CHAPTER VIII.—ESTIMATION OF STRENGTH OF INDIVIDUAL MEMBERS*

This chapter contains approved methods of estimating the strength of individual members, together with schedules of strengths and stresses. It is not possible to give a general ruling governing the circumstances in which appeal to an *ad hoc* strength test is (i) essential or (ii) permissible. The strength of some types of built up members cannot be calculated with confidence from the scheduled material properties. In such cases the strength should be determined from an *ad hoc* test made under the conditions described in chapter I. The strength of standardised members such as solid drawn tubular struts should be obtained from the formulæ and tables given in this chapter and not from *ad hoc* tests. The generalised information given in this chapter will often prove inadequate for calculating the strength of fittings, particularly thin plate fittings incorporating bolts, pins or rivets loaded in shear and bearing. Generalised test information from other sources is therefore admissible, provided it is supported by adequate and satisfactory tests and is not inconsistent with the data given in this chapter. Instances of serious inconsistency should be brought to the attention of the Airworthiness Department. Alternatively the strength of a fitting may be determined from an *ad hoc* destruction test on a specimen identical in all essential respects with the fitting concerned, the test being carried out and interpreted in accordance with chapter I.

Section I.—Tubular struts

1. Strength formulæ.—It can generally be assumed that compliance with the ultimate factor requirement will automatically ensure compliance with the proof factor requirement in the case of tubular struts.

The strength of axially loaded hard drawn tubular struts with diameter/thickness less than 80 is to be obtained from the following formula. The strength of struts with diameter/thickness greater than 80 should in general be determined from an *ad hoc* test.

$$p_2 = \frac{P}{A} + \frac{P\,e\,h \sec\alpha}{A\,k^2}$$

where p_2 = 0·2 per cent. proof stress of the material.

P = maximum load which the strut will withstand (*i.e.*, the crippling load).

e = equivalent eccentricity of end-load (*i.e.*, eccentricity due to manufacture).

$$\alpha = \frac{l}{2}\sqrt{\frac{P}{EI}}$$

A, I and k are respectively the area, the moment of inertia and minimum radius of gyration of the cross section.

h is the distance from the normal position of the neutral axis to the most highly stressed fibre.

l is the length of the strut.

This formula is applicable to any long strut of uniform cross-section of any elastic material. The formula gives reasonable values for short struts of uniform section and its use is therefore, general for all struts irrespective of length.

* For spars, *see* chapter VI.

2. Eccentricity.—The equivalent eccentricity, e, of end load given in table I below, is the sum of two terms :—

(*a*) The crookedness of the tube, *i.e.* the maximum deflection of any point on the centre line of the tube from the straight line joining the centres of areas of the end sections. The maximum allowable crookedness is usually given in the specification as $l/600$ or $l/300$, l being the length of the tube.

(*b*) A quantity representing the eccentricity of bore or neutral axis of the tube relative to the centre of the external diameter, expressed as a fraction of the nominal internal diameter.

If the end load is intentionally offset, the amount of this intentional offset should be added to e found as above.

In the case of unwelded seamless and cold-drawn circular steel tubes loaded in pure compression, two-thirds only of the equivalent eccentricity need be taken. This ruling does not apply to welded tubes.

In calculating the eccentricity of bore from the tolerances given in the specification, the following assumptions have been made :—

(*a*) The external and internal perimeters are truly circular.

(*b*) The mean thickness is the minimum permitted by the specification, *i.e.* the nominal thickness.

(*c*) The linear eccentricity (*i.e.* the distance between the centres of the external and internal perimeters) is such that at one point the tube has the minimum thickness permitted by the specification.

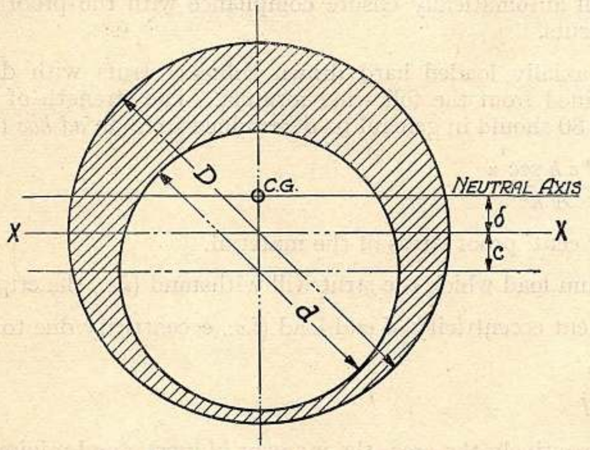


FIG. 1.—CHAP. VIII., Section I.

Referring to fig. 1

D and d are the external and internal diameters respectively.

c is the linear eccentricity.

δ is the eccentricity of bore, *i.e.* the eccentricity of the neutral axis relative to XX its nominal position.

Let t_0 be the minimum mean thickness permitted by the specification then $D - d = 2t_0$

Taking moments about XX, difference of areas of external and internal circles $\times\ \delta$

$$= \text{area of internal circle} \times c$$

A.L.2.

so that $$\delta = \frac{\frac{\pi\, d^2}{4} \times c}{\frac{\pi}{4}\,(D^2 - d^2)}$$

or $$\delta = \frac{cd^2}{D^2 - d^2}$$

Now $$D = d + 2t_0$$

hence $$\delta = \frac{cd}{4t_0} \text{ approx.}$$

In the majority of relevant specifications the minimum thickness permitted is $\cdot 9t_0$.
Thus $$c = \cdot 1t_0.$$
With this value of c
$$\delta = \frac{d}{40}$$

d = internal diameter having regard to the tolerance allowed, but for all practical purposes eccentricity of bore = (nominal internal diameter)/40.

The equivalent eccentricity is $\frac{d}{40} + \frac{l}{600}$ for the following specifications :—

B.S.I.	D.T.D.	
2 T.1.	41	102
2 T.2.	~~89A~~	105
T.5.	91A	113
T.45	97	167

A.L.3

For B.S. Specification 3 T.4 for tubes of $\frac{3}{4}$ in. outside diameter or more, the eccentricity is as above. For tubes less than $\frac{3}{4}$ in. outside diameter, $e = \frac{d}{40} + \frac{l}{300}$.

3. Solution of strength formulæ.—The formula given in para. 1 may be written
$$p_2 = p\left\{1 + \lambda \sec \tfrac{1}{2}\,\frac{l}{k}\sqrt{\frac{p}{E}}\right\}$$

where $$p = \frac{P}{A}$$

and $$\lambda = \frac{eh}{k^2}$$

If $$e = \frac{d}{40} + \frac{l}{600}$$
then it may be shown that
$$\lambda = \frac{d_m{}^2 - t^2}{10\,(d_m{}^2 + t^2)} + \frac{\sqrt{2}}{600}\,\frac{d_m + t}{\sqrt{(d_m{}^2 + t^2)}}\,\frac{l}{k}$$
where d_m = mean diameter
and t = thickness

Having regard to the dimensions of the tubes in practical use as struts, it can be demonstrated that with the above value of e the following faired value of λ
$$\lambda = \frac{1}{10} + \frac{1}{400}\,\frac{l}{k}$$
will be applicable to tubes of all diameters and gauges in practical use with a maximum error of about 2 per cent.

For $$e = \frac{d}{40} + \frac{l}{300}$$

the appropriate faired value of λ will be

$$= \frac{1}{10} + \frac{1}{200}\,\frac{l}{k}$$

For an eccentricity $\frac{2}{3}e$ the corresponding values of λ will be $\frac{2}{3}\lambda$. With such values of λ, p becomes a function of $\frac{l}{k}$ only and hence strength curves may be compiled on a limiting stress-$\frac{l}{k}$ basis which are applicable to all tubes in practical use with a sufficient degree of accuracy. Such curves for various B.S.I. and D.T.D. tubes may be obtained on application to the Air Ministry.† In the absence of such curves the analytical solution of the formula can be avoided by the use of the diagram given in fig. 2.

Procedure in using the diagram (fig. 2).—(*a*) Calculate, from the known conditions of the strut, the following values :—

(1) q = Euler failing stress

$= \frac{\pi^2 E k^2}{l^2}$ where k = minimum radius of gyration of cross section of strut.

l = length of strut between pin centres.

E = Young's Modulus, taken from section VI of this chapter.

(2) $\beta = p_2/q$ where p_2 = $0{\cdot}2$ per cent. proof stress of the material, taken from section VI of this chapter.

(3) $\lambda = \frac{eh}{k^2}$ where h = distance from neutral axis of cross-section to the most highly stressed fibre.

e = equivalent eccentricity of end load taken from table below for tubes to B.S. and D.T.D. specifications. In the case of unwelded seamless and cold-drawn circular steel tubes loaded in pure compression, two-thirds only of the equivalent eccentricity need be taken. This ruling does not apply to welded tubes.

(*b*) Deduce the value β/λ and $\frac{\beta - 1}{\lambda}$.

(*c*) Draw a straight line across the diagram by joining appropriate points on the scales β, β/λ and $\frac{\beta - 1}{\lambda}$. Three points are known; the most convenient two may be used. Where the straight line cuts the curve read off the value of x.

(*d*) Then the crippling load P for the strut is given by Axq where A = area of cross-section of strut. To bring the diagram within a reasonable compass, two curves are given. Curve Y can be used for small values of β/λ and curve Z, which has a vertical scale $\frac{1}{10}$th that of Y, for larger values of β/λ.

A.L.3

Example.

Required to find the crippling load of a pin-ended unwelded strut $1\frac{1}{4}$ in. outside diameter, 20 gauge thick, 30 in. pin-centres to Specification ~~D.T.D. 80A.~~ T.45

For this strut $\begin{cases} A = \cdot 1373 \text{ sq. ins.*} \\ k = \cdot 4295 \text{ ins.*} \\ l = 30 \text{ ins.} \end{cases}$ $\quad l/k = 69{\cdot}8$, $\quad d = 1{\cdot}178$ ins.

From section VI of this chapter

$0{\cdot}2$ per cent. proof stress = 40 tons per sq. in.

* These are nominal values which differ from the minimum values permitted by the specification by inappreciable amounts ; this applies only to steel tubes.

† 737155/38

Hence $p_2 = 40$ tons per sq. in.

$E = 30 \times 10^6$ lb. per sq. in.

$$\therefore q = \frac{\pi^2 Ek^2}{l^2} = \frac{\pi^2 \times 30 \times 10^6}{(69 \cdot 8)^2} \text{lb. per sq. in.} = 60{,}800 \text{ lb. per sq. in.}$$

$$\text{and } \beta = p_2/q = \frac{40 \times 2{,}240}{60{,}800} = 1 \cdot 474$$

$$e = d/40 + l/600.$$

Only two-thirds of this eccentricity need be assumed as this is an unwelded seamless and cold-drawn circular steel tube loaded in pure compression.

$$\therefore e = \frac{2}{3}\left(\frac{1 \cdot 178}{40} + \frac{30}{600}\right) \text{ins.} = \cdot 053 \text{ ins.}$$

$$\therefore \lambda = \frac{eh}{k^2} = \frac{\cdot 053 \times \cdot 625}{(\cdot 4295)^2} = \cdot 179$$

$$\therefore \beta/\lambda = \frac{1 \cdot 474}{\cdot 179} = 8 \cdot 23 \text{ and } \frac{\beta - 1}{\lambda} = 2 \cdot 65$$

Hence, by drawing a straight line across the diagram joining the above points on the β/λ and $\frac{\beta - 1}{\lambda}$ scales, the value of $x = \cdot 768$.

$$\begin{aligned} \therefore \text{Crippling load} &= A\, x\, q \\ &= \cdot 1373 \times \cdot 768 \times 60{,}800 \text{ lb.} \\ &= 6{,}410 \text{ lb.} \end{aligned}$$

4. Note on thickness of 22 gauge tubes.—Only tubes of nominal thickness ·028 in. are to be referred to on drawings and in schedules as " 22 gauge ". Tubes of ·025 in. nominal thickness are to be labelled " ·025 in. thick " and are not to be called 22 gauge.

5. Note on use of non-corrosive steel tubes.—Tests carried out at the Royal Aircraft Establishment on a number of austenitic non-corrosive steel tubes show that this type of tube is likely to give low values of maximum compression stress. For those tubes which are " soft ", the ratio maximum compression strength to maximum tensile strength is of the order of ·5. For those tubes which are " hard " a higher ratio of the order of ·75–·85 is obtained. Further details relating to the tests are given in Royal Aircraft Establishment Report No. 06/9956.

Section II.—Tubes in bending, torsion and bearing

1. Bending.—It will usually be acceptable to calculate the ultimate strength in pure bending of members of circular section on a stress equal to the 0·1 per cent. proof stress increased by half the difference between the 0·1 per cent. proof stress and the ultimate stress. Compliance with the proof factor condition will then follow automatically.

Alternatively the strength of such members may be based upon an *ad hoc* test carried to the proof factor load only. If undamaged at this load it will usually be safe to assume, without continuing the test, that the ultimate factor condition will also be complied with.

The above is not applicable to tubes of diameter/thickness greater than 80 (eighty). For such tubes an *ad hoc* test will usually be required, carried to complete failure.

2. Torsion.—The available information on the strength of tubes in torsion is summarised in fig. 1. The strength needs to be calculated under both the proof factor condition and the ultimate factor condition, the allowable stresses for each of these conditions being shown in fig. 1.

When calculating the stress corresponding to the proof factor loads the following formula is to be used

$$\text{Shear stress} = \frac{16\,\text{D}}{\pi\,(D^4 - d^4)} \times \text{torque}$$

where D and d are the external and internal diameters. This assumes linear variation of stress with distance from neutral axis.

When calculating the stress corresponding to the ultimate factor loads the following formula is to be used

$$\text{Shear stress} = \frac{12}{\pi\ (D^3 - d^3)} \times \text{torque}$$

which assumes uniform stress throughout the thickness of the tube wall.

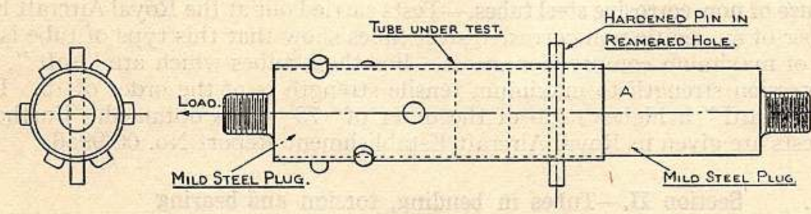
These two assumptions give the most consistent interpretation of test results, and have been adopted in preparing fig. 1.

The proof stress given in fig. 1 corresponds to a permanent set of ·001 radians on a length equal to the outside radius.

The dotted lines on fig. 1 indicate that the curves in these regions are tentative only, pending further tests.

3. Bearing.—The formulæ given below are derived from tests on the type of specimen shown in fig. 2. The plugs were made a sliding fit and the pins a push fit in the tubes. Steel and duralumin tubes 1 in. × 17 gauge and $1\frac{1}{2}$ in. × 22 gauge to Specification T.1, T.5, T.4 and D.T.D. Specification 89 were used in conjunction with high tensile steel pins $\frac{1}{4}$ in. and ·185 in. dia. (2.B.A.), the pins being in effect part of the test apparatus. This investigation and the formulæ given below deal only with the bearing strength of the tube wall. In any actual joint the strength of the pin or bolt in shear will also need consideration. (*See* section IV of this chapter.)

BEARING OF PINS IN TUBES.



TYPE OF TEST PIECE.

FIG. 2.—CHAP. VIII., Section II.

The above sizes of tubes and pins give a range of ratios thickness of tube/pin diameter from 0·1 to 0·3.

The allowable bearing pressure based on the above tests is as follows (for notation *see* below) :—

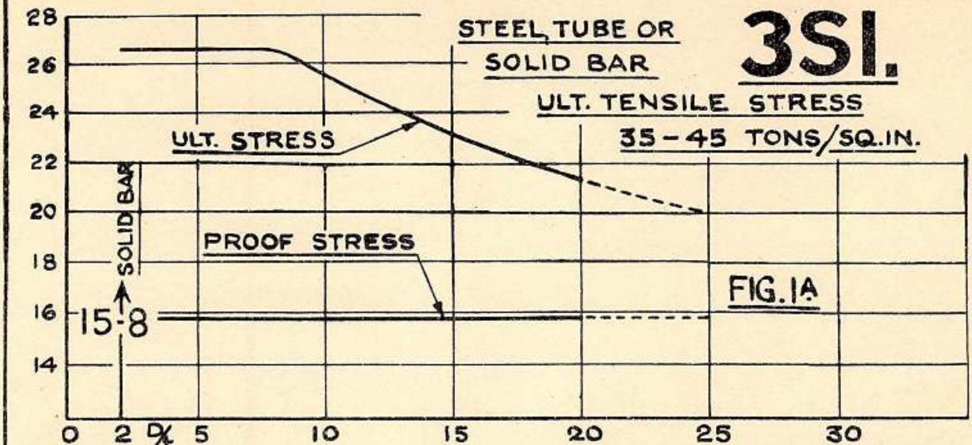
(*a*) For joints, not subject to serious vibration, where the design load is always applied in the same direction

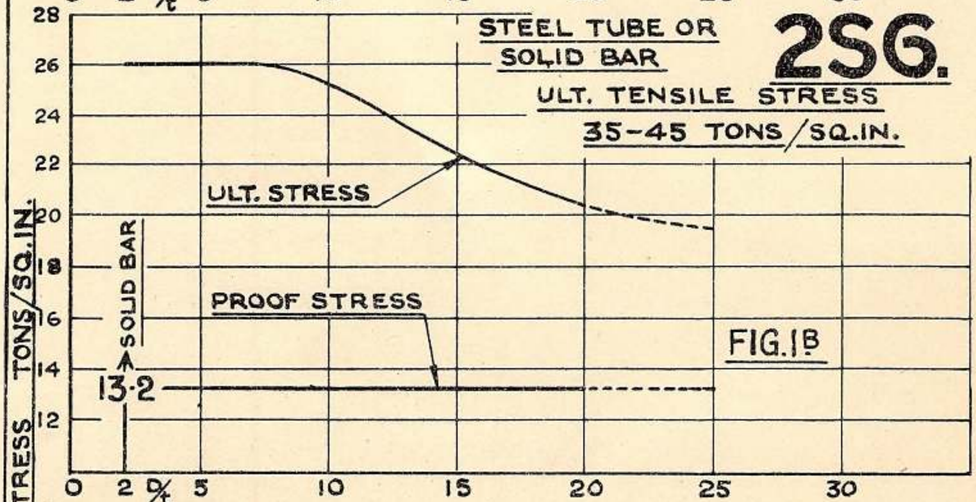
$$f_b\ (\text{ult.}) = f_t\left(4{\cdot}45\frac{t}{d} + 1{\cdot}22\right) \text{ for steels.}$$

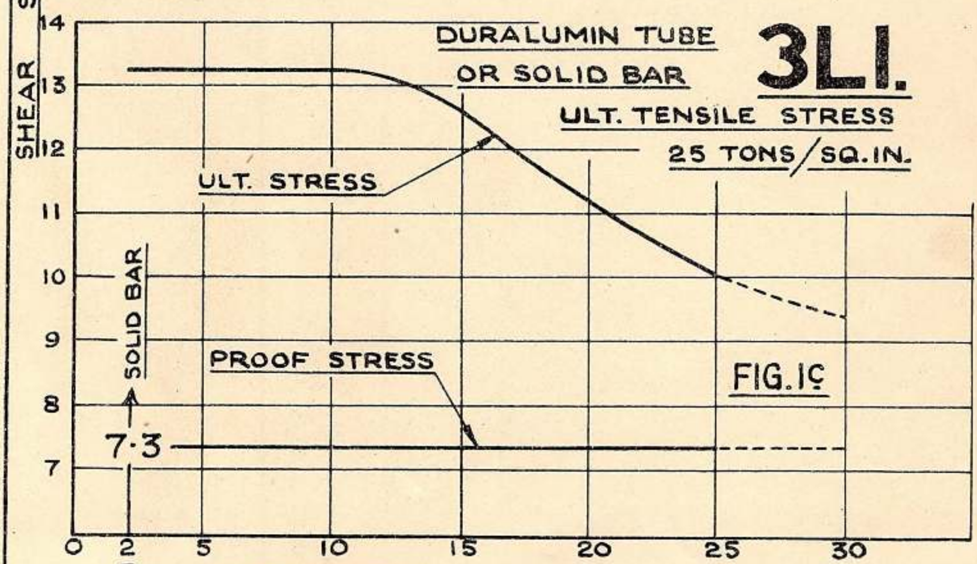
$$f_b\ (\text{proof}) = 1{\cdot}4\,f_t \text{ for steels.}$$

$$f_b\ (\text{ult.}) = f_t\left(1{\cdot}9\,\frac{t}{d} + 1{\cdot}63\right) \text{ for duralumin.}$$

$$f_b\ (\text{proof}) = 1{\cdot}2\,f_t \text{ for duralumin.}$$







$D/t = \dfrac{\text{TUBE DIAMETER}}{\text{WALL THICKNESS}}$ FIGS. 1A, 1B, 1C. CHAP. VIII. SECTION II.

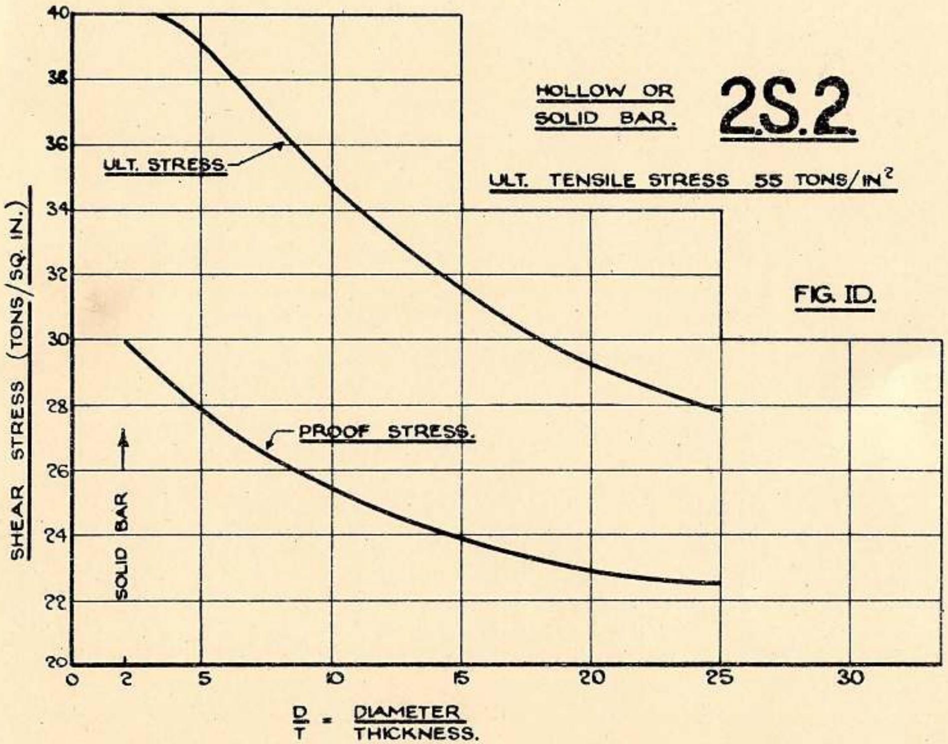
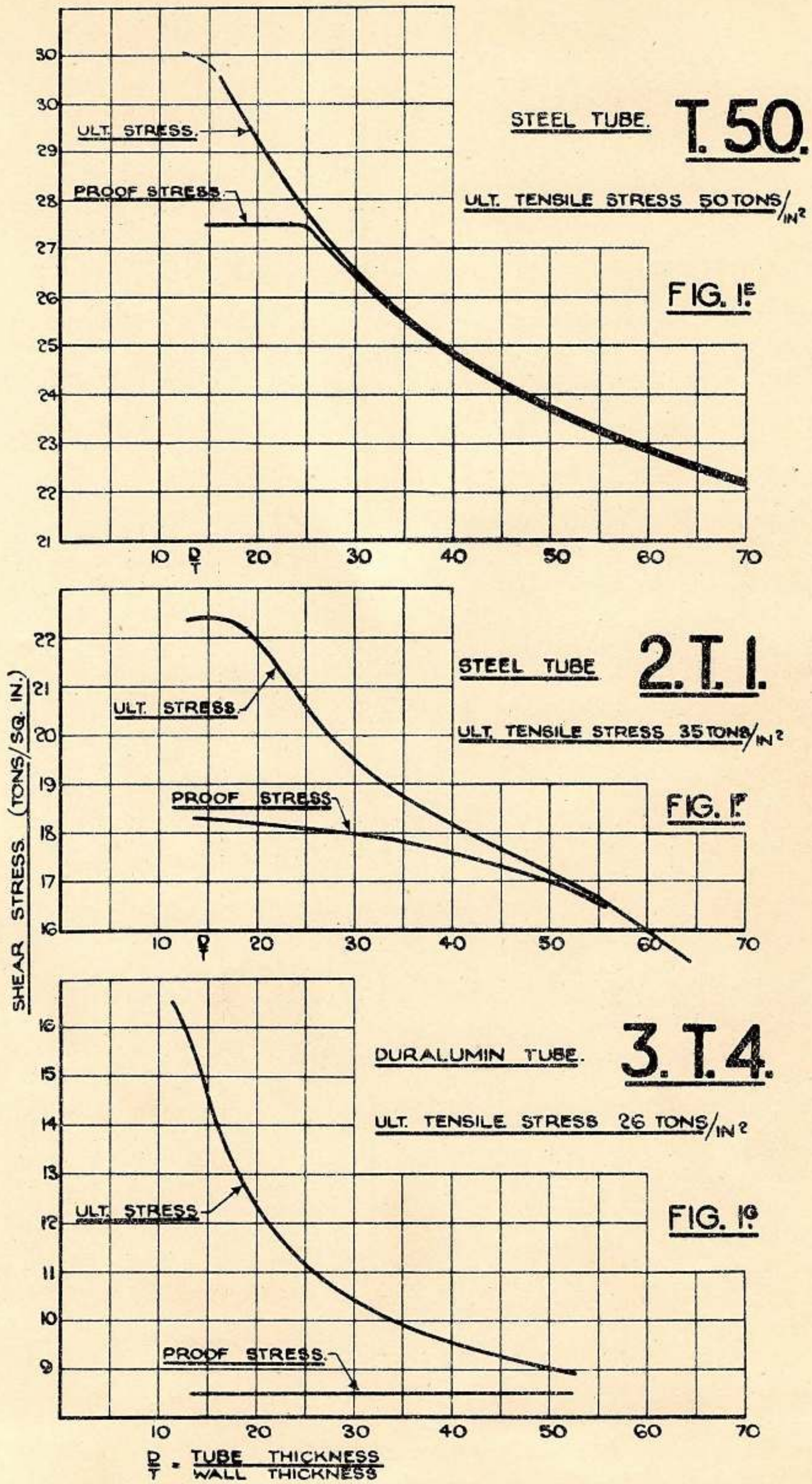
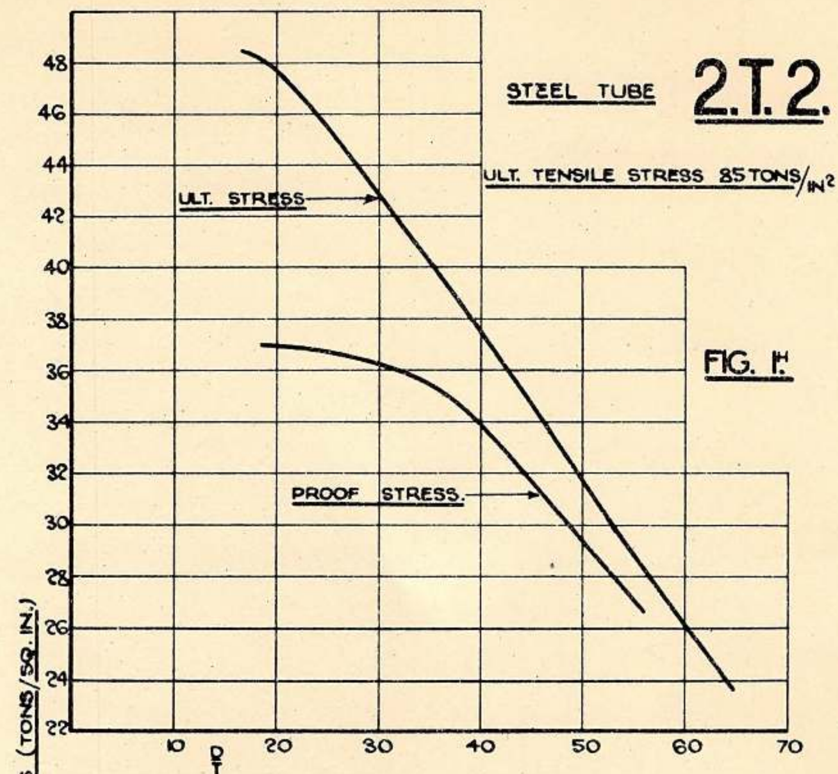
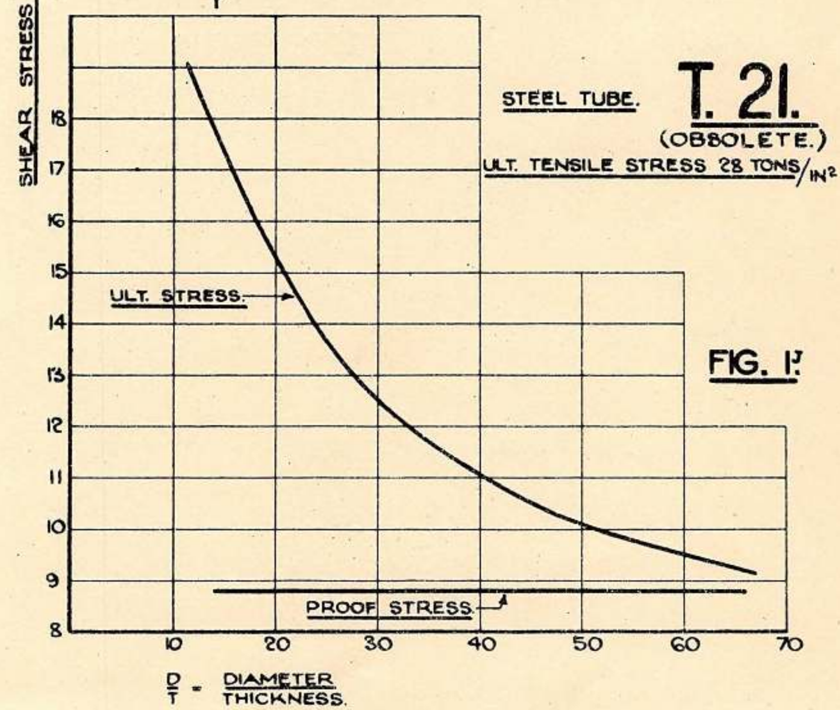


FIG. I^{D} CHAP. VIII SECTION II.

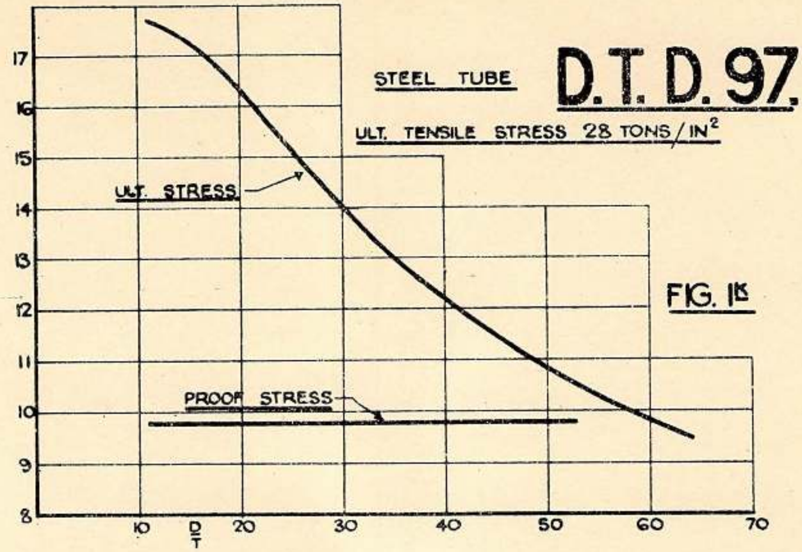


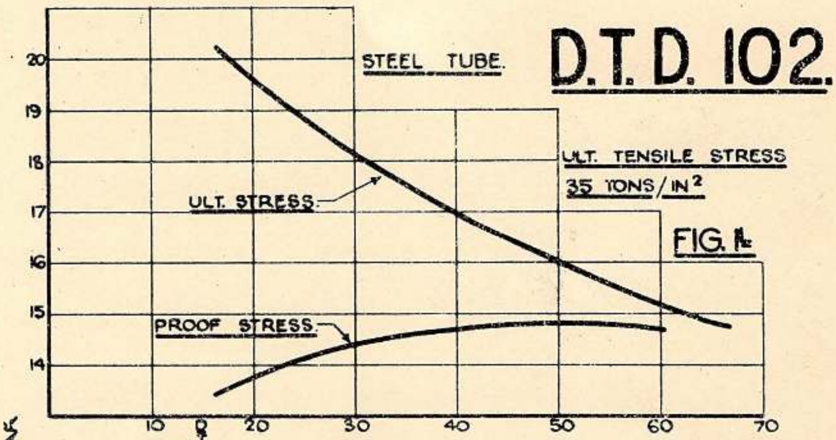
FIGS. 1E 1F & 1G. CHAP. VIII SECTION II.

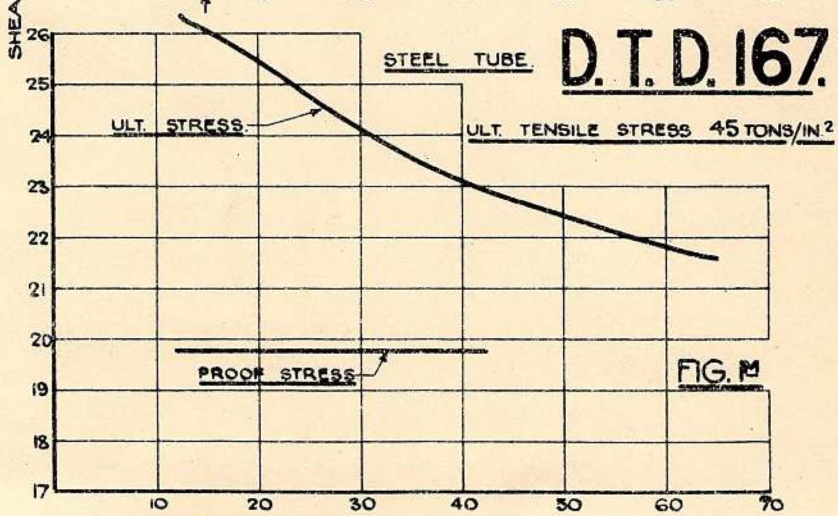




FIGS. 1H. & 1J. CHAP. VIII. SECTION II.

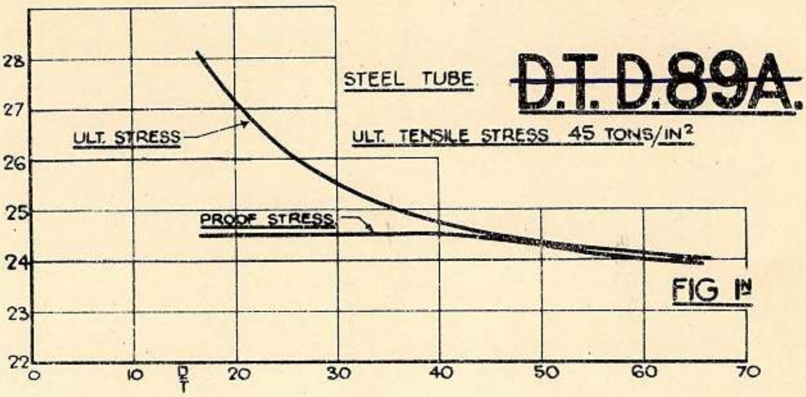






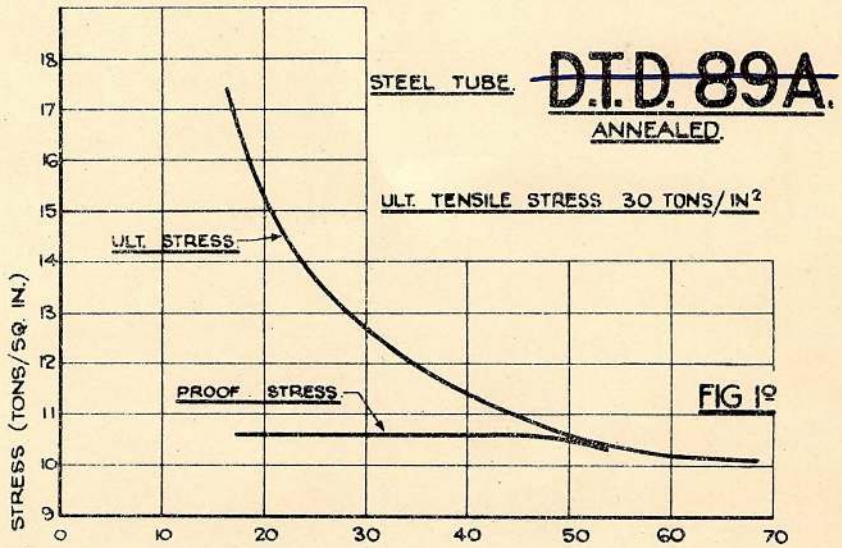
$\frac{D}{T} = \frac{\text{TUBE DIAMETER.}}{\text{WALL THICKNESS.}}$

FIG. 1K 1L & 1M CHAP. VIII. SECTION II.



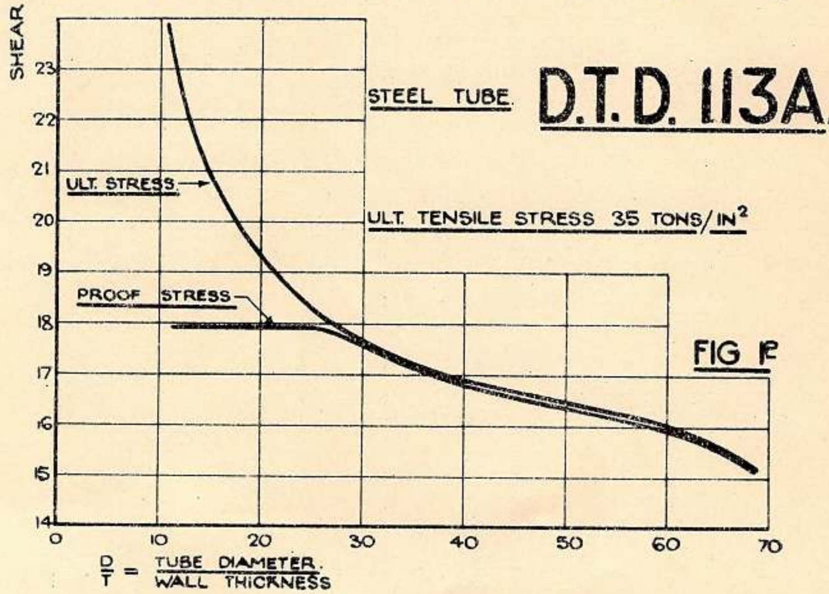
T.45

A.L.3

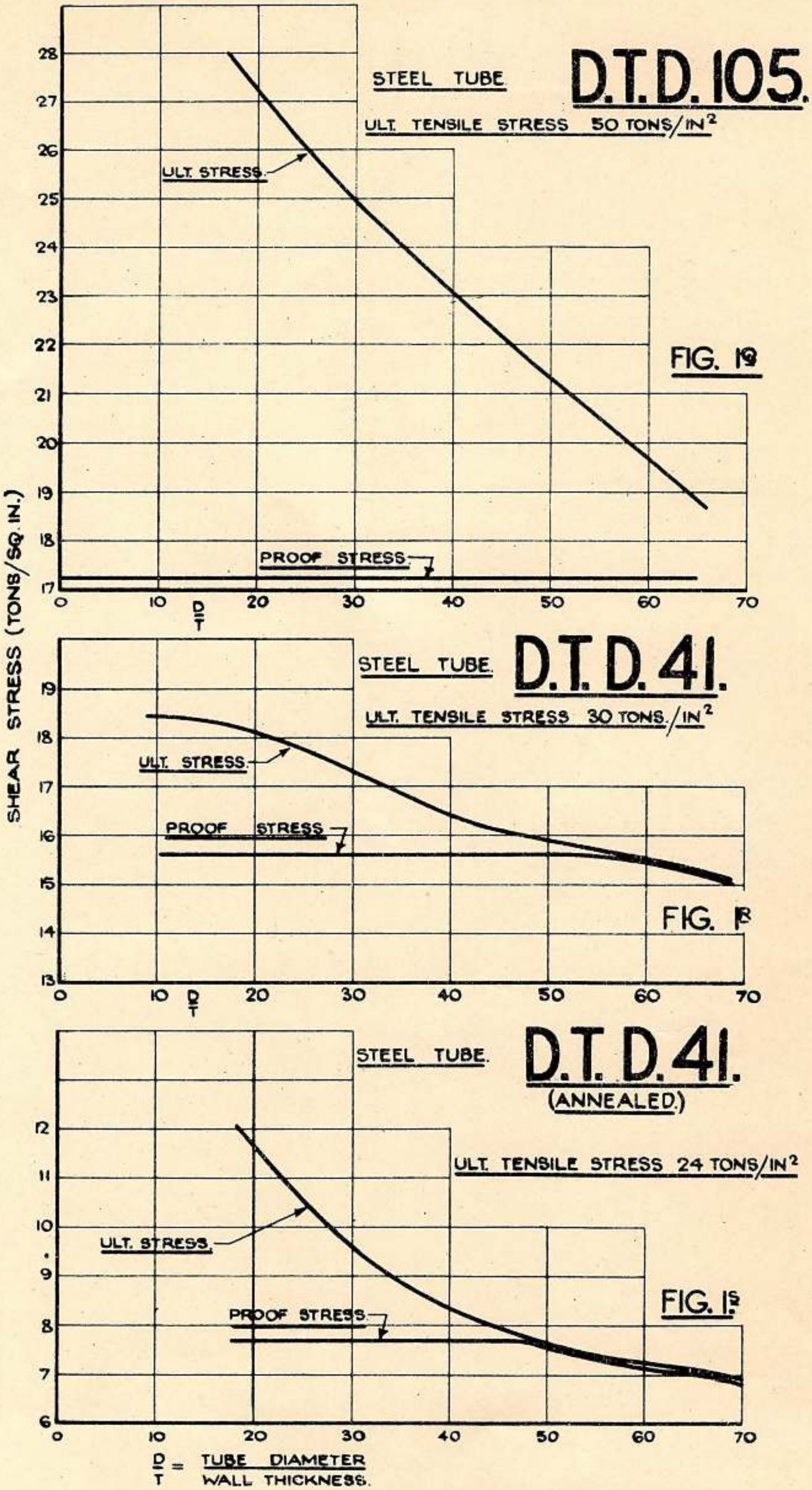


T.45

A.L.3



FIGS. 1N, 1O & 1P CHAP VIII SECTION II



FIGS. 1a, 1b & 1c CHAP. IX. SECTION II.

(*b*) For joints subject to serious vibration, and joints where the design load may be applied in either direction :—

f_b (ult.) as in (*a*) above for steels.

f_b (proof) $= f_t$ for steels.

f_b (ult.) as in (*a*) above for duralumin.

f_b (proof) $= \cdot 8 \, f_t$ for duralumin.

In the above expressions f_b and f_t refer respectively to the allowable bearing pressure and the minimum specified ultimate tensile stress, t is the thickness of the tube walls and d the diameter of the pin or bolt.

In any joint of the type considered the bearing pressure corresponding to the ultimate factor load transmitted by the joint should not exceed f_b (ult.) and the bearing pressure corresponding to the proof factor load should not exceed f_b (proof). For most values of t/d the proof factor formula will be critical since the strength given by the ultimate factor formula usually exceeds $1\frac{1}{3}$ times that given by the proof factor formula.

The above data are directly applicable only to tube joints similar to that shown in fig. 2 with proportions of pin diameter to tube thickness within the specified range. It will, however, give some guidance in other cases until more precise information is available but must be used with increasing caution as conditions depart more and more from those of these tests.

The above figures refer to joints transmitting the load via a single pin or bolt. The information at present available indicates that when two or more bolts are used it is impossible to ensure that the load is uniformly distributed among them. If, therefore, it is necessary to use the preceding formulæ for calculating the strength of joints with two or more bolts it is necessary to make some allowance for this unequal distribution of load between them. The test evidence at present available indicates that the strength of a two-bolt joint under proof factor conditions should not be assumed to be more than 80 per cent. of the sum of the strength contributed by each bolt acting alone calculated from the preceding formulæ.

Test data on multi-bolt joints from other sources, as mentioned at the beginning of this chapter, may be used for calculating the strength of multi-bolt joints instead of calculating the strength from the preceding formulæ with the above 80 per cent. assumed distribution of load between bolts.

Under ultimate factor conditions the load can usually be assumed to be more uniformly distributed between the bolts. No general ruling, however, can yet be given.

Section III.—Torsional stresses—General formulæ

1. General.—The formulæ and methods of calculation described below give a rough guide to the maximum shear stress in any member corresponding to a given applied torque. Except in the case of metal members of circular section (*see* section II of this chapter) the maximum shear stress which the member can develop before failure will usually have to be obtained from an *ad hoc* test. For such members, therefore, the formulæ are of little practical value as the *ad hoc* test will give a direct measure of torsional strength and the corresponding stress need not be calculated. The formulæ may, however, be of interest as a rough preliminary guide. *See* also R. & M. 1393.

2. Pure torsion.

T = torque in lb. in.

q = shear stress in lb. per sq. in.

Type of Section.	Formulæ for Stress.	Position of Shear Stress.
Solid circle Diameter D	$q = \dfrac{16T}{\pi D^3}$	At boundary.
Hollow circle Outside diameter D Inside diameter d	$q = \dfrac{16TD}{\pi (D^4 - d^4)}$	At boundary.
If thickness t is small compared with the outside diameter D.	$q = \dfrac{2T}{\pi t D^2}$	At boundary.
Solid square Side S	$q = 4{\cdot}8 \dfrac{T}{S^3}$	At middle of sides.
Solid ellipse Major axis $2a$ Minor axis $2b$	$q = \dfrac{2T}{\pi ab^2}$	At end of minor axis.
	$q = \dfrac{2T}{\pi a^2 b}$	At end of major axis.
Hollow ellipse Outer major axis $2a$ Outer minor axis $2b$ Inner major axis $2a'$ Inner minor axis $2b'$ such that $a^2 - a'^2 = b^2 - b'^2$	$q = \dfrac{2T\dfrac{a^2b}{a^2+b^2}}{\pi\left\{\dfrac{a^3b^3}{a^2+b^2} - \dfrac{a'^3b'^3}{a'^2+b'^2}\right\}}$	At end of minor axis.
	$q = \dfrac{2T\dfrac{ab^2}{a^2+b^2}}{\pi\left\{\dfrac{a^3b^3}{a^2+b^2} - \dfrac{a'^3b'^3}{a'^2+b'^2}\right\}}$	At end of major axis.
Solid rectangle Long side a Short side b	Approximately $q = \dfrac{T}{ab^2}\left(3 + 1{\cdot}8\,\dfrac{b}{a}\right)$	At middle of long side.
Any hollow section Thickness t small compared with smallest outside dimensions. A is area bounded by mean perimeter.	Approximately $q = \dfrac{T}{2tA}$	Any point on boundary where thickness = t.

Any irregular solid section (*see* R. & M. 334, A.R.C. Report 1917/18, Vol. 3).—(i) Draw the largest possible inscribed circle and measure its radius $= a$ (fig. 1).

(ii) If there be a sharp projecting corner A, round it off by an arc of radius r, where r is found from the curve of r/a in R. & M. 334 for values of $\frac{\theta}{\pi}$.

Note.—r/a is given sufficiently well for values of $\frac{\theta}{\pi}$ from 0 to $0{\cdot}8$ by the formula :—

$$r/a = 1 - 0{\cdot}96\,\frac{\theta}{\pi}.$$

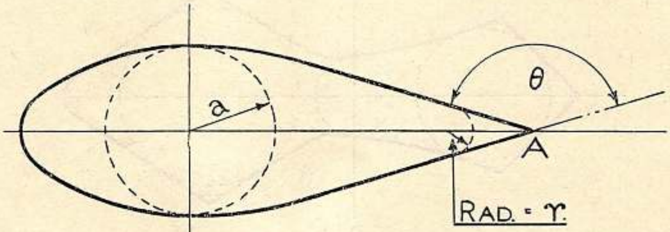


FIG. 1.—CHAP. VIII., Section III.

(iii) Let the area of the original section be A and its perimeter P : let the area and perimeter of the section be modified according to (ii) be A_1 and P_1. Let $h = \frac{2A}{P}$. Find K from the curve in the above R. & M. for values of a/h.

Note.—K is given sufficiently well for values of a/h from $0{\cdot}5$ to $1{\cdot}0$ by the formula :—

$$K = 3{\cdot}09\,\frac{a}{h} - 1{\cdot}614\left(\frac{a}{h}\right)^2 - 0{\cdot}476$$

Then find the quantity C given by :—

$$C = \frac{1}{2}\,KA\left(\frac{2A_1}{P_1}\right)^2$$

This formula gives C correct within 1 per cent. for sections such as triangles, ovals, etc., where only one value of a is possible. Other sections must be divided into component parts in the manner to be described below.

(iv) Having found C the shear stress can be estimated. The following remarks apply either to the whole of a simple section or to each component of a compound section, such as the spar section below. The maximum stress occurs at or near one of the points of contact of the largest inscribed circle which can be drawn in the section. The stresses at the three or more points of contact of the inscribed circle of maximum radius are given by :—

$$\frac{2aT}{(1 + m^2)\,C}\left\{1 + 0{\cdot}15\left(m^2 - \frac{a}{\rho}\right)\right\}$$

when $m = \frac{\pi a^2}{A}$ and ρ is the radius of curvature of the boundary at the point in question. (If the boundary is straight $\frac{a}{\rho} = 0$; if the boundary is concave $\frac{a}{\rho}$ is negative). The mean stress round the boundary of any component is given accurately by :—

$$\frac{2TA}{CP}$$

By means of these two formulæ the stresses all round the boundary can be estimated fairly accurately. The first of the two does not, strictly, apply to points where the boundary is concave, but it is found to agree with soap-film experiments for many sections, such as H, L and T sections, where the re-entrant angle is approximately a right angle, and is not very small.

In the division of compound solid sections there are really two classes to consider. In sections where there are two or more positions of the inscribed circle which give three or more points of contact, division is made by a straight line through the points of contact of the minimum inscribed circle which lies between two maximum inscribed circles.

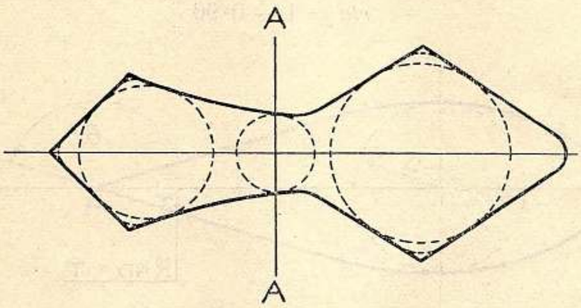


FIG. 2.—CHAP. VIII., Section III.

In fig. 2 the division line is AA. For sections such as H, T or L beams, the figure must be divided as follows :—In each of the sections, the points where the straight part of a boundary meets the curved parts are marked A. (*See* fig. 3.) The lines of division B are to be drawn at a distance from the commencement of the straight part, equal to half the thickness of the straight part. Each component can be treated separately (the spar and L section have three, the other has five components) ; each must have its corners rounded off as described above. The inscribed circles are arcs for rounding off the corners are shown dotted. The value of C can then be found for each component and the total C found by addition, but for each component, P must stand for only that part of the perimeter which forms part of the boundary of the whole section.

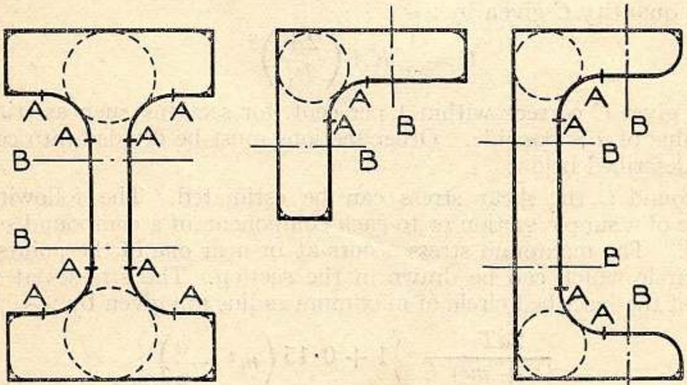


FIG. 3.—CHAP. VIII., Section III.

3. Torsion combined with other types of loading.—*Torsion combined with lateral load.*—In aeroplane structures torsion is usually accompanied by lateral load. The shear stress due to the lateral load occurs in a plane normal to the axis and in horizontal planes through the section (assuming the load vertical). Torsional shear stresses occur in a plane normal to the axis and in all planes through the axis of the section. Where a horizontal plane through the axis cuts the boundaries of the section the shear stresses due to twisting and bending will be algebraically additive. On one side they will add, on the other subtract. At the top and bottom of the section the shear stresses will be those due to torsion alone. Both cases may need consideration. In wooden beams the points on the side will probably be of the greater importance.

Combination of shear and direct stress.—On an element of material subject to a direct stress p and shear stress q, the following two formulæ giving the principal direct stresses and the maximum shear stresses are well known.

(*a*) Principal direct stress $= \frac{1}{2}p \pm \sqrt{(\frac{1}{4}\,p^2 + q^2)}$

(*b*) Maximum shear stress $= \sqrt{(\frac{1}{4}\,p^2 + q^2)}$

The shear stress may be due to torsion or lateral load or both, and the direct stress may be due to bending or axial load, or both. In beams subjected to combinations of these loads the section should be explored by means of the above formulæ.

N.B.—Owing to the fact that timber is neither homogeneous nor isotropic the above does not apply to timber members.

Section IV.—Shear and bearing of bolts, pins and rivets

1. Shear.—The strength in shear of bolts, pins and rivets depends on the associated bearing pressure, and decreases with increasing bearing pressure.

Until the bearing stress exceeds the shear stress this effect is very slight, but with high bearing stresses failure by shearing occurs at much lower shear stresses. The information available concerning the relationship between shear failure, shear stress, and bearing stress for pins in double shear can most conveniently be conveyed by recording allowable shear stresses against the geometrical proportions of the fitting under consideration.

If d = diameter of pin

and t = thickness of the centre plate of the three by which load is applied (*see* fig. 1)

then low d/t ratios will correspond to low bearing pressures and high ratios to high pressures.

Investigations have only been carried up to $\frac{d}{t} = 4$ at present.

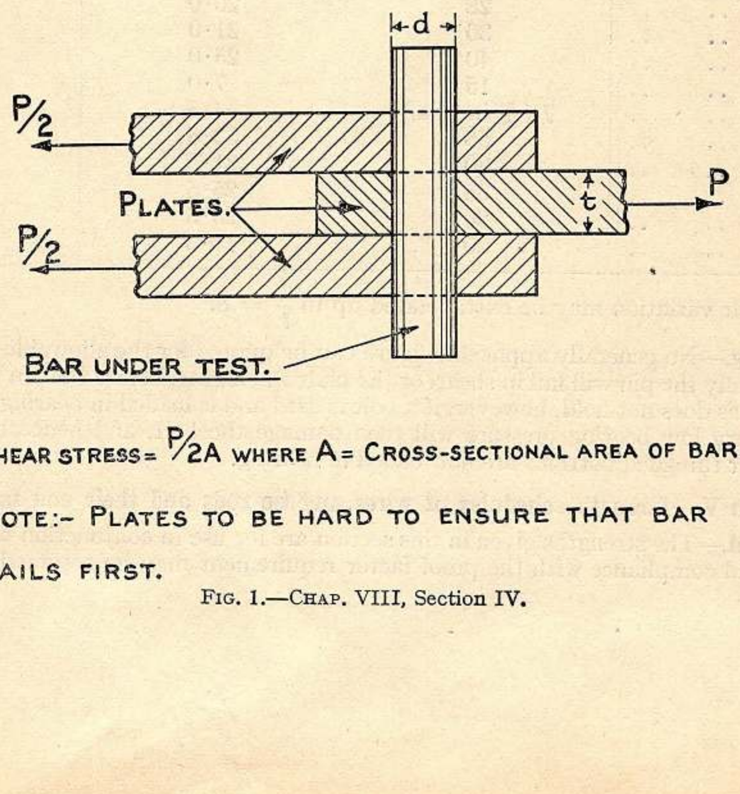


FIG. 1.—CHAP. VIII, Section IV.

The allowable shear stress, s, is given in the following table. This shearing stress is to be associated with the specified ultimate factor. Compliance with the proof factor requirement may then be assumed.

Allowable Shear Stress for Pins in Double Shear

(To be associated with the specified ultimate factor)

Material.	Minimum specification ultimate tensile stress (tons/sq. in.).	Shear stress (tons/sq. in.). Up to $\frac{d}{t} = 1{\cdot}75$	Shear stress (tons/sq. in.). At $\frac{d}{t} = 4{\cdot}0$
		(Linear variation from $\frac{d}{t} = 1{\cdot}75$ to $4{\cdot}0$.)	
B.2	16	15·0	12·5*
B.13	25	12·8	9·8*
3L.1	25	12·5	10·0
L.32	10	6·5	5·2*
L.35	14	10·0	8·0*
S.1	35	23·0	17·0
S.2	55	35·5	26·5
S.6	35	23·5	18·0
S.11	55	35·0	26·0
S.21	25	17·5	13·0
S.61	35	24·0	17·5
S.62	46	30·0	22·0
S.71	25	17·1	12·8
S.76	40	27·0	19·5
S.77	30	21·0	14·0
S.80	55	34·0	25·0
D.T.D.24A (non-magnetic)	30	22·0	18·5
D.T.D.24A (magnetic) ..	30	21·0	15·0
D.T.D.53A	28	20·0	15·0
D.T.D.78A	30	21·0	15·5
D.T.D.126	40	25·0	18·5
D.T.D.142	15	7·0	5·2
D.T.D.148	7 ($\frac{5}{16}$ in. dia.)	4·5	2·7
D.T.D.153	55	33·5	24·0
D.T.D.161	30	21·5	16·0
D.T.D.176	35	25·5	21·0
D.T.D.189	30	23·5	18·5
D.T.D.198	20	11·3	8·5

* The linear variation may be extrapolated up to $\frac{d}{t} = 8$.

2. Bearing.—No generally applicable figure can be quoted for the allowable bearing pressure on a pin. Usually the pin will fail in shear, or the plate in bearing, before the pin itself is damaged in bearing. This does not hold, however, if a bolt is used and is loaded in bearing on the threaded portion. A very low bearing pressure will then damage the bolt, and hence bolts must be so fitted that their threaded portions are not loaded in bearing.

Section V.—Strength schedules of wires and tie rods and their end fastenings

1. General.—The strengths given in this section are for use in conjunction with the ultimate factor load, and compliance with the proof factor requirement may be assumed without further investigation.

2. Streamline wires to Specification 5.W.3 and swaged tie rods to Specification 5.W.8.

Table I

Size.	Strength (lb.).
4.B.A.	1,050
2.B.A.	1,900
$\frac{7}{32}$ in.	2,600
$\frac{1}{4}$ in.	3,450
$\frac{9}{32}$ in.	4,650
$\frac{5}{16}$ in.	5,700
$\frac{11}{32}$ in.	7,150
$\frac{3}{8}$ in.	8,500
$\frac{13}{32}$ in.	10,250
$\frac{7}{16}$ in.	11,800
$\frac{15}{32}$ in.	13,800
$\frac{1}{2}$ in.	15,500
$\frac{9}{16}$ in.	19,300
$\frac{5}{8}$ in.	23,630
$\frac{11}{16}$ in.	29,610
$\frac{3}{4}$ in.	34,520
$\frac{7}{8}$ in.	48,190
1 in.	63,850
$1\frac{1}{8}$ in.	81,270
$1\frac{1}{4}$ in.	103,500

Note.—The strength of both streamline wires and swaged tie rods is the same for corresponding sizes.

3. High tensile steel wires to Specification 3.W.1.

TABLE II

Gauge.	Strength (lb.).
8	3,603
9	2,919
10	2,450
11	2,131
12	1,713
13	1,339
14	1,070
15	866
16	721
17	552
18	405
19	282
20	229
21	179
22	137
23	102
24	84

Note.—The strengths tabulated above are for the wire only. The maximum load will, however, usually be limited by the type of end fastening. *See* para. 4.

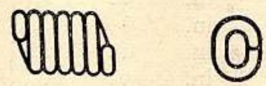
4. Strength of terminal connections of solid wires.—The end connections of solid wires have usually less strength than the wires themselves. The weakest point may be in—

(*a*) the fastening of the loop,

(*b*) the loop of the wire,

(*c*) the lug or eye through which the wire is threaded.

(*a*) *The fastening of the loop.*—The standard method of securing a loop of wire is by a ferrule or close coil of about six turns of wire, as shown in fig. 1.



(See A.G.S. 156)

FIG. 1.—CHAP. VIII., Section V.

The tail of the loop is normally bent back over the ferrule, *see* fig. 2. It has been found, especially with the thinner wires, that unless special precautions are taken the bent portion opens out under load and the wire pulls through the ferrule. Because of this only 45 per cent. of the nominal strength of the wire can be relied upon for end fastenings such as fig. 2. If, however, the

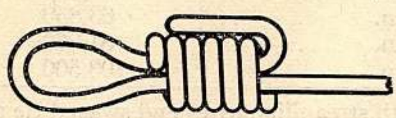
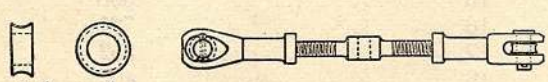


FIG. 2.—CHAP. VIII., Section V.

turned back end is bound to the ferrule with—for example—a soft iron wire, pulling through is prevented and some other type of failure supervenes. Sometimes sweating as well as binding is employed, but it is doubtful if this further precaution adds to the effective strength. If the turned back end is bound to the ferrule 60 per cent. of the nominal wire strength can be relied upon, failure then probably being in the loop.

(*b*) *Failure of loop.*—Tests show a considerable variation of strength of loops even between examples of the same design. Failure usually occurs in the neck at the entrance of the ferrule. Because of the variability of results it is unsafe to allow for the loop more than 60 per cent. of the nominal strength of the wire.

The form and diameter of the pin or roller round which the wire passes has little effect on the loop strength. Rollers used with turnbuckles (fig. 3) give practically the same failing load in the loop as when plain round pins are used instead of rollers.

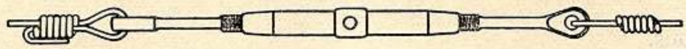


Enlarged

(See A.G.S. 702) (See A.G.S. 702)

FIG. 3.—CHAP. VIII., Section V.

(*c*) *Lug or eye.*—No general rule can be made for strength of lugs loaded by a wire, but warning is given that the intense bearing stress under the wire may cause the wire to cut through the lug at loads much less than the nominal failing load of the eye. Special attention is drawn to the reduction of strength of standard barrel type turnbuckles (fig. 4) when used with solid high tensile steel wires.



(See A.G.S. Nos. 490, 491, 492)

FIG. 4.—CHAP. VIII., Section V.

The following table gives allowable strength of the turnbuckles and wires in combination. The wires are assumed to be fastened with standard ferrules and the ends effectively turned back and tied. Strength is determined by failing load of loop or turnbuckle eye, whichever is the lower.

TABLE III

Turnbuckle.		Wire.		Strength of Combination.
A.G.S. No.	Nominal strength lb.	Gauge.	Nominal strength lb.	
490	560	18	405	243
		16	721	432
		14	1,070	530
491	1,120	16	721	432
		14	1,070	642
		12	1,713	1,028
		10	2,450	1,240
492	2,240	12	1,713	1,028
		10	2,450	1,470

5. Loop splices in straining cord and steel wire rope.—The efficiency of a loop splice in flexible steel wire rope of the standard length of $4\frac{1}{2}$ tucks is to be assumed not greater than 80 per cent.

For standard straining cord, where the loop is made by means of four bindings of copper wire sweated in position, the efficiency can be taken as 100 per cent.

Section VI.—Schedule of strength of materials.

1. General.—It is emphasised that the standard material properties quoted below should be used with considerable caution. If failure due to instability, or change in distribution of stress due to deformation under load, is anticipated the scheduled material properties may give no reliable indication of strength and an appeal to test will be necessary.

When compliance with requirements is based upon strength calculations the following rulings will generally apply.

(*a*) *Members in tension.*—The ultimate factor is to be based upon the ultimate stress, the proof factor on the 0·1 per cent. proof stress.

(*b*) *Members in compression.*—The ultimate factor is to be estimated from strut formulæ based upon the 0·2 per cent. proof stress (*see* section I, para. 1 of this chapter). It can generally be assumed that compliance with the ultimate factor requirement will automatically ensure compliance with the proof factor requirement.

2. Abbreviations.

p_1 0·1 per cent. proof stress, tons/sq. in., tensile unless otherwise stated.
p_2 0·2 per cent. proof stress, tons/sq. in., tensile unless otherwise stated.
p_5 0·5 per cent. proof stress, tons/sq. in., tensile unless otherwise stated.
Ult. Ultimate tensile stress, tons/sq. in.
y Yield stress, tons/sq. in.
E Young's Modulus/10^6 lb./sq. in.

Figures in heavy type are extracted direct from the material specifications.

A "?" preceding a figure indicates that it is a tentative value only and requires confirmation.

When upper and lower limits are quoted in a material specification the lower limit should always be used in strength calculations.

3. Strength of B.S.I. Materials.

Note.—Specifications marked * admit material of less than the nominal size. The most adverse tolerances must be taken into account in strength calculations. *See* Chapter I, para. 3.

*4.A.1. Bolts and Nuts (Low Tensile). *See* S.1 or S.61 stainless.

2.B.2. Bronze (Gun Metal) Castings. Ult. **16,** $p_1$7, $p_2$7·5, E 12·5.

*3.B.13. Brass Bars. Ult. **25,** $p_1$11, $p_2$11·5, E 14.

†4.L.1. Light Aluminium Alloy Bar (Dural.) up to 3 in. diameter. Ult. **25,** p_1**15,** $p_2$15·5, E 10·5.

*3.L.3. Light Aluminium Alloy Sheets (Dural.). Ult. **25,** p_1 **15,** E 10·5. *See* chapter IV, paras. 27 and 34.

2.L.32. Aluminium Bars. Ult. **10,** $p_1$3, $p_2$3·3, E 9. Cancelled.

L.35 Y Aluminium Alloy Castings (Heat-Treated). Ult. **14,** $p_1$12, $p_2$13, E 9·5.

*3.S.1. Bright Steel Bars. Ult. **35–45,** $p_1$27, $p_2$28·5, E 28·5.

2.S.2. 55-Ton Alloy Steel Bars. Ult. **55–65,** $p_1$45·5, $p_2$46, E 28·5.

2.S.3. Hot Rolled Mild Steel Sheets (for welding). Ult. **28,** $p_1$16·5, E 29·5.

2.S.4. 5 per cent. Nickel Steel Sheets (not for welding). Ult. **48,** $p_1$40, E 29.

2.S.6. "40" Carbon Steel Bars, etc. Ult. **35–45,** $p_1$20, $p_2$20·5, E 28·5.

3.S.11. 55-Ton Nickel-Chrome Steel Bars, etc. Ult. **55–65,** $p_1$43, $p_2$45, E ? 28.

† *Note.*—This material should preferably be used with the direction of the principal tensile or shear stress in the direction of the "grain" of the material, i.e., parallel to the axis of the bar. If the stress is transverse to the grain, the above allowable values must be reduced by 25 per cent. on proof stresses and 33 per cent. on ultimate stress.

Strength of B.S.I. Materials—*contd.*

S.61. High Chromium Steel Bars, etc. (Non-corrosive). Ult. **35–45,** $p_1$20, $p_2$21, E ? 29. *See* A.D.M. 206.

S.62. High Chromium Steel Bars, etc. (Non-corrosive). Ult. **46–52,** $p_1$30, $p_2$32, E ? 29. *See* A.D.M. 206.

S.77. " 30 " Carbon Steel, Hardened and Tempered. Ult. **30–40,** $p_1$19, $p_2$19, E 28·5.

S.80. 55-Ton Nickel-Chrome Steel Bars, etc. Ult. **55,** $p_1$45, $p_2$47, E 28.

*2.T.1. 35-Ton Steel Tubes. Ult. **35,** $p_1$29, $p_2$30, y **30,** E 28.

*2.T.2. Nickel-Chrome Steel Axle Tubes. Ult. **85–110,** $p_1$68, p_2**78,** E 29. *See* chapter IV, para. ~~21~~. 20.

*3.T.4. Duralumin Tubes, as supplied by manufacturers. Ult. **26,** p_1**18,** $p_2$19, E 10·5. *See* Chapter IV, para. 34.

*3.T.4. Duralumin Tubes, subsequently heat-treated. Ult. **25,** p_1**15,** $p_2$17, E 10·5. *See* Chapter IV, para. 34.

*T.5. Carbon Steel Tubes. Ult. **45,** $p_1$40·0, p_2**40·0,** E 29. Cancelled, *see* T.50.

*2.T.18. Hard Drawn Brass Tubes. Ult. **25–35,** $p_1$17·6, E ? 15.

T.21. Annealed Carbon Steel Tubes (obsolete). Ult. **28,** $p_1$18, $p_2$18, y **18,** E 28·5.

*T.35. 35-Ton Steel Tubes, suitable for welding.

(i) Before welding, Ult. **35,** $p_1$30, p_2**30,** E 27·4.

(ii) After welding, Ult. **30,** p_2**25,** E 27·4.

*T.45. 45-Ton Steel Tubes, suitable for welding.

(i) Before welding, Ult. **45,** $p_1$40, p_2**40,** E 28·8.

(ii) After welding, Ult. **30,** p_2**25,** E 28·8.

*T.50. 50-Ton Steel Tubes. Ult. **50,** $p_1$44·5, $p_2$45, E 28·5.

*4.V.3. All-birch plywood (3-ply)—

Tension : With outer grains at 0° to load 10,000 lb./sq. in.
,, ,, ,, ,, 45° ,, ,, 4,000 lb./sq. in.
,, ,, ,, ,, 90° ,, ,, 6,500 lb./sq. in.

Shear : With outer grains parallel to longitudinal axis of member: 1,800 lb./sq. in.

With grains at 45° to longitudinal axis of member: 2,100 lb./sq. in.

3.V.4. Ash E **1·5**; modulus of rupture **10,500** lb./sq. in.; end grain tension 12,700 lb./sq. in.; end grain compression **5,800** lb./sq. in.

3.V.5. Walnut E **1·5**; modulus of rupture **11,500** lb./sq. in. end grain compression **7,000** lb./sq. in.

4.V.7. Mahogany E **1·5**; modulus of rupture **10,000** lb./sq. in.; end grain compression **6,250** lb./sq. in.

Strength of D.T.D. Materials.

24A. Non-Corrosive Steel Rivets and Split Pin Wire. Ult. **30,** $p_1$15, E ? 30. Cancelled, *see* D.T.D. 161, 185, 189.

A.L.3. 36A. Silver spruce or approved substitutes. E **1·5** ; modulus of rupture 8,000 lb./sq. in. ; end grain compression **5,000** lb./sq. in. ; end grain tension 10,000 lb./sq. in. ; combined bending and compression 5,500 lb./sq. in. ; shear 900 ~~1,600~~ lb./sq. in. ; crushing on side grain 600 lb./sq. in.

*41. Weldable Mild Steel Tube (before welding). Ult. **30,** $p_1$26·5, $p_2$27, y **28,** E 28.

*41. Weldable Mild Steel Tube (after welding). Ult. **24,** $p_1$14, $p_2$14, y **17,** E 27.

42. Chromium Nickel Non-Corrosive Steel Sheets (Hard). Ult. **55,** $p_1$31, y **35,** E 27. Cancelled, *see* D.T.D. 144.

42. Chromium Nickel Non-Corrosive Steel Sheets (Soft). Ult. **40,** $p_1$13, y **15,** E 27. Cancelled, *see* D.T.D. 144.

43. Non-Corrosive Steel Bars (Hard) up to 2 in. diameter. Ult. **50,** y **22,** E ? 28·5. Cancelled, *see* D.T.D. 156.

43. Non-Corrosive Steel Bars (Hard) above 2 in. diameter. Ult. **45,** y 22, E ? 28·5. Cancelled, *see* D.T.D. 156.

43. Non-Corrosive Steel Bars (Soft). Ult. **40,** y **15,** E ? 30. Cancelled, *see* D.T.D. 156.

*46A. Non-Corrosive Steel Strip, Hardened and Tempered. Ult. 86·7, p_1**65,** E 29.

*53. Non-Corrosive Low Tensile Steel Bar. Ult. **28–35,** $p_1$13, $p_2$14, E 26·5.

*54A. High Tensile Nickel Chromium Steel Strip (Hard). Ult. 86·7, p_1**65,** E 29.

*57B. Chromium Nickel Non-Corrosive Steel Sheet and Strip. Ult. **54,** $p_1$37, p_5**50–60,** E ? 26·5. Cancelled, *see* D.T.D. 166.

60A. High Chromium Non-Corrosive Steel Sheet and Strip. Ult. **55,** $p_1$40, $p_2$43, p_5**45,** E 29·0.

78A. Hard Drawn Phosphor Bronze Bars, up to 2 in. dia. Ult. **30,** p_1**15,** $p_2$16, E 17·0.

78A. Hard Drawn Phosphor Bronze Bars, above 2 in. dia. Ult. **28,** p_1**15.**

*89A. Weldable Steel Tubes (before welding). Ult. **45,** $p_1$40, $p_2$40, y **40,** E 29. Cancelled, *see* B.S. T.45.

*89A. Weldable Steel Tubes (after welding). Ult. **30,** $p_2$18·5, y **25,** E 29. Cancelled, *see* B.S. T.45.

*91A. 50-Ton Steel Tubes (Hard Drawn and Blued). Ult. **50,** y **45,** E ? 30. Cancelled, *see* B.S. T.50.

*97. Low Tensile Non-Corrosive Tubes. Ult. **28,** $p_1$15·5, $p_2$16, y **18,** E 28·5.

Strength of D.T.D. Materials—*contd.*

*99. Nickel-Chrome Steel Strip (Hardened and Tempered). p_1**55,** E ? 27.

*102. 35-Ton Non-Corrosive Tubes. Ult. **35,** $p_1$28, $p_2$29, y **30,** E 29.

*105. 50-Ton Non-Corrosive Tubes. Ult. **50,** $p_1$32, $p_2$36, y **40,** E 29·5.

*111. "Alclad" Aluminium Coated Light Alloy Sheet. Ult. **24,** $p_1$13·5, E ? 9·7. *See* chapter IV, para. 27.

*113. Steel Tubes suitable for welding (before welding). Ult. **35,** $p_1$30, $p_2$30, y **30,** E 28·5. Cancelled, *see* B.S. T.35.

*113. Steel Tubes suitable for welding (after welding). Ult. **30,** $p_2$19, y **25,** E 29. Cancelled, *see* B.S.T.35.

*118. Magnesium Alloy Sheets, Ult. **11.**

*124. 40-Ton Proof Carbon Steel Strip (before welding). Ult. 42, p_1**40,** E ? 28.

*124. 40-Ton Proof Carbon Strip (after welding). Ult. **30,** E ? 28.

126. Carbon Steel Bars, suitable for welding. Ult. **40,** $p_1$31, $p_2$34, E 28·0.

129. Magnesium Alloy Bars. Up to 2 in. dia. Ult. **20,** p_1**12,** $p_2$14, E 6·4.

*141. Cold Rolled, Close Annealed, Mild Steel Sheets, Ult. **20–28.**

142. Magnesium Alloy Bars. Ult. **15,** p_1**8,** $p_2$10, E 6·0.

*146A. High Chromium, Non-Corrodible Steel Sheet and Strip, Ult. **40,** p_1**30.**

153. Bright Steel Bars (for Pins and H.T. Bolts). Ult. **55,** $p_1$44, $p_2$49, E 27.

161. Non-Corrodible Steel Rods, Ult. **30,** $p_1$14, $p_2$14·5, E 26.

*166A. Chromium Nickel Non-Corrosive Steel Sheet. Ult. **52–70,** p_1**40–50,** E ? 25·4.

*167. 45-Ton Steel Tubes. Ult. **45,** $p_1$35·5, $p_2$39, y **40,** E 29·5.

*168. High Chromium Non-Corrodible Steel Sheet and Strip.
(i) Soft, Ult. **50.**
(ii) Hardened and Tempered, Ult. **60.**

*171A. Chromium Nickel Non-Corrodible Steel Sheet and Strip, Ult. **35,** p_1**15.**

176A. Chromium Nickel Non-Corrodible Steel, Ult. **35,** p_1**15,** $p_2$16·0, E 28·0.

185A. High Chromium Non-Corrodible Steel Rod, Ult. **30–50,** $p_1$17·5, $p_2$19, E 24·0.

Strength of D.T.D. Materials—*contd.*

*186A. 7 per cent. Magnesium-Aluminium Alloy Tubes (hard).
(i) 12 s.w.g. and under, Ult. **26**, p_1**18.**
(ii) Over 12 s.w.g., Ult. **25**, p_1**17.**

189. Chromium Nickel Non-Corrodible Steel Rod, Ult. **30**, $p_1 10\cdot5$, $p_2 11\cdot0$, E $28\cdot0$.

*190. 7 per cent. Magnesium-Aluminium Alloy Tubes (annealed), Ult. **20–23**, p_1**10.**

198. 7 per cent. Magnesium Aluminium Alloy Rivets. Ult. **20**, $p_1 13\cdot5$, $p_2 14\cdot0$, E $10\cdot0$.

*199. 50-Ton High Chromium Non-Corrodible Steel Tubes, Ult. **50**, $p_1 40$, p_2**45**, E $30\cdot5$.

*203. 50-Ton Non-Corrodible Steel Tubes, Ult. **50**, p_2**45.**

*207. 35-Ton Chromium Nickel Non-Corrodible Steel Tubes, Ult. **35**, $p_1 14$, $p_2 16$, E 27.

*211. 50-Ton Chromium Nickel Non-Corrodible Steel Tubes, Ult. **50**, $p_1 37$, p_2**45**, E $25\cdot5$.

*220. Wrought Light Aluminium Alloy Tubes, Ult. **27**, p_1**22.**

*225. High Chromium, Non-Corrodible Steel Sheets and Strips, Ult. **35–40**, p_1**20.**

Section VII—Direction of grain in fittings

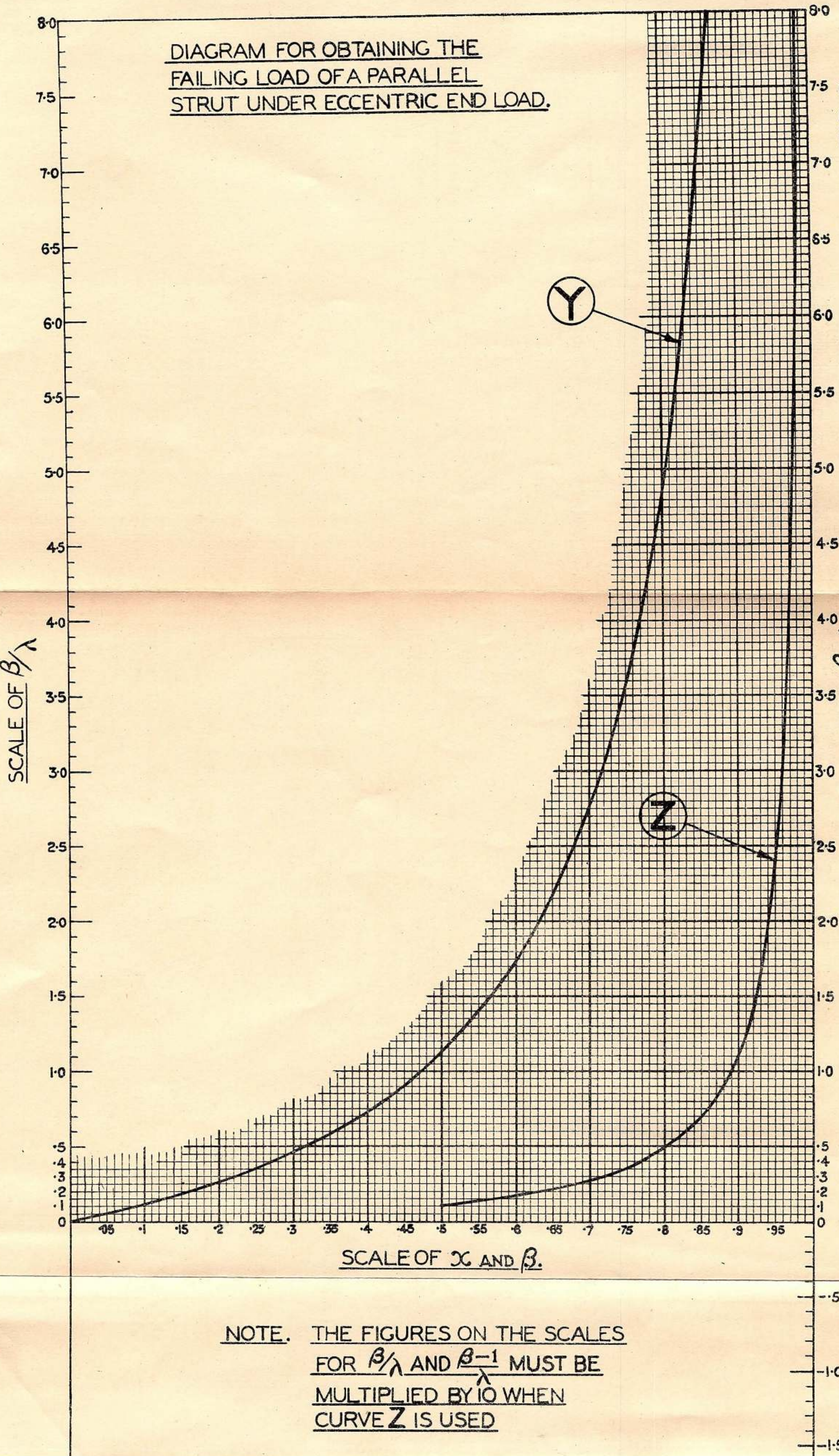
The importance of grain direction as affecting the ability of parts to withstand shock loading is clearly indicated by the Izod test, the Izod figure being considerably reduced when the direction of the blow is parallel to the grain.

This is of special importance in the case of machined parts, and generally the best disposition of grain can be obtained by suitable forging or stamping. It may be that considerations of cost will sometimes preclude this course, and in such cases the design of the part should be carried out * in such a way as best to dispose the grain of the metal to resist shock. This especially applies to B.S. Specification S.80. This material must not be used for machined fittings unless the grain is suitably disposed and the drawings of the parts are to show the grain direction.

In the case of bars of $2\frac{1}{2}$ in. diameter and over, attention is drawn to the note with regard to the use of heat-treated bars of large diameter now issued with B.S. Specifications for aircraft steels. This note affords an added reason for adopting the course outlined in the preceding paragraph.

FIG. 2.
CHAP. VIII. SECTION I.

DIAGRAM FOR OBTAINING THE FAILING LOAD OF A PARALLEL STRUT UNDER ECCENTRIC END LOAD.



NOTE. THE FIGURES ON THE SCALES FOR β/λ AND $\dfrac{\beta-1}{\lambda}$ MUST BE MULTIPLIED BY 10 WHEN CURVE Z IS USED

CHAPTER IX.—AIRSCREWS

Section I.—Calculation of performance and stresses

1. Estimation of aerodynamic performance

The aerodynamic performance of an airscrew can be calculated by the "Vortex Theory" of R. & M. 786 and 869 or by the later theory of R. & M. 1521, and the torque and thrust coefficients and the efficiency over the normal working range of advance per revolution obtained. Such analysis is, however, somewhat protracted and it is usually not necessary to calculate the torque and thrust coefficients at more than two or three values of advance per revolution in order to determine whether the strength requirements are satisfied. For this reason, it is usual to employ a method of calculating, by successive approximation, the aerodynamic performance at certain predetermined values of advance per revolution.

By this method, the inflow factor a, for the particular value of advance per revolution under consideration, is first estimated and the subsequent analysis follows the well-known method of Lanchester and Drzewiecki. In determining the inflow factor, it is assumed that $a = \frac{b}{2}$, which is in accordance with the "Vortex Theory," where

$V(1 + a)$ = velocity of the inflowing air at the disc, and

$V(1 + b)$ = maximum velocity of the outflowing air behind the disc, both relative to the screw;

and V = velocity of advance of the screw with reference to still air.

The inflow factor is assumed to be constant over the annulus bounded by circles of radii $\frac{D}{4}$ and $\frac{D}{2}$ and is assumed to fall linearly to zero from radius $\frac{D}{4}$ to radius $\frac{D}{6}$.

In the analysis, due allowance must be made for the effect of the actual density of the air in which the screw is working, both on the airscrew characteristics and on the B.H.P. developed by the engine.

For air at standard sea level temperature and pressure, the density ρ is equal to $0 \cdot 002378$ slugs per cu. ft. and the density at any other altitude is given by fig. 1 where the density ratio σ is plotted against altitude.

In the case of normally aspirated engines, if.—

H_h = B.H.P. developed by the engine at the altitude under consideration and at the crankshaft speed permitted by the airscrew,

and H_o = B.H.P. developed by the engine at the same crankshaft speed at ground level,

then $H_h = f\,H_o$

where f is the power factor and may be taken as equal to the pressure ratio (also given in fig. 1).

In the case of ground boosted or supercharged engines, the B.H.P. developed below the rated altitude will be obtained directly from power curves. Above the rated altitude the power may be taken as proportional to the ratio of the atmospheric pressure to that at the rated altitude.

2. Estimation of radial fibre stresses

(i) *General.*—The selection of the particular conditions of flight in which the maximum steady radial fibre stresses are likely to occur is a matter requiring careful consideration in each case. Generally, with wooden airscrews, the stresses are greatest in the static or "take-off" condition when the engine is at full throttle or maximum permissible boost. In addition, the conditions must be considered of full throttle climb at the rated height, full throttle level flight at the maximum power altitude and high speed dives engine on or off.

With metal airscrews, each of the above-mentioned conditions must also be considered, but, in general, the maximum steady radial stresses will occur in the static or " take-off " condition or in high speed dives.

A fairly rigid determination of the radial stresses in airscrew blades can be made by use of either of the two methods of R. & M. 420. The estimation of the stresses by either of these methods is, however, somewhat involved, and experience has shown that the following approximate method is sufficiently accurate for the investigation of the strength of blades of conventional design under normal conditions of use.

The important steady radial stresses occurring at any blade section are as follow.—

(a) Direct tension due to centrifugal force,

(b) Fibre stress due to bending resulting from the air reactions of torque and thrust,

(c) Fibre stress due to bending resulting from centrifugal force,

each of which is calculated separately and summed algebraically to give the total stress in the tabular form shown in fig. 2.

At the blade roots and at the hub sockets or boss, these stresses will be accompanied by important longitudinal shear stresses which will usually be greatest for the high speed dive condition of flight and which must be determined and considered in the design.

(ii) *Calculation of tension due to centrifugal force.*—Consider a strip of the blade of radial length dr at a mean radius r and let the area of cross-section of the blade at this point be A. Then the centrifugal force of the element is

$$dF = A\frac{\lambda}{g}\,\Omega^2\,rdr$$

where λ is the density of the material of which the blade is constructed, and Ω is the angular velocity in radians per second. Hence the total centrifugal force acting at a section of the blade distant r_1 from the centre is

$$F_{r1} = \frac{\lambda}{g}\,\Omega^2\int_{r_1}^{D/2} Ardr$$

The centrifugal tensile stress at any radius r_1 is then given by

$$f_{r1} = \frac{F_{r1}}{A_{r1}}$$

where A_{r1} is the area of the cross-section. The areas of cross-section can be obtained by graphical means or by use of the formulæ given in fig. 3, which will be found sufficiently accurate in most cases.

For wooden airscrews with blades to which metal sheaths are fitted, the additional tensile stress due to the sheath is calculated on the assumption that any section of the blade has to withstand the centrifugal force of the portion of the sheath between that section and the tip of the blade.

(iii) *Calculation of fibre stresses due to bending resulting from the air reactions of torque and thrust.*—Consider an element of the blade of radial length dr at a mean radius r, where the chord width is c and the air pressure p. Then the load on the element is $pcdr$ and the bending moment at radius r_1, caused thereby, is

$$dM = pc\,(r - r_1)\,dr$$

Assuming that the resultant forces on the blade act normal to the chord and neglecting the variation of angle along the length of the blade, the total aerodynamic bending moment on any section is the sum of all such elemental moments over that portion of the blade between the section under consideration and the tip, i.e.,

$$Mr_1 = \int_{r_1}^{D/2} pc\,(r - r_1)\,dr.$$

Hence the aerodynamic bending moments at all radii can be evaluated by integrating curves of $pc\ (r - r_1)$ plotted against r (fig. 4).

The air pressure p is obtained directly from the formula

$$p = \frac{\rho\ k_L\ W^2}{\cos \gamma}$$

where W is the velocity of the air relative to the element, and

$$\gamma = \cot^{-1} \frac{k_L}{k_D}$$

As γ is usually small, $\cos \gamma$ in general approximates to unity. The values of lift and drag coefficients used must be appropriate to the aerofoil sections of the element considered and must have been corrected to infinite aspect ratio (*see* Appendix A).

If, however, the " Vortex Theory " is used, then according to R. & M. 869

$$W = r\Omega\ (1 - a_2) \sec \phi,$$

where a_2 is the rotational interference factor, so that p can be obtained from the equation

$$p = \frac{\rho k_L r^2 \Omega^2\ (1 - a_2)^2 \sec^2 \phi}{\cos \gamma}$$

where ρ, r, Ω, a_2 and γ are as previously defined and ϕ is the angle which W makes with the plane of rotation.

Employing the engineer's theory of bending, and assuming the major principal axis of the cross-section of the blade to be parallel to the chord, then the fibre stresses due to the bending moments calculated as described, follow immediately when the moduli of resistance of the sections Z_c and Z_r, for compression and tension respectively, are known. These moduli are determined about an axis through the centroid of the section and parallel to the chord. Their exact values can be determined by graphical means, but for most purposes the approximate values given in fig. 3 will be found sufficiently accurate. In any case of doubt or where the sections are of appreciably different form from those illustrated in fig. 3, the true moduli should be obtained graphically.

(iv) *Calculation of bending stresses due to centrifugal force.*—Unless the locus of the centroids of the cross-sections of the blade is a radial line contained in a plane parallel to that of rotation, bending stresses arise from centrifugal force. As in para. 2 (ii) the centrifugal force of each element of the blade is

$$dF = \frac{\lambda}{g}\ \Omega^2\ Ar\ dr$$

which force lies in a plane parallel to that of rotation and acts radially through the centroid of the element under consideration. If the line of action of each such elemental centrifugal force be projected towards the centre of the screw and z be the departure of the point of intersection of each such line with respect to the centroid of the section about which the bending moment is to be found, then

$$dM = \frac{\lambda}{g}\ \Omega^2\ Arz\ dr \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(fig. 5).}$$

is the bending moment at that section due to the centrifugal force of the outer element considered. This elemental bending moment can be resolved into two components

$$dM_1 = \frac{\lambda}{g}\ \Omega^2\ Arz \sin\ \beta\ dr$$

and

$$dM_2 = \frac{\lambda}{g}\ \Omega^2\ Arz \cos\ \beta\ dr$$

where β is the angle between z and the principal axis of the section considered.

The former moment, which produces bending about the assumed major principal axis, thereby inducing radial fibre stresses on the pressure and suction faces of the blade, need alone be considered. The latter moment can be neglected as the fibre stresses occurring near the leading and trailing edges which result therefrom are small.

The bending moment at any section at radius r_1 about the assumed major principal axis, due to the centrifugal forces of all outer elements, is

$$M_{r1} = \frac{\lambda}{g}\,\Omega^2\int_{r_1}^{D/_2} Arz \sin\,\beta\, dr.$$

The component departures $z \sin\,\beta$ are best found graphically on a true-to-scale diagram of the blade, whence curves of $Arz \sin\,\beta$ are plotted against r (fig. 6), and the areas enclosed thereby determined. For wooden airscrews with blades to which metal sheaths are fitted, similar measurements are made of the component departures of the centrifugal forces of elements of the metal sheathing. The bending moments at any section at radius r_1 about the assumed major principal axis, due to the centrifugal forces of all outer elements of the sheathing, is then obtained in a similar manner and added algebraically to the bending moments due to the centrifugal force of the outer part of the blade.

(v) *Effect of blade deflection.*—The stresses in the blades will be affected by their deflection under load. For wood, however, the deflection cannot be calculated with accuracy and, as the effect of deflection will usually be to decrease the calculated stresses, the neglect of this factor errs on the safe side.

For solid metal blades it is possible to estimate the deflection under load, and where necessary such deformation, which will reduce radial stresses, can be taken into account in strength determination. The deflection at any section of the blade at radius r_1 is given by

$$\frac{1}{E}\iint_{r_1}^{D/_2}\frac{M}{I}\,dr\,dr$$

where M is the nett bending moment, I the minor moment of inertia of the section and E is Young's Modulus. As I varies along the length of the blade in a manner which cannot be simply expressed as a function of r, and since the nett bending moment varies with the deflection, the integration cannot be performed analytically, and graphical methods of successive approximation have to be adopted.

For this purpose, the blade is regarded as flat at an angle corresponding to that obtaining at 0·7 of the tip radius and a probable deflection curve assumed, the deflection being taken normal to the assumed flat blade. The centrifugal bending moments corresponding to this deflected state are determined in the manner previously described, and added algebraically to the aerodynamic bending moments, thus giving the resultant bending moment at each section. The deflection at any section of the blade at radius r_1 is then found from the formula given above.

The method is continued as a series of successive approximations until the assumed and calculated deflection curves are in reasonable agreement. Unless the actual stresses are required it is not necessary to carry the process further than the stage at which it can be seen that the stresses will not exceed the maximum permitted. Deflection need only be taken into account in cases where the nett bending moments calculated for the undeflected blade lead to stresses in excess of the maximum permissible.

3. Blade stiffness of wooden airscrews

It is not yet possible to issue precise rules for the prevention of airscrew blade flutter, but the following information supplements that already published in R. and M. Nos. 1258 and 1518.

R. and M. 1258 gives the variation of thickness-chord ratio with radius and the minimum values of this ratio recommended for mahogany blades of normal chord-diameter ratio. Experience has shown, further, that a heavy blade tip may induce a tendency to flutter unless the stiffness of the other parts of the blade is above normal.

The values of maximum thickness and chord, therefore, when plotted against radius, sh... give faired curves and these values should in general decrease with increase of radius ove... outer part of the blade. This is consistent with the principle, which is important for airs... of all types, that change in section and in stiffness throughout the length of the blades shou... gradual.

Section II.—Design requirements

1. General requirements

The following requirements apply to airscrews irrespective of the material of construction.

(i) The design of fixed-pitch airscrews and also of adjustable-pitch airscrews, with the blades set at the appropriate pitch, must be such that.—

(*a*) in full throttle level flight at the maximum power altitude of the engine, the maximum emergency crankshaft speed is not exceeded ;

(*b*) in a full throttle climb at the best climbing speed at the rated height of the engine, the maximum crankshaft speed established for the type engine in climbing flight is not exceeded ;

(*c*) at take-off, the crankshaft speed is not less than the minimum nor more than the maximum take-off r.p.m. established for the type engine at the take-off power or boost.

The design of variable-pitch airscrews must be such that the crankshaft speeds can be regulated as above by means of the pitch or governor speed control.

(ii) The maximum total steady radial stress, including any stress due to metal sheathing and protective covering, must not exceed, in all conditions of use, the allowable value for the material of construction (*see* paras. 2 (ii) and 3 (i)).

(iii) For blades of a material or of a form of construction with which a high margin of torsional strength is not obtainable, the plan form must be geometrically symmetrical, or approximately so, about a radial line extending from the centre of rotation to the tip. Moreover, the axis of symmetry of each blade, when viewed in elevation, must be straight or approximately so, and must not be tilted forward of a line normal to the axis of rotation by an amount appreciably in excess of that required to avoid the stresses exceeding the allowable value for the material.

(iv) The blades must be sufficiently rigid to resist flutter in all conditions of use. The first or a similar airscrew of any conventional design may be required satisfactorily to complete test (*a*) and, where applicable, test (*b*) of para. 1 (viii) if doubt exists whether the blades are sufficiently stiff to resist flutter.

(v) The design must be such that static, dynamic and aerodynamic balance, within permissible limits, can be attained during manufacture of the airscrew and, in the case of detachable blades, the design must be suitable for the production of blades which are interchangeable to a reasonable extent with a standard blade of that design.

(vi) The diameter and (in the case of fixed-pitch airscrews) the pitch are to be quoted in the drawings in feet and in decimals of a foot, the pitch being calculated by the formula.

$$P = \text{pitch in feet} = 2 \cdot 2D \tan \theta,$$

where D is the diameter of the airscrew in feet and θ is the angle in degrees with the plane of rotation of a section of the blade distant $0 \cdot 7$ of the tip radius from the axis of rotation. The angle θ is that included between the plane of rotation and the chord of the section, when the pressure face is curved, or between the plane of rotation and the pressure face of the section when this face is flat over the greater part of the width.

(vii) The blades and hubs of adjustable-pitch airscrews must be provided with suitable marks so that the blades can be set at the same pitch within the range of pitches required in flight.

(viii) The first or a similar airscrew which incorporates untried features, whether of design or construction, or which incorporates detachable blades of new design may be required satisfactorily to complete the test (*a*) detailed below. Where the pitch is adjustable or variable, the airscrew must satisfactorily complete tests (*b*) and (*c*) following the satisfactory completion of test (*a*), and the design is to be such that the blades can be set and locked at the pitches required to enable tests (*a*) and (*b*) to be made. Where the pitch is fixed, the airscrew may be required satisfactorily to complete endurance bench tests on an engine of the type for which it is designed, details of the tests being arranged in consultation with the designer. Tests (*a*), (*b*) and (*c*) will normally be made under static conditions of running. When the airscrew is provided with a detachable spinner and/or nose cap these parts are to be fitted to the airscrew for tests (*a*) and (*c*).

(*a*) Spinning, by means of electric motors at a rotational speed 5 per cent. in excess of that at which is absorbed the established minimum take-off power of the engine for which the airscrew is designed. Where the pitch is adjustable or variable this test will be made with the blades set to absorb the established take-off power when the airscrew is running at a speed corresponding to the minimum take-off r.p.m. This test will normally be of 30 min. duration.

(*b*) Spinning, by means of electric motors at a rotational speed 5 per cent. in excess of that corresponding to the established maximum diving speed of the engine for which the airscrew is designed, except for variable-pitch airscrews with automatic governor control, where the speed of the test will be 5 per cent. in excess of the highest speed in flight permitted by the automatic control. The pitch of the blades for this test may be determined by the designer and will normally be such that, during the test, not more than the established maximum climbing power of the engine is absorbed. The duration of this test will normally be 1 hour.

(*c*) Endurance bench test of 50 hr. duration on an engine of the type for which the airscrew is designed (*see* para. 4).

(ix) Adjustable or variable-pitch airscrews which have previously been approved for use on an engine of specified B.H.P. and r.p.m. may, after investigation, be approved for flight tests, without further spinning or bench tests, for use on another type of engine of similar power rating.

2. Particular requirements for fixed-pitch wooden airscrews

(i) The designs are to be suitable for construction in mahogany, B.S. Specification V.7 and must satisfy strength requirements in this material.

(ii) The allowable stresses for the design of mahogany airscrews are.—

Compression	2,500 lb./sq. in.
Tension	4,000 lb./sq. in.
Longitudinal shear	500 lb./sq. in.

(iii) Blades must be designed to be capable of carrying, under all conditions of use, a metal sheath of approved design suitable for the type of aeroplane and the size of the airscrew.

(iv) Airscrew drawings must comply with the requirements of S.I.S. No. 145 and must specify the type of protective finish to be used.

(v) All laminae are to be continuous at the boss except that, when essential to the design, discontinuous outer laminae may be permitted up to a maximum total thickness of one-hundredth of the diameter of the airscrew on each side.

(vi) Except where specially conceded, four-blade airscrews must be of two-part construction.

(vii) Where metal spinners are provided the drawings are to call for discs of emery, glued back to back, or of other approved friction material to be interposed between the adjacent metal surfaces of spinner plates and hub flanges.

3. Particular requirements for metal airscrews

(i) The allowable tensile or compressive stresses for the design of metal airscrews, subject to the provisions of sub-paras. (ii) and (iii) are.—

(*a*) Duralumin to Specification No. D.T.D.147 or D.T.D.150 6·75 tons/sq. in.

(*b*) Mild Steel sheet to B.S. Specification S.3 .. .. 10·0 tons/sq. in.

(ii) In blades of hollow construction, the maximum steady compressive stress, in any condition of use, must not exceed one-half of the steady compressive stress at which instability of the section occurs.

(iii) In order to reduce the risk of fatigue failure, a margin of safety, additional to that indicated by the allowable stresses given in sub-para. (i), must be allowed at the roots and over the inner parts of the blades, particularly where abrupt changes of section are necessary. The allowable values for the stresses in these parts of the blade and, in the case of detachable blades, the corresponding values for shear and bearing stresses, will be determined in consultation with the designer.

(iv) The drawings must be fully dimensioned and must give particulars of the material and finish. Furthermore they must either call for manufacture to be in accordance with Specification G.E.139 and give particulars of the manufacturing tolerance and processes, or refer to an approved specification which contains full instructions for manufacture.

(v) The surface of metal airscrews, unless manufactured from non-corrodible material, must be protected in an approved manner against corrosion, both internal and external surfaces being treated when hollow construction is employed. For duralumin, mild steel and magnesium alloy airscrews, the approved protectives are anodic treatment, stove enamel and chromate treatment, respectively.

4. Engine bench tests of adjustable or variable pitch airscrews

(i) For test (*c*) of para. 1 (viii), the airscrew is to be fitted to the engine and the undermentioned schedule of tests is to be followed, the pitch or pitch controls and the engine throttle or boost pressure being adjusted to permit the engine to run under the specified conditions. If the airscrew has already been submitted to engine tests during the type trials of the engine, the tests so made need not be repeated for approval of the airscrew, provided the same airscrew is used for any remaining bench tests.

(ii) A proof of two hours' duration is to be made with the engine running at the maximum r.p.m. and power established for continuous cruising flight at sea level. The airscrew and pitch controlling mechanism, if any, are then to be removed from the engine and after balance, blade angles and track dimensions of the airscrew have been checked, are to be completely dismantled for examination and the recording of the condition and the clearances between any parts in relative motion one with another.

(iii) On re-assembly and re-fitting of the airscrew, the engine is to be run, at the r.p.m. and power specified in sub-para. (ii), for 20 hours in two 10-hour non-stop runs. During the last few minutes of every two hours of this test the airscrew pitch controls, if any, are to be operated through the normal range without change of engine throttle opening provided the r.p.m. are not reduced below the established minimum take-off r.p.m. or raised above the maximum emergency r.p.m. At the end of each 10 hours the airscrew hub and blades are to be examined in position for excessive wear or other defects.

(iv) The engine is then to be run, at the maximum r.p.m. and power established for climbing flight at sea level, for 20 hours in convenient periods, the airscrew pitch controls, if any, being operated during the last few minutes of each two hours as specified in sub-para. (iii). At the end of each 10 hours and also at other convenient periods, the hub and blades are to be examined in position for excessive wear or other defects.

(v) The following non-stop runs are then to be made.—

(*a*) Two hours at the established minimum take-off conditions.

(*b*) Two hours at the established maximum take-off conditions.

(*c*) Two hours at r.p.m. intermediate between the minimum and maximum take-off r.p.m. and at the established take-off boost or power.

(*d*) Four quarter-hour runs to cover the normal cruising range of r.p.m. and power, at 80, 85, 90 and 95 per cent. of the maximum r.p.m. established for continuous cruising flight at approximately 50, 60, 70 and 85 per cent., respectively, of the power established for continuous cruising at sea level.

(*e*) One hour at 5 per cent. in excess of the maximum emergency r.p.m. at approximately 30 per cent. of the rated power or, if necessary, at the higher power associated with the minimum pitch obtainable. For variable-pitch airscrews with automatic governor speed control, this test will be made at the highest speed permitted by the governor control in flight and at the minimum power required to run the engine and airscrew at this speed.

(vi) With the engine running under the conditions specified in sub-para. (ii) for the proof test, the throttle setting is to be reduced until the engine runs at 25 per cent. of the maximum r.p.m. established for continuous cruising flight. At the end of five minutes the engine is to be accelerated rapidly to full throttle or maximum boost for level flight at least five times.

(vii) Finally, the airscrew and pitch controlling mechanism, if any, are to be removed from the engine and, after balance, blade angles and track dimensions of the airscrew have been checked, are to be completely dismantled for examination and the recording of the condition and the clearances between any parts in relative motion one with another.

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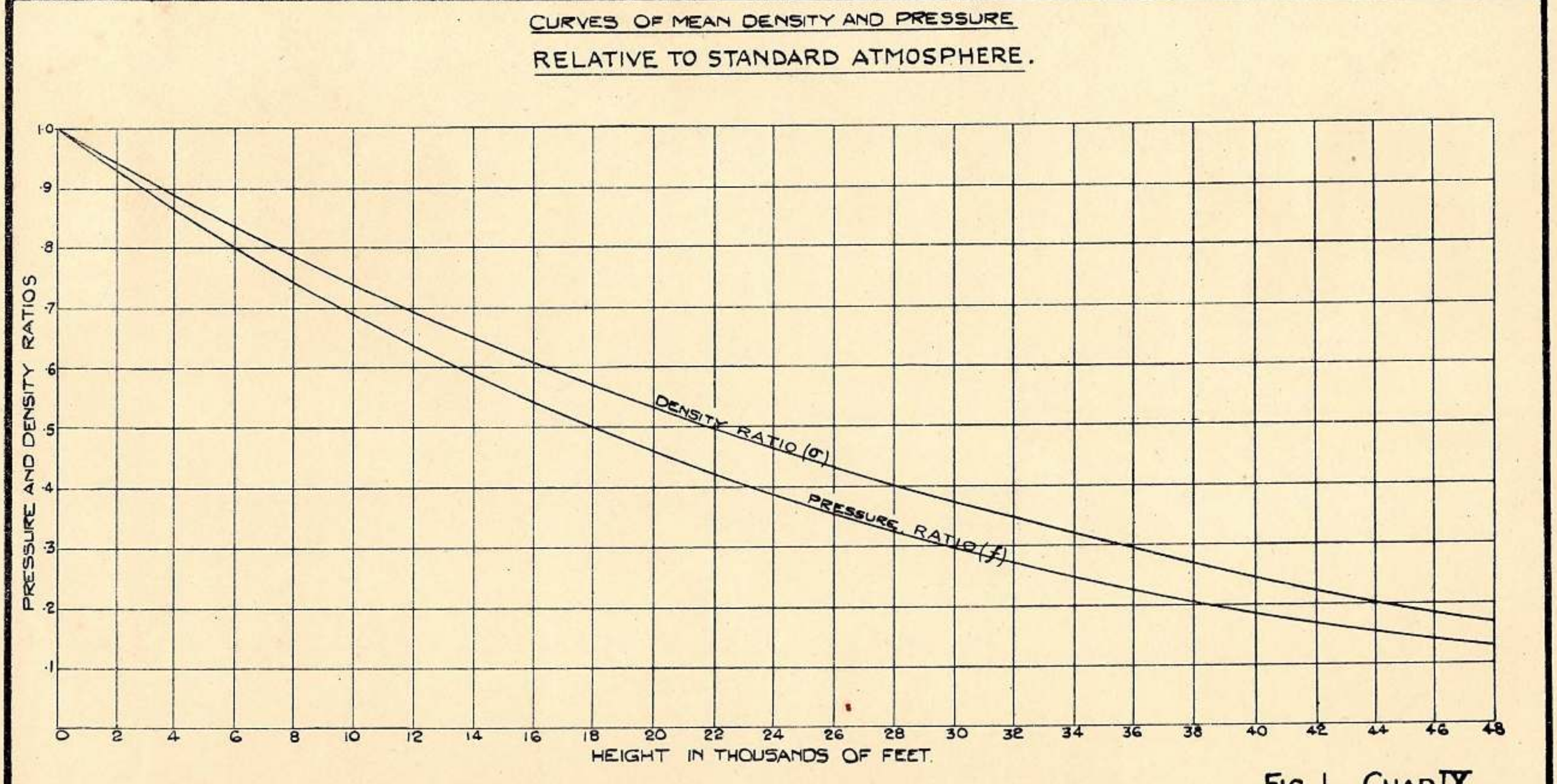

FIG. I. CHAP. IX.

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SUMMARY OF AIRSCREW CALCULATIONS.

AIRCRAFT:-

ENGINE:- GEAR RATIO:-

DRG Nº:- DIA.FT:- PITCH.FT:- Nº OF BLADES:-

ALTITUDE FT:- M.PH:- R.P.M:- =R.P.S:-

B.H.P.:- η :-

AEROFOIL DATA USED:-

MATERIAL:-

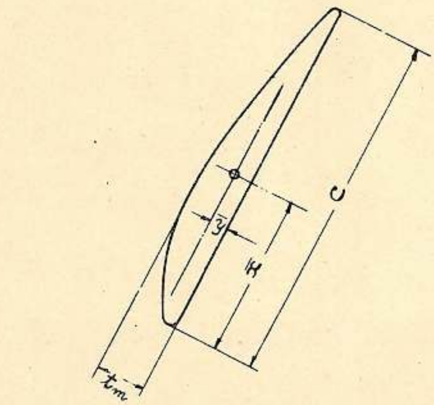
RADIUS r FEET								
CHORD C INS.								
THICKNESS t_m INS.								
* DEFLECTION INS.								
Z INS.^{3} COMPRESSION								
Z INS.^{3} TENSION								
AERO.B.M. LB.INS.								
CENT.B.M. LB.INS.								
NETT.B.M. LB. INS.								
NETT B.M. STRESS LB./INS.^{2} COMPN								
NETT B.M. STRESS LB./INS.^{2} TENSION								
CENTRIFUGAL FORCE								
AREA OF SECTION INS.^{2}								
CENTRIFUGAL TENSILE STRESS LB./INS.^{2}								
TOTAL STRESS LB./INS.^{2} COMPN								
TOTAL STRESS LB./INS.^{2} TENSION								

* SOLID METAL AIRSCREWS ONLY.

FIG. 2. CHAP.IX.

APPROX: FORMULAE FOR POSITIONS OF CENTROIDS, AREAS AND MODULI OF RESISTANCE OF TYPICAL AIRSCREW SECTIONS.

THE CONSTANTS EMPLOYED IN THESE FORMULAE HAVE BEEN DERIVED FROM AN ACCURATE COMPUTATION OF A LARGE NUMBER OF SIMILAR SECTIONS.



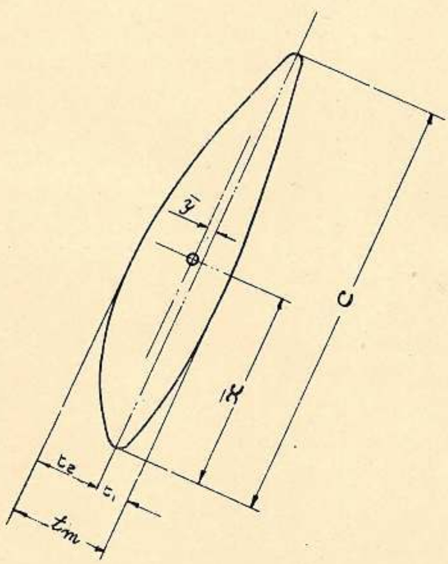
$\bar{x} = \cdot 46\,C$

$\bar{y} = \cdot 41\,t_m$

AREA OF SECTION $= \cdot 7\,C\,t_m$

$Z_C = \cdot 08\,C\,t_m^2$

$Z_t = \cdot 12\,C\,t_m^2$



$\bar{x} = \cdot 46\,C$

$\bar{y} = \cdot 41\,(t_2 - t_1)$

AREA OF SECTION $= \cdot 7\,C\,t_m$

$Z_C = \cdot 08\,C\,t_m^2$

$Z_t = K\,C/t_m^2$ THE VALUE OF K BEING OBTAINED FOR ANY VALUE OF t_1/t_2 FROM THE CURVE BELOW.

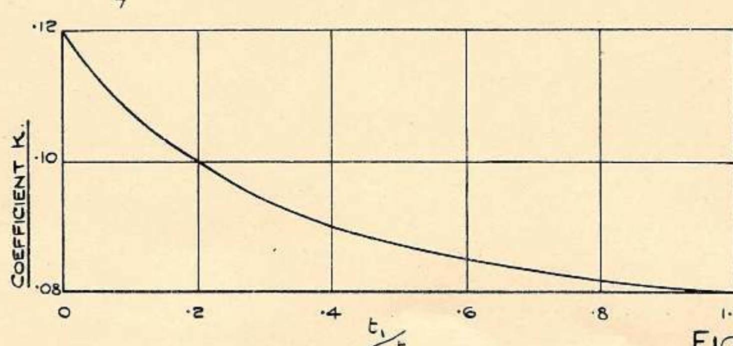


FIG. 3. CHAP. IX

AERODYNAMIC BENDING MOMENT.

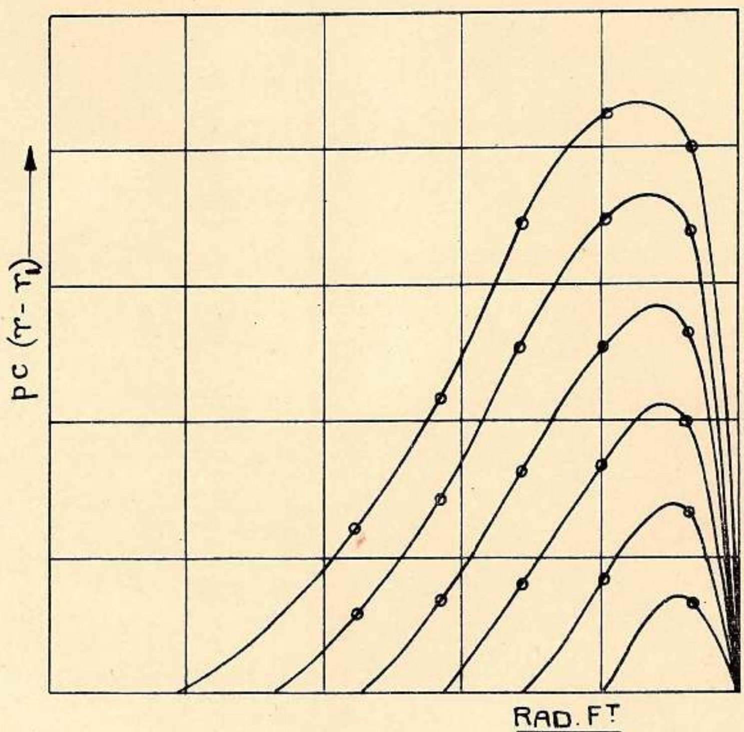


FIG. 4. CHAP. IX.

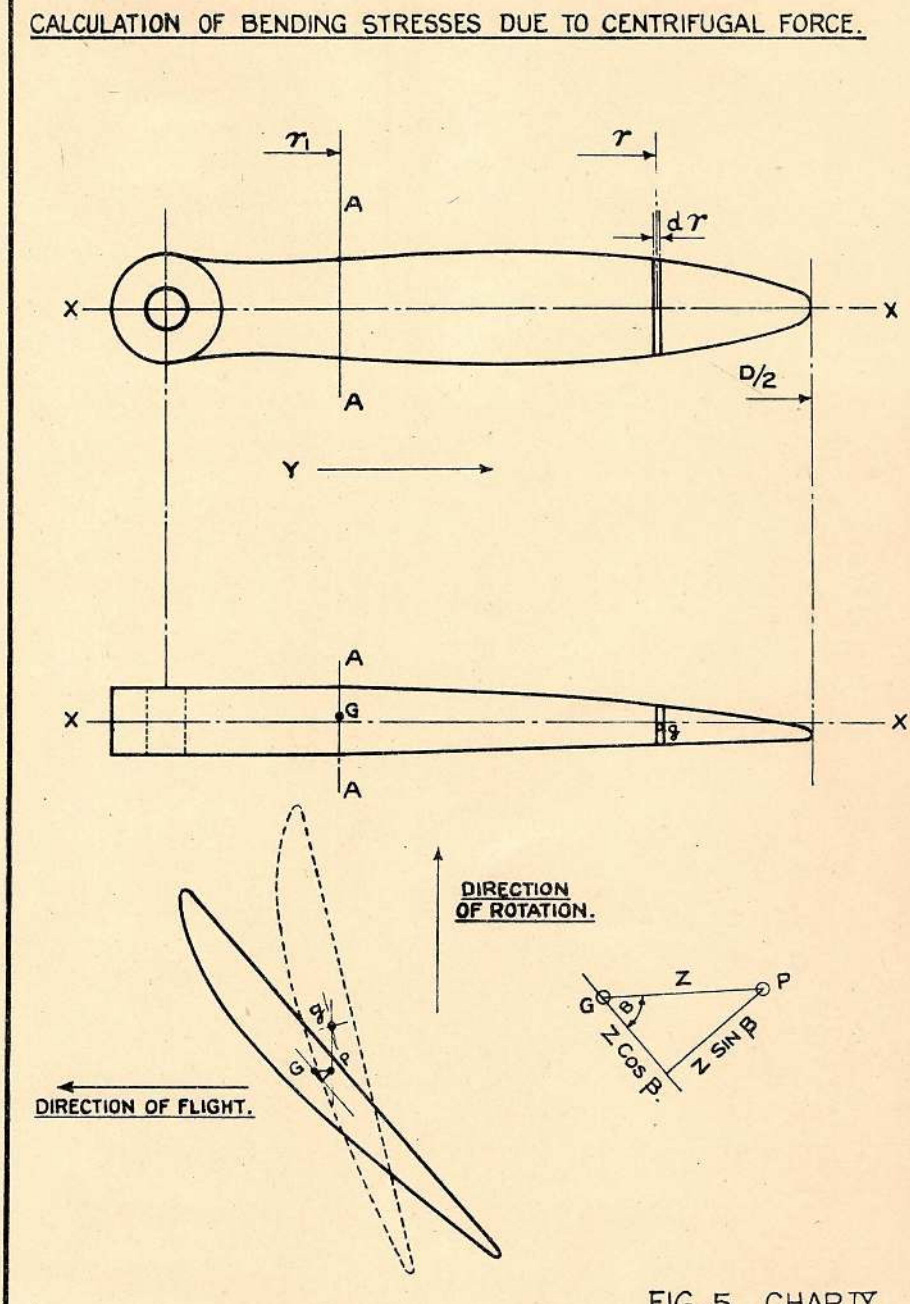

CALCULATION OF BENDING STRESSES DUE TO CENTRIFUGAL FORCE.
r_1
r
dr
A
A
X
X
D/2
Y
A
G
g
X
X
A
DIRECTION OF ROTATION.
Z
G
P
B
Z COS B.
Z SIN B
g
G
P
DIRECTION OF FLIGHT.

FIG. 5. CHAP. IX.

CALCULATION OF BENDING STRESSES DUE TO CENTRIFUGAL FORCE.

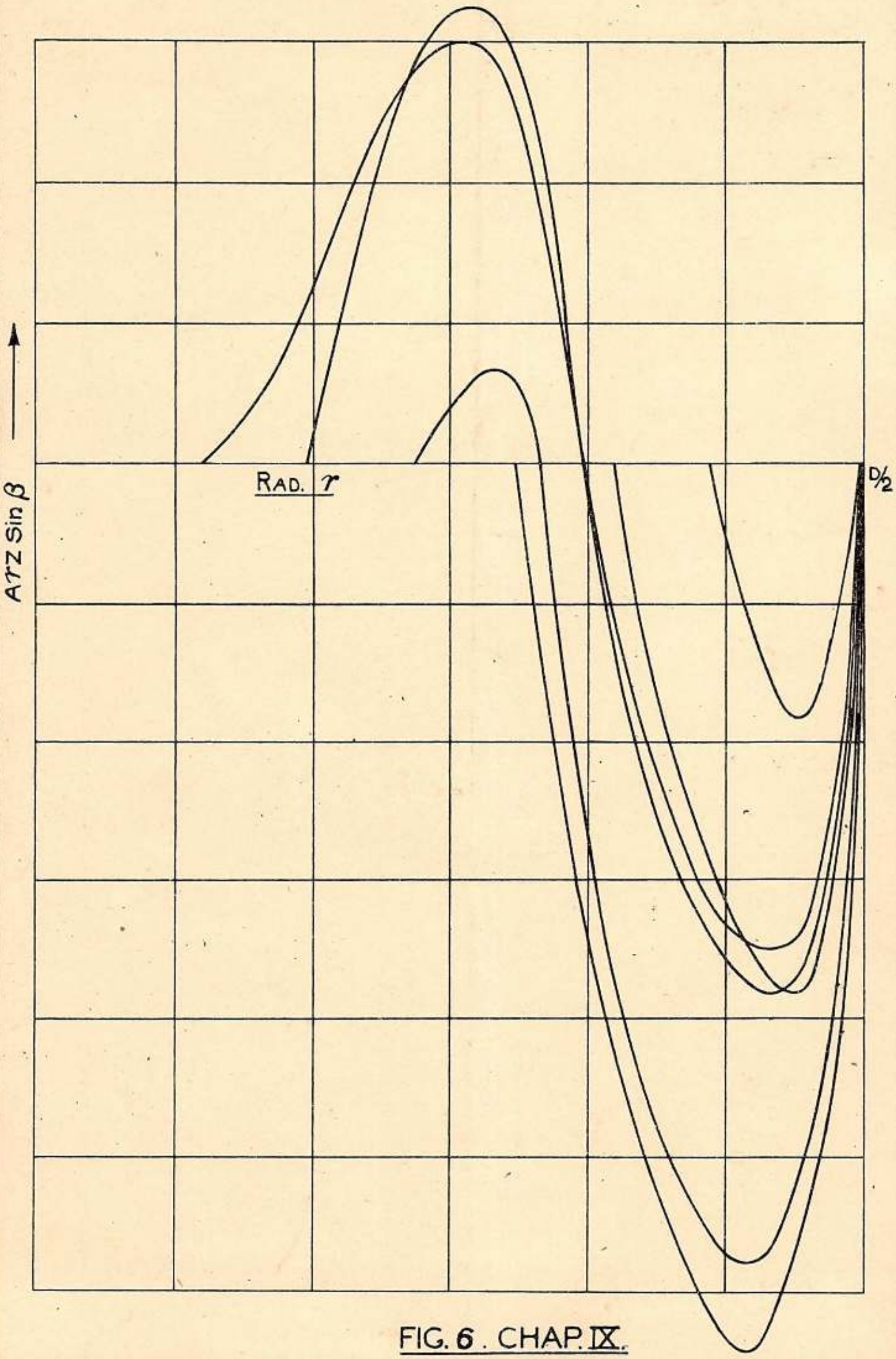


FIG. 6. CHAP. IX.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