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APPENDIX B

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APPENDIX B
MATHEMATICAL NOTES

I. TRIGONOMETRICAL FORMULAE

1. The complement of an angle is the angle which must be added to it to make a right angle. Thus the complement of 60° is 30°, and of $\frac{\pi}{6}$ radians is $\frac{\pi}{3}$ radians. The trigonometrical ratios of an angle and its complement are related as follows.

$$\sin (90 - A) = \cos A \qquad \cos (90 - A) = \sin A$$
$$\tan (90 - A) = \cot A \qquad \cot (90 - A) = \tan A$$
$$\operatorname{cosec} (90 - A) = \sec A \qquad \sec (90 - A) = \operatorname{cosec} A.$$

These formulae are valid for all values of A, whether greater or less than 90°.

2. The supplement of an angle is the angle which must be added to it to make two right angles. Thus the supplement of 60° is 120°, and of $\frac{\pi}{4}$ radians is $\frac{3}{4}\pi$ radians. The trigonometrical ratios of an angle and its supplement are related as follows.

$$\sin (180 - A) = \sin A \qquad \cos (180 - A) = -\cos A$$
$$\tan (180 - A) = -\tan A \qquad \cot (180 - A) = -\cot A$$
$$\operatorname{cosec} (180 - A) = \operatorname{cosec} A \qquad \sec (180 - A) = -\sec A.$$

3. The following are known as the Addition Formulae.

$$\sin (A + B) = \sin A \cos B + \sin B \cos A$$
$$\sin (A - B) = \sin A \cos B - \sin B \cos A$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

4. By the addition and subtraction of various pairs of the above the following relations are obtained.

$$\sin (A + B) + \sin (A - B) = 2 \sin A \cos B$$
$$\sin (A + B) - \sin (A - B) = 2 \cos A \sin B$$
$$\cos (A - B) + \cos (A + B) = 2 \cos A \cos B$$
$$\cos (A - B) - \cos (A + B) = 2 \sin A \sin B.$$

5. By putting $(A + B) = P$, $(A - B) = Q$,

$$\sin P + \sin Q = 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}$$
$$\sin P - \sin Q = 2 \cos \frac{P+Q}{2} \sin \frac{P-Q}{2}$$
$$\cos Q + \cos P = 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}$$
$$\cos Q - \cos P = 2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}.$$

6. The following properties of the tangent are also important.

$$\tan (A + B) = \frac{\sin (A + B)}{\cos (A + B)} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

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7. Many other useful relations may be deduced from the Addition Formulae; amongst those most frequently used are:—

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 1 - 2 \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

$$\frac{1 - \cos A}{1 + \cos A} = \tan^2 A$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

II.—HYPERBOLIC FUNCTIONS

1. In dealing with transmission lines, both for radio and audio frequencies, considerable use is made of hyperbolic functions. The direct hyperbolic functions of a quantity x are its hyperbolic sine, hyperbolic cosine, hyperbolic tangent, etc., and these have corresponding inverse functions. The hyperbolic functions are so called because they are connected with the geometry of the rectangular hyperbola in a manner resembling the relation between the circular functions, sine, cosine, etc., and the angle subtended by a circular arc, but this geometrical relation is of no interest from the engineering point of view. They are most easily understood if treated merely as combinations of exponential functions.

2. The direct hyperbolic functions are written *sinh x*, *cosh x*, etc., and are defined as follows :—

$$\sinh x = \frac{\varepsilon^{x} - \varepsilon^{-x}}{2}$$

$$\cosh x = \frac{\varepsilon^{x} + \varepsilon^{-x}}{2}$$

$$\tanh x = \frac{\varepsilon^{x} - \varepsilon^{-x}}{\varepsilon^{x} + \varepsilon^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

(" *Sinh* " is pronounced as " shin," " *tanh* " as " than," and " *sech* " as " shec ").

3. The circular functions $\sin\varphi$, $\cos\varphi$, may be expressed in the forms of infinite series, thus

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} \cdots\cdots$$

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} \cdots\cdots$$

Similarly, the hyperbolic functions may be represented by infinite series, for example,

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\cdots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} \cdots\cdots$$

4. The values of $\cosh x$, $\sinh x$, and $\tanh x$, from $x = 0$ to $x = 2{\cdot}5$, are shown graphically in fig. 1. It is obvious from the formulae already given that $\sinh 0 = 0$, and $\cosh 0 = 1$. When x exceeds about 3, ε^{-x} becomes small compared with ε^{x}. Both $\sinh x$ and $\cosh x$ then approach more closely to the value $\frac{\varepsilon^{x}}{2}$ as x increases. The functions ε^{x}, ε^{-x}, are given in many books of Tables, and may also be evaluated with sufficient accuracy for most purposes by means of a Log-log slide rule. The following relations between circular and hyperbolic functions are of primary importance.

5. In Chapter V it is shown that

$$\varepsilon^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\varepsilon^{-j\varphi} = \cos\varphi - j\sin\varphi$$

Hence

$$\cos\varphi = \frac{\varepsilon^{j\varphi} + \varepsilon^{-j}}{2} = \cosh j\varphi$$

$$\sin\varphi = \frac{\varepsilon^{j\varphi} - \varepsilon^{-j\varphi}}{2j} = \frac{\sinh j\varphi}{j}$$

6. If instead of φ we write $j\varphi$,

$$\cos j\varphi = \frac{\varepsilon^{j(j\varphi)} + \varepsilon^{-j(j\varphi)}}{2} = \frac{\varepsilon^{-\varphi} + \varepsilon^{+\varphi}}{2},$$

i.e.

$$\cos j\varphi = \cosh \varphi$$

Also

$$\sin j\varphi = \frac{\varepsilon^{j(j\varphi)} - \varepsilon^{-j(j\varphi)}}{2j} = \frac{\varepsilon^{-\varphi} - \varepsilon^{+\varphi}}{2j}$$

But

$$\frac{\varepsilon^{-\varphi} - \varepsilon^{+\varphi}}{2j} = j\,\frac{\varepsilon^{\varphi} - \varepsilon^{-\varphi}}{2}$$

Hence

$$\sin j\varphi = j \sinh \varphi$$

Again,

$$\tan j\varphi = \frac{\sin j\varphi}{\cos j\varphi} = j\,\frac{\sinh \varphi}{\cosh \varphi} = j \tanh \varphi$$

$$\cot j\varphi = \frac{\cos j\varphi}{\sin j\varphi} = \frac{\cosh \varphi}{j \sinh \varphi} = -j \coth \varphi.$$

7. Collecting and extending the above results,

$$\begin{aligned}
\sinh j\varphi &= j \sin \varphi & \qquad \sin j\varphi &= j \sinh \varphi \\
\cosh j\varphi &= \cos \varphi & \qquad \cos j\varphi &= \cosh \varphi \\
\tanh j\varphi &= j \tan \varphi & \qquad \tan j\varphi &= j \tanh \varphi \\
\coth j\varphi &= -j \cot \varphi & \qquad \cot j\varphi &= -j \coth \varphi \\
\operatorname{sech} j\varphi &= \sec \varphi & \qquad \sec j\varphi &= \operatorname{sech} \varphi \\
\operatorname{cosech} j\varphi &= -j \operatorname{cosec} \varphi & \qquad \operatorname{cosec} j\varphi &= -j \operatorname{cosech} \varphi.
\end{aligned}$$

8. The Addition Formulae for hyperbolic functions are as below :—

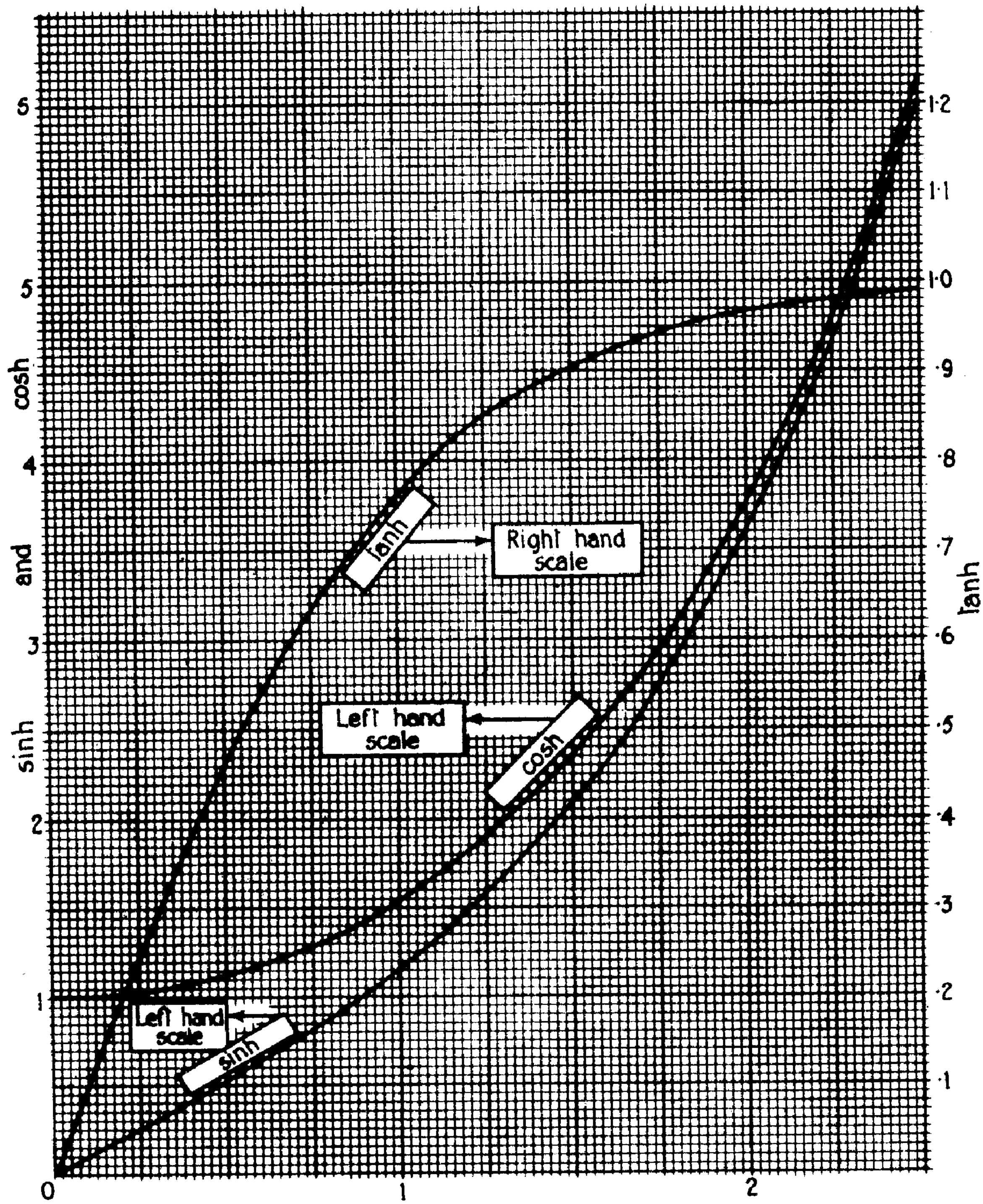
$$\begin{aligned}
\sinh (A + B) &= \sinh A \cosh B + \cosh A \sinh B \\
\sinh (A - B) &= \sinh A \cosh B - \cosh A \sinh B \\
\cosh (A + B) &= \cosh A \cosh B + \sinh A \sinh B \\
\cosh (A - B) &= \cosh A \cosh B - \sinh A \sinh B.
\end{aligned}$$

9. By addition and subtraction of various pairs of the above, the following relations are obtained :

$$\begin{aligned}
\sinh (A + B) + \sinh (A - B) &= 2 \sinh A \cosh B \\
\sinh (A + B) - \sinh (A - B) &= 2 \cosh A \sinh B \\
\cosh (A + B) + \cosh (A - B) &= 2 \cosh A \cosh B \\
\cosh (A + B) - \cosh (A - B) &= 2 \sinh A \sinh B
\end{aligned}$$

10. By putting $(A + B) = P$, $(A - B) = Q$,

$$\begin{aligned}
\cosh P + \cosh Q &= 2 \cosh \frac{P + Q}{2} \cosh \frac{P - Q}{2} \\
\cosh P - \cosh Q &= 2 \sinh \frac{P + Q}{2} \sinh \frac{P - Q}{2} \\
\sinh P + \sinh Q &= 2 \sinh \frac{P + Q}{2} \cosh \frac{P - Q}{2} \\
\sinh P - \sinh Q &= 2 \cosh \frac{P + Q}{2} \sinh \frac{P - Q}{2}.
\end{aligned}$$



HYPERBOLIC FUNCTIONS

FIG. I
APPENDIX B

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11. Other useful relations are :—

$$\tanh (A + B) = \frac{\sinh A \cosh B + \cosh A \sinh B}{\cosh A \cosh B + \sinh A \sinh B}$$

$$= \frac{\tanh A + \tanh B}{1 + \tanh A \tanh B}$$

$$\tanh (A - B) = \frac{\tanh A - \tanh B}{1 - \tanh A \tanh B}$$

$$\cosh^2 A - \sinh^2 A = 1$$

$$\sinh 2A = 2 \sinh A \cosh A$$

$$\cosh 2A = \cosh^2 A + \sinh^2 A$$

$$= 1 - 2 \sinh^2 A$$

$$= 2 \cosh^2 A - 1$$

$$\tanh 2A = \frac{2 \tanh A}{1 + \tanh^2 A} = \frac{2 \coth A}{1 + \coth^2 A} = \frac{2}{\coth A + \tanh A}.$$

12. Expressions such as $\sinh (A + jB)$, which are met with in the theory of telephonic transmission, are dealt with in the following manner :—

$$\sinh (A + jB) = \sinh A \cosh jB + \cosh A \sinh jB$$

$$\cosh jB = \cos B$$

$$\sinh jB = j \sin B$$

therefore

$$\sinh (A + jB) = \sinh A \cos B + j \cosh A \sin B$$

Similarly

$$\cosh (A + jB) = \cosh A \cos B + j \sinh A \sin B$$

$$\sinh (A + jB) = \sinh A \cos B - j \cosh A \sin B$$

$$\cosh (A - jB) = \cosh A \cos B - j \sinh A \sin B$$

and

$$\tanh (A + jB) = \frac{\sinh (A + jB)}{\cosh (A + jB)}$$

$$\coth (A + jB) = \frac{\cosh (A + jB)}{\sinh (A + jB)}$$

Thus, the hyperbolic functions of complex numbers are also complex.

Example.—Evaluate $\sinh (A + jB)$ when $A = 0{\cdot}7$ and $B = 1$ radian $= 57$ degrees approx. From tables of circular and hyperbolic functions

$$\sin B = 0{\cdot}8387 \qquad \cos B = 0{\cdot}5446$$

$$\sinh A = 0{\cdot}7586 \qquad \cosh A = 0{\cdot}6044$$

$$\sinh (A + jB) = 0{\cdot}7586 \times 0{\cdot}5446 + j\,0{\cdot}6044 \times 0{\cdot}8387$$

$$= 0{\cdot}4131 + j\,0{\cdot}5069.$$

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III.—HARMONIC ANALYSIS

1. When the graphical representation of one or more cycles of a complex periodic wave is given, the analysis of the waveform into its constituent harmonics, up to and including the sixth, is easily performed by a procedure known as the twelve-ordinate method. The mathematical theory will not be given; it is sufficient to say that the process has been reduced to simple arithmetic by means of the schedules given below. As a concrete example, the waveform of fig. 2 will be analysed.

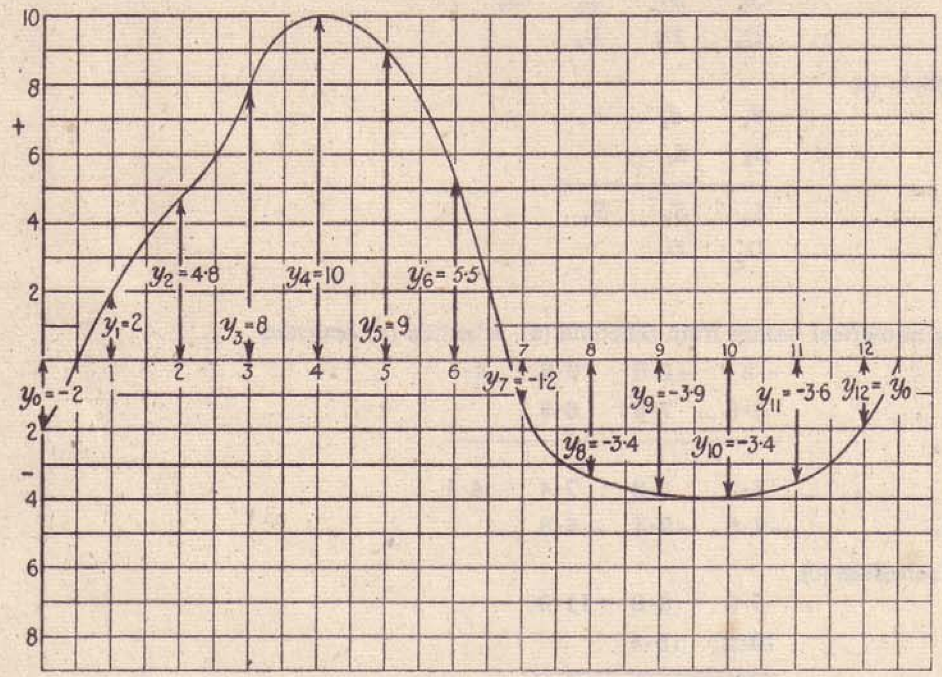

FIG. 2, APPENDIX B.—Analysis of complex wave-form.

2. In the diagram, one complete cycle of 360° has been divided into 12 equal parts, and the corresponding ordinates erected. Their values are then tabulated as below:—

y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
$-2{\cdot}0$	$2{\cdot}0$	$4{\cdot}8$	$8{\cdot}0$	$10{\cdot}0$	$9{\cdot}0$	$5{\cdot}5$	$-1{\cdot}2$	$-3{\cdot}4$	$-3{\cdot}9$	-4	$-3{\cdot}6$

The computation is performed as follows. First, enter the values of the ordinates upon the following schedule (a), finding the sums (Σ) and differences (Δ) as indicated; $s_1 = y_1 + y_{11}$, $d_1 = y_1 - y_{11}$ and so on.

Schedule (a)

		y_0	y_1	y_2	y_3	y_4	y_5	y_6
			y_{11}	y_{10}	y_9	y_8	y_7	
(Σ)	.. ..	s_0	s_1	s_2	s_3	s_4	s_5	s_6
(Δ)	.. ..		d_1	d_2	d_3	d_4	d_5	

3. Substituting actual values of $y_0, y_1, \ldots \ldots y_6$ from the table, Schedule (a) becomes

		-2	$2{\cdot}0$	$4{\cdot}8$	$8{\cdot}0$	$10{\cdot}0$	$9{\cdot}0$	$5{\cdot}5$
			$-3{\cdot}6$	$-4{\cdot}0$	$-3{\cdot}9$	$-3{\cdot}4$	$-1{\cdot}2$	
(Σ)	.. ..	-2	$-1{\cdot}6$	$0{\cdot}8$	$4{\cdot}1$	$6{\cdot}6$	$7{\cdot}8$	$5{\cdot}5$
(Δ)	.. ..		$5{\cdot}6$	$8{\cdot}8$	$11{\cdot}9$	$13{\cdot}4$	$10{\cdot}2$	

i.e.

$s_0 = -2{\cdot}0,\ s_1 = -1{\cdot}6,\ s_2 = 0{\cdot}8,\ s_3 = 4{\cdot}1,\ s_4 = 6{\cdot}6,\ s_5 = 7{\cdot}8,\ s_6 = 5{\cdot}5.$

$d_1 = 5{\cdot}6,\ d_2 = 8{\cdot}8,\ d_3 = 11{\cdot}9,\ d_4 = 13{\cdot}4,\ d_5 = 10{\cdot}2.$

4. Next, find the sums and differences in the schedules (b) and (c) :—

Schedule (b)

	s_0	s_1	s_2	s_3
	s_6	s_5	s_4	
(Σ)	S_0	S_1	S_2	S_3
(Δ)	D_0	D_1	D_2	

Schedule (c)

	d_1	d_2	d_3
	d_5	d_4	
(Σ)	S_4	S_5	S_6
(Δ)	D_3	D_4	

5. Inserting numerical values from schedule (a), schedule (b) becomes

	−2	−1·6	0·8	4·1
	5·5	7·8	6·6	
(Σ)	3·5	6·2	7·4	4·1
(Δ)	−7·5	−9·4	−5·8	

and schedule (c),

	5·6	8·8	11·9
	10·2	13·4	
(Σ)	15·8	22·2	11·9
(Δ)	−4·6	−4·6	

i.e. $S_0 = 3 \cdot 5$, $S_1 = 6 \cdot 2$, $S_2 = 7 \cdot 4$, $S_3 = 4 \cdot 1$, $S_4 = 15 \cdot 8$

$S_5 = 22 \cdot 2$, $S_6 = 11 \cdot 9$

$D_0 = -7 \cdot 5$, $D_1 = -9 \cdot 4$, $D_2 = -5 \cdot 8$, $D_3 = -4 \cdot 6$, $D_4 = -4 \cdot 6$

6. Finally complete the schedules (d) and (e) below

Schedule (d)

	S_0	S_1
	S_2	S_3
(Σ) ..	S_7	S_8

Schedule (e)

	S_4	D_0
	S_6	D_2
(Δ) ..	D_5	D_6

7. Inserting numerical values from previous results

Schedule (d) becomes

	3·5	6·2
	7·4	4·1
(Σ) ..	10·9	10·3

Schedule (e) becomes

	15·8	−7·5
	11·9	−5·8
(Δ) ..	3·9	−1·7

i.e. $S_7 = 10 \cdot 9$, $S_8 = 10 \cdot 3$, $D_5 = 3 \cdot 9$, $D_6 = -1 \cdot 7$.

The coefficients $S_0 \ldots . S_8$, $D_0 \ldots . D_6$ are the quantities actually required for the analysis.

8. The equation of the wave-form is

$$y = A_0 + A_1 \cos \omega t + A_2 \cos 2\,\omega t + A_3 \cos 3\,\omega t + A_4 \cos 4\,\omega t + A_5 \cos 5\,\omega t + A_6 \cos 6\,\omega t + B_1 \sin \omega t + B_2 \sin 2\,\omega t + B_3 \sin 3\,\omega t + B_4 \sin 4\,\omega t + B_5 \sin 5\,\omega t$$

where

$$A_0 = \frac{S_7 + S_8}{12}$$

$$A_1 = \frac{D_0 + \frac{\sqrt{3}}{2}D_1 + \frac{1}{2}D_2}{6} \quad ; \quad B_1 = \frac{\frac{1}{2}S_4 + \frac{\sqrt{3}}{2}S_5 + S_6}{6}$$

$$A_2 = \frac{S_0 + \frac{1}{2}(S_1 - S_2) - S_3}{6} ; \quad B_2 = \frac{\frac{\sqrt{3}}{2}(D_3 + D_4)}{6}$$

$$A_3 = \frac{D_6}{6} \quad ; \quad B_3 = \frac{D_5}{6}$$

$$A_4 = \frac{S_0 - \frac{1}{2}(S_1 + S_2) + S_3}{6} ; \quad B_4 = \frac{\frac{\sqrt{3}}{2}(D_3 - D_4)}{6}$$

$$A_5 = \frac{D_0 - \frac{\sqrt{3}}{2}D_1 + \frac{1}{2}D_2}{6} \quad ; \quad B_5 = \frac{\frac{1}{2}S_4 - \frac{\sqrt{3}}{2}S_5 + S_6}{6}$$

$$A_6 = \frac{S_7 - S_8}{12}$$

9. The term A_0 is a constant displacement, e.g. the mean anode current in the case of a valve circuit. The amplitude H_1 of the fundamental is $\sqrt{A_1^2 + B_1^2}$ and of the n^{th} harmonic $\left(n \leqq 6\right)$ is $\sqrt{A_n^2 + B_n^2}$. The instantaneous value of the n^{th} harmonic is

$$h_n = \sqrt{A_n^2 + B_n^2} \sin (n\,\omega t + \varphi_n)$$

where

$$\varphi_n = \tan^{-1}\frac{A_n}{B_n}.$$

10. Returning to the analysis of fig. 2,

$$A_0 = \frac{10{\cdot}9 + 10{\cdot}3}{12} = 1{\cdot}77$$

$$A_1 = \frac{-\,7{\cdot}5 + (0{\cdot}866 \times -\,9{\cdot}4) - 2{\cdot}9}{6} = \frac{-\,18{\cdot}5}{6} = -\,3{\cdot}1.$$

$$B_1 = \frac{(\frac{1}{2} \times 15{\cdot}8) + (0{\cdot}866 \times 22{\cdot}2) + 11{\cdot}9}{6} = 6{\cdot}5.$$

$$h_1 = 7{\cdot}2 \sin \left(\omega t - \tan^{-1}\frac{3{\cdot}1}{6{\cdot}5}\right)$$

$$A_2 = \frac{3{\cdot}5 + \frac{1}{2}(6{\cdot}2 - 7{\cdot}4) - 4{\cdot}1}{6} = \frac{-\,1{\cdot}2}{6} = -\,0{\cdot}2$$

$$B_2 = \frac{{\cdot}866\,(-\,4{\cdot}6 - 4{\cdot}6)}{6} = -\,1{\cdot}33$$

$$A_3 = \frac{-\,1{\cdot}7}{6} = -\,0{\cdot}28$$

$$B_3 = \frac{3{\cdot}9}{6} = 0{\cdot}65$$

$$A_4 = \frac{3{\cdot}5 - \frac{1}{2}(6{\cdot}2 + 7{\cdot}4) + 4{\cdot}1}{6} = \frac{0{\cdot}8}{6} = 0{\cdot}133$$

$$B_4 = \frac{{\cdot}866\,(-\,4{\cdot}6 + 4{\cdot}6)}{6} = 0.$$

$$A_5 = \frac{-\,7{\cdot}5 + (0{\cdot}866 \times 0{\cdot}94) - 2{\cdot}9}{6} = \frac{-\,2{\cdot}26}{6} = -\,0{\cdot}38$$

$$B_5 = \frac{7{\cdot}9 - (0{\cdot}866 \times 22{\cdot}2) + 11{\cdot}9}{6} = \frac{0{\cdot}4}{6} = 0{\cdot}067$$

$$A_6 = \frac{10{\cdot}9 - 10{\cdot}3}{12} = 0{\cdot}05$$

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11. The following checks should be applied to ensure arithmetical accuracy. For the A terms :—

$$A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = y_0$$

i.e.

$$1{\cdot}77 - 3{\cdot}1 - 0{\cdot}2 - 0{\cdot}28 + 0{\cdot}133 - 0{\cdot}38 + 0{\cdot}05 = y_0$$
$$\text{or } y_0 = 2{\cdot}007.$$

This is well within the accuracy with which the value of y_0 can be determined from the graph.

The accuracy of the B terms is checked by

$$(B_1 + B_5) + \sqrt{3}\,(B_2 + B_4) + 2\,B_3 = y_1 - y_{11}$$

i.e.

$$(6{\cdot}5 + 0{\cdot}067) + \sqrt{3}\,(-\,1{\cdot}33) + (2 \times 0{\cdot}65) = y_1 - y_{11}$$
$$\text{or } y_1 - y_{11} = 2 - (-\,3{\cdot}567)$$
$$= 5{\cdot}567$$

which is again satisfactory. From the table, $y_1 - y_{11} = 5{\cdot}6$.

12. The amplitudes of the various harmonics are

$$H_1 = \sqrt{A_1^2 + B_1^2} = 7{\cdot}2$$
$$H_2 = \sqrt{A_2^2 + B_2^2} = 1{\cdot}35$$
$$H_3 = \sqrt{A_3 \ + B_3} \ = 0{\cdot}707$$
$$H_4 = \sqrt{A_4^2 + B_4^2} = 0{\cdot}133$$
$$H_5 = \sqrt{A_5^2 + B_5^2} = 0{\cdot}38$$
$$H_6 = \sqrt{A_6^2 + B_6^2} = 0{\cdot}05$$

The percentage of second harmonic is $\dfrac{H_2}{H_1} \times 100 = 18{\cdot}7$,

of third harmonic, $\dfrac{H_3}{H_1} \times 100 = 9{\cdot}8$,

and so on.

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