CHAPTER VII.—THE AERODYNAMIC LOAD DISTRIBUTION ON TAPERED AND TWISTED WINGS

(Monoplanes only. *See* chapter V, section I, para. 1.)

1. General.—(i) This chapter deals with the calculation of the shape of the curve of load distribution along the span of a wing, the chord of which varies from wing root to wing tip. An acceptable alternative method involving the use of "induction factors" is given in R. & M. No. 1643. A method is also given for calculating the pitching moment of such a wing.

(ii) Prior to the development of the Prandtl theory, tapered wings were commonly dealt with on the assumption that each section was independent of those on either side, this method being known as the "Strip Theory." It can be shown that in many cases calculations based on this theory may lead to an over-estimation of the strength of the wing structure, and hence the more accurate method described in this chapter should be used in preference to the "Strip Theory."

(iii) A twisted wing is defined as one in which the no-lift lines of all the sections are not parallel. This may be due to a geometrical twist which tilts the chord lines of the various sections relative to each other, or it may be due to an aerodynamic twist caused by a variation in the no-lift angles of the aerofoils forming the wing at different points along the span. Both these effects are often present simultaneously. For the purpose of the calculations described in this chapter the total twist only is of importance, and it is immaterial how much of this is due to the one cause and how much to the other.

(iv) For an untwisted wing the shape of the load distribution curve is the same for both normal flight cases and for the terminal velocity dive and inverted flight cases. This is not so when the wing is twisted as then the load distribution curve may vary considerably with incidence, particularly at low angles of incidence.

(v) The calculations described below deal with a wing the root of which is on the centre line of the aeroplane. In para. 3 (ix) a correction is given to take account of the loss in lift due to the presence of the fuselage which masks, or replaces, a portion of the wing at the centre. The root chord, c_0, of the wing is found by producing the leading and trailing edges of the actual wing up to the centre line of the aeroplane. The characteristics of the aerofoil section at the wing root (*i.e.*, no-lift angle, slope of lift-incidence curve, etc.) may be found by plotting the known characteristics for the various sections along the actual wing and producing by eye the curves so formed beyond the point at which the wing merges into the fuselage up to the centre line of the aeroplane.

2. Notation.—(i) The notation employed is as shown in fig. 1, together with the following :—

α = angle of incidence of any section of the wing measured (*in radians*) from the no-lift line of that particular section.

a = slope of the curve of lift coefficient against angle of incidence (radians) for the section considered, corrected to infinite aspect ratio (*see* below).

c = chord of section.

$\left.\begin{matrix} a_0 \\ \alpha_0 \\ c_0 \end{matrix}\right\}$ similar quantities at the wing root, *i.e.*, on the centre line of the aeroplane (*see* above).

$$u = \frac{ac}{4s}$$

$$\cos\theta = \frac{x}{s} \text{ (\textit{see} fig. 1).}$$

S = area of complete wing (port and starboard) including that portion of the wing masked or replaced by the fuselage, referred to in the text as the " assumed wing."

λ = tip chord/root chord for straight tapered wing with maximum chord at the wing root.

k_L = mean lift coefficient of the whole wing, assuming the centre (fuselage) portion of the wing to be a lifting surface.

A = aspect ratio of wing $= \frac{(2s)^2}{S.}$

s = semi-span.

w = loading in pounds per inch run along span.

(ii) *Determination of " a."*—The slope a, of the lift curve for two-dimensional flow is always very close to π. In the absence of any data the value 3·0 should be used. Generally, tests will be available of finite wings of aspect ratio A the slope of the lift curve being a_A. If the results have not been corrected for wind tunnel interference this must first be done.

(*a*) If a' is the observed slope then a_A the true slope for the aspect ratio A is given by

$$a_A = \frac{a'}{1 + \cdot 274 \frac{S_T a'}{C}}$$

where S_T is the area of the model wing and C the cross sectional area of the tunnel. This formula applies to square closed tunnels and to the duplex tunnel when the model is tested horizontally.

(*b*) a_A is now corrected to infinite aspect ratio by the curve given in fig. 2. This shows the ratio $\frac{a}{a_A}$ as a function $\frac{A}{a_A}$, where A is the aspect ratio of the tested wing.

(*c*) The no-lift angle of the finite wing is the same as that of the infinite wing.

3. Lift coefficient.—(i) *General method.*—The method consists in expressing the load distribution curve as a Fourier's series and determining the coefficients by fitting the Fourier expression to the characteristics of the wing at selected points. To reduce the labour involved in calculating the load distribution curve for any given wing the analysis has been carried as far as possible in general terms, the point at which the general treatment must give place to detailed numerical treatment being dependent upon the type of wing concerned.

(ii) It is convenient to divide tapered wings into the following groups :—

(*a*) Straight tapered untwisted wings. In this type the leading and trailing edges are straight, the chord decreasing uniformly from the root to the tip.

(*b*) Straight tapered twisted wings.

(*c*) Curved tapered untwisted wings.

(*d*) Curved tapered twisted wings.

These four types are dealt with below in this order, the simplest type being considered first.

(iii) *Straight tapered untwisted wings.*—The load distribution curve for a straight tapered wing can be obtained directly from fig. 3. This gives several curves corresponding to a series of values of λ, where

$$\lambda = \frac{\text{chord at wing tip.}}{\text{chord at wing root.}}$$

In practice the wing tip will usually be rounded to some extent, in which case the tip chord is determined by producing the leading and trailing edges to cut the tangent at the wing tip.

The top half of fig. 4 is derived from fig. 3 by cross plotting and is convenient for interpolation. The values of the ordinates in fig. 4 correspond to a root incidence of 1 radian from no-lift, the linear relationship between k_L and α being assumed to hold up to this incidence.

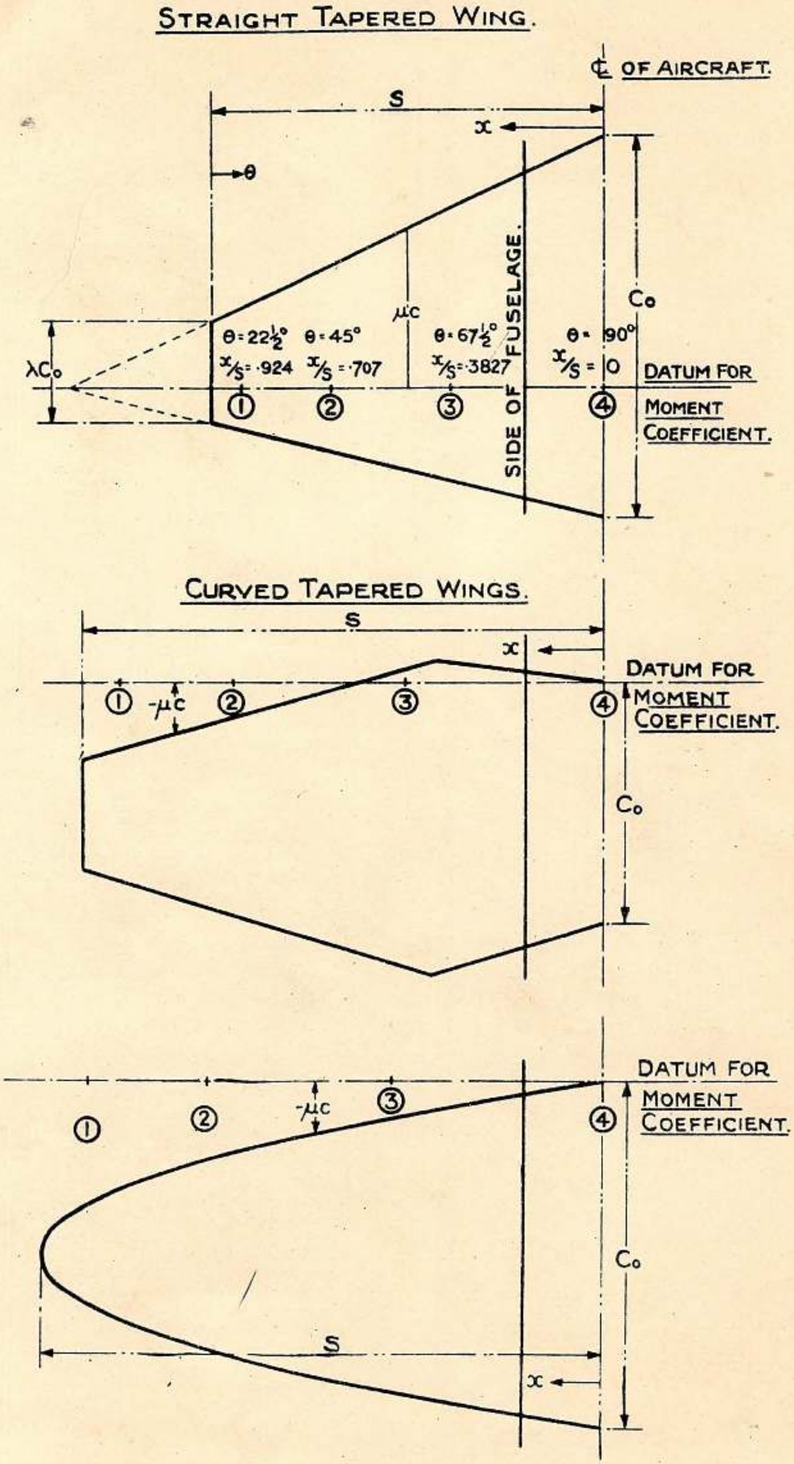


FIG. 1 Chap. VII.

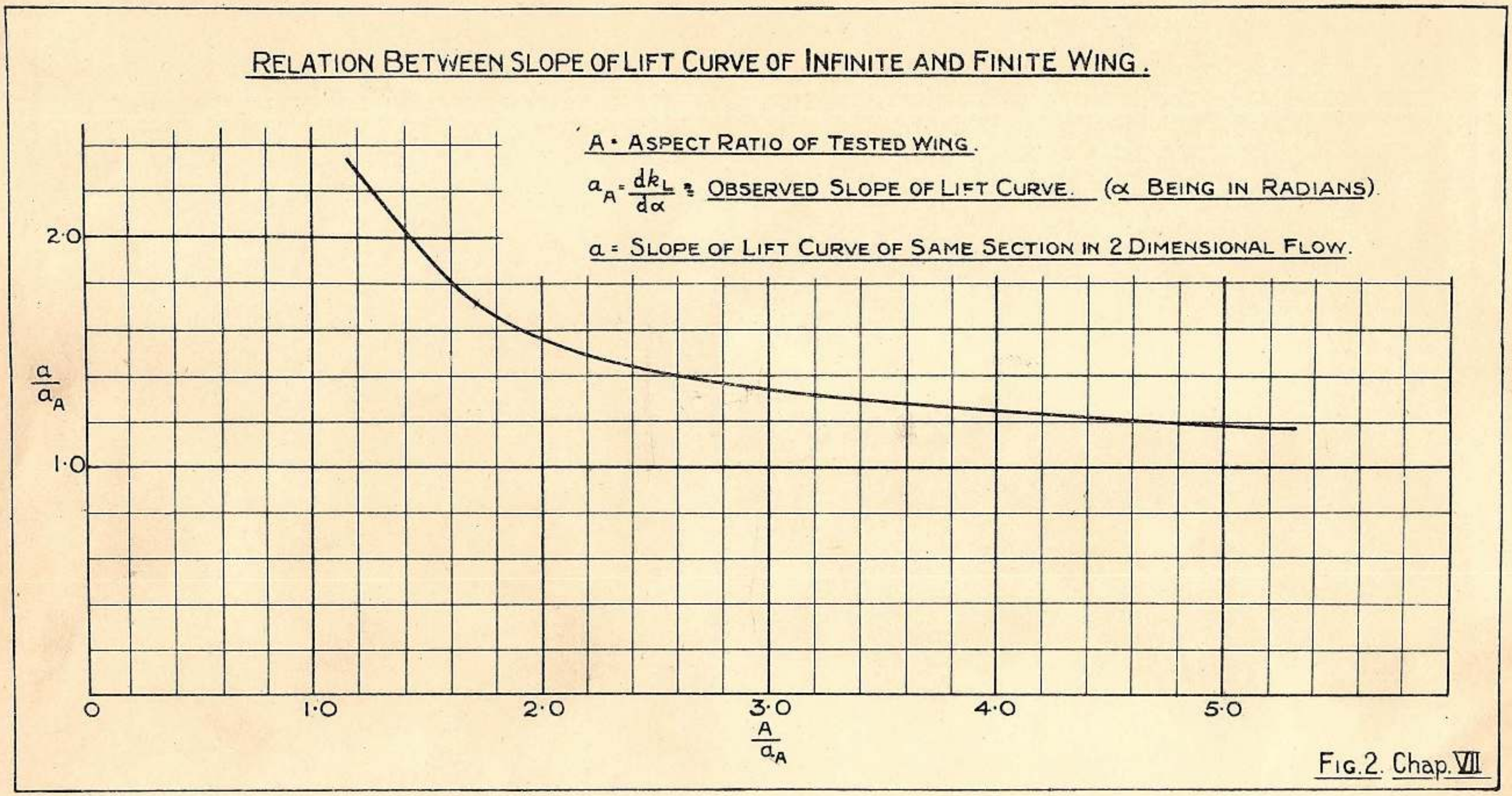


FIG. 2. Chap. VII

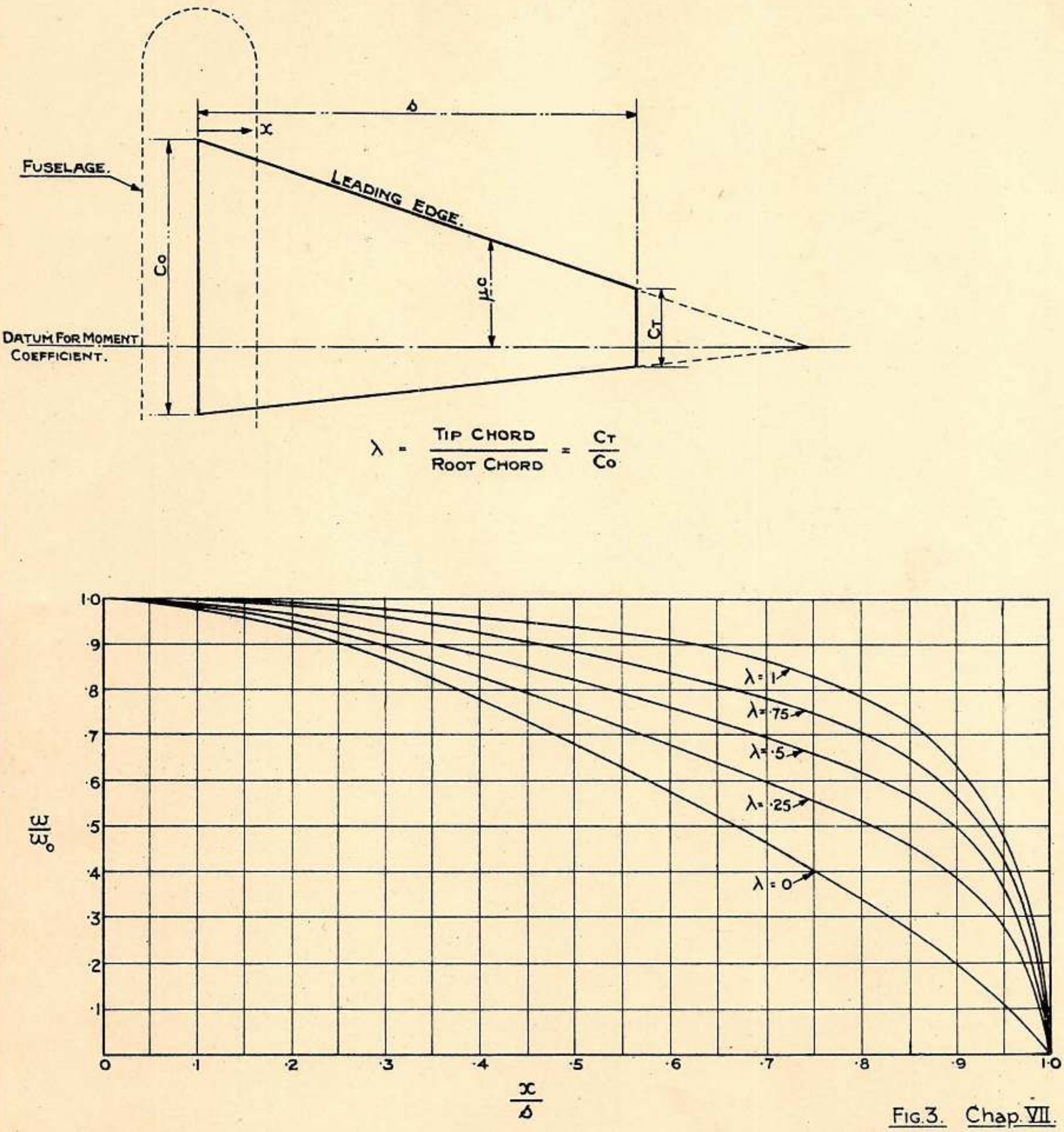


FIG. 3. Chap. VII.

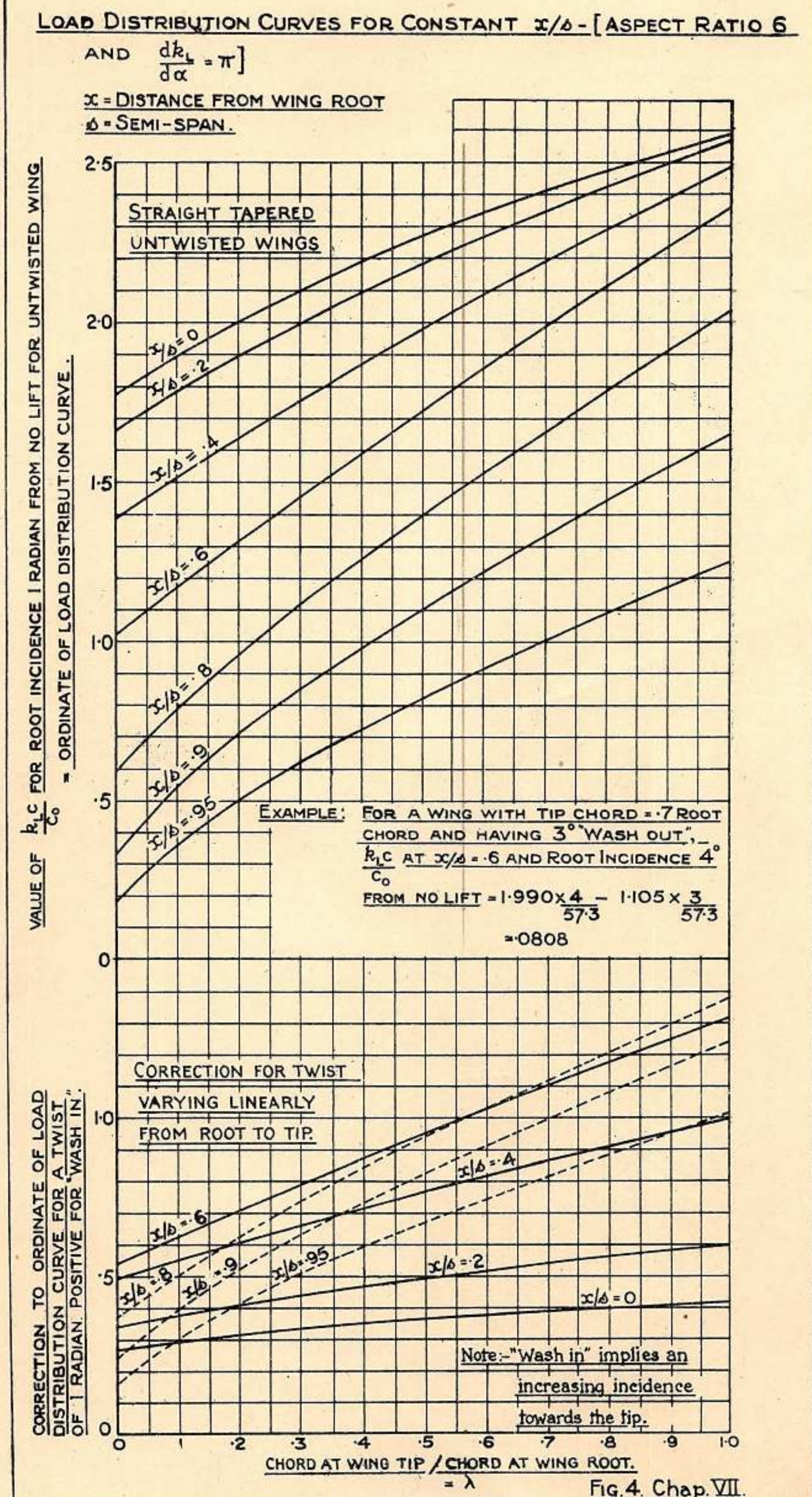
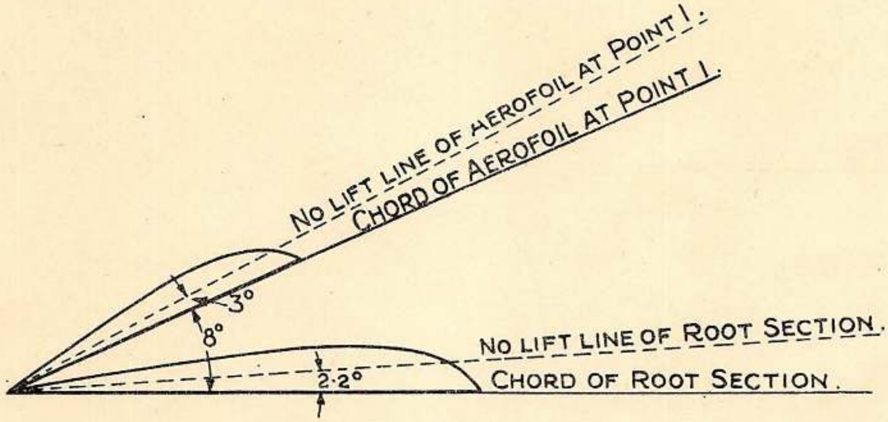


FIG. 4. Chap. VII.

TWISTED WING.

DIAGRAM TO ILLUSTRATE METHOD OF DETERMINING THE INCIDENCE OF ANY SECTION FROM NO LIFT (α) WHEN THE ROOT SECTION IS AT NO LIFT ($\alpha_0 = 0$)



THE "NO LIFT LINE" GIVES THE DIRECTION OF THE WIND FOR ZERO LIFT.

	SECTION 1	2	3	4 ROOT.
ANGLE BETWEEN CHORD OF SECTION AND CHORD OF CENTRE SECTION. POSITIVE WHEN ROOT CHORD IS AT LESS INCIDENCE THAN THE CHORD OF THE SECTION CONCERNED.	+ 8°			0
NO LIFT ANGLE OF SECTION.	−3°			−2·2°
INCIDENCE OF SECTION (MEASURED FROM ITS OWN NO LIFT LINE) WHEN THE ROOT SECTION IS AT NO LIFT; I.E. ANGLE BETWEEN THE NO LIFT LINES OF THE SECTION CONCERNED AND THE ROOT SECTION.	8°+3°−2·2° = 8·8°			0
IN RADIANS.	$\alpha_1 = \cdot 1535$	$\alpha_2 =$	$\alpha_3 =$	$\alpha_4 = \alpha_0 = 0$

FIG. 5. Chap. VII.

Fig. 3 only gives the shapes of the curves. The curves of figs. 3 and 4 are calculated for a wing of aspect ratio of 6 and $\frac{dk_L}{d\alpha} = \pi$. The effect of small changes in these quantities may be ignored.

The load grading curve for a rectangular wing ($\lambda = 1$) should only be used for monoplanes. For a rectangular wing biplane the load distribution is assumed to be uniform from the centre line of the aircraft to a point distant $1 \cdot 2c$ from the wing tip, the loading falling away at the wing tip according to the curve given in chapter *V*, section I, fig. 1.

(iv) *Straight tapered twisted wings in which the twist varies linearly along the span.*—It may sometimes happen that the twist of a straight tapered wing at any point along the span is a linear function of the distance of that point from the wing root. A tabular method for determining the aerodynamic twist at points along the wing is given in fig. 5, and linear distribution of twist is indicated by α_1, α_2, α_3, and α_4 being proportional to the distances of sections 1, 2, 3 and 4 from the wing root. The load distribution curve for such a wing can be obtained directly from fig. 4, the curves on the lower portion of the diagram giving the correction to be applied to the ordinates of the loading curve of an untwisted wing given in the upper part of the diagram. This correction must be multiplied by the twist appropriate to the wing under consideration, as shown in the example given on the figure. It will be seen at once from this construction that the shape of the curve varies with incidence, since a constant quantity, representing the correction due to twist, is added to a quantity which is a function of the incidence.

(v) *General case of a curved tapered untwisted wing.*—The load distribution curve for this type of wing should be calculated as explained below. The possible variations in plan form are too large for it to be feasible to give generalised curves. The method is as follows :—

(*a*) Choose four points along the span of the wing such that the characteristics of the aerofoils at these points are known. An arithmetical simplification is obtained by taking the points given in table I.

TABLE I

Point.	$\frac{x}{s}$	chord, c .	a (*see* para. 2).	$\frac{ac}{4s} = u$
4	0			$u_4 =$
3	·3827			$u_3 =$
2	·7071			$u_2 =$
1	·9239			$u_1 =$

and it will be assumed to begin with that the wing characteristics at these points are known. Methods are given in sub-para. (vii) for dealing with any four points, and in sub-para. (viii) for taking into account a greater number of points than four.

(*b*) Determine the characteristics of the aerofoils forming the wing at the points given in table I, and fill in the table.

(*c*) Insert the values of u calculated in table I into the following four simultaneous equations, taking α_4 (and hence α_1, α_2 and α_3) equal to 1 :—

$$(0{\cdot}3827 + u_1)\, A_1 + 2{\cdot}414\,(0{\cdot}3827 + 3u_1)\, A_3 + 2{\cdot}414\,(0{\cdot}3827 + 5u_1)\, A_5 + (0{\cdot}3827 + 7u_1)\, A_7 = u_1\, \alpha_1 \quad (1,$$

$$(0{\cdot}7071 + u_2)\, A_1 + (0{\cdot}7071 + 3u_2)\, A_3 - (0{\cdot}7071 + 5u_2)\, A_5 - (0{\cdot}7071 + 7u_2)\, A_7 = u_2\, \alpha_2 \quad (2)$$

$$(0{\cdot}9239 + u_3)\, A_1 - 0{\cdot}4142\,(0{\cdot}9239 + 3u_3)\, A_3 - 0{\cdot}4142\,(0{\cdot}9239 + 5u_3)\, A_5 + (0{\cdot}9239 + 7u_3)\, A_7 = u_3\, \alpha_3 \quad (3)$$

$$(1 + u_4)\,A_1 - (1 + 3u_4)\,A_3 + (1 + 5u_4)\,A_5 - (1 + 7u_4)\,A_7 = u_4\,\alpha_4 \quad \ldots \quad (4)$$

Solve these equations for A_1, A_3, A_5 and A_7. A convenient tabular method of solution is given in table V at the end of this chapter. Columns 6 and 7 in this table refer only to twisted wings. The load distribution curve over the span is given by the equation :—

$$\frac{k_{LC}}{4s} = A_1 \sin\theta + A_3 \sin 3\,\theta + A_5 \sin 5\,\theta + A_7 \sin 7\,\theta \quad \ldots \quad (5)$$

and can be drawn out by giving a number of values to θ corresponding to various points along the span. Values of $\sin n\theta$ at seven points along the span are given below.

TABLE II

x/s	θ	$\sin\theta$	$\sin 3\theta$	$\sin 5\theta$	$\sin 7\theta$
0	90°	1·0	−1·0	1·0	−1·0
0·2	78° 28′	·9798	— ·8231	·5348	— ·161
0·4	66° 25′	·9164	— ·3307	— ·4624	·9663
0·6	53° 08′	·8001	·3518	— ·9971	·2068
0·8	36° 52′	·60	·9360	— ·0756	— ·9783
0·9	25° 51′	·4361	·9765	·7853	— ·017
0·95	18° 12′	·3123	·8151	·9998	·7945
1·0	0	0	0	0	0

(*d*) The load distribution curve corresponding to equation (5) can be converted into a curve of k_L by multiplying the ordinates by $\frac{4s}{c}$, c having the value appropriate to each individual point considered. The value so obtained will correspond to an angle of incidence of one radian from no lift. k_L is directly proportional to incidence, so that its value for any given angle is easily calculated. The centre of pressure position at each point along the span can then be calculated from the known relation between lift coefficient and centre of pressure as given by standard wind tunnel tests on rectangular aerofoils, and hence the spar loading curves can be obtained.

(*e*) The shape of the wing loading curve is the same for all angles of incidence, the scale alone varying. The mean lift coefficient is dependent upon A_1 only and is given by the expression

$$\bar{k}_L = \frac{2\pi s^2}{S} A_1 \quad \ldots \quad (6)$$

The value of A_1 for a straight tapered untwisted wing can be obtained from fig. 6 by putting $\phi = 0$. This mean lift coefficient applies to the assumed wing which extends to the centre line of the aeroplane. The correction to $\bar{k}_L$ to allow for the centre (fuselage) portion of the wing is given in para. 3 (ix).

(vi) *General case of curved tapered twisted wing.*—The shape of the load distribution curve for a twisted wing varies with incidence and hence it is necessary to calculate values of A_1, A_3, A_5 and A_7 for more than one value of root incidence. The work is conveniently arranged as follows :—

(*a*) Using the same four standard points as for an untwisted wing, insert appropriate values in table III for the wing characteristics at each point and work out values of u and $u\,\alpha$, taking three different values of α_0.

VALUE OF A_1 FOR STRAIGHT TAPERED WING WITH LINEAR DISTRIBUTION OF TWIST.

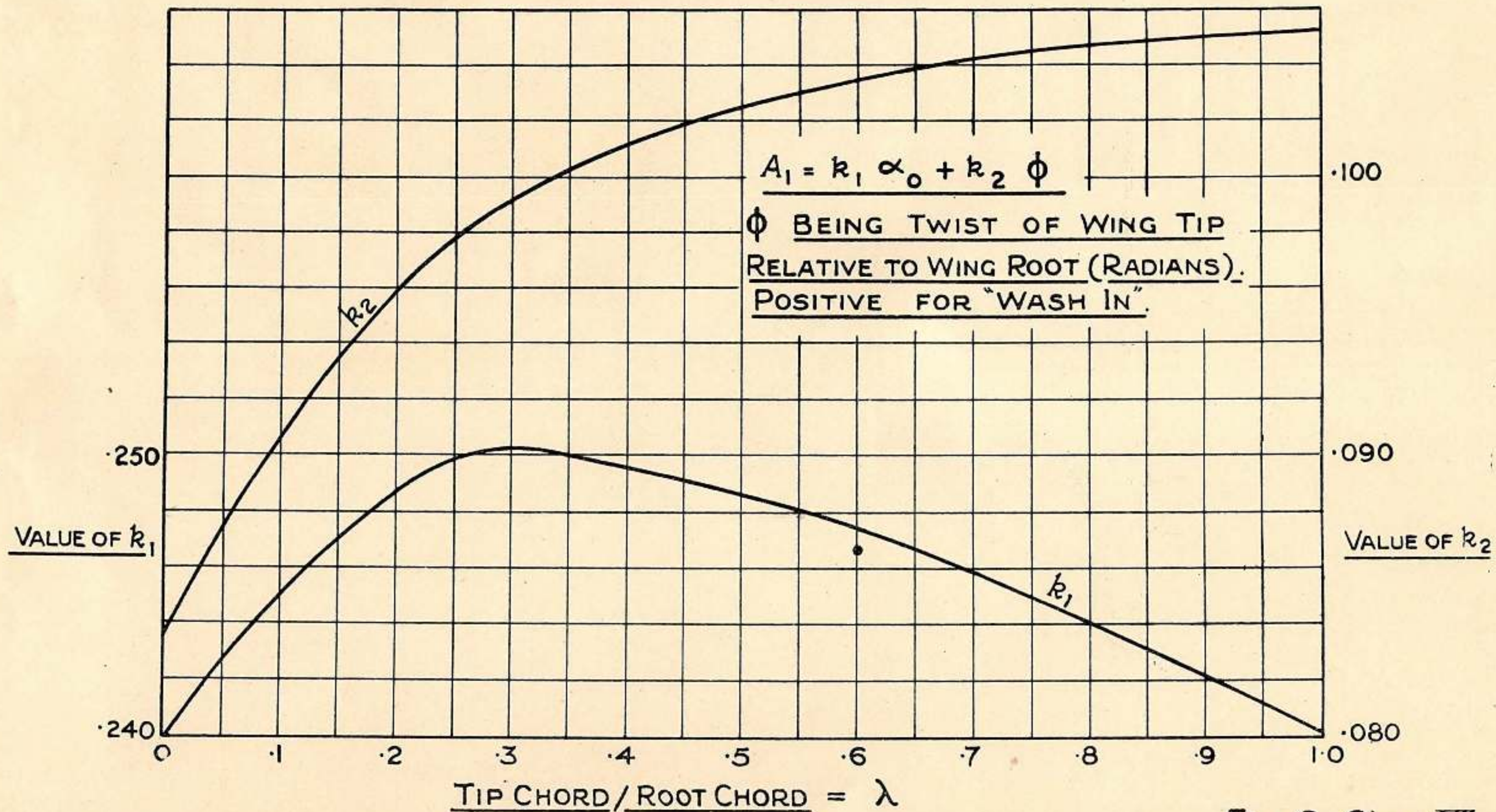


Fig. 6. Chap. VII

TABLE III

Point.	$\frac{x}{s}$	Incidence from no-lift in radians when $\alpha_0 = 0$	Chord c	a (*see* para. 2)	$\frac{ac}{4s} = u$	$u\alpha$ (α in radians)		
						$\alpha_0 = 0$	$\alpha_0 = \cdot 5$	$\alpha_0 = 1 \cdot 0$
4	0	$\alpha_4 = \alpha_0 = 0$			$u_4 =$	$u_4\,\alpha_4 =$		
3	·3827	$\alpha_3 =$ } *See* Fig. 5			$u_3 =$	$u_3\,\alpha_3 =$		
2	·7071	$\alpha_2 =$ } *See* Fig. 5			$u_2 =$	$u_2\,\alpha_2 =$		
1	·9239	$\alpha_1 =$ } *See* Fig. 5			$u_1 =$	$u_1\,\alpha_1 =$		

(*b*) Insert these values of u and $u\alpha$ into equations (1) to (4). This will give three sets of four simultaneous equations, since there are three values of $u\,\alpha$ corresponding to three different root incidences at each point. These three sets of equations can be solved as shown in the numerical example given in para. 6 with little more labour than is involved in solving one set. The values of A_1, A_3, A_5 and A_7 should be linear functions of the root incidence, and hence the three values of α_0 taken above give a partial check upon the work.

(*c*) Express these constants in terms of α_0 :—

$$\left.\begin{aligned} A_1 &= h_1\,\alpha_0 + h_2 \\ A_3 &= h_3\,\alpha_0 + h_4 \\ A_5 &= h_5\,\alpha_0 + h_6 \\ A_7 &= h_7\,\alpha_0 + h_8 \end{aligned}\right\} \qquad \ldots \quad (7)$$

α_0 being in radians.

(*d*) To draw the load distribution curves representing stalling, maximum speed, fast glide and terminal velocity dive conditions, calculate the mean lift coefficients (based upon the true wing area, *i.e.*, the $\bar{k}_L$ of para. 3 (ix)) corresponding to these speeds, and hence determine the root incidence for each case from the relation

$$\bar{k}_L = \frac{2\pi\, s^2}{S}\,(h_1\,\alpha_0 + h_2) \qquad \ldots \quad (8)$$

The connection between the mean lift coefficient on the true wing area, $\bar{k}'_L$, and $\bar{k}_L$ is given in para. 3 (ix).

(*e*) Knowing α_0, the values of A_1, A_3, A_5, and A_7, are found from equation (7), and inserting these values in equation (5) gives the load distribution curve.

(vii) *Aerofoil characteristics not known at the four standard points.*—In the case of a wing the aerofoil section of which varies along the span, it may not always be convenient to take the four standard points given in tables I and III. Four other points may be taken if desired, at distances x_1, x_2, x_3 and x_4 from the wing root, where

$$\frac{x_1}{s} = \cos\ \theta_1$$

$$\frac{x_2}{s} = \cos\ \theta_2 \qquad \text{etc.}$$

The four equations (1) to (4) must then be replaced by the following :—

$$\sin\theta_1\,(\sin\theta_1 + u_1)\,A_1 + \sin 3\theta_1\,(\sin\theta_1 + 3u_1)\,A_3 + \sin 5\theta_1\,(\sin\theta_1 + 5u_1)\,A_5 + \sin 7\theta_1\,(\sin\theta_1 + 7u_1)\,A_7 = u_1\,\alpha_1\,\sin\theta_1 \quad (9)$$

$$\sin\theta_2\,(\sin\theta_2 + u_2)\,A_1 + \sin 3\theta_2\,(\sin\theta_2 + 3u_2)\,A_3 + \sin 5\theta_2\,(\sin\theta_2 + 5u_2)\,A_5 + \sin 7\theta_2\,(\sin\theta_2 + 7u_2)\,A_7 = u_2\,\alpha_2\,\sin\theta_2 \quad (10)$$

$$\sin\theta_3\,(\sin\theta_3 + u_3)\,A_1 + \sin 3\theta_3\,(\sin\theta_3 + 3u_3)\,A_3 + \sin 5\theta_3\,(\sin\theta_3 + 5u_3)\,A_5 + \sin 7\theta_3\,(\sin\theta_3 + 7u_3)\,A_7 = u_3\,\alpha_3\,\sin\theta_3 \quad (11)$$

$$\sin\theta_4\,(\sin\theta_4 + u_4)\,A_1 + \sin 3\theta_4\,(\sin\theta_4 + 3u_4)\,A_3 + \sin 5\theta_4\,(\sin\theta_4 + 5u_4)\,A_5 + \sin 7\theta_4\,(\sin\theta_4 + 7u_4)\,A_7 = u_4\,\alpha_4\,\sin\theta_4 \quad (12)$$

In choosing the positions of the four points it is advisable to arrange for them to be more closely grouped towards the tip of the wing, as the loading curve is varying more rapidly in this region.

(viii) *Aerofoil characteristics known at more than four points along the span.*—A method is described below for taking into account the aerofoil characteristics at more than four points along the span of the wing, without retaining more than four terms of the Fourier's series of equations (9) to (12).

If more terms in the series were to be retained no theoretical difficulty would arise; thus, if five points in the wing were considered, and the first five terms of the series retained, this would lead to five unknowns ($A_1 - A_9$) with five simultaneous equations for their determination. This method, however, soon becomes unmanageable, owing to the difficulty of solving a large number of simultaneous equations of this type with sufficient arithmetical accuracy. The labour involved is, in addition, usually unnecessary, as for most tapered wings the fifth and subsequent terms of the series have a negligible effect upon the shape of the load distribution curve. A type of wing in which these higher terms may be of importance is one in which the maximum chord is not at the wing root. In such a case sufficient terms should be calculated to ensure that subsequent terms may safely be ignored.

Dealing with a normal wing, in which the first four terms only need be considered, a method of taking into account more than four points on the wing is as follows. Suppose that ten points along the wing are taken, leading to ten equations of the type (9) to (12). These equations can be written in tabular form thus :—

A_1	A_3	A_5	A_7	
a_1	b_1	c_1	d_1	$= e_1$
a_2	b_2	c_2	d_2	$= e_2$
—	—	—	—	= —
—	—	—	—	= —
a_{10}	b_{10}	c_{10}	d_{10}	$= e_{10}$

Then by the method of least squares the best values of A_1, A_3, A_5 and A_7 are defined by the four simultaneous equations :—

A_1	A_3	A_5	A_7	
(aa)	(ab)	(ac)	(ad)	$= (ae)$
(ab)	(bb)	(bc)	(bd)	$= (be)$
(ac)	(bc)	(cc)	(cd)	$= (ce)$
(ad)	(bd)	(cd)	(dd)	$= (de)$

where the brackets indicate " the sum of all such terms," *i.e.*, $(ab) = a_1b_1 + a_2b_2 + a_3b_3 + \ldots\ldots + a_{10}b_{10}$. The majority of the summation terms in these equations are repeated once, thus reducing the labour of calculation.

This method can be generalised by replacing sums by integrals. Thus the summations (aa) (ab), etc., can be obtained by plotting the values of aa, ab, etc., along the span, and measuring the area enclosed by these curves by planimeter.

(ix) *Effect of fuselage on lift of wing.*—In the calculations described above the assumption has been made that the wing extends to the centre line of the aeroplane. In practice the centre part of the wing will be replaced by the body of the aeroplane, and hence the lift as calculated above will exceed the true lift by an amount equal to that taken by this non-existent central portion of the wing.

This discrepancy will not affect the shape of the load distribution curve, and hence in the case of untwisted wings no complication arises. That portion of the load distribution curve between the wing tip and the point at which the wing merges into the fuselage is all that is required for strength calculations, the scale being fixed by the consideration that the area under the load distribution curve must represent the known lift of the wing.

In the case of a twisted wing, however, where the load distribution curve varies with incidence, it is necessary to determine what root incidences will correspond to the various conditions of flight for which strength calculations are made. Hence the relation between $\overline{k}_L$ and $\overline{k}_L'$ is required, where $\overline{k}_L$ is as defined in para. 2 and $\overline{k}_L'$ is the mean lift coefficient over the actual wing (*i.e.*, as terminated by the fuselage) based upon the true wing area.

This relation is given approximately by the following expression :—

$$\overline{k}_L' = \overline{k}_L\left\{1 - \frac{S_1}{S - S_1}\left(\frac{4}{\pi} - 1\right)\right\} \quad \ldots \quad (13)$$

where S_1 is the amount by which the area of the assumed wing S exceeds the area of the actual wing. The approximations involved in the above formulæ are :—

(1) That for the assumed wing under consideration the ratio of the k_L on the centre line of the aeroplane to $\overline{k}_L$ is the same as for an elliptically tapered wing, and

(2) That on the assumed wing this centre k_L acts over the centre area S_1.

4. Drag coefficient.—(i) *Distribution of drag along the span.*—The wing drag coefficient at each point along the span is the sum of the induced drag, k_{D_1}, and the profile drag, k_{D_0}. The induced drag at the point $x/s = \cos\theta$ is given by the expression

$$\frac{k_{D_1} c}{4s} = \left\{\frac{k_L c}{4s}\right\}\frac{1}{\sin\theta}\left\{A_1 \sin\theta + 3\,A_3 \sin 3\theta + 5\,A_5 \sin 5\theta + 7\,A_7 \sin 7\theta\right\} \quad \ldots \quad (14)$$

where $\frac{k_L c}{4s}$ has the value already calculated in equation (5).

(ii) Values of A_1, A_3, A_5 and A_7 for a straight tapered untwisted wing, corresponding to an angle of incidence of 1 radian from no-lift, are given below.

λ	A_1	A_3	A_5	A_7
0	·240	− ·051	·002	− ·005
·25	·250	− ·012	·010	− ·002
·50	·249	·007	·010	− ·001
·75	·245	·020	·008	0
1·0	·240	·029	·006	·001

(iii) For strength calculations it will usually be sufficiently accurate to take the profile drag coefficient for most aerofoils in common use as a constant, $k_{D_0} = \cdot 006$. Strictly speaking, k_{D_0} should vary at each section with the aerofoil and the angle of incidence of the section, but for strength calculations this variation may usually be ignored and the above mean value assumed.

(iv) The wing drag at any point along the span is given by $D = (k_{D_0} + k_{D_1})\ c\rho\ V^2$ lb. per unit length along the span, and hence a drag load distribution curve can be drawn out as has been done in the case of the lift load.

5. Moment coefficient.—(i) *General.*—Let

k_m = moment coefficient of any section referred to its leading edge.

$\bar{k}_m$ = mean moment coefficient of wing referred to the datum line (*see* below).

$$= \frac{M}{Sc_0\ \rho\ V^2}$$

μc = distance between the datum and the leading edge at any point, positive when the datum is behind the leading edge.

e and b are constants such that

$$k_m = ek_L + b$$

e and b can be taken from wind tunnel tests on wings of finite aspect ratio.

A datum line is chosen arbitrarily. For the general wing the perpendicular to the line of symmetry through the leading edge of the root section is most convenient. For straight tapered wings it is convenient to choose the perpendicular which passes through the intersection of the leading and trailing edges produced, as for this datum μ is constant.

The mean moment coefficient of a tapered wing is given (neglecting the contribution of k_D) by the definite integral.

$$\bar{k}_m = \frac{2}{Sc_0} \int_0^s (k_m + \mu k_L)\ c^2\ dx \qquad \ldots \qquad (15)$$

$$\text{or } \bar{k}_m = \frac{2}{Sc_0} \int_0^s \left\{ (e + \mu)\ k_L + b) \right\} c^2\ dx \qquad \ldots \qquad (16)$$

The evaluation of this integral for the various types of tapered wing is dealt with in the subsequent paragraphs.

As in the case of the lift coefficient, the moment coefficient $\bar{k}_m$ will refer to the assumed wing (*see* para. 2). The correct coefficient for the actual wing, $\bar{k}_{m'}$ is related to $\bar{k}_m$ by the formula :

$$\bar{k}'_m = \bar{k}_m \left(1 + \frac{S_1}{S - S_1} \right) - \left\{ b + (e + \mu) \frac{4}{\pi} \bar{k}_L \right\} \frac{S_1}{S - S_1} \qquad \ldots \qquad (17)$$

Here e and b have the values appropriate to the root section, *i.e.*, the section on the centre line of the aeroplane. This expression involves the same assumptions as already made in the case of the lift coefficient (*see* para. 3 (ix) (*d*)).

(ii) *Straight tapered untwisted wing.*—For this type of wing a datum is again chosen so that μ is constant (*see* sub-para. (i)). Curves are given in fig. 7 from which the moment coefficient for a straight tapered untwisted wing can be obtained directly. These curves are based upon the following assumptions :—

(1) The moment coefficient of the aerofoil forming the wing is a linear function of the lift coefficient.

(2) The aspect ratio, A, is 6. The aspect ratio appears in the expression for $\bar{k}_m$ given in fig. 7 only in order to define the geometry of the wing (in place of using the semi-span).

Moment Coefficient of Untwisted Straight Tapered Wings.

$$\bar{k}_m = \left[(e+\mu)\left(1 - A f_1(\lambda)\right)\right] \bar{k}_L + b f_2(\lambda)$$

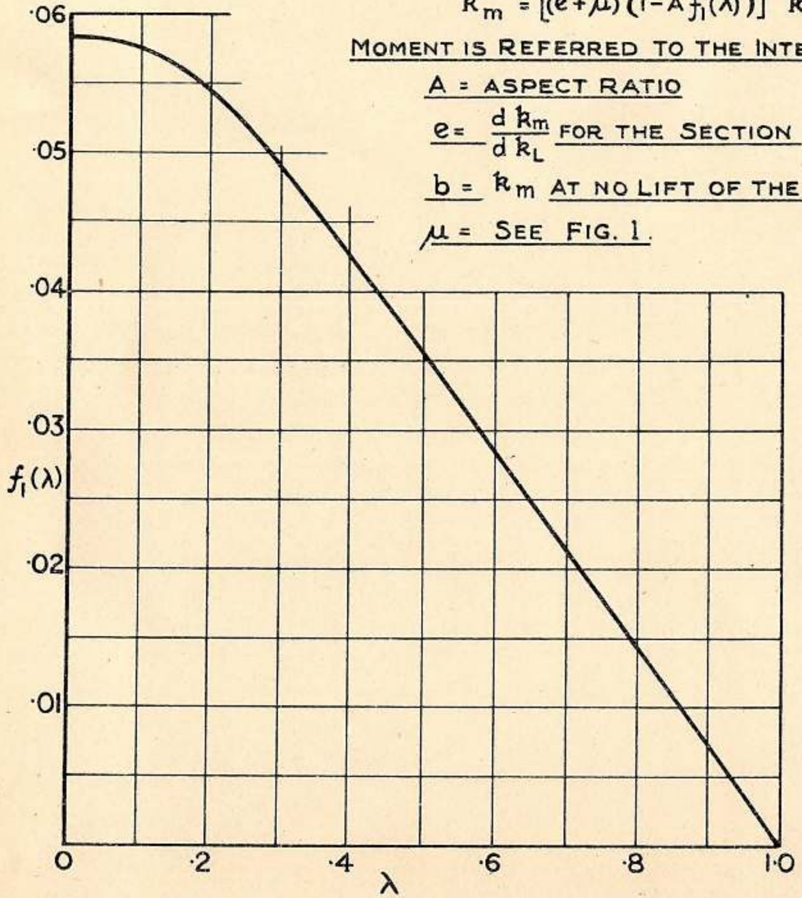
Moment is Referred to the Intersections of the Leading and Trailing Edges. (See Fig. 1.)

A = Aspect Ratio

$e = \dfrac{d k_m}{d k_L}$ for the Section

b = k_m at no Lift of the Section.

μ = See Fig. 1.



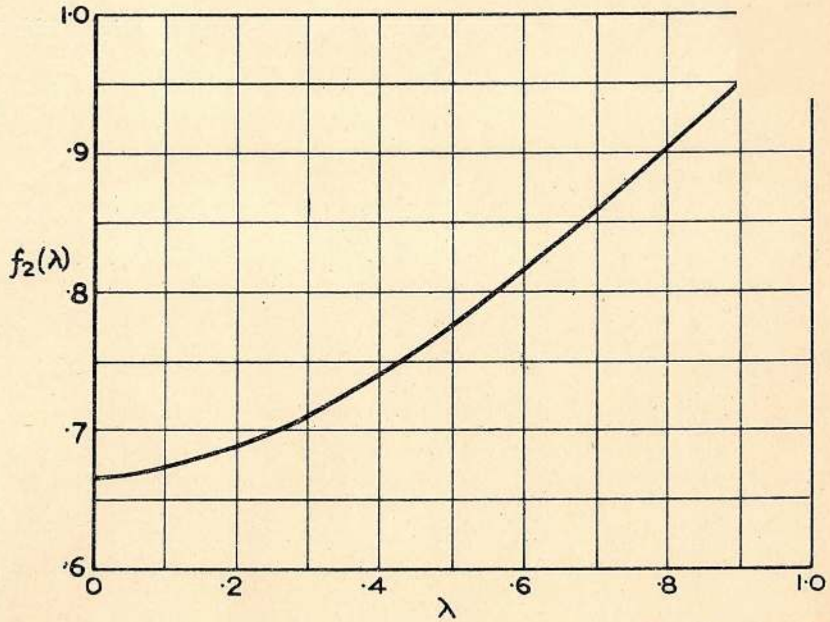


Fig. 7. Chap. VII.

Calculations on aspect ratios 6 and 8 have indicated that the shape of the load distribution curve is not greatly affected by small changes in aspect ratio, and hence the curves given will apply to the majority of wings met with in practice.

(3) The shape of the load distribution curves for different values of λ is as given in fig. 3.

These curves (fig. 7) give $\bar{k}_m$ in terms of $\bar{k}_L$, and hence, by equation (6), in terms of A_1. A_1 is a linear function of α_0, given for a straight tapered wing in fig. 6, and hence $\bar{k}_m$ can be expressed as a linear function of α_0 (α_0 in fig. 6 is in radians). In determining the tail load it must be remembered that $\bar{k}_m$ as calculated above is referred to the datum chosen and not to the leading edge, and that the chord c_0 is as defined in para. 2.

(iii) *Curved tapered untwisted wing.*—The loading curve obtained as described in para. 3 (v) is a curve of $\frac{k_{LC}}{4s}$ along the span corresponding to an incidence of one radian from no-lift. From this curve another curve representing $(e + \mu)\, k_{LC}{}^2$ can be constructed, and integrating this curve graphically will give the first term of equation (16). This value will correspond to $\alpha_0 = 57{\cdot}3°$, and since this term is directly proportional to the incidence it can be expressed in terms of α_0 (in degrees) by dividing by $57{\cdot}3$.

The second term, $\int_O^S bc^2dx$, can be integrated either graphically or analytically according to the shape of the wing, and hence the moment coefficient can be expressed in the form

$$\bar{k}_m = K_1\, \alpha_0 + K_2 \quad \ldots \qquad (18)$$

(iv) *Straight and curved tapered twisted wings.*—Inserting the values of A_1, A_3, A_5, A_7, given by equation (7) in equation (5), at the four standard points, will lead to the following equations :—

Point No. 1

$$\frac{k_{LC}}{4s} = {\cdot}3827\,(h_1 + h_7)\,\alpha_0 + {\cdot}9239\,(h_3 + h_5)\,\alpha_0 + {\cdot}3827(h_2 + h_8) + {\cdot}9239\,(h_4 + h_6)$$

Point No. 2

$$\frac{k_{LC}}{4s} = {\cdot}7071\,(h_1 + h_3 - h_5 - h_7)\,\alpha_0 + {\cdot}7071\,(h_2 + h_4 - h_6 - h_8)$$

Point No. 3

$$\frac{k_{LC}}{4s} = {\cdot}9239\,(h_1 + h_7)\,\alpha_0 - {\cdot}3827\,(h_3 + h_5)\,\alpha_0 + {\cdot}9239\,(h_2 + h_8) - {\cdot}3827\,(h_4 + h_6)$$

Point No. 4

$$\frac{k_{LC}}{4s} = (h_1 - h_3 + h_5 - h_7)\,\alpha_0 + (h_2 - h_4 + h_6 - h_8) \quad \ldots \qquad (19)$$

k_{LC} is now represented at four points along the span as the sum of two curves, one in terms of α_0 and one independent of α. Hence $(e + \mu)\, k_{LC}{}^2$ will be represented by two similar curves, the areas of which will give $\int_O^S (e + \mu)\, k_{LC}{}^2dx$ in the form of equation (18). To this integral must be added the term $\int_O^S bc^2dx$ which is independent of α_0 and contributes to the constant term K_2. The final expression for $\bar{k}_m$ will be in the form of equation (18).

6. Numerical example.—(i) A typical numerical example is given below to illustrate a convenient tabular method of solution and to indicate the magnitude of the constants involved. In the case of a wing with maximum chord not at the root the terms A_3, A_5 and A_7, will probably be larger in comparison with A_1 than in the case given. The example has not been carried further than the determination of A_1, A_3, A_5, and A_7, the work from this point onwards being quite straightforward. The particulars of the twisted wing dealt with are given in table IV (in the form of table III, para. 3 (vi)).

TABLE IV

Point.	$\frac{x}{s}$	Incidence from no-lift in radians when $\alpha_0 = 0$	Chord c	a (*see* para 2).	$\frac{ac}{4s} = u$	$u\alpha$ (α in radians) $\alpha_0 = 0$	$\alpha_0 = \cdot 5$	$\alpha_0 = 1\cdot 0$
4	0	$\alpha_4 = \alpha_0 = 0$	10 ft.	3·142	·2990	$u_4\,\alpha_4 =$ 0	·1495	·2990
3	·3827	$\alpha_3 = -\cdot 0334$	9·043	3·142	·2702	$u_3\,\alpha_3 = -\cdot 009025$	·12607	·26117
2	·7071	$\alpha_2 = -\cdot 0617$	8·233	3·142	·2460	$u_2\,\alpha_2 = -\cdot 01518$	·10782	·23082
1	·9239	$\alpha_1 = -\cdot 0806$	7·690	3·142	·2300	$u_1\,\alpha_1 = -\cdot 01854$	·09646	·21146

(ii) Putting the above values of u and $u\alpha$ into equations (1) to (4)gives the following three sets of four simultaneous equations :—

$$\cdot 6127\,A_1 + 2\cdot 5900\,A_3 + 3\cdot 7010\,A_5 + 1\cdot 9927\,A_7 = -\cdot 01854 \quad \text{I.}$$
$$\cdot 9531\,A_1 + 1\cdot 4451\,A_3 - 1\cdot 9371\,A_5 - 2\cdot 4291\,A_7 = -\cdot 01518 \quad \text{II.}$$
$$1\cdot 1941\,A_1 - \cdot 7185\,A_3 - \cdot 9425\,A_5 + 2\cdot 8153\,A_7 = -\cdot 009025 \quad \text{III.}$$
$$1\cdot 2990\,A_1 - 1\cdot 8970\,A_3 + 2\cdot 4950\,A_5 - 3\cdot 0930\,A_7 = 0 \quad \text{IV.}$$

$$\cdot 6127\,A_1 + 2\cdot 5900\,A_3 + 3\cdot 7010\,A_5 + 1\cdot 9927\,A_7 = \cdot 09646 \quad \text{V.}$$
$$\cdot 9531\,A_1 + 1\cdot 4451\,A_3 - 1\cdot 9371\,A_5 - 2\cdot 4291\,A_7 = \cdot 10782 \quad \text{VI.}$$
$$1\cdot 1941\,A_1 - \cdot 7185\,A_3 - \cdot 9425\,A_5 + 2\cdot 8153\,A_7 = \cdot 12607 \quad \text{VII.}$$
$$1\cdot 2990\,A_1 - 1\cdot 8970\,A_3 + 2\cdot 4950\,A_5 - 3\cdot 0930\,A_7 = \cdot 1495 \quad \text{VIII.}$$

$$\cdot 6127\,A_1 + 2\cdot 5900\,A_3 + 3\cdot 7010\,A_5 + 1\cdot 9927\,A_7 = \cdot 21146 \quad \text{IX.}$$
$$\cdot 9531\,A_1 + 1\cdot 4451\,A_3 - 1\cdot 9371\,A_5 - 2\cdot 4291\,A_7 = \cdot 23082 \quad \text{X.}$$
$$1\cdot 1941\,A_1 - \cdot 7185\,A_3 - \cdot 9425\,A_5 + 2\cdot 8153\,A_7 = \cdot 26117 \quad \text{XI.}$$
$$1\cdot 2990\,A_1 - 1\cdot 8970\,A_3 + 2\cdot 4950\,A_5 - 3\cdot 0930\,A_7 = \cdot 2990 \quad \text{XII.}$$

(iii) It will be seen that these three sets only differ from each other in the terms on the right-hand side. A convenient way of solving these three sets together is shown in table V.

To make the explanation of the table clear, neglect to begin with the figures printed in italics. The remaining figures in columns 2, 3, 4 and 5 are then the coefficients of A_1, A_3, A_5 and A_7 respectively, as given at the head of each column. Column 6 contains the terms on the right-hand side of equations I to IV ; column 7 the terms on the right-hand side of equations V to VIII ; and column 8 the terms on the right-hand side of equations IX to XII.

Equation I is written down at the head of the table (line 1) and the appropriate values from equations V and IX are written in columns 7 and 8. Equations II, III, and IV are written down in a similar way, several lines being left between each equation (equation II on line 23, III on line 42, and IV on line 55). Each equation is then divided through by the coefficient of A_1, giving equations IA, IIA, IIIA and IVA (lines 2, 24, 43 and 56). These are subtracted from one another as shown in column 1. In choosing which two equations to use together it is advisable to avoid equations which have the two coefficients of the same unknown nearly equal, as otherwise the solution may become arithmetically indeterminate.

The three equations given as a result of the subtractions are shown on lines 7, 29 and 48, and bringing the coefficients of A_3 to unity gives the three " B " equations, lines 8, 30 and 49.

This process is repeated, giving two " C " equations (lines 14 and 36) and one " D " equation (line 20), this latter equation giving the numerical value of A_7 when $\alpha_0 = 0$, $\cdot 5$ and $1 \cdot 0$. It is advisable to check at this point that A_7 is a linear function of α_0.

The figures in italics refer to the calculation of A_5, A_3 and A_1.

Since A_7 is known, A_5 can be determined from equation Ic (line 14). Under the coefficient of A_7 in equation Ic ($- 1 \cdot 170$) write its values when A_7 has the values corresponding to $\alpha_0 = 0$, $\cdot 5$ and $1 \cdot 0$. A_5 is then determined by subtracting in turn the numerical values of A_7 from the terms in columns 6, 7 and 8. As a check upon the work determine A_5 in a similar way from equation IIc (line 36) and take the mean of the two values thus found for each of the three values of α_0.

Similarly three determinations of A_3 can be made from equations IB, IIB and IIIB (lines 8, 30 and 49) and four determinations of A_1 from equation IA, IIA, IIIA and IVA (lines 2, 24, 43, 56). It is advisable to calculate these in every case and to take the mean, as it gives a valuable check on the arithmetic and tends to reduce discrepancies due to small sliderule errors.

The mean of the values calculated in table V are as follows :—

	A_1	A_3	A_5	A_7
$\alpha_0 = 0$	$-\cdot 00914$	$-\cdot 00498$	$\cdot 000277$	$-\cdot 000536$
$\alpha_0 = \cdot 5$	$\cdot 1130$	$\cdot 00488$	$\cdot 000421$	$-\cdot 000487$
$\alpha_0 = 1 \cdot 0$	$\cdot 2351$	$\cdot 01475$	$\cdot 00815$	$-\cdot 000442$

and hence equations (7) para. 3 (vi) (c) are as follows :—

$$A_1 = \cdot 2442\alpha_0 - \cdot 00914$$
$$A_3 = \cdot 01973\alpha_0 - \cdot 00498$$
$$A_5 = \cdot 007873\alpha_0 + \cdot 000277$$
$$A_7 = \cdot 000094\alpha_0 - \cdot 000536$$

(iv) In the above calculations A_7 has been evaluated first. It will usually be advisable to do this, as the probable error in A_1 is generally of the same order of magnitude as A_7, so that if the latter is made to depend upon the accuracy of the calculation of A_1 erratic results will probably be obtained.

TABLE V

	1	2	3	4	5	6	7	8
						Right Hand Side of Equation when $\alpha_0 =$		
		A_1	A_3	A_5	A_7	0	·5	1·0 radian.
1	I	·6127	2·590	3·701	1·9927	− ·01854	·09646	·21146
2	IA	1	4·2272	6·0405	3·2523	− ·03026	·15743	·34513
3	$\alpha_0 = 0$		− ·02092	·00167	− ·00174	$A_1 =$ − ·00926		
4	$\alpha_0 = ·5$		·02061	·02545	− ·00158	$A_1 =$ ·11296		
5	$\alpha_0 = 1·0$		·06234	·04920	− ·00143	$A_1 =$ ·23502		
6	IVA	1	−1·4603	1·9207	−2·3816	0	·11509	·23018
7	IA—IVA		5·6875	4·1198	5·6339	− ·03026	·04234	·11495
8	IB		1	·7244	·9906	− ·00532	·00744	·02021
9	$\alpha_0 = 0$			·00016	− ·00053	$A_3 =$ − ·00495		
10	$\alpha_0 = ·5$			·00305	− ·00048	$A_3 =$ ·00488		
11	$\alpha_0 = 1·0$			·00590	− ·00044	$A_3 =$ ·01475		
12	IIIB		1	1·4127	·1852	− ·00470	·01073	·02617
13	IIIB—IB			·6883	− ·8054	·00062	·00329	·00596
14	IC			1	−1·170	·000901	·00478	·00866
15	$\alpha_0 = 0$				·00062	$A_5 =$ ·000276		
16	$\alpha_0 = ·5$				·00057	$A_5 =$ ·004213		
17	$\alpha_0 = 1·0$				·00051	$A_5 =$ ·008146		
18	IIC			1	1·2459	− ·000391	·003604	·00760
19	IIC—IC				2·4159	− ·001292	− ·001176	− ·00106
20	ID	Final Equation for A_7			1	− ·000536	− ·000487	− ·000442
21								
22								
23	II	·9531	1·4451	−1·9371	−2·4291	− ·01518	·10782	·23082
24	IIA	1	1·5162	−2·0324	−2·5486	− ·01589	·11314	·24218
25	$\alpha_0 = 0$		− ·00756	− ·00056	·00136	$A_1 =$ − ·00912		
26	$\alpha_0 = ·5$		·00740	− ·00856	·00124	$A_1 =$ ·11304		
27	$\alpha_0 = 1·0$		·02235	− ·01656	·00113	$A_1 =$ ·23525		
28	IA	1	4·2272	6·0405	3·2523	− ·03026	·15743	·34513
29	IA—IIA		2·7110	8·0729	5·8009	− ·01437	·04429	·10295
30	IIB		1	2·9778	2·1398	− ·00531	·01638	·03806
31	$\alpha_0 = 0$			·00082	− ·00115	$A_3 =$ − ·00499		
32	$\alpha_0 = ·5$			·01254	− ·00104	$A_3 =$ ·00488		
33	$\alpha_0 = 1·0$			·02426	− ·00094	$A_3 =$ ·01474		
34	IIIB		1	1·4127	·1852	− ·00470	·01074	·02617
35	IIB—IIIB			1·5651	1·9546	− ·00061	·00564	·01189
36	IIC			1	1·2459	− ·000391	·003604	·00760
37	$\alpha_0 = 0$				− ·00067	$A_5 =$ ·000277		
38	$\alpha_0 = ·5$				− ·00061	$A_5 =$ ·004212		
39	$\alpha_0 = 1·0$				− ·00055	$A_5 =$ ·00815		
40								
41								
42	III	1·1941	− ·7185	− ·9425	2·8153	− ·009025	·12607	·26117
43	IIIA	1	− ·6017	− ·7893	2·3576	− ·00756	·10558	·21872
44	$\alpha_0 = 0$		·00300	− ·00022	− ·00126	$A_1 =$ − ·00908		
45	$\alpha_0 = ·5$		− ·00293	− ·00332	− ·00115	$A_1 =$ ·11299		
46	$\alpha_0 = 1·0$		− ·00887	− ·00643	− ·00104	$A_1 =$ ·23507		
47	IA	1	4·2272	6·0405	3·2523	− ·03026	·15743	·34513
48	IA—IIIA		4·8289	6·8298	·8947	− ·02270	·05185	·12641
49	IIIB		1	1·4127	·1852	− ·00470	·01073	·02617
50	$\alpha_0 = 0$			·00039	− ·00010	$A_3 =$ − ·00499		
51	$\alpha_0 = ·5$			·00595	− ·00009	$A_3 =$ ·00487		
52	$\alpha_0 = 1·0$			·01151	− ·00008	$A_3 =$ ·01475		
53								
54								
55	IV	1·2990	−1·8970	2·4950	−3·093	0	·1495	·2990
56	IVA	1	−1·4603	1·9207	−2·3816	0	·11509	·23018
57	$\alpha_0 = 0$		·00729	·00053	·00128	$A_1 =$ − ·00910		
58	$\alpha_0 = ·5$		− ·00712	·00809	·00116	$A_1 =$ ·11295		
59	$\alpha_0 = 1·0$		− ·02153	·01565	·00105	$A_1 =$ ·23501