I_1 = I_2 = I$, say, I divides out from equation (1) and can be omitted.

If the loadings are the same in the two members, so that $w_1 = w_2$, the right-hand side of equations (1) and (2) can be slightly simplified.

It is important to notice that when once the three functions which occur in (1) and (2) have been tabulated, their mathematical expression can be completely ignored ; all that is wanted in any numerical case is to take the values directly from the tables. Tables for $f(\alpha)$, $\phi(\alpha)$ and $\psi(\alpha)$ have been carried up to $2 \cdot 36$ radians (*see* table I), the values of the functions between $1 \cdot 52$ and $1 \cdot 62$ radians being omitted as they are varying very rapidly in this region and linear interpolation is no longer sufficiently accurate. Approximate formulae are given at the appropriate place in table I from which the functions over this range can easily be calculated. The calculations should be carried out with special care when α in any bay approaches or exceeds $\pi/2$ radians. This point is dealt with later.

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If both members are in tension the generalised equation of three moments becomes

$$\frac{a_1}{I_1} M_A\, F(\alpha_1) + \frac{a_2}{I_2} M_C\, F(\alpha_2) + 2M_B \left\{ \frac{a_1}{I_1} \Phi(\alpha_1) + \frac{a_2}{I_2} \Phi(\alpha_2) \right\} + \frac{3E}{2a_1}(\delta_B - \delta_A)$$
$$+ \frac{3E}{2a_2}(\delta_B - \delta_C) = \frac{w_1 a_1^3}{I_1} \Psi(\alpha_1) + \frac{w_2 a_2^3}{I_2} \Psi(\alpha_2) \quad \ldots \quad \ldots \quad (3)$$

where $F(\alpha) = \frac{3}{2}\left\{\frac{1 - 2\,\alpha \operatorname{cosech} 2\,\alpha}{\alpha^2}\right\} = 2\,\Phi(\alpha) - \Phi\left(\frac{\alpha}{2}\right)$

$$\Phi(\alpha) = \frac{3}{4}\left\{\frac{2\,\alpha \coth 2\,\alpha - 1}{\alpha^2}\right\}$$

$$\Psi(\alpha) = 3\left\{\frac{\alpha - \tanh \alpha}{\alpha^3}\right\} = \frac{1}{3}\,\Phi\left(\frac{\alpha}{2}\right)\left\{4\,\Phi(\alpha) - \Phi\left(\frac{\alpha}{2}\right)\right\}$$

Tables for these hyperbolic functions and for $\tanh \alpha$ are given in tables II and III respectively.

If the end load, P, in one member, say AB, is zero the corresponding functions all become unity.

If one member, say AB, is in tension and the other BC, in compression, the three moments equation becomes

$$\frac{a_1}{I_1} M_A\, F(\alpha_1) + \frac{a_2}{I_2} M_C\, f(\alpha_2) + 2M_B \left\{ \frac{a_1}{I_1} \Phi(\alpha_1) + \frac{a_2}{I_2} \phi(\alpha_2) \right\} + \frac{3E}{2a_1}(\delta_B - \delta_A)$$
$$+ \frac{3E}{2a_2}(\delta_B - \delta_C) = \frac{w_1 a_1^3}{I_1} \Psi(\alpha_1) + \frac{w_2 a_2^3}{I_2} \psi(\alpha_2) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

(ii) *Maximum bending moment in bays.*—The bending moment at any point in a member AB *under compression* is given by

$$M = \frac{1}{2}(M_A - M_B)\frac{\sin \mu x}{\sin \alpha} + \frac{1}{2}(M_A + M_B)\frac{\cos \mu x}{\cos \alpha} + \frac{w}{\mu^2}\left(1 - \frac{\cos \mu x}{\cos \alpha}\right) \quad \ldots \quad (5)$$

$$\text{or } M = \frac{wa^2}{\alpha^2} - \left(\frac{wa^2}{\alpha^2} - \frac{M_A + M_B}{2}\right)\frac{\cos \mu x}{\cos \alpha} + \frac{M_A - M_B}{2}\frac{\sin \mu x}{\sin \alpha} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

For the bending moment M to be a maximum

$$\tan \mu x = -\frac{\frac{1}{2}(M_A + M_B)\cot \alpha}{\frac{wa^2}{\alpha^2} - \frac{1}{2}(M_A - M_B)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

in which x is measured from the middle point of AB.

This equation is for a mathematical maximum or minimum of M. It should be noted therefore that although μx (which may be positive or negative) given by equation (7) may be numerically less than α it does not necessarily follow that the bending moment for the corresponding point is the greatest (ignoring sign) in the span as one or both of the fixing moments may be numerically greater than this value.

If μx given by equation (7) is numerically greater than α then there is no true maximum or minimum for the bending moment in the bay AB and in these circumstances M increases steadily from A to B or vice versa, the worst stresses in the bay occurring either at A or B.

The maximum value of M is given by :—

$$M_{max} = \frac{wa^2}{\alpha^2} - \frac{\frac{wa^2}{\alpha^2} - \frac{1}{2}(M_A + M_B)}{\cos \mu x \cos \alpha} \quad \ldots \quad (8)$$

μx being given by equation (7).

Taking M_{max} positively, the maximum fibre stress in the bay is given by the usual formula

$$f_s = \frac{M_{max}}{Z} + \frac{P}{A} \quad \ldots \quad (9)$$

where Z is the modulus of section and A is the area.

The bending moment at any point in a member AB *in tension* is

$$M = \tfrac{1}{2}(M_A - M_B)\frac{\sinh \mu x}{\sinh \alpha} + \tfrac{1}{2}(M_A + M_B)\frac{\cosh \mu x}{\cosh \alpha} - \frac{w}{\mu^2}\left(1 - \frac{\cosh \mu x}{\cosh \alpha}\right) \quad \ldots \quad (10)$$

and the maximum bending moment is

$$M_{max} = \frac{\frac{1}{2}(M_A + M_B) + \frac{wa^2}{\alpha^2}}{\cosh \mu x \cosh \alpha} - \frac{wa^2}{\alpha^2} \quad \ldots \quad (11)$$

where μx is given by

$$\tanh \mu x = -\frac{\frac{1}{2}(M_A - M_B)\coth \alpha}{\frac{1}{2}(M_A + M_B) + \frac{wa^2}{\alpha^2}} \quad \ldots \quad (12)$$

(iii) *Offsets.*—In the case of a spar with the lift-wires offset the usual modifications must be made in the above equations.

For example, if there is an offset at B, equation (1) becomes

$$\frac{a_1}{I_1} M_{AL} f(\alpha_1) + \frac{a_2}{I_2} M_{CR} f(\alpha_2) + 2\left\{\frac{a_1}{I_1} M_{BR}\,\phi(\alpha_1) + \frac{a_2}{I_2} M_{BL}\,\phi(\alpha_2)\right\} = \frac{w_1 a_1^3}{I_1}\psi(\alpha_1) + \frac{w_2 a_2^3}{I_3}\psi(\alpha_2) \quad \ldots \quad (13)$$

In the formulae for the bay AB under compression M_{AL} should be taken for M_A, and M_{BR} for M_B.

(iv) When the beam carries concentrated as well as lateral loads (*e.g.* the front spar of a slotted plane) or when the lateral loading changes at a point between two points of support, equation (1) becomes

$$\frac{a_1}{I_1} M_A f(\alpha_1) + 2M_B\left\{\frac{a_1}{I_1}\phi(\alpha_1) + \frac{a_2}{I_1}\phi(\alpha_2)\right\} + \frac{a_2}{I_2} M_C f(\alpha_2)$$

$$= \frac{3a_1^2}{I_1\alpha_1^2}\Big[R'_{BR} - \frac{a_1}{2\alpha_1}(F_1 \sec \alpha_1 + H_1 \operatorname{cosec} \alpha_1) + \tfrac{1}{2}(G_1 \sec \alpha_1 - K_1 \operatorname{cosec} \alpha_1)$$

$$+ \frac{a_1}{2\alpha_1}\Big\{(w_n - w_o)\cot \alpha_1 + (w_n + w_o)\tan \alpha_1\Big\}\Big]$$

$$+ \frac{3a_2^2}{I_2\alpha_2^2}\Big[R'_{BL} - \frac{a_2}{2\alpha_2}(F_2 \sec \alpha_2 - H_2 \operatorname{cosec} \alpha_2) + \tfrac{1}{2}(G_2 \sec \alpha_2 + K_2 \operatorname{cosec} \alpha_2)$$

$$- \frac{a_2}{2\alpha_2}\Big\{(w_n - w_o)\cot \alpha_2 - (w_n + w_o)\tan \alpha_2\Big\}\Big] \quad \ldots \quad (14)$$

Where (*see* fig. 6) :—

$$\theta_r = \mu x_r$$

$$F = \sum_{r=1}^{r=n} \left\{ (w_r - w_{r-1}) \sin \theta_r \right\}$$

$$H = \sum_{r=1}^{r=n} \left\{ (w_r - w_{r-1}) \cos \theta_r \right\}$$

$$G = \sum_{r=1}^{r=n} (W_r \cos \theta_r)$$

$$K = \sum_{r=1}^{r=n} (W_r \sin \theta_r)$$

R'_A and R'_B are the reactions at A and B due to lateral loads when the continuity of the beam is neglected. The sign convention is that loads and reactions are positive upwards, so that R'_A and R'_B will usually be negative. It should be noted that the expression on the right-hand side referring to Bay 1 is not identical with that referring to Bay 2. If the loading is symmetrical about the centre line in any bay H, K and $(w_n - w_o)$ vanish. The application of this equation to consecutive bays will give sufficient equations for the calculation of the bending moments at the points of support. When the bending moments at the points of support have been calculated as above, the bending moment diagram can be drawn as described in R. & M. 1233.

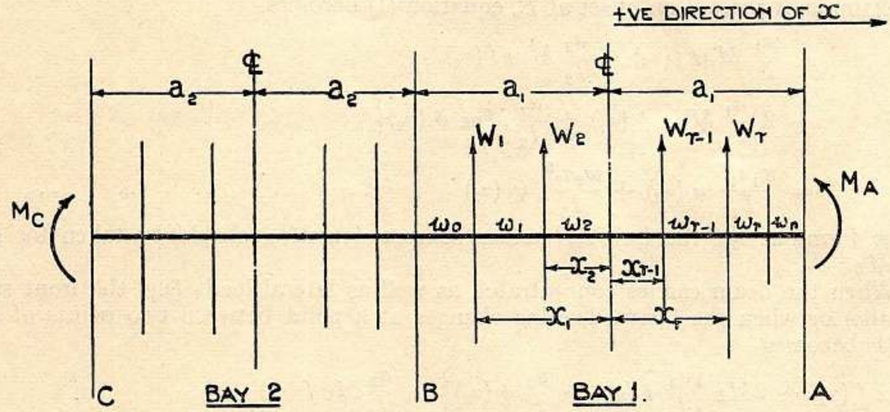

FIG. 6.—CHAP. VI.

TABLE I

$$f(\alpha) = \frac{3\,(2\,\alpha \operatorname{cosec} 2\alpha - 1)}{2\,\alpha^2} \qquad \phi(\alpha) = \frac{3\,(1 - 2\,\alpha \cot 2\,\alpha)}{4\,\alpha^2} \qquad \psi(\alpha) = \frac{3\,(\tan \alpha - \alpha)}{\alpha^3}$$

α	$f(\alpha)$	Diff.	$\phi(\alpha)$	Diff.	$\psi(\alpha)$	Diff.
Radians.						
0·00	1·000	·001	1·000	·000	1·000	·001
0·04	1·001	·002	1·000	·002	1·001	·002
0·08	1·003	·004	1·002	·002	1·003	·003
0·12	1·007	·005	1·004	·003	1·006	·004
0·16	1·012	·007	1·007	·004	1·010	·006
0·20	1·019	·009	1·011	·005	1·016	·008
0·24	1·028	·010	1·016	·006	1·024	·008
0·28	1·038		1·022		1·032	
0·28	1·038	·006	1·022	·003	1·032	·005
0·30	1·044	·006	1·025	·003	1·037	·006
0·32	1·050	·007	1·028	·004	1·043	·005
0·34	1·057	·007	1·032	·004	1·048	·006
0·36	1·064	·008	1·036	·005	1·054	·007
0·38	1·072	·008	1·041	·004	1·061	·007
0·40	1·080	·009	1·045	·005	1·068	·008
0·42	1·089	·009	1·050	·006	1·076	·008
0·44	1·098	·010	1·056	·005	1·084	·009
0·46	1·108	·011	1·061	·006	1·093	·009
0·48	1·119	·011	1·067	·007	1·102	·009
0·50	1·130	·012	1·074	·007	1·111	·010
0·52	1·142	·013	1·081	·007	1·121	·011
0·54	1·155	·014	1·088	·007	1·132	·012
0·56	1·169	·014	1·095	·008	1·144	·012
0·58	1·183	·015	1·103	·008	1·156	·012
0·60	1·198	·016	1·111	·009	1·168	·014
0·62	1·214	·017	1·120	·010	1·182	·014
0·64	1·231	·018	1·130	·010	1·196	·016
0·66	1·249	·019	1·140	·010	1·212	·016
0·68	1·268	·020	1·150	·011	1·228	·017
0·70	1·288	·022	1·161	·012	1·245	·017
0·72	1·310	·022	1·173	·012	1·262	·020
0·74	1·332	·023	1·185	·013	1·282	·020
0·76	1·355	·025	1·198	·014	1·302	·021
0·78	1·380	·028	1·212	·015	1·323	·023

TABLE I—*cont.*

α	f (α)	Diff.	φ (α)	Diff.	ψ (α)	Diff.
Radians.		28		15		23
0·80	1·408	·029	1·227	·016	1·346	·024
0·82	1·437	·030	1·243	·016	1·370	·025
0·84	1·467	·032	1·259	·017	1·395	·027
0·86	1·499	·034	1·276	·019	1·422	·029
0·88	1·533	·037	1·295	·020	1·451	·031
0·90	1·570	·039	1·315	·021	1·482	·033
0·92	1·609	·043	1·336	·022	1·515	·034
0·94	1·652	·046	1·358	·025	1·549	·039
0·96	1·698	·049	1·383	·026	1·588	·042
0·98	1·747	·053	1·409	·027	1·630	·042
1·00	1·800	·056	1·436	·030	1·672	·047
1·02	1·856	·061	1·466	·033	1·719	·050
1·04	1·917	·067	1·499	·034	1·769	·052
1·06	1·984	·072	1·533	·037	1·821	·058
1·08	2·056	·078	1·570	·042	1·879	·069
1·10	2·134	·085	1·612	·045	1·948	·071
1·12	2·219	·093	1·657	·049	2·019	·080
1·14	2·312	·103	1·706	·054	2·099	·087
1·16	2·415	·116	1·760	·059	2·186	·090
1·18	2·531	·128	1·819	·066	2·276	·102
1·20	2·659		1·885		2·378	
1·200	2·659	·034	1·885	·017	2·378	·028
1·205	2·693	·035	1·902	·018	2·406	·028
1·210	2·728	·036	1·920	·018	2·434	·028
1·215	2·764	·037	1·938	·020	2·462	·032
1·220	2·801	·039	1·958	·020	2·494	·033
1·225	2·840	·039	1·978	·020	2·527	·033
1·230	2·879	·040	1·998	·021	2·560	·033
1·235	2·919	·043	2·019	·022	2·593	·034
1·240	2·962	·044	2·041	·022	2·627	·036
1·245	3·006	·044	2·063	·023	2·663	·038
1·250	3·050	·046	2·086	·024	2·701	·040
1·255	3·096	·047	2·110	·024	2·741	·040
1·260	3·143	·048	2·134	·025	2·781	·040
1·265	3·191	·050	2·159	·026	2·821	·041
1·270	3·241	·052	2·185	·027	2·862	·041
1·275	3·293	·055	2·212	·027	2·903	·043
1·280	3·348	·057	2·239	·029	2·946	·046
1·285	3·405	·059	2·268	·030	2·992	·050
1·290	3·464	·061	2·298	·031	3·042	·050
1·295	3·525	·063	2·329	·032	3·092	·051

TABLE I—*cont.*

α	$f\ (\alpha)$	Diff.	$\phi\ (\alpha)$	Diff.	$\psi\ (\alpha)$	Diff.
Radians.						
1·300	3·588	·065	2·361	·033	3·143	·053
1·305	3·653	·066	2·394	·034	3·196	·055
1·310	3·719	·070	2·428	·036	3·251	·056
1·315	3·789	·073	2·464	·037	3·307	·059
1·320	3·862	·077	2·501	·039	3·366	·062
1·325	3·939	·080	2·540	·040	3·428	·066
1·330	4·019	·084	2·580	·043	3·494	·071
1·335	4·103	·087	2·623	·045	3·565	·073
1·340	4·190	·091	2·668	·046	3·638	·074
1·345	4·281	·095	2·714	·048	3·712	·075
1·350	4·376	·099	2·762	·050	3·787	·076
1·355	4·475	·102	2·812	·051	3·863	·081
1·360	4·577	·107	2·863	·054	3·944	·089
1·365	4·684	·113	2·917	·058	4·033	·097
1·370	4·797		2·975		4·130	
1·3700	4·797	·062	2·975	·031	4·130	·049
1·3725	4·859	·063	3·006	·031	4·179	·049
1·3750	4·922	·064	3·037	·032	4·228	·050
1·3775	4·986	·065	3·069	·033	4·278	·050
1·3800	5·051	·065	3·102	·033	4·328	·052
1·3825	5·116	·066	3·135	·034	4·380	·057
1·3850	5·182	·068	3·169	·035	4·437	·060
1·3875	5·250	·071	3·204	·036	4·497	·062
1·3900	5·321	·073	3·240	·037	4·559	·062
1·3925	5·394	·077	3·277	·039	4·621	·062
1·3950	5·471	·078	3·316	·039	4·683	·062
1·3975	5·549	·080	3·355	·040	4·745	·064
1·4000	5·629	·082	3·395	·041	4·809	·064
1·4025	5·711	·085	3·436	·043	4·873	·067
1·4050	5·796	·089	3·479	·045	4·940	·073
1·4075	5·885	·090	3·524	·046	5·013	·078
1·4100	5·975	·093	3·570	·047	5·091	·079
1·4125	6·068	·096	3·617	·048	5·170	·081
1·4150	6·164	·103	3·665	·052	5·251	·082
1·4175	6·267	·104	3·717	·052	5·333	·083
1·4200	6·371	·105	3·769	·053	5·416	·084
1·4225	6·476	·107	3·822	·054	5·500	·086
1·4250	6·583	·112	3·876	·056	5·586	·094
1·4275	6·695	·119	3·932	·061	5·680	·103

TABLE I—*cont.*

α	$f(\alpha)$	Diff.	$\phi(\alpha)$	Diff.	$\psi(\alpha)$	Diff.
Radians.						
1·4300	6·814	·122	3·993	·062	5·783	·105
1·4325	6·936	·127	4·055	·064	5·888	·106
1·4350	7·063	·131	4·119	·066	5·994	·107
1·4375	7·194	·136	4·185	·068	6·101	·109
1·4400	7·330	·143	4·253	·071	6·210	·110
1·4425	7·473	·147	4·324	·074	6·320	·115
1·4450	7·620	·153	4·398	·077	6·435	·122
1·4475	7·773	·160	4·475	·080	6·557	·130
1·4500	7·933	·167	4·555	·084	6·687	·136
1·4525	8·100	·174	4·639	·087	6·823	·140
1·4550	8·274	·182	4·726	·091	6·963	·140
1·4575	8·456	·190	4·817	·095	7·103	·150
1·4600	8·646	·198	4·912	·099	7·253	·161
1·4625	8·844	·208	5·011	·105	7·414	·171
1·4650	9·052	·217	5·116	·109	7·585	·178
1·4675	9·269	·229	5·225	·115	7·763	·187
1·4700	9·498	·240	5·340	·120	7·950	·193
1·4725	9·738	·253	5·460	·127	8·143	·203
1·4750	9·991	·267	5·587	·134	8·346	·218
1·4775	10·258		5·721		8·564	
1·4775	10·26	·28	5·721	·142	8·564	·236
1·4800	10·54	·30	5·863	·149	8·800	·245
1·4825	10·84	·31	6·012	·157	9·045	·256
1·4850	11·15	·33	6·169	·166	9·301	·263
1·4875	11·48	·36	6·335	·177	9·564	·280
1·4900	11·84		6·512		9·844	
1·4900	11·84	·37	6·512	·189	9·84	·30
1·4925	12·21	·41	6·701	·202	10·14	·33
1·4950	12·62	·43	6·903	·214	10·47	·36
1·4975	13·05	·46	7·117	·230	10·83	·37
1·5000	13·51	·49	7·347	·246	11·20	·40
1·5025	14·00	·53	7·593	·266	11·60	·42
1·5050	14·53	·57	7·859	·287	12·02	·45
1·5075	15·10	·62	8·146	·310	12·47	·50
1·5100	15·72	·67	8·456	·337	12·97	·55
1·5125	16·39	·74	8·793	·367	13·52	·60
1·5150	17·13	·80	9·160	·402	14·12	·66
1·5175	17·93	·88	9·562	·442	14·78	·72
1·5200	18·81		10·004		15·50	

For values of α between 1·52 and 1·62 the functions $f(\alpha)$, $\phi(\alpha)$ and $\psi(\alpha)$ can be calculated with sufficient accuracy from the approximations

$$f(\alpha) = \frac{6}{\pi(\pi - 2\alpha)} = \frac{1{\cdot}910}{\pi - 2\alpha}$$

$$\phi(\alpha) = \frac{3}{\pi(\pi - 2\alpha)} + \frac{6}{\pi^2} = \frac{{\cdot}955}{\pi - 2\alpha} + {\cdot}608$$

$$\psi(\alpha) = \frac{48}{\pi^3(\pi - 2\alpha)} + \frac{144}{\pi^4} - \frac{12}{\pi^2} = \frac{1{\cdot}547}{\pi - 2\alpha} + {\cdot}262$$

TABLE I—*cont.*

α	f (α)	Diff.	φ (α)	Diff.	ψ (α)	Diff.
Radians.						
1·6200	−19·42		−9·092		−15·48	
		·94		·478		·76
1·6225	−18·48		−8·624		−14·72	
		·85		·426		·69
1·6250	−17·63		−8·198		−14·03	
		·77		·389		·63
1·6275	−16·86		−7·809		−13·40	
		·71		·355		·57
1·6300	−16·15		−7·454		−12·83	
		·66		·327		·53
1·6325	−15·49		−7·127		−12·30	
		·60		·305		·49
1·6350	−14·89		−6·825		−11·81	
		·57		·285		·47
1·6375	−14·32		−6·540		−11·34	
		·50		·252		·40
1·6400	−13·82		−6·288		−10·94	
1·640	−13·82		−6·288		−10·936	
		·93		·466		·755
1·645	−12·89		−5·822		−10·181	
		·81		·405		·655
1·650	−12·08		−5·417		−9·526	
		·72		·360		·584
1·655	−11·36		−5·057		−8·942	
1·655	−11·364		−5·057		−8·942	
		·635		·316		·515
1·660	−10·729		−4·739		−8·427	
		·567		·285		·462
1·665	−10·162		−4·454		−7·965	
		·509		·255		·414
1·670	−9·653		−4·199		−7·551	
		·461		·231		·375
1·675	−9·192		−3·968		−7·176	
		·418		·211		·340
1·680	−8·774		−3·757		−6·836	
		·382		·191		·311
1·685	−8·392		−3·566		−6·525	
		·350		·177		·281
1·690	−8·042		−3·389		−6·244	
		·320		·161		·263
1·695	−7·722		−3·228		−5·981	
		·298		·149		·243
1·700	−7·424		−3·079		−5·738	
1·70	−7·424		−3·079		−5·732	
		·986		·496		·812
1·72	−6·438		−2·583		−4·920	
		·749		·381		·596
1·74	−5·689		−2·202		−4·324	
		·589		·300		·481
1·76	−5·100		−1·902		−3·843	
		·478		·243		·390
1·78	−4·622		−1·659		−3·453	
		·393		·202		·322
1·80	−4·229		−1·457		−3·131	
		·328		·169		·270

TABLE I—*cont.*

α	f (α)	Diff.	ϕ (α)	Diff.	ψ (α)	Diff.
Radians.						
1·82	−3·901	·278	−1·288	·145	−2·861	·226
1·84	−3·623	·239	−1·143	·125	−2·635	·200
1·86	−3·384	·208	−1·018	·109	−2·435	·174
1·88	−3·176	·180	−0·909	·096	−2·261	·150
1·90	−2·996	·159	−0·813	·086	−2·111	·133
1·92	−2·837	·141	−·727	·077	−1·978	·119
1·94	−2·696	·126	−·650	·069	−1·859	·108
1·96	−2·570	·111	−·581	·063	−1·751	·093
1·98	−2·459	·102	−·518	·058	−1·658	·088
2·00	−2·357	·092	−·460	·053	−1·570	·081
2·02	−2·265	·081	−·407	·048	−1·489	·070
2·04	−2·184	·075	−·359	·046	−1·419	·066
2·06	−2·109	·069	−·313	·041	−1·353	·064
2·08	−2·040	·061	−·272	·040	−1·289	·055
2·10	−1·979	·056	−·232	·037	−1·234	·052
2·12	−1·923	·052	−·195	·035	−1·182	·049
2·14	−1·871	·047	−·160	·033	−1·133	·046
2·16	−1·824	·042	−·127	·032	−1·087	·043
2·18	−1·782	·040	−·095	·030	−1·044	·040
2·20	−1·742	·035	−·065	·028	−1·004	·035
2·22	−1·707	·032	−·037	·028	−·969	·034
2·24	−1·675	·029	−·009	·026	−·935	·032
2·26	−1·646	·026	+·017	·026	−·903	·031
2·28	−1·620	·023	+·043	·025	−·872	·029
2·30	−1·597	·021	+·068	·025	−·843	·028
2·32	−1·576	·019	+·093	·025	−·815	·025
2·34	−1·557	·016	+·118	·024	−·790	·025
2·36	−1·541		+·142		−·765	

TABLE II

$$F(\alpha) = \frac{3(1 - 2\alpha \operatorname{cosech} 2\alpha)}{2\alpha^2} \qquad \Phi(\alpha) = \frac{3(2\alpha \coth 2\alpha - 1)}{4\alpha^2} \qquad \Psi(\alpha) = \frac{3(\alpha - \tanh \alpha)}{\alpha^3}$$

α	$F(\alpha)$	$\Phi(\alpha)$	$\Psi(\alpha)$	α	$F(\alpha)$	$\Phi(\alpha)$	$\Psi(\alpha)$
Radians.				Radians.			
·00	1·0000	1·0000	1·0000	·40	·9301	·9598	·9399
·01	0·9999	1·0000	0·9999	·41	·9268	·9579	·9371
·02	·9998	0·9999	·9998	·42	·9234	·9559	·9342
·03	·9996	·9998	·9996	·43	·9200	·9539	·9312
·04	·9992	·9996	·9994	·44	·9165	·9519	·9282
·05	·9988	·9993	·9990	·45	·9129	·9499	·9252
·06	·9983	·9990	·9986	·46	·9094	·9478	·9221
·07	·9977	·9987	·9981	·47	·9057	·9456	·9189
·08	·9970	·9983	·9975	·48	·9020	·9435	·9157
·09	·9962	·9979	·9968	·49	·8983	·9413	·9125
·10	·9953	·9973	·9960	·50	·8945	·9391	·9092
·11	·9944	·9968	·9952	·51	·8906	·9369	·9059
·12	·9933	·9962	·9943	·52	·8868	·9346	·9026
·13	·9922	·9955	·9933	·53	·8828	·9323	·8992
·14	·9909	·9948	·9922	·54	·8789	·9300	·8957
·15	·9896	·9940	·9910	·55	·8748	·9276	·8922
·16	·9882	·9932	·9898	·56	·8708	·9253	·8887
·17	·9867	·9924	·9886	·57	·8667	·9229	·8851
·18	·9851	·9915	·9872	·58	·8626	·9204	·8815
·19	·9834	·9905	·9857	·59	·8584	·9180	·8779
·20	·9816	·9895	·9842	·60	·8542	·9155	·8743
·21	·9798	·9884	·9826	·61	·8500	·9130	·8706
·22	·9779	·9873	·9810	·62	·8457	·9105	·8669
·23	·9758	·9862	·9793	·63	·8414	·9080	·8632
·24	·9738	·9850	·9775	·64	·8371	·9054	·8595
·25	·9716	·9837	·9756	·65	·8328	·9028	·8557
·26	·9693	·9824	·9736	·66	·8284	·9003	·8519
·27	·9670	·9811	·9717	·67	·8240	·8977	·8481
·28	·9646	·9797	·9696	·68	·8196	·8950	·8442
·29	·9621	·9783	·9675	·69	·8151	·8924	·8403
·30	·9595	·9768	·9653	·70	·8107	·8897	·8364
·31	·9569	·9753	·9630	·71	·8062	·8871	·8325
·32	·9542	·9737	·9607	·72	·8017	·8844	·8286
·33	·9514	·9721	·9583	·73	·7972	·8817	·8247
·34	·9486	·9705	·9558	·74	·7927	·8790	·8207
·35	·9457	·9688	·9533	·75	·7881	·8762	·8167
·36	·9427	·9671	·9507	·76	·7835	·8735	·8127
·37	·9396	·9653	·9481	·77	·7790	·8708	·8087
·38	·9365	·9635	·9454	·78	·7744	·8680	·8047
·39	·9333	·9617	·9427	·79	·7698	·8653	·8007

TABLE II—*cont.*

α	F (α)	Φ (α)	Ψ (α)	α	F (α)	Φ (α)	Ψ (α)
Radians.				Radians.			
·80	·7652	·8625	·7967	1·20	·5843	·7499	·6360
·81	·7606	·8597	·7927	1·21	·5801	·7472	·6322
·82	·7560	·8569	·7887	1·22	·5759	·7444	·6284
·83	·7513	·8541	·7847	1·23	·5717	·7417	·6246
·84	·7467	·8513	·7807	1·24	·5675	·7390	·6208
·85	·7421	·8485	·7766	1·25	·5633	·7363	·6170
·86	·7374	·8457	·7725	1·26	·5592	·7336	·6133
·87	·7328	·8429	·7684	1·27	·5551	·7309	·6096
·88	·7282	·8400	·7643	1·28	·5510	·7282	·6059
·89	·7235	·8372	·7601	1·29	·5469	·7255	·6022
·90	·7189	·8344	·7560	1·30	·5429	·7229	·5985
·91	·7143	·8315	·7519	1·31	·5389	·7202	·5948
·92	·7096	·8287	·7478	1·32	·5349	·7175	·5912
·93	·7050	·8259	·7437	1·33	·5309	·7149	·5875
·94	·7004	·8230	·7396	1·34	·5269	·7123	·5839
·95	·6958	·8202	·7355	1·35	·5230	·7097	·5803
·96	·6912	·8173	·7314	1·36	·5191	·7071	·5767
·97	·6866	·8145	·7273	1·37	·5152	·7045	·5732
·98	·6820	·8117	·7232	1·38	·5114	·7019	·5697
·99	·6774	·8088	·7192	1·39	·5075	·6993	·5662
1·00	·6728	·8060	·7152	1·40	·5037	·6967	·5627
1·01	·6683	·8031	·7112	1·41	·4999	·6942	·5593
1·02	·6637	·8003	·7072	1·42	·4962	·6916	·5559
1·03	·6592	·7975	·7031	1·43	·4924	·6891	·5525
1·04	·6547	·7946	·6991	1·44	·4887	·6866	·5491
1·05	·6501	·7918	·6950	1·45	·4851	·6840	·5457
1·06	·6456	·7890	·6910	1·46	·4814	·6815	·5423
1·07	·6411	·7861	·6870	1·47	·4778	·6790	·5389
1·08	·6367	·7833	·6830	1·48	·4742	·6766	·5355
1·09	·6322	·7805	·6790	1·49	·4706	·6741	·5321
1·10	·6278	·7777	·6750	1·50	·4670	·6716	·5288
1·11	·6233	·7749	·6711	1·51	·4635	·6692	·5255
1·12	·6189	·7721	·6672	1·52	·4600	·6668	·5222
1·13	·6145	·7693	·6633	1·53	·4565	·6643	·5189
1·14	·6102	·7665	·6594	1·54	·4530	·6619	·5157
1·15	·6058	·7637	·6555	1·55	·4496	·6595	·5125
1·16	·6015	·7609	·6516	1·56	·4462	·6571	·5093
1·17	·5972	·7582	·6477	1·57	·4428	·6547	·5061
1·18	·5929	·7554	·6438	1·58	·4395	·6524	·5030
1·19	·5886	·7527	·6399	1·59	·4361	·6500	·4999
				1·60	·4328	·6476	·4968

TABLE III

Tanh α

α	Tanh α	α	Tanh α	α	Tanh α	α	Tanh α
Radians.		Radians.		Radians.		Radians.	
0·00	·00000	0·40	·37995	0·80	·66404	1·20	·83365
·01	·01000	·41	·38847	·81	·66959	1·21	·83668
·02	·02000	·42	·39693	·82	·67507	1·22	·83965
·03	·02999	·43	·40532	·83	·68048	1·23	·84258
·04	·03998	·44	·41364	·84	·68581	1·24	·84546
·05	·04996	·45	·42190	·85	·69107	1·25	·84828
·06	·05993	·46	·43008	·86	·69626	1·26	·85106
·07	·06989	·47	·43820	·87	·70137	1·27	·85380
·08	·07983	·48	·44624	·88	·70642	1·28	·85648
·09	·08976	·49	·45422	·89	·71139	1·29	·85913
0·10	·09967	0·50	·46212	0·90	·71630	1·30	·86172
·11	·10956	·51	·46995	·91	·72113	1·31	·86427
·12	·11943	·52	·47770	·92	·72590	1·32	·86678
·13	·12928	·53	·48538	·93	·73059	1·33	·86925
·14	·13909	·54	·49299	·94	·73522	1·34	·87167
·15	·14888	·55	·50052	·95	·73978	1·35	·87405
·16	·15865	·56	·50798	·96	·74428	1·36	·87639
·17	·16838	·57	·51536	·97	·74870	1·37	·87869
·18	·17808	·58	·52267	·98	·75307	1·38	·88095
·19	·18775	·59	·52990	·99	·75736	1·39	·88317
0·20	·19738	0·60	·53705	1·00	·76159	1·40	·88535
·21	·20697	·61	·54413	1·01	·76576	1·41	·88749
·22	·21652	·62	·55113	1·02	·76987	1·42	·88960
·23	·22603	·63	·55805	1·03	·77391	1·43	·89167
·24	·23550	·64	·56490	1·04	·77789	1·44	·89370
·25	·24492	·65	·57167	1·05	·78181	1·45	·89569
·26	·25430	·66	·57836	1·06	·78566	1·46	·89765
·27	·26362	·67	·58598	1·07	·78946	1·47	·89958
·28	·27291	·68	·59152	1·08	·79320	1·48	·90147
·29	·28213	·69	·59798	1·09	·79688	1·49	·90332
0·30	·29131	0·70	·60437	1·10	·80050	1·50	·90515
·31	·30044	·71	·61068	1·11	·80406	1·51	·90694
·32	·30951	·72	·61691	1·12	·80757	1·52	·90870
·33	·31852	·73	·62307	1·13	·81102	1·53	·91043
·34	·32748	·74	·62915	1·14	·81441	1·54	·91212
·35	·33638	·75	·63515	1·15	·81775	1·55	·91379
·36	·34521	·76	·64108	1·16	·82104	1·56	·91542
·37	·35399	·77	·64693	1·17	·82427	1·57	·91703
·38	·36271	·78	·65271	1·18	·82745	1·58	·91860
·39	·37136	·79	·65841	1·19	·83058	1·59	·92015

TABLE III—*cont.*

α	Tanh α	α	Tanh α	α	Tanh α	α	Tanh α
Radians.		Radians.		Radians.		Radians.	
1·60	·92167	1·70	·93541	1·80	·94681	1·90	·95624
1·61	·92316	1·71	·93665	1·81	·94783	1·91	·95709
1·62	·82462	1·72	·93786	1·82	·94884	1·92	·95792
1·63	·92606	1·73	·93906	1·83	·94983	1·93	·95873
1·64	·92747	1·74	·94023	1·84	·95080	1·94	·95953
1·65	·92886	1·75	·94138	1·85	·95175	1·95	·96032
1·66	·93022	1·76	·94250	1·86	·95268	1·96	·96109
1·67	·93155	1·77	·94361	1·87	·95359	1·97	·96185
1·68	·93286	1·78	·94470	1·88	·95449	1·98	·96259
1·69	·93415	1·79	·94576	1·89	·95537	1·99	·96331
						2·00	·96403

ii.—Generalised Theorem of Three Moments extended to include shear deflection

The generalised theorem of three moments given above takes account of the deflection due to bending moment only. Some cases have arisen where there is a considerable discrepancy between the calculated deflection of a spar and the deflection actually measured on test. If this discrepancy is due to shear deflection it can be taken into account as follows :—

In general it will not be possible to calculate the shear deflection of such metal spars as are met with in aircraft without first ascertaining the value of a constant by means of tests on the particular spar section in question. For simple solid rectangular sections an approximate method of calculating the shear deflection is given in " The Strength of Materials ", by John Case, chapter XIV, but at the present time no simple method has been evolved for dealing with the complicated sections usually employed in metal spars.

In the work that follows it will be assumed that tests have been carried out to determine a constant " r " such that :—

$$\frac{dy_s}{dx} = -rS$$

where S is the shear and y_s is the deflection due to shear. Suitable tests for this purpose are described later.

(i) *Notation.*—The notation is the same as that used in section A with the following additions :—

$r =$ see above.

$y_b =$ deflection of the spar at any point due to bending.

$y_s =$ deflection of the spar at any point due to shear.

$y = y_b + y_s$.

$S =$ shear force at any point.

$$\mu^2 = \frac{P}{EI\ (1 - rP)}$$

$$\alpha^2 = \frac{Pa^2}{EI\ (1 - rP)}$$

(ii) *Mathematical theory.*

$$\frac{dy_s}{dx} = -rS = -r\frac{dM}{dx}$$

$$\frac{d^2y}{dx^2} = -\frac{rd^2M}{dx^2} + \frac{M}{EI}$$

i.e. $$EI\frac{d^2y}{dx^2} = M - r\,EI\frac{d^2M}{dx^2}$$

This is the fundamental equation, reducing to the ordinary form when $r = 0$, *i.e.* when the shear deflection is zero.

By taking moments about any point of a member AB (*see* fig. 5) under compression, we have

$M = -Py + \frac{1}{2}w(a-x)^2 + S_A(x-a) + M_A$, where S_A is the shear to the left of A.

Hence $$\frac{d^2M}{dx^2} = -\frac{P\,d^2y}{dx^2} + w$$

or $$EI\frac{d^2y}{dx^2} = -\frac{EI}{P}\frac{d^2M}{dx^2} + \frac{EIw}{P}$$

Thus $$EI(1-rP)\frac{d^2M}{dx^2} + PM = EIw$$

or $$\frac{d^2M}{dx^2} + \mu^2M = \frac{w}{1-rP}$$

This equation is identical with that obtained neglecting shear, except for the modified definition of μ^2, and that w is replaced by $\frac{w}{1-rP}$.

When forming the three moment equation $\frac{dy_b}{dx}$ should be equated for the two bays and not $\frac{dy}{dx}$. This leads to the following expression for the three moment equation :—

$$\left\{\frac{a_1}{I_1}f(\alpha_1) - \frac{3Er_1}{2a_1}\right\}M_A + \left\{\frac{a_2}{I_2}f(\alpha_2) - \frac{3Er_2}{2a_2}\right\}M_C + 2M_B\left\{\frac{a_1}{I_1}\phi(\alpha_1) + \frac{a_2}{I_2}\phi(\alpha_2) + \frac{3E}{4}\left(\frac{r_1}{a_1} + \frac{r_2}{a_2}\right)\right\} = \frac{w_1\,a_1^3}{I_1(1-rP)}\ \psi(\alpha_1) + \frac{w_2\,a_2^3}{I_2(1-rP)}\ \psi(\alpha_2)$$

The expression for the bending moment at any point and for the maximum bending moment can be obtained from equations (6) and (8), section A, by using the above definitions of μ^2 and α^2 and replacing w by w' where

$$w' = \frac{w}{1-rP}$$

(iii) *Experimental determination of the shear constant r.*—The following two tests have been found to give a satisfactory determination of this constant. In order that the deflection may be easily measurable, the length of the specimen should be about 20 times its depth.

(*a*) *Pure bending.*—The object of this test is to obtain the correct value of EI for the specimen, so that any error in calculating I or any variation in E from the specified value will not have any influence upon the value of r.

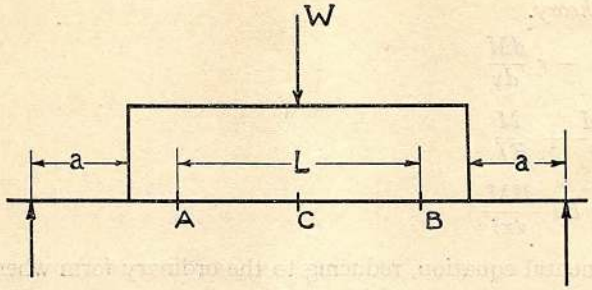

FIG. 7.—CHAP. VI.

The test is shown diagrammatically in fig. 7. The deflection of the centre point C relative to A and B is noted for a range of values of W and hence, by plotting, the ratio $\frac{W}{y_b}$ is found.

Then $$EI = \frac{aL^2W}{16y_b}$$

(*b*) *Bending and shear*.—This test is shown diagrammatically in fig. 8.

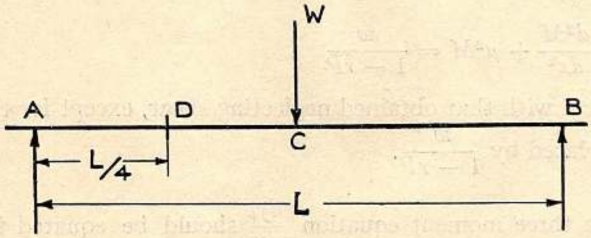

FIG. 8.—CHAP. VI.

The deflection of C relative to A and B is noted as before for a range of values of W.

Hence, by plotting, $\frac{y}{W}$ is obtained.

$$\text{Then } y_s = y - y_b$$
$$= \left\{\frac{y}{W} - \frac{L^3}{48\ EI}\right\} W$$

$$\text{But } y_s = \frac{rLW}{4}.$$

$$\text{Hence } r = \frac{4}{L}\left\{\frac{y}{W} - \frac{L^3}{48\,EI}\right\}$$

It is a useful check to take readings at another point D. If D be distant $\frac{L}{4}$ from one support the total deflection of D should be :—

$$\frac{11\ WL^3}{768\ EI} + \frac{r\,WL}{8}$$

iii.—Miscellaneous applications of the Generalised Theorem of Three Moments

(i) *Case 1. Generalised three moment equations where the compressive end load in one bay approximates to the Euler failing load for that bay.*—When the value of α in any bay approaches $\pi/2$ (corresponding to the Euler failing load for that bay considered as a pin-jointed strut) the Berry equations become unworkable because of the rapid variation of the Berry functions with α. The case is dealt with in the "Transactions of the Royal Aeronautical Society," No. 1, by writing $\theta = \pi/2 - \alpha$ and replacing the Berry functions by approximate expressions in terms of θ and π.

The three moment equation for the spars AB and BC, where α_1, corresponding to the bay AB, approximates to $\frac{\pi}{2}$, is then as follows :—

$$a_1 M_A + M_B \left[a_1 + 1{\cdot}047\ \theta_1 \left\{ 1{\cdot}216\ a_1 + 2a_2\ \phi\,(\alpha_2) \right\} \right] + 1{\cdot}047\ \theta_1\ a_2 f(\alpha_2)\ M_C$$

$$= {\cdot}811\ w_1 a_1^3\ (1 + 1{\cdot}273\ \theta_1) + 1{\cdot}047\ \theta_1 \left\{ w_2'\ a_2^{\ 3} \psi\,(\alpha_2) - {\cdot}723\ w_1\ a_1^{\ 3} \right\}$$

where $\theta_1 = \frac{\pi}{2} - \alpha_1$, θ_1 and α_1 being in radians.

The bending moment at any point in the bay AB distant x from its midpoint is given by the following expression :—

$$M = \frac{w\ a_1^{\ 2}}{\alpha_1^{\ 2}} + S \cos \mu x + \frac{M_A - M_B}{2}\ \frac{\sin \mu x}{\sin \alpha_1}$$

where

$$S = {\cdot}524 \left[\frac{w_2\ a_2^{\ 3}\ \psi\,(\alpha_2)}{a_1} - {\cdot}723\ w_1 a_1^{\ 2} - \left\{ 1{\cdot}216 + \frac{2a_2\ \phi\,(\alpha_2)}{a_1} \right\} M_B - \frac{a_2 f(\alpha_2)\ M_C}{a_1} \right]$$

The maximum bending moment occurs at x_1 where

$$\tan \mu x_1 = \frac{M_A - M_B}{2S}$$

and is given by

$$M_{max} = \frac{w a_1^{\ 2}}{\alpha_1^{\ 2}} + S \sec \mu x_1$$

It will be seen that in the particular case when $\alpha_1 = \frac{\pi}{2}$, *i.e.* when $\theta_1 = 0$, the term in M_C disappears from the three moment equation so that this equation only connects the moments at either end of the weaker span. This does not mean that M_C is indeterminate.

(ii) *Case 2. Instability criteria for continuous spars with end load.*—As will be seen from the previous case, a continuous spar subjected to end load will not, in general, fail when the end load in any one bay is equal to the Euler failing load for that bay considered as a pin-jointed strut.

The indication of instability given by the Berry equations is that the fixing moment between two bays becomes infinite or indeterminate as shown by the denominator of the expression for the fixing moment being zero.

For a spar with only one unknown fixing moment, as occurs, for instance, in a two bay aeroplane with a pin joint at the centre, the three moment equation is :—

$$\frac{a_1}{I_1} f\,(\alpha_1)\ M_A + \frac{a_2}{I_2} f\,(\alpha_2)\ M_C + 2M_B \left\{ \frac{a_1}{I_1}\ \phi\,(\alpha_1) + \frac{a_2}{I_2}\ \phi\,(\alpha_2) \right\}$$

$$= \frac{w_1\ a_1^{\ 3}}{I_1}\ \psi\,(\alpha_1) + \frac{w_2\ a_2^{\ 3}}{I_2}\ \psi\,(\alpha_2)$$

where M_A and M_C are known, M_A being the bending moment at the overhang and M_C a pin joint (or vice versa). This equation, therefore, gives M_B directly, the denominator of the expression for M_B being :—

$$2\left\{\frac{a_1}{I_1}\phi(\alpha_1) + \frac{a_2}{I_2}\phi(\alpha_2)\right\}$$

Thus, M_B will become infinite or indeterminate when :—

$$\frac{a_1}{I_1}\phi(\alpha_1) + \frac{a_2}{I_2}\phi(\alpha_2) = 0.$$

This relation between α_1 and α_2 is shown graphically in fig. 9. For any given value of α_1 the value of α_2 is given, corresponding to any particular value of $\frac{a_2/I_2}{a_1/I_1}$ that will make the bending moment at B (and hence at every other point in the spans ABC) indeterminate or infinite. Hence, if α_2 has a value less than this the spar will be stable, the moment at any point being calculable in the ordinary way, even though in one bay α may exceed 90°. The form of the usual equations being unsuitable for values of α between $1 \cdot 52$ and $1 \cdot 62$ radians the modified form given above (Case 1) should be used.

In a similar way it can be shown that in a single bay aircraft with the spar continuous over the centre section, the condition for instability is :—

$$2\left\{\frac{a_1}{I_1}\phi(\alpha_1) + \frac{a_2}{I_2}\phi(\alpha_2)\right\} + \frac{a_2}{I_2}f(\alpha_2) = 0$$

This relation is shown graphically in fig. 10.

In more complicated cases where there are two or more unknown fixing moments, the conditions for instability become too cumbersome to be conveniently represented on a graph. In any particular case the required condition is easily found by solving for one fixing moment and equating the denominator of the expression to zero.

The presence of offsets does not affect the conditions of instability. This is easily seen by replacing in equation (6)

$$M_{BL} \text{ by } M_B$$
$$\text{and } M_{BR} \text{ by } M_B + Ph$$

h being the amount by which the wire is offset. Then the coefficient of M_B is as given above.

Instability criteria for a continuous strut divided into a number of bays could be obtained on these lines as the instability is not affected by the magnitude of the lateral load.

(iii) *Case 3. An aeroplane spar in which the dihedral starts from a point between two supports.* —A convenient method of dealing with this case is to treat the point at which the dihedral starts as a support which has deflected below the line joining the supports on either side, and inserting the condition that the reaction at this support is zero. At each undeflected support the end load is resolved along the line joining that point to the next undeflected support, the other component being taken by the reaction at the support.

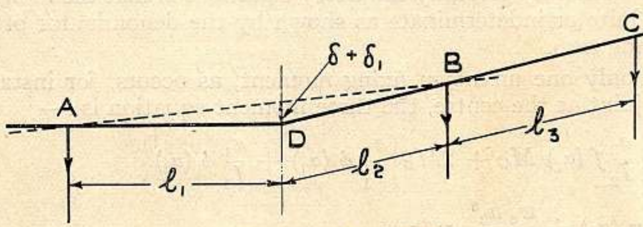

FIG. 11.—CHAP. VI.

INSTABILITY CRITERIA FOR SPARS ACCORDING TO THE GENERALISED THEOREM OF THREE MOMENTS.

For any given value of M and $\left\{ \begin{smallmatrix} \alpha_1 \\ \alpha_2 \end{smallmatrix} \right\}$ the graph shows the value of $\left\{ \begin{smallmatrix} \alpha_2 \\ \alpha_1 \end{smallmatrix} \right\}$ which corresponds to instability. It is necessary to check that this value has not been exceeded, as it may not be immediately apparent from the three-moment equations that the instability point has been passed.

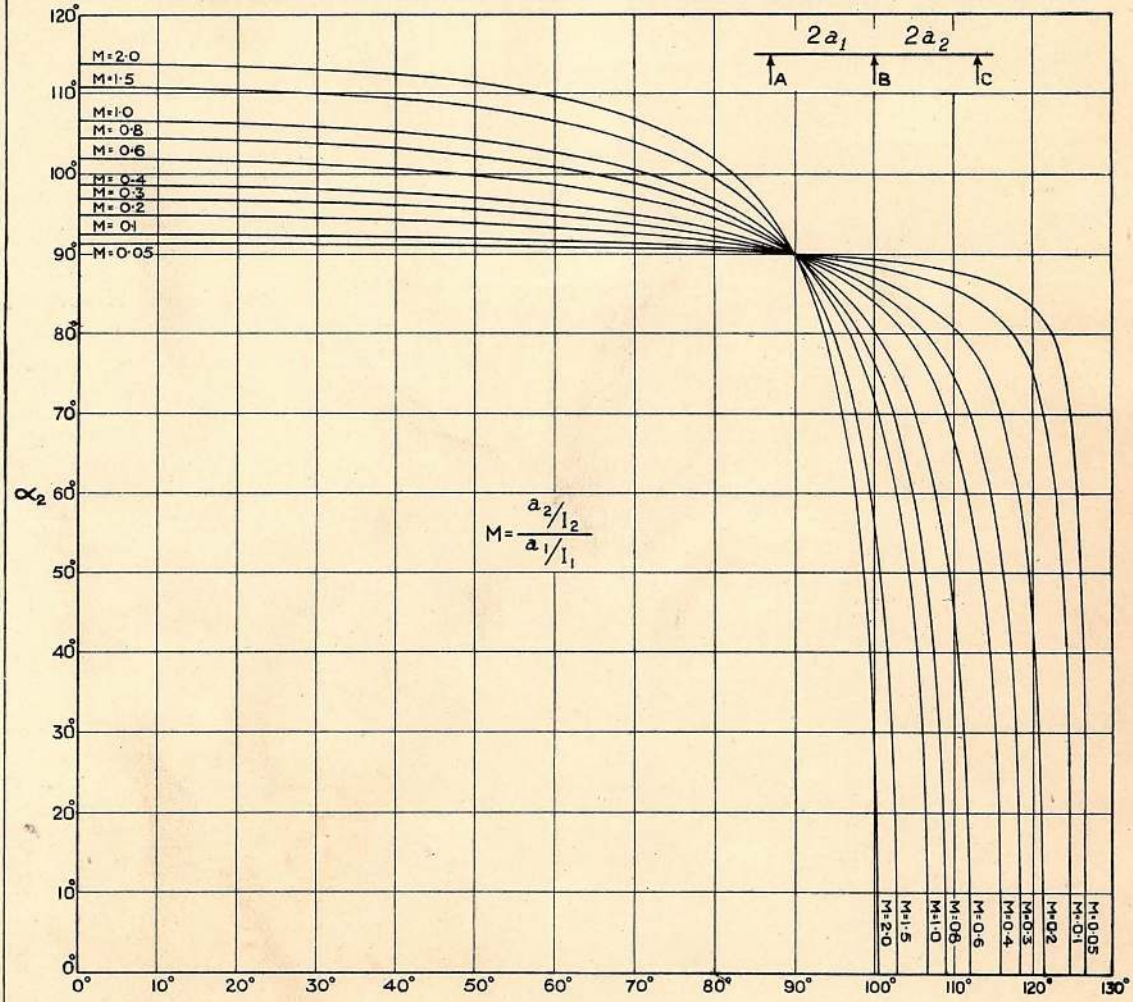

FIG. 9. CHAP. VI.

INSTABILITY CRITERIA FOR SPARS ACCORDING TO THE GENERALISED THEOREM OF THREE MOMENTS.

For any given value of M and $\left\{\frac{\alpha_1}{\alpha_2}\right\}$ the graph shows the value of $\left\{\frac{\alpha_2}{\alpha_1}\right\}$ which corresponds to instability. It is necessary to check that this value has not been exceeded, as it may not be immediately apparent from the three-moment equations that the instability point has been passed.

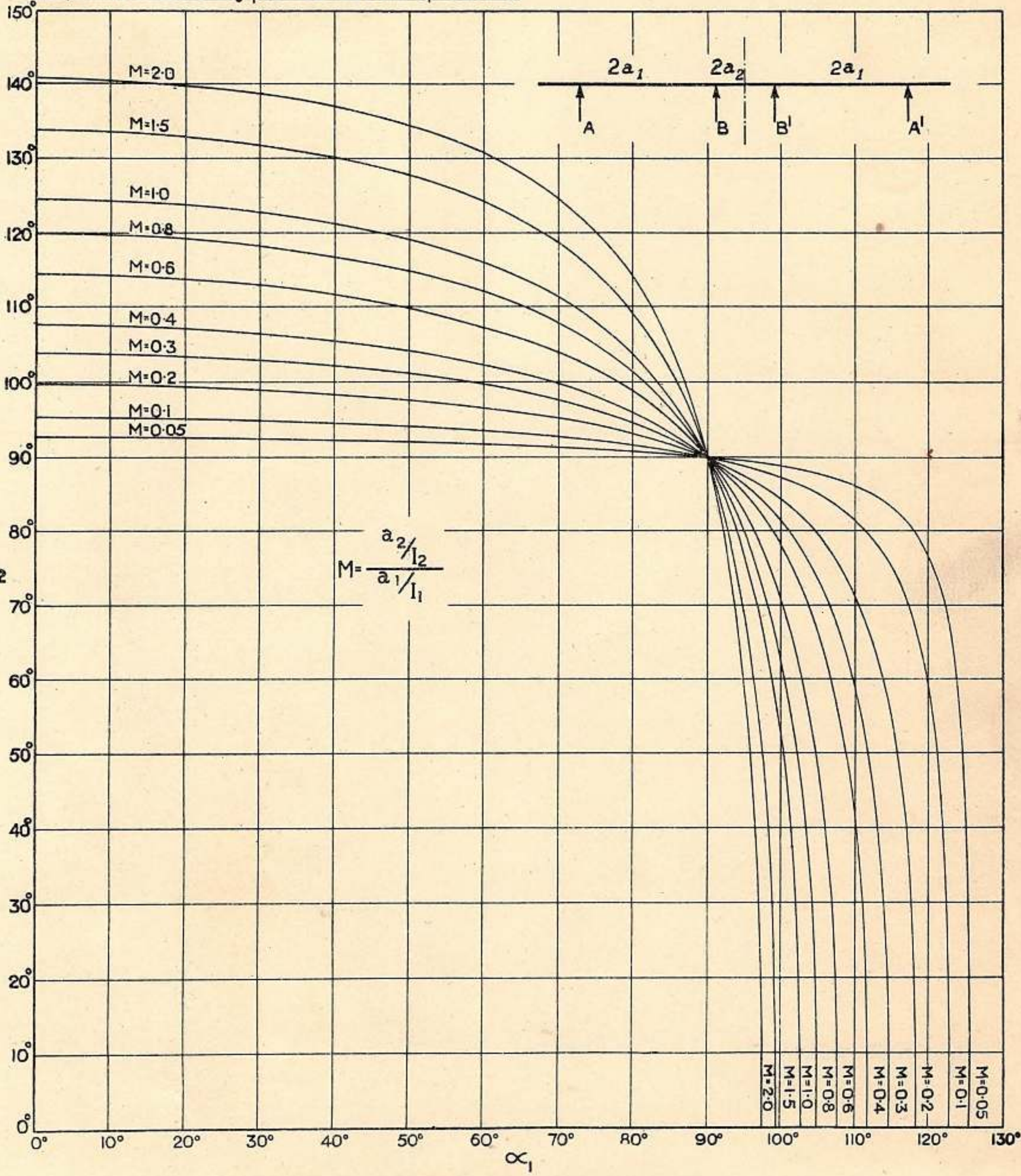

FIG. 10. CHAP. VI.

Referring to fig. 11 let

w_1 and P_1 be uniform up load and end load respectively in bay l_1.

w_2 and P_2 be uniform up load and end load respectively in bay l_2.

Assume reactions positive when acting in the direction shown.

The deflection of D (positive when D is above AB) is the sum of two terms $(\delta + \delta_1)$, where δ_1 is the deflection when the beam is unloaded and δ is the increased deflection under load. δ_1 is known from the geometry of the structure (it will be negative for the configuration of fig. 11), and the five unknowns δ, M_A, M_B, M_C, M_D are given by the following three equations together with two end conditions :—

$$\frac{l_1}{I_1} M_A f(\alpha_1) + \frac{l_2}{I_2} M_B f(\alpha_2) + 2\, M_D \left\{ \frac{l_1}{I_1} \phi(\alpha_1) + \frac{l_2}{I_2} \phi(\alpha_2) \right\} + 6E\, \delta \left(\frac{1}{l_1} + \frac{1}{l_2} \right)$$

$$= \frac{w_1\, l_1^3}{4\, I_1}\, \psi(\alpha_1) + \frac{w_2\, l_2^3}{4\, I_2}\, \psi(\alpha_2) \qquad \ldots \quad (1)$$

$$\frac{l_2}{I_2} M_D f(\alpha_2) + \frac{l_3}{I_3} M_C f(\alpha_3) + 2\, M_B \left\{ \frac{l_2}{I_2} \phi(\alpha_2) + \frac{l_3}{I_3} \phi(\alpha_3) \right\} - \frac{6E\delta}{l_2}$$

$$= \frac{w_2\, l_2^3}{4\, I_2}\, \psi(\alpha_2) + \frac{w_3\, l_3^3}{4\, I_3}\, \psi(\alpha_3) \qquad \ldots \quad (2)$$

$$0 = \frac{w_1\, l_1 + w_2\, l_2}{2} + \frac{M_D - M_A}{l_1} + \frac{M_D - M_B}{l_2} + \left\{ \frac{P_1}{l_1} + \frac{P_2}{l_2} \right\} (\delta + \delta_1) \qquad \ldots \quad (3)$$

This treatment assumes that the end load acts along the straight line joining the two undeflected supports on each side of the deflected supports. This is not strictly accurate when part of the end load is due to the drag bracing, as this will come on to the spar at the drag bay nodes which will usually have deflected above or below the straight line joining the undeflected supports. In most cases, however, the drag bracing contribution at any deflected drag bay node will be a small percentage of the total end load and so the error involved in the above assumption will be small. It will usually be sufficiently accurate to take the mean end load so that instead of dealing with P_1 and P_2, the mean load P could be taken as acting over both bays l_1 and l_2, where

$$P = \tfrac{1}{2}(P_1 + P_2)$$

(iv) *Case* 4. *Eccentrically loaded struts with concentrated lateral loads.—(a) One lateral load only* :—

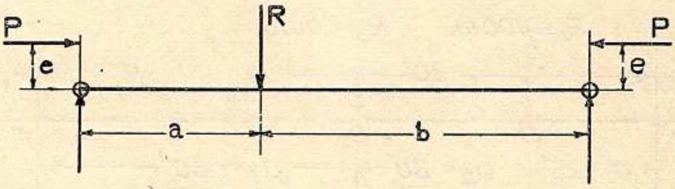

FIG. 12.—CHAP. VI.

Referring to fig. 12.

M_{max} occurs at a point distant x from the outside end of the longer bay (bay b), where x (measured positive towards the concentrated load) is given by :—

$$\tan \mu x = \tan \frac{\mu (a + b)}{2} + \frac{R \sin \mu a}{Pe\, \mu \sin \mu (a + b)}$$

and

$$M_{max} = Pe \sec \mu x.$$

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If x is greater than $\frac{b}{2}$, M_{max} occurs at the concentrated load, and is given by :—

$$M_{max} = \frac{R \sin \mu a \sin \mu b}{\mu \sin \mu (a + b)} + \frac{Pe (\sin \mu a + \sin \mu b)}{\sin \mu (a + b)}$$

(*b*) *Several equal lateral loads arranged symmetrically.*

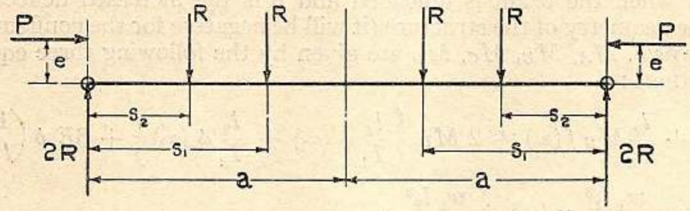


FIG. 13.—CHAP. VI.

Referring to fig. 13.

For 2 pairs of equal loads.—M_{max} occurs at the mid-point of the strut and is given by :—

$$M_{max} = \frac{R}{\mu} \sec \mu a (\sin \mu s_1 + \sin \mu s_2) + Pe \sec \mu a.$$

For n pairs of equal loads :—

$$M_{max} = \frac{R}{\mu} \sec \mu a (\sin \mu s_1 + \sin \mu s_2 + \ldots + \sin \mu s_{n-1} + \sin \mu s_n) + Pe \sec \mu a.$$

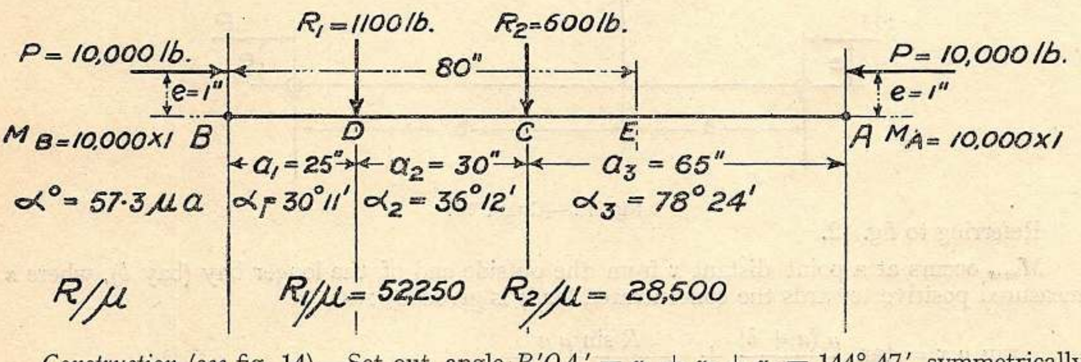
(*c*) *Two unequal loads unsymmetrically placed.*—The easiest way of determining the maximum bending moment and bending moment diagram is by the polar diagram method described in R. & M. 1233. A numerical example is given below.

Example.—Required to draw the bending moment diagram for a strut 120 in. long, end load 10,000 lb., eccentricity of loading 1 in. and concentrated lateral loads of 1,100 and 600 lb. distant 25 in. and 55 in. respectively from one end.

$$E = 1 \cdot 5 \times 10^6 \text{ lb. per sq. in. } I = 15 \text{ ins.}^4.$$

Tabulate the necessary data as follows :—

$$\mu = \sqrt{\frac{P}{EI}} = \cdot 02105 \quad \frac{1}{\mu} = 47 \cdot 5''.$$



Construction (*see* fig. 14).—Set out angle $B'OA' = \alpha_1 + \alpha_2 + \alpha_3 = 144° 47'$ symmetrically about a vertical. Set off angle $B'OD' = \alpha_1 = 30° 11'$ and angle $A'OC' = \alpha_3 = 78° 24'$. Mark off OB and $OA = M_B$ and M_B respectively, in this case 10,000 lb. ins.

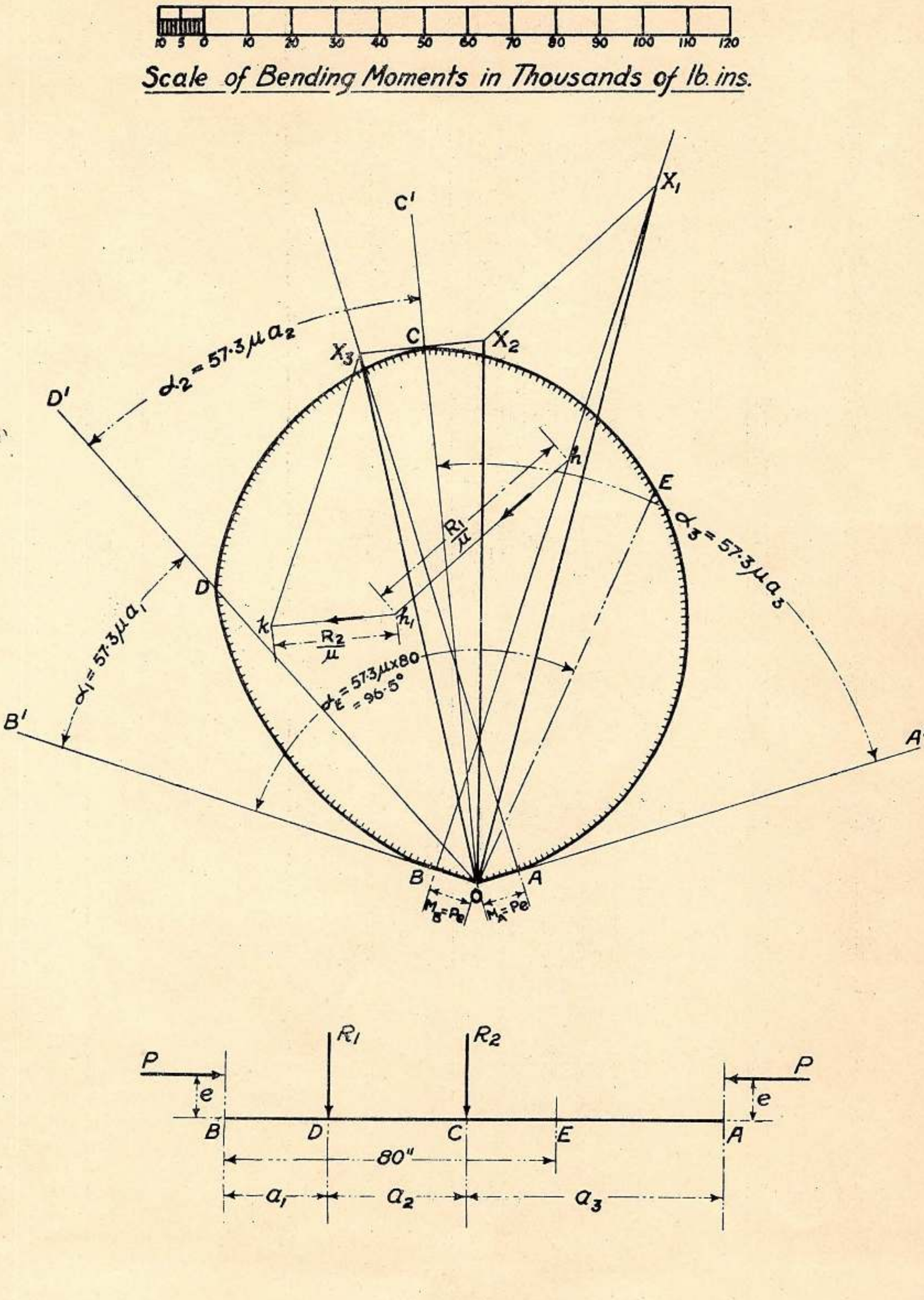


FIG. 14, CHAP. VI.

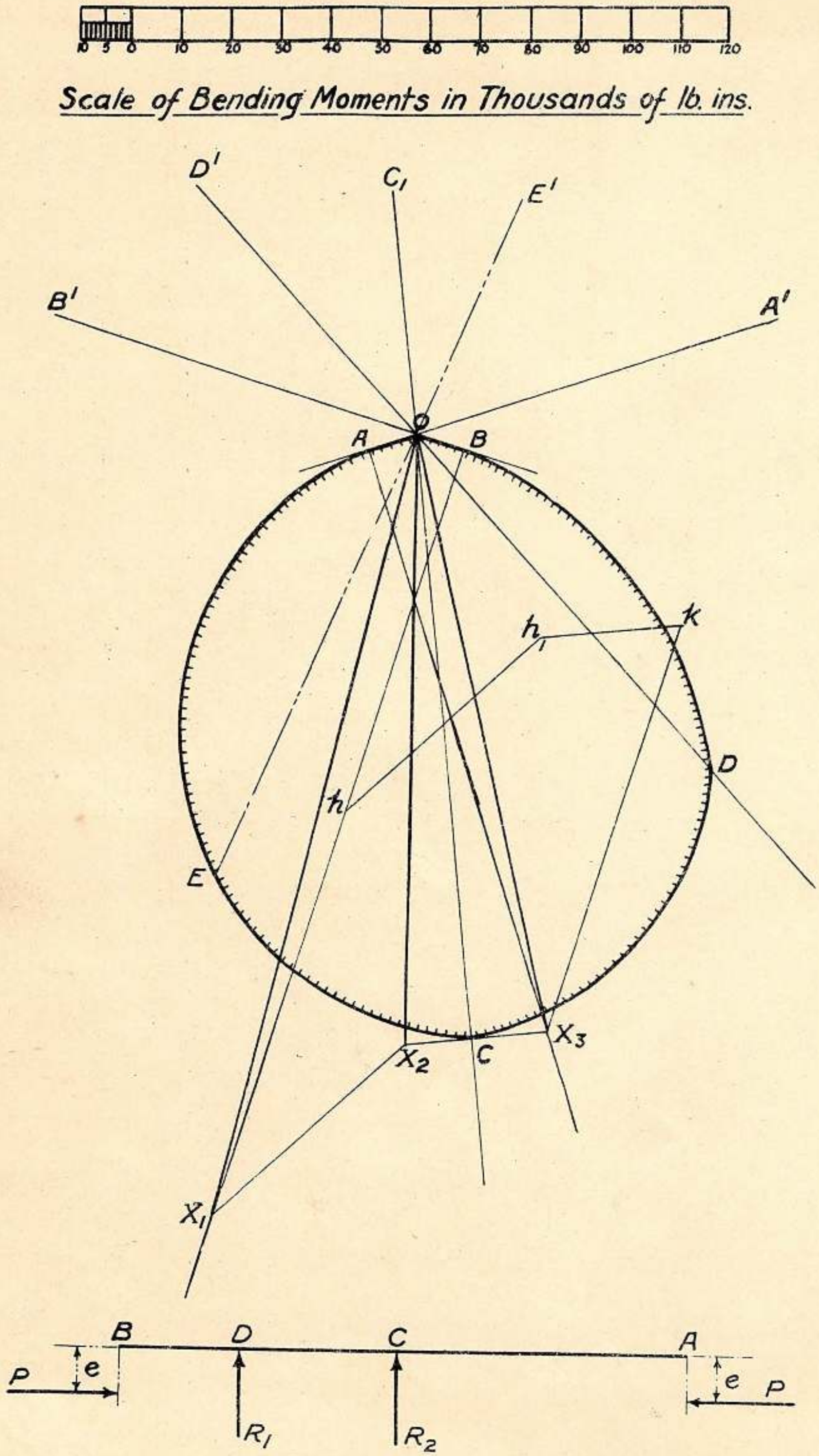
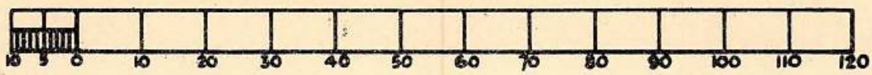


FIG. 15, CHAP. VI.



Scale of Bending Moments in Thousands of lb. ins.

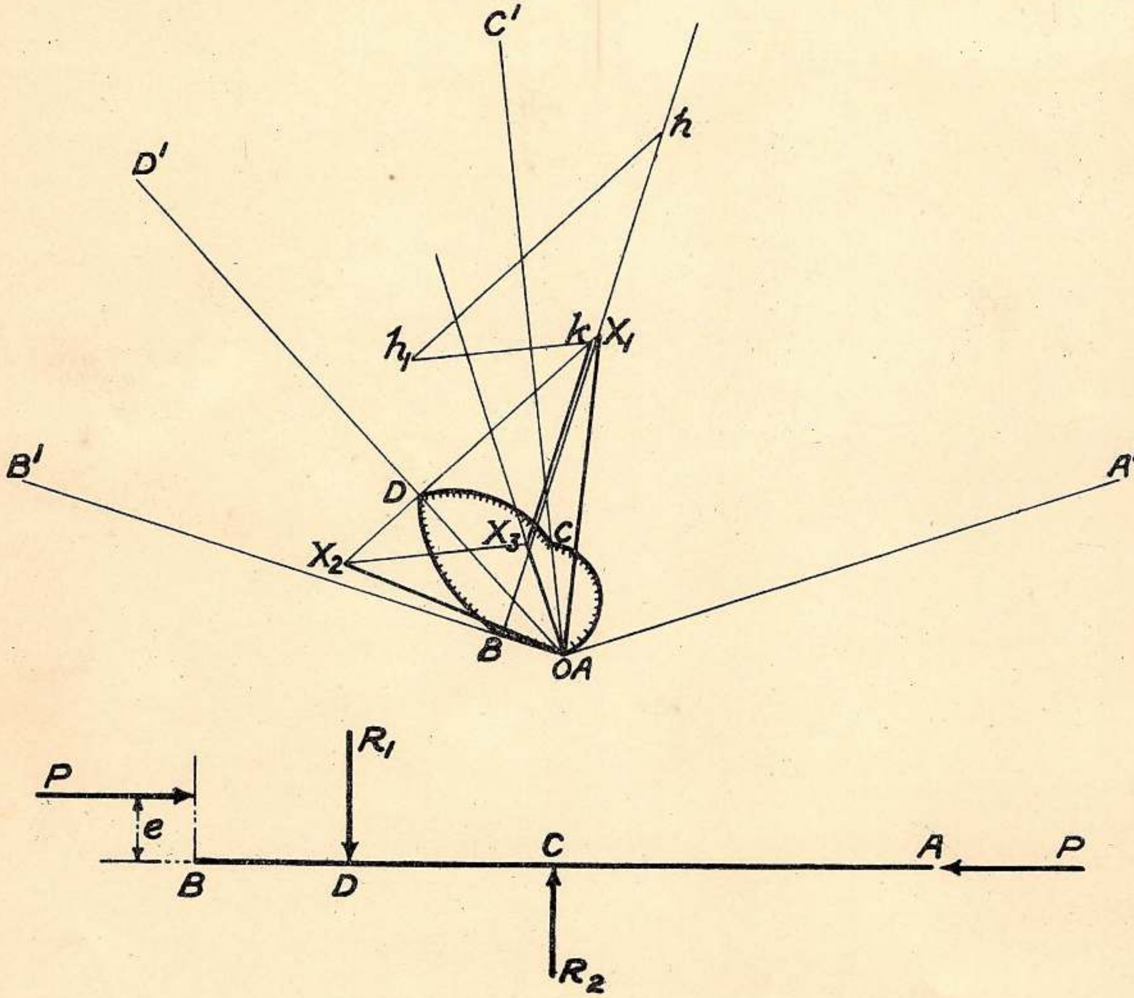


FIG. 16, CHAP. VI.

The following sign conventions should be rigidly adhered to. Bending moments tending to make the beam deflect thus $\smile$ are considered positive and should be marked off along the radius vector *upwards* from 0. Up loads on the beam are considered positive and should be marked off (*see* below) at right angles to the radius vector and *to the right* when looking upwards along the radius vector.

Erect perpendiculars at B and A and through any point h on perpendicular at B draw hh_1 at right angles to OD' and equal to R_1/μ, *i.e.* 52,250. R_1 being a down load, hh_1 is drawn to the left when looking upwards along OD'. Draw h_1k at right angles to OC' and equal to R_2/μ, *i.e.* 28,500. Again h_1k is drawn to the left because R_2 is a down load. Draw kX_3 parallel to Bh, meeting perpendicular at A in X_3. Draw X_3X_2 parallel, equal to, and in the same sense as kh_1 and X_2X_1 parallel, equal to, and in the same sense as h_1h so that X_1 falls on Bh produced. On OX_1 as diameter, describe an arc starting from B, moving in clockwise direction and ending on OD at D (D is on X_1X_2 produced). On OX_2 as diameter, describe an arc from D in clockwise direction and ending on OC' at C (C is on X_2X_3). On OX_3 as diameter, describe an arc from C to A. These three arcs form the bending moment diagram. Thus the bending moment at a point E distant 80 ins. from B is represented in fig. 14 by OE, the angle $B'OE$ being $57 \cdot 3$ $\mu \times 80 = 96 \cdot 5°$. The maximum bending moment occurs in this case at C and is equal to OC.

As a further illustration of the method of drawing the diagram, fig. 15 shows the form it takes when R_1 and R_2 act upwards and the eccentricity is in the opposite direction.

The bending moment diagram is of course identical but of different sign.

A third example, fig. 16 shows the shape of the diagram when R_1 and R_2 are in opposite directions and the eccentricity at A is zero.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