CHAPTER XV.—AERIAL ARRAYS AND TRANSMISSION LINES

AERIALS AND AERIAL ARRAYS

Introductory

1. From the earliest days of radio communication, the advantages of directional transmission and reception, particularly for the purpose of point-to-point communication, have been fully appreciated. The earliest attempts in this direction were made at comparatively low frequencies (of the order of 15 to 30 kc/s) using L aerials having a great horizontal length—some ten to twenty times the height. Such aerials were very expensive in first cost and maintenance. With the development of high frequency communication, the employment of highly directional aerial systems proceeded rapidly. For any energy-radiating system to possess directional properties, its dimensions must be at least comparable with the wavelength of radiation in the particular medium. For example, suppose it is desired to radiate a beam of sound waves at a frequency of 500 cycles per second by means of a horn. As the speed of sound in air is about 1,120 feet per second, the wavelength is $\frac{1{,}120}{f}$ or 2·24 feet, and the mouth of the horn, if square, should be at least 2 feet by 2 feet, and if circular or elliptical it should have an area of at least 4 square feet. In the same way, directional electro-magnetic radiation requires that the aerial system shall have dimensions of at least the same order as the wavelength, and this requirement is obviously more easily met at high frequencies (short wavelengths) than at the low frequencies formerly employed.

2. A combination of radiating members designed for the purpose of directional transmission or reception is called an aerial array. The object of an aerial array is to produce some particular distribution of field strength in space, according to the nature of the service, the distance of the receiving station and other factors. The spatial distribution of field strength may be shown by horizontal and vertical polar diagrams as in the case of single aerials. Aerial arrays are for the most part employed in long distance point-to-point communication, and for this service the horizontal polar diagrams should be long and narrow. Since the propagation is dependent upon reflection from the ionosphere the vertical polar diagram should be such that most energy is radiated at a low angle to the horizontal, usually between ten and fifteen degrees. It is found that the apparent direction from which the strongest radiation arrives at a receiver depends partly upon the state of the ionosphere, and may vary through several degrees from hour to hour or from day to day. It is therefore not desirable to aim at an extremely directive polar diagram. At the receiver, a fairly sharp vertical diagram is an advantage, provided that the optimum angle can be decided, because under these conditions less trouble is caused by echo phenomena. In general, however, the optimum angle also varies with the time of day, season, etc. Certain special types of communication, e.g. ground to air and *vice versa*, may require types of array very different from those used for long distance point-to-point communication.

Reciprocal properties

3. The properties of any aerial, when used for reception, are in most respects similar to the corresponding properties of the same aerial when used for transmission. In particular, the directional characteristic is practically unchanged. The current distribution and effective impedance are not quite the same because the current is due to a field spatially distributed over the whole aerial (not necessarily in a uniform manner) instead of an E.M.F. applied between two feeding points, and the impedance changes slightly owing to an indirect effect of the current distribution. The fact that the directional characteristic is substantially the same enables the merits of a given aerial or array for transmitting purposes to be deduced from its behaviour as a receiver and *vice versa*.

Current distribution

4. The current and voltage distributions along an aerial wire were dealt with briefly in Chapter VII. It is now necessary to discuss the current distribution somewhat more fully. Consider an aerial suspended above the earth in any manner whatever, e.g. as shown in fig. 1a, and its lower end to be connected to one terminal of a high frequency generator, the other terminal of which is earthed. In order to measure the amplitude of the current at different points of the wire, we may use a thermo-ammeter, inductively coupled to the aerial by means of a loop of wire. This device is in fact in constant use for the adjustment of aerial arrays, and suitable dimensions are given later. If arrangements are made to draw this loop along the aerial we

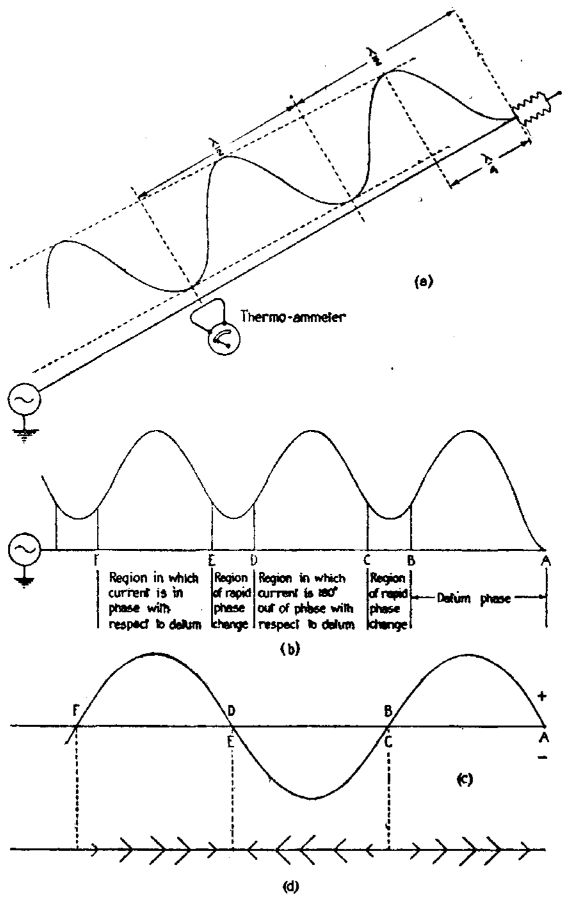


FIG. 1, CHAP. XV.—Current distribution along wire.

may obtain an indication of the R.M.S. current at different points. At the end remote from the generator, the current in the wire will be zero. As the loop is moved towards the generator the current increases, and will be found to pass through a maximum at a distance exactly $\frac{\lambda}{4}$ from the open end. It then decreases, and passes through a minimum value at a distance of $\frac{\lambda}{2}$ from the end. It will then start to increase again and will pass through another maximum when the ammeter is $\frac{3}{4}\lambda$ from the end, this maximum being slightly greater in amplitude than the previous one. If at each point in the wire a perpendicular is drawn, and its length is proportional to the current at that point the ends of these lines will lie upon a curve as in fig 1a. This curve then gives the current distribution, so far as its magnitude is concerned, but it will give no information as to the relative phase of the current at different points in the wire. If, however, steps were taken to measure this it would be found that starting from the far end, the current at all points in a length A B, i.e. over a distance of nearly $\frac{\lambda}{2}$, is very nearly of the same phase. Over a short length B C, fig. 1b, in the vicinity of the current minimum, the phase changes very rapidly, and in passing from B to C a total change of 180° takes place. In the distance C D, the current is syn-phased at all points and is therefore 180° out of phase with the current in A B. This process of phase reversal again occurs in the length D E, so that the current in E F is in phase with that in A B, but opposite in phase to that in C D.

5. In Chapter VII it is assumed that the lengths B C, D E, etc., are so small that they may be represented by geometrical points, and also that the current at these points, instead of being a minimum, falls to zero. This assumption is often made in theoretical work, because under these conditions both the magnitude and the phase of the current can be shown by a sine curve, fig. 1c; at points lying above the wire the current is of the same phase throughout, and at points lying below the wire the current is 180° out of phase with the points lying above it. Alternatively, we may say that if at any instant the current is flowing from B to A, the current in B C is flowing towards B, in the length C D from C to D and so on. We may therefore show the distribution of current along the wire by drawing a series of arrows of varying sizes as in fig. 1d. This method is of great assistance when considering the distribution in an array consisting of a number of conductors connected in series.

Nature of input impedance

6. (i) The distribution of the current maxima and minima is entirely independent of circuit adjustment at the transmitter. The tuning adjustments at this end may greatly alter the magnitude of the current, but will not affect the relative amplitudes or phases at any two points in the wire. The impedance offered to the transmitter will, however, vary greatly with the distance of the supply point from the far end. In fig. 2a the transmitter is connected at a distance of $\frac{\lambda}{4}$, and in fig. 2b, a distance of $\frac{3}{4}\lambda$, from the far end. In both cases the supply is connected at

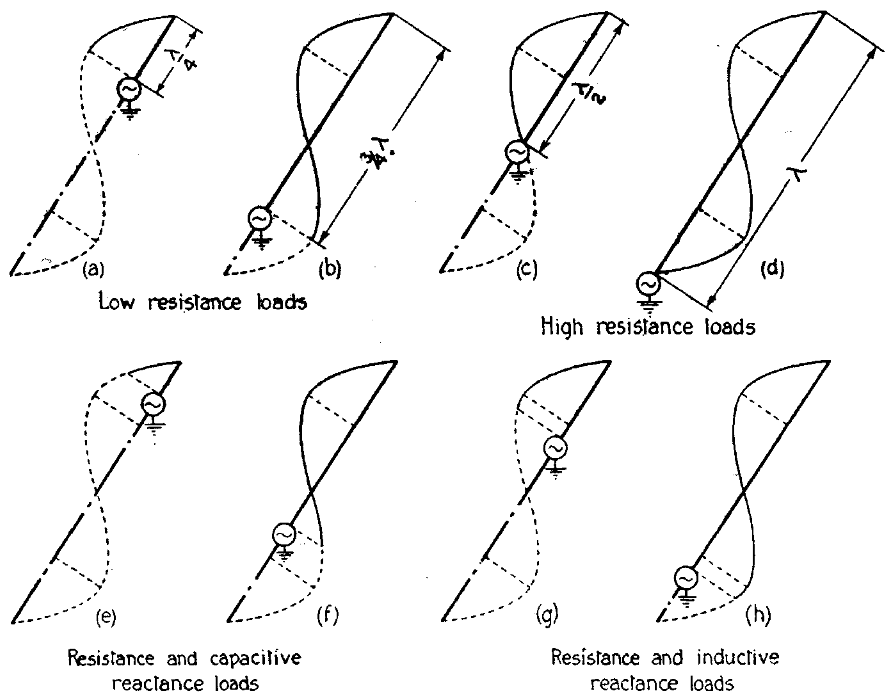


FIG. 2, CHAP. XV.—Nature of impedance of wires of various lengths a current maximum, and whenever this is so, the wire offers an approximately non-reactive impedance of the order of 35 to 100 ohms. In fig. 2c, however, the transmitter is connected at a distance of $\frac{\lambda}{2}$, and in fig 2c, a distance of λ, from the far end. In both cases the supply is conneeted at a current minimum and the wire offers an approximately non-reactive impedance of the order of 2,000 to 8,000 ohms.

(ii) Now suppose the generator to be connected at some intermediate point. In order to be quite clear it is preferable to draw a considerable portion of line, insert the theoretical current distribution and afterwards insert the generator. For example in figs. 2e and 2f, the generator has been inserted at a point less than $\frac{\lambda}{4}$ after a current ***minimum***, measured from the far end. Under these conditions the impedance offered by the wire is equivalent to that of a condenser and resistance in series. In fig. 2g the generator has been inserted at a point between $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$ and in fig. 2h, at a point between $\frac{3}{4}\lambda$ and λ, from the far end. In both these cases, the supply is connected at a point less than $\frac{\lambda}{4}$ after a current ***maximum*** and the input impedance of the wire is equivalent to that of an inductance and resistance in series.

7. (i) Theoretically, a wire in free space has zero reactance whenever its length is a multiple of $\frac{\lambda}{4}$. Actually, the velocity of wave propagation along copper wires located near to a conductive plane is about four per cent. less than the velocity of electro-magnetic waves in free space, and if a length of wire is to be non-reactive its length should be only about 0·96 of the theoretical value. Thus an aerial having an electrical length of $\frac{\lambda}{4}$ should be about $0{\cdot}24\lambda$ in actual length and so on. The exact location with respect to the ground, and the variation in permittivity and conductivity of the latter, must necessitate a slight variation of this figure in certain instances. In practice it may be found necessary to reduce the lengths of radiating members as much as ten per cent. below their nominal length, because discontinuities such as sharp bends, suspension insulators, etc., all tend to reduce the velocity of propagation along the wire.

(ii) In the early days of radio communication the frequencies employed were very much lower than those now in general use, and it was as a rule, only possible to employ aerials having a length very much less than $\frac{\lambda}{4}$. In these circumstances, the most important electrical constants of the aerial are (a) its effective resistance and (b) its effective capacitance. With the higher frequencies now in use, however, the input impedance may be equivalent to that of an inductive or capacitive resistance, or purely resistive, depending upon the ratio of length to λ.

Radiation resistance

8. The radiation resistance of an aerial is defined in Chapter VII ; expressions giving the radiation resistance in certain simple cases are also contained therein. The conventional method of finding the radiation resistance of any given aerial is to develop an equation giving the field strength at all points in space. In Chapter I it is shown that the energy density of a uniform electric field of strength Γ is $\frac{\kappa\Gamma^2}{8\pi}$ ergs per cubic centimetre. If the calculated value of field strength is inserted in this expression, and the result multiplied by the velocity of propagation, we obtain the energy per second, i.e. the power, which is passing through any unit area in a plane perpendicular to

the direction of propagation. The total power passing through a sphere surrounding the radiator is obviously equal to that radiated, hence, on summing up the total power passing through every unit area on the surface of this sphere, and then dividing by the R.M.S. loop current, the quotient is the radiation resistance. Another method is to sum up the energy passing through a cylinder of unit thickness immediately surrounding the wire. This method gives both the radiated power and the wattless volt-amperes required to maintain the induction field, and therefore gives the aerial impedance as the vector sum of the radiation resistance and the reactance of the aerial. In this manner the impedance of a $\frac{\lambda}{2}$ dipole in free space is found to be $73 \cdot 3 + j42 \cdot 5$ ohms, and the impedance of a vertical $\frac{\lambda}{4}$ aerial over a perfectly conductive earth, $36 \cdot 6 + j21 \cdot 25$ ohms. When tuned to resonance with the frequency of the supply, the reactance of the aerial is annulled, although of course the induction field is still maintained.

9. (i) In practice, the radiation resistance is affected by the proximity of the ground, to an extent depending upon the permittivity and conductivity of the soil. The radiation resistance of a vertical $\frac{\lambda}{2}$ dipole, with its lower end at ground level on a perfectly conductive earth, is approximately 100 ohms, whereas in free space it would be $73 \cdot 3$ ohms. The nature of the variation with height above ground level is shown in fig. 3. Over moist earth of permittivity $\kappa = 25$ and conductivity $\sigma = 10^8$ E.S.U. the actual radiation resistance is found to be very close to the theoretical value given by this curve, which may therefore be used for practical purposes. Application of the image theorem of paragraph 35 shows that a horizontal $\frac{\lambda}{2}$ dipole very close indeed to the earth's surface will radiate infinitesimal energy, i.e. the radiation resistance of such an aerial approaches zero. When far above the surface, however, its radiation resistance is $73 \cdot 3$ ohms, the theoretical nature of the variation with height being also shown in fig. 3. Over the ground specified above, however,

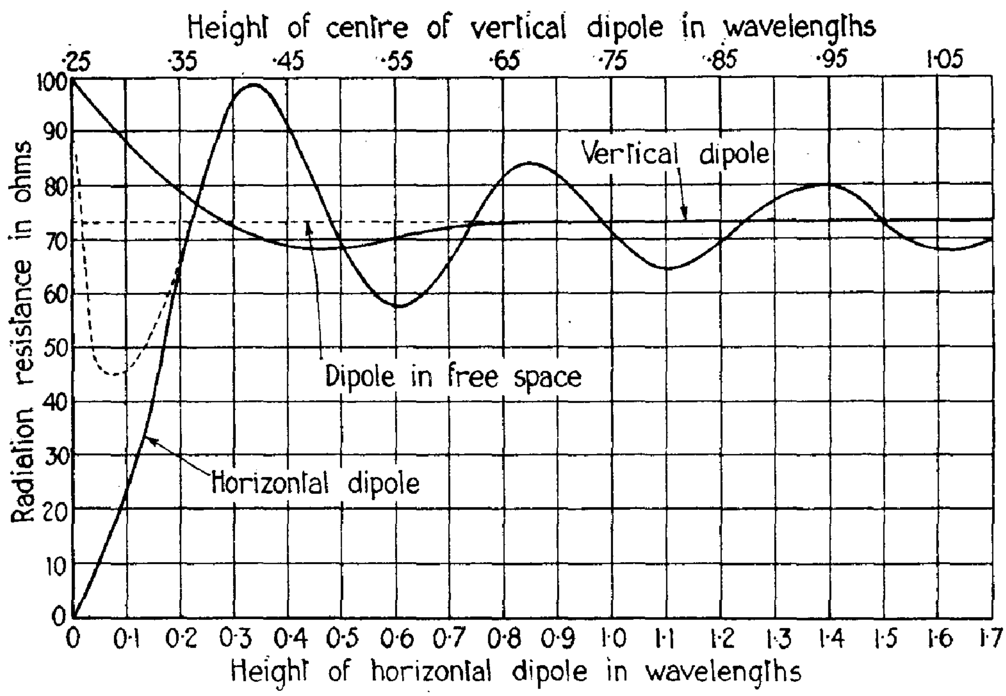


FIG. 3, CHAP. XV.—Radiation resistance of dipole.

the resistance at heights less than $\frac{\lambda}{4}$ was found experimentally to be given by the curve shown in dotted line. It will be observed that in both solid-line curves, the radiation resistance approaches its free-space value in an oscillating manner.

(ii) The radiation resistance of an aerial array can be calculated by either of the methods previously referred to, but except for very simple arrays, the labour is prohibitive. In any event, it is impossible to define the radiation resistance of an array of which the various members carry currents of different magnitudes, except by the somewhat arbitrary method of referring it to the current in some particular member. Subject to this limitation, however, it is possible to compute the radiation resistance of a simple array from that of each member and the mutual impedance between the various members.

Radiation from hertzian doublet

10. (i) As an introduction to the theory of aerial arrays we may first consider the radiation field, γ_0, in the equatorial plane of a single hertzian doublet of length l, situated in free space, fig. 4a. In this theoretical radiator, a conduction current $i = \mathcal{I} \cos \omega t$ is assumed to flow in the wire joining the two capacitance areas, and a corresponding displacement current in the dielectric between them, but the elementary portions of the conductor itself are supposed to have no capacitance, so that the amplitude $\mathcal{I}$ of the current is the same at all points in the wire. The conduction current consists of a number N of electrons of charge e E.S.U. $= q$ coulombs, the average instantaneous velocity being, say, b centimetres per second. Let the cross-section of the conductor be A cm^2; the total volume of wire is then Al cm^3 and the density of the moving charge inside the wire is $\frac{Nq}{Al}$ coulombs per cm^3. The current density will therefore be $\frac{Nq}{Al} \frac{\text{coulomb}}{\text{cm}^3} \times b \frac{\text{cm}}{\text{sec}} = \frac{Nqb}{Al}$ coulomb per second per cm^2 or amperes per cm^2, and the

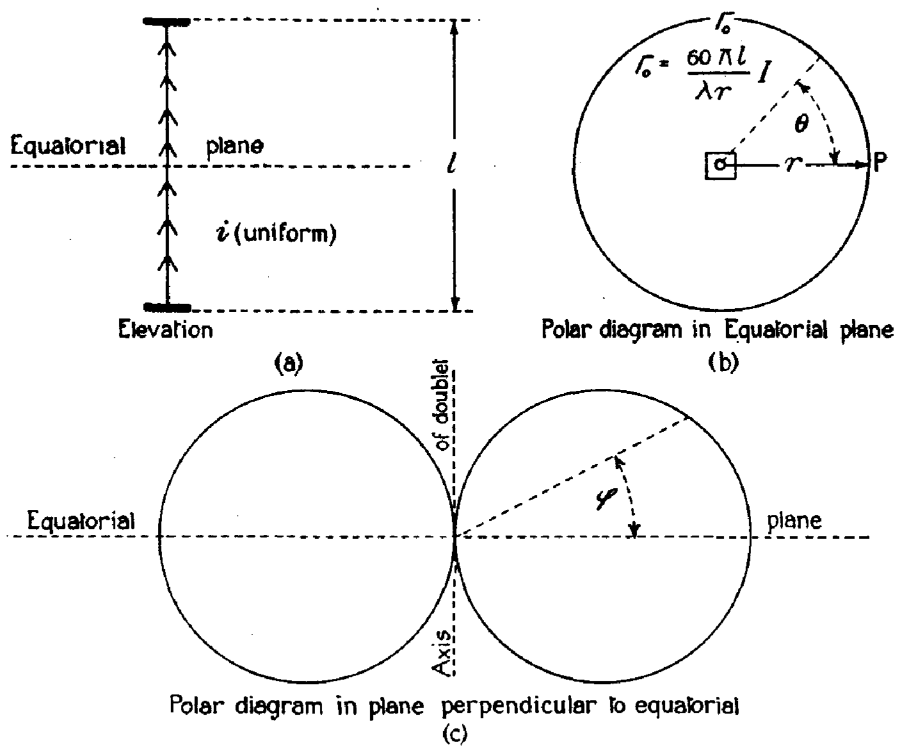


FIG. 4, CHAP. XV.—Radiation from hertzian doublet.

total instantaneous current $\frac{Nqb}{l}$ amperes. We may therefore write

$$i = \frac{Nqb}{l} = \mathcal{I} \cos \omega t,$$

$$\therefore b = \frac{l\,\mathcal{I}}{Nq} \cos \omega t. \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

The radiation depends upon the acceleration $a = \frac{db}{dt}$ of the electrons. Since b is of sinusoidal form,

$$a = -\frac{\omega l\,\mathcal{I}}{Nq} \sin \omega t$$

$$= \frac{\omega l\,\mathcal{I}}{Nq} \cos\left(\omega t + \frac{\pi}{2}\right). \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

(ii) The object of keeping the above expression in cosine form is to bring into prominence the relative phase of the radiation and the current. In Chapter VII it is shown that the electric field strength γ due to a single accelerated electron at a point P, distant r centimetres from the centre 0 of the doublet, and in the equatorial plane, is equal to $\frac{ae}{c^2 r}$ dynes per unit charge. If then the electrons in the wire are sinusoidally accelerated, the field at the point P is also sinusoidal, but will lag behind the acceleration producing it by an angle δ, owing to the finite velocity of propagation, c. Instead of expressing the field strength in dynes per unit charge, it is more convenient to express it in practical units. The field set up by the oscillating charge of q coulombs is therefore given by substituting, in the formula $\gamma = \frac{ae}{c^2 r}$, $\frac{9 \times 10^{11} \times \omega l\,\mathcal{I}}{Nq} \cos\left(\omega t + \frac{\pi}{2}\right)$ for the acceleration a, and Nq for the charge e, giving

$$\gamma_0 = \frac{9 \times 10^{11}\,\omega l\,\mathcal{I}}{Nq} \cos\left(\omega t + \frac{\pi}{2} - \delta\right). \qquad \ldots \quad \ldots \quad (3)$$

It is now convenient to put $c = 3 \times 10^{10}$, $\omega = 2\pi f = \frac{2\pi c}{\lambda}$, giving the amplitude $\hat{\Gamma}_0$ as

$$\hat{\Gamma}_0 = 9 \times 10^{11} \times \frac{2\pi c}{\lambda} \times \frac{l\,\mathcal{I}}{c^2 r}$$

$$= \frac{60\,\pi l}{\lambda r}\,\mathcal{I}.$$

Thus the complete expression for the electric field in volts per centimetre is

$$\gamma_0 = \frac{60\,\pi l}{\lambda r}\,\mathcal{I} \cos\left(\omega t + \frac{\pi}{2} - \delta\right). \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

Note that the lengths l, λ, and r are all measured in centimetres. If these are given in metres the field strength is expressed in volts per metre; or if the constant 60 is replaced by 37·25, and r is measured in miles (l and λ still in metres) the field strength is in millivolts per metre.

11. The amplitude of the field is seen to vary inversely as the distance r, but is independent of the angle θ. Hence the polar diagram in the equatorial plane is a circle with the axis of the

dipole as centre, fig. 4b. The phase angle δ obviously depends upon the distance r, for the wave travels this distance in a time $\frac{r}{c}$, hence $\delta = \frac{\omega r}{c} = \frac{2\pi}{\lambda} r$, and therefore

$$\gamma_0 = \frac{60\,\pi l}{\lambda r} \mathcal{J} \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right). \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

If the point P, instead of being in the equatorial plane, is situated at an angle φ above it, the field strength will be proportional to $\cos\varphi$ and is

$$\gamma_\varphi = \frac{60\,\pi l}{\lambda r} \mathcal{J}.\cos\varphi.\cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right). \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Considering the amplitude only, we see that this varies with the angle φ; in the equatorial plane, $\varphi = 0$, and the amplitude of the field strength is $\frac{60\,\pi l}{\lambda r} \mathcal{J}$, while in the polar direction, $\varphi = 90°$, the amplitude is zero. The variation of γ with φ is easily shown by means of a polar diagram as in fig. 4c in which the radius vector is proportional to $\cos\varphi$. The diagram therefore consists of two circles of unit diameter which are in contact at the axis of the doublet. By rotating the diagram round this axis we obtain a solid surface giving the relative field strengths in all directions in space. It is most important to remember that this solid surface does not represent the wave front, the latter being a spherical surface.

Radiation from half-wave dipole

12. The half-wave aerial differs from the hertzian doublet in its current distribution. Let the current at the centre of the dipole be $i = \mathcal{J}_0 \cos \omega t$. If the distance, $y = OA$, fig. 5, is measured from an origin O at the centre of the wire, the peak current $\mathcal{J}_y$ at the point A is $\mathcal{J}_0 \cos \frac{2\pi y}{\lambda}$. To find the field strength at a point P, distant r from the origin and in the equatorial plane, we consider the field set up by an elementary length dy of conductor, distant

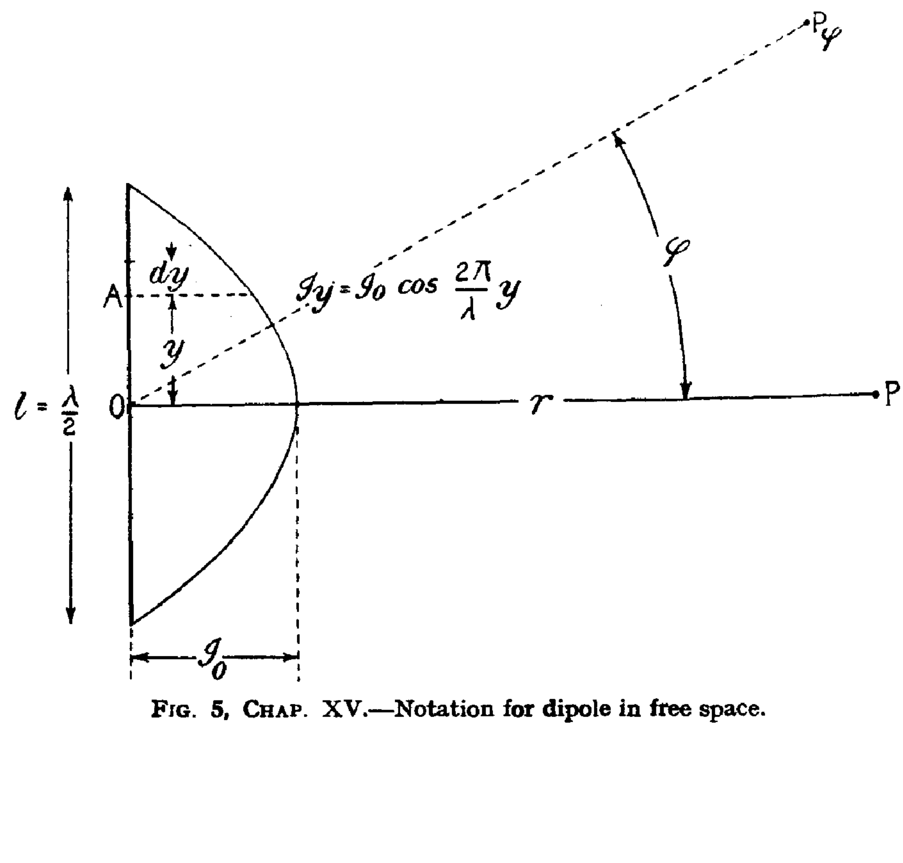


FIG. 5, CHAP. XV.—Notation for dipole in free space.

y from the origin; the current over this short length is practically uniform and we may therefore treat the element of conductor as a hertzian doublet of length dy. Obviously the field $d\gamma_0$ at P, due to this element of conductor, is

$$d\gamma_0 = \left(\frac{60\pi\, dy}{\lambda r}\, \mathcal{J}_0 \cos \frac{2\pi y}{\lambda}\right) \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right).$$

Now the fields produced at P by all the elements of conductor will be in phase, and the total field is that contributed by all such elements above and below the origin, i.e. from $y = +\frac{\lambda}{4}$ to $y = -\frac{\lambda}{4}$, and

$$\gamma_0 = \frac{60\pi}{\lambda r}\, \mathcal{J}_0 \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right) \int_{y=-\frac{\lambda}{4}}^{y=+\frac{\lambda}{4}} \cos \frac{2\pi y}{\lambda}.\, dy$$

$$= \left[\frac{60\pi}{\lambda r}\, \mathcal{J}_0 \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)\right] \times \frac{\lambda}{\pi}$$

$$= \frac{60}{r}\, \mathcal{J}_0 \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right). \qquad \dots \quad \dots \quad \dots \quad (7)$$

At a point P_φ above or below the equatorial plane, subtending with the latter an angle φ, the field is not now merely proportional to $\cos\varphi$ as in the case of the hertzian doublet. Instead, a factor $\dfrac{\cos\left(\frac{\pi}{2} \sin\varphi\right)}{\cos\varphi}$ must be introduced, giving

$$\gamma_\varphi = \frac{60}{r}\, \frac{\cos\left(\frac{\pi}{2} \sin\varphi\right)}{\cos\varphi}\, \mathcal{J}_0 \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right). \qquad \dots \quad \dots \quad \dots \quad (8)$$

The factor $\dfrac{\cos\left(\frac{\pi}{2} \sin\varphi\right)}{\cos\varphi}$ is plotted in fig. 6. It is seen to consist of two approximately elliptical figures, in contact on the axis of the dipole. By rotating this figure we obtain the solid polar diagram as in the previous instance.

13. Collecting the principal formulae so far developed, we have the following expressions for the amplitude $\hat{\Gamma}_\varphi$ of the electric field at an angle φ with respect to the equatorial plane.

(i) For the hertzian doublet

$$\hat{\Gamma}_\varphi = \frac{60\,\pi\, l}{\lambda\, r}\, \mathcal{J}_0 \cos\varphi,$$

(ii) For the $\frac{\lambda}{2}$ dipole

$$\hat{\Gamma}_\varphi = \frac{60}{r}\, \mathcal{J}_0 \frac{\cos\left(\frac{\pi}{2} \sin\varphi\right)}{\cos\varphi}.$$

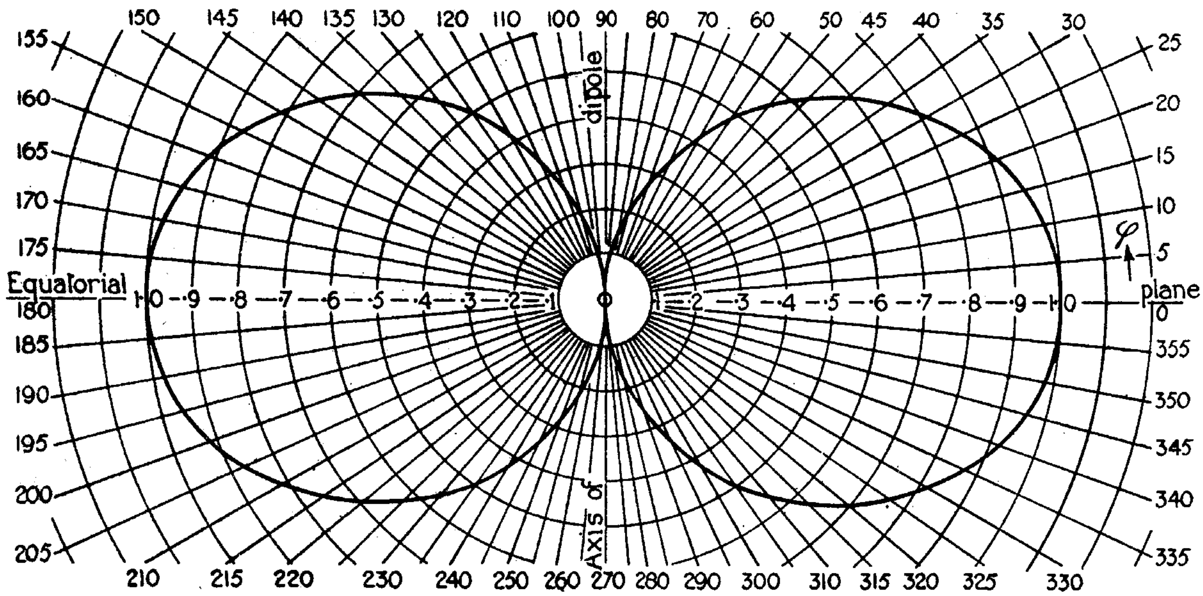


FIG. 6, CHAP. XV.—Current Distribution Factor for $\frac{\lambda}{2}$ dipole.

Each of these expressions may be divided into three factors,

(*a*) $\frac{60}{r}\,\vartheta_0$, which is common to both.

(*b*) In (i), $\frac{\pi l}{\lambda}$. The corresponding factor in (ii) is unity. These factors depend upon the distribution of capacitance along the aerial, and may be called the respective Form Factors. Thus the Form Factor of a hertzian doublet is $\frac{\pi l}{\lambda}$ and of a $\frac{\lambda}{2}$ dipole is unity.

(*c*) In (i), $\cos\varphi$. The corresponding factor in (ii) is $\dfrac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}$.

These factors take into account the effect of the current distribution upon the relative phases of the elements of field strength at points above and below the equatorial plane, and may be termed the respective Current Distribution Factors. In general, if the Form Factor is denoted by F, and the Current Distribution Factor by $f(\varphi)$, the amplitude of the electric field at a distance r may be written

$$\hat{\Gamma}_\varphi = \frac{60}{r}\,\vartheta_0 . F . f(\varphi).$$

Power input

14. Suppose it is required to produce a certain field strength at a point P, distant r miles from the radiator, where r is sufficiently great to justify the neglect of the induction field but so near that the effects of attenuation are negligible. Working in R.M.S. values, we have from the previous discussion

$$\Gamma = \frac{37\cdot 25}{r}\,F . f(\varphi)\ I \text{ (millivolts/metre)}$$

and if the radiation resistance of the aerial is R_r ohms the power radiated is P_r watts, where

$$P_r = I^2 R_r.$$

If P_r is given, then, the required current is

$$I = \sqrt{\frac{P_r}{R_r}}$$

and the field strength at a distance r is

$$\Gamma = \frac{37 \cdot 25}{r} F.f(\varphi)\sqrt{\frac{P_r}{R_r}}$$

$$P_r = \left(\frac{r\Gamma}{37 \cdot 25 \times F \times f(\varphi)}\right)^2 R_r.$$

Also, the input power will be given by

$$P_i = \left(\frac{r\Gamma}{37 \cdot 25 \times F \times f(\varphi)}\right)^2 R_A$$

where R_A is the total resistance of the aerial. For example let the point P be in the equatorial plane of a $\frac{\lambda}{2}$ dipole. Then $F = 1, f(\varphi) = 1$. Let the radiation resistance be 73 ohms and the loss resistance 10 ohms. Then to produce a field of 100 millivolts per metre at a distance of one mile, the power input must be

$$P_i = \left(\frac{1 \times 100}{37 \cdot 25}\right)^2 \times 83$$

$$= 593 \text{ watts.}$$

The power actually radiated will be $\frac{73}{83}$ of this or 522 watts.

Field due to two parallel dipoles

15. Let us now consider the field strength produced by a simple array consisting of two parallel half-wave dipoles in free space; these are spaced apart by a distance d as shown in plan and elevation in fig. 7, where A and B are the wires and O a point which will be regarded as an origin. We will first calculate the radiation in the equatorial plane perpendicular to the wires, each of which is assumed to carry a current $i = \mathcal{I} \cos \omega t$, i.e. the currents in the two wires are in phase. As before, consider the field at a point P, distant r from the origin O, and let $r >> \lambda$. Then AP, OP and BP are practically parallel to each other, and OP $= r$, AP $=$ $r + \frac{d}{2}\cos\theta$, BP $= r - \frac{d}{2}\cos\theta$.

The field produced by the current in the wire will be

$$\gamma_A = K\,\mathcal{I}_0 \cos\left\{\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r + \frac{d}{2}\cos\theta\right)\right\} \quad \ldots \quad \ldots \quad \ldots \quad (9a)$$

where $K = \frac{60}{r + \frac{d}{2}\cos\theta} \div \frac{60}{r}$; similarly the current in B will produce a field

$$\gamma_B = K\,\mathcal{I}_0 \cos\left\{\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r - \frac{d}{2}\cos\theta\right)\right\} \quad \ldots \quad \ldots \quad \ldots \quad (9b)$$

and the total field $\gamma_0 = \gamma_A + \gamma_B$ or

$$\gamma_0 = K\,\mathcal{I}_0\left[\cos\left\{\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}r - \frac{\pi d}{\lambda}\cos\theta\right\} + \cos\left\{\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}r + \frac{\pi d}{\lambda}\cos\theta\right\}\right]. \quad .. \quad (9c)$$

By a formula developed in Chapter V, this may be written

$$\gamma_0 = 2K\,\mathcal{I}_0\cos\left(\frac{\pi d}{\lambda}\cos\theta\right)\left\{\cos\,\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}r\right\}. \quad .. \quad .. \quad .. \quad (10)$$

Thus the amplitude of the electric field is

$$\hat{\Gamma}_0 = \frac{60}{r}\mathcal{I}_0 \times 2\cos\left(\frac{\pi d}{\lambda}\cos\theta\right).$$

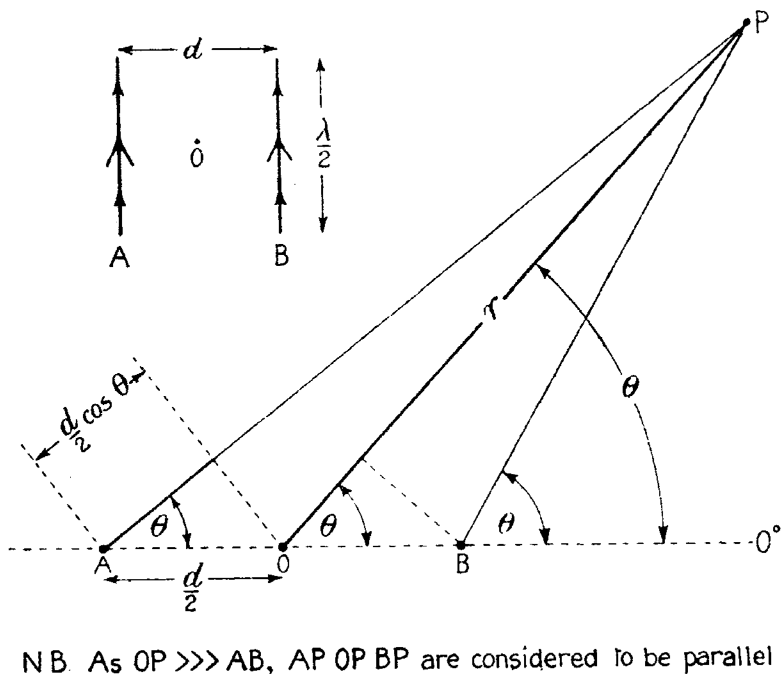


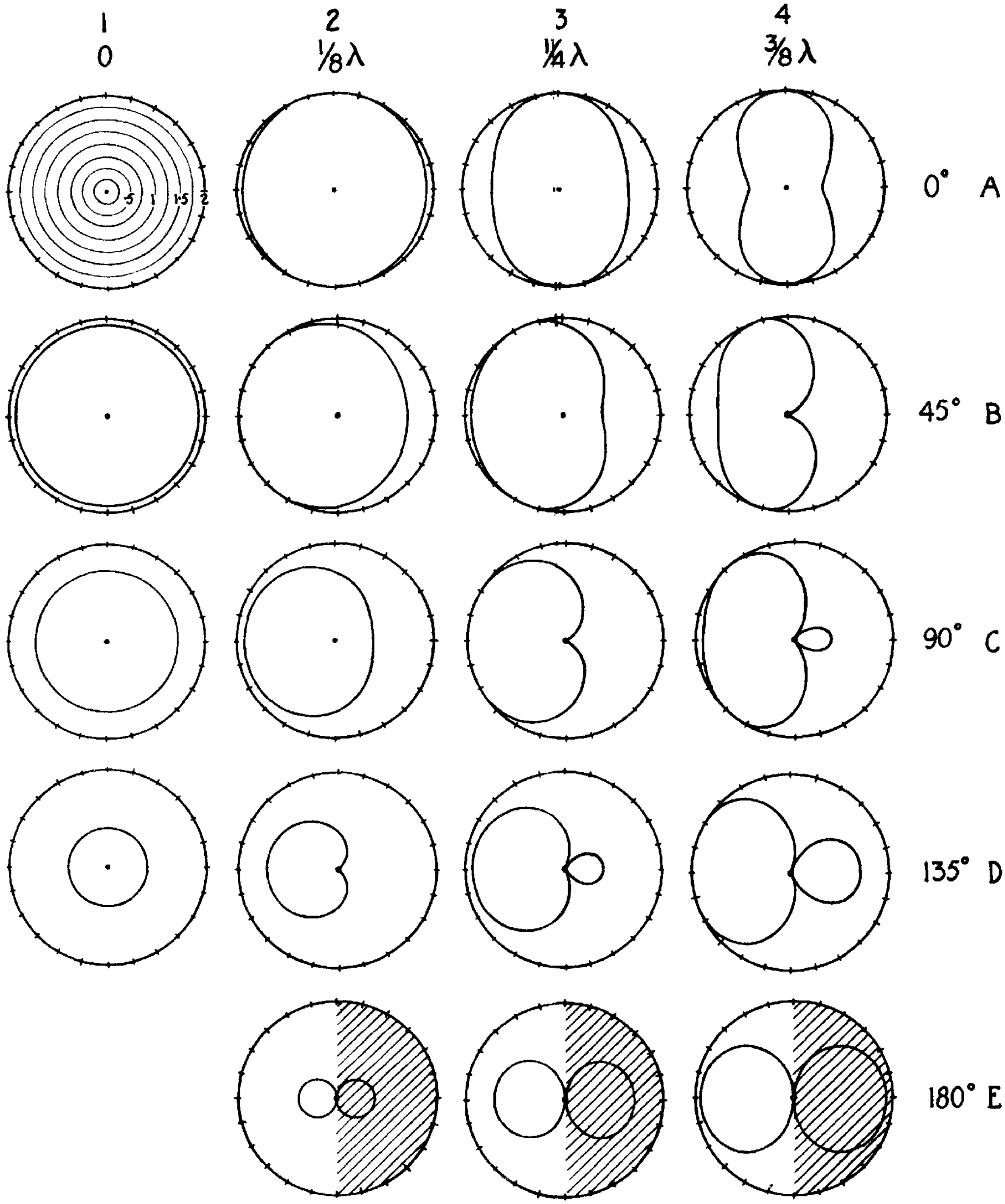
FIG. 7, CHAP. XV.—Notation for parallel dipoles in free space.

Grating factor

16. The amplitude of the field due to a single dipole situated at the origin O would be

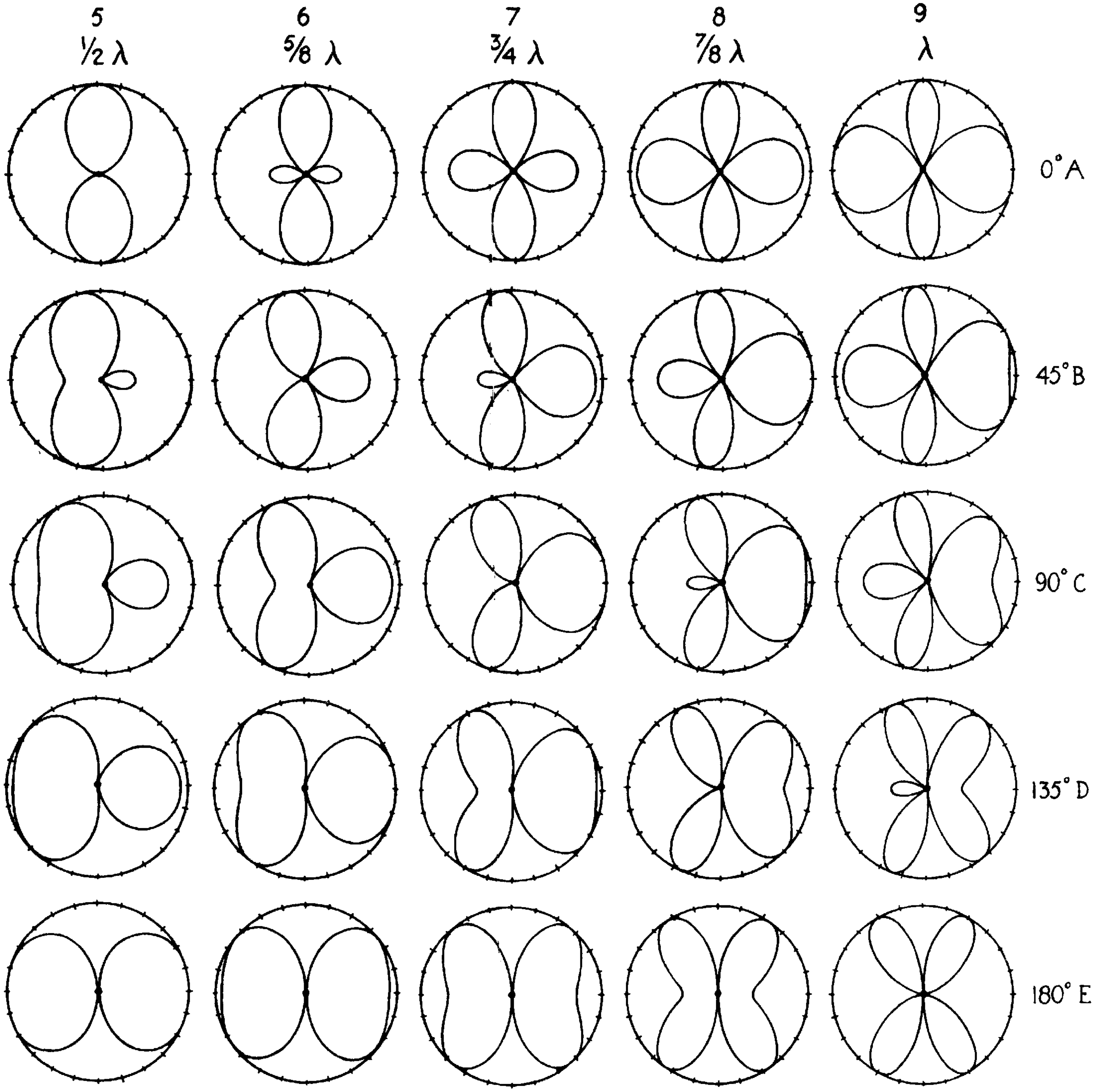
$$\hat{\Gamma}_0 = \frac{60}{r}\mathcal{I}_0$$

and we see that the field due to two parallel dipoles, separated by a distance d, and carrying equal, syn-phased currents, is obtained by multiplying that of a single dipole by the factor $2\cos\left(\frac{\pi d}{\lambda}\cos\theta\right)$, which is called the equatorial plane Grating Factor for a pair of dipoles. Its value obviously lies between the limits zero and 2, and is plotted in polar co-ordinates in fig. 8, line A, for various values of $\frac{d}{\lambda}$ from 0 to 4. The circle surrounding each diagram has a radius of 2 units, representing the upper limiting value of the Grating Factor. The first diagram, fig. 8, A 1, is for $d = 0$, i.e. two superimposed dipoles each carrying unit current, which are



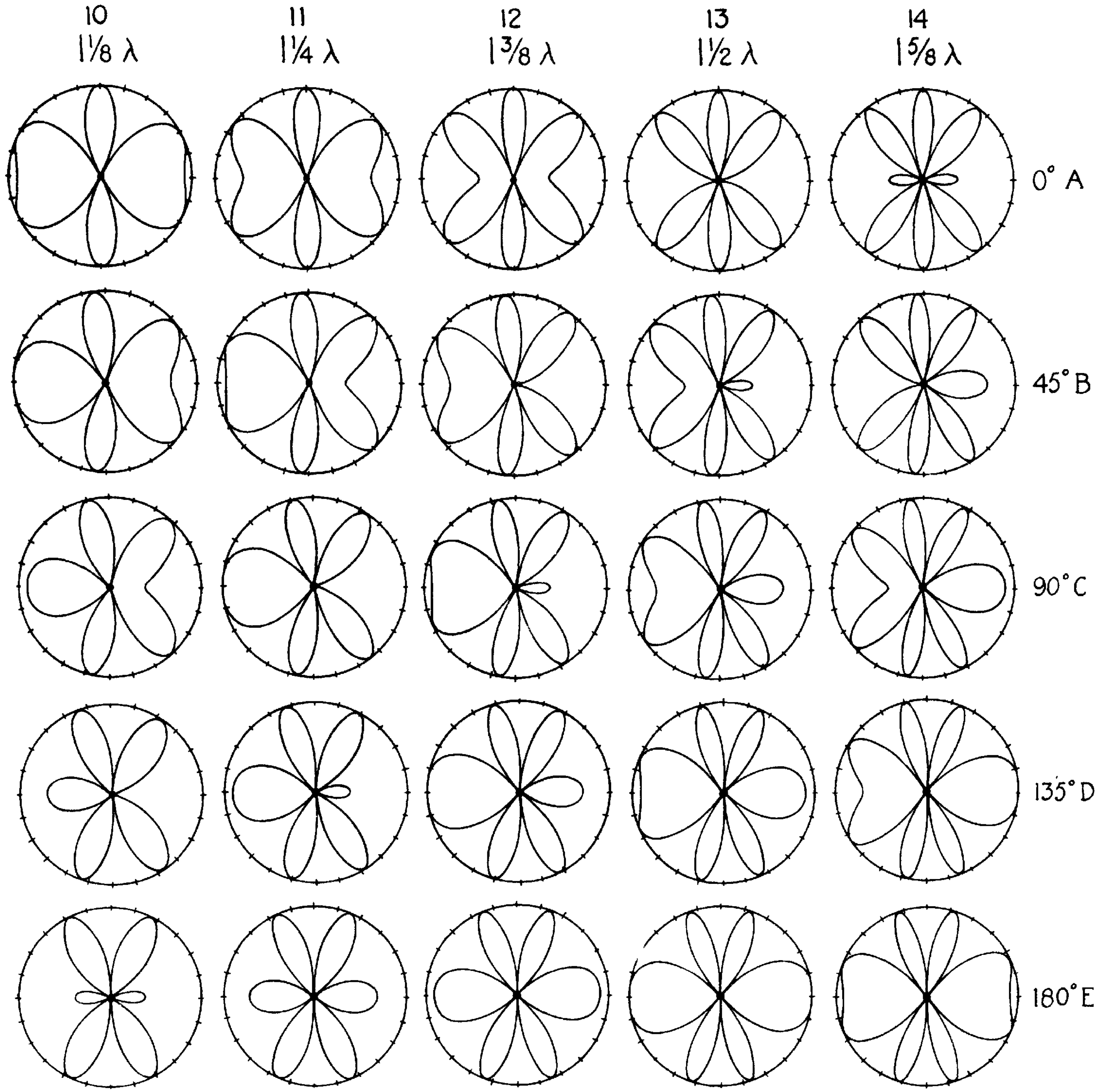
GRATING FACTOR (SHEET 1)

FIG. 8,
CHAP. XV



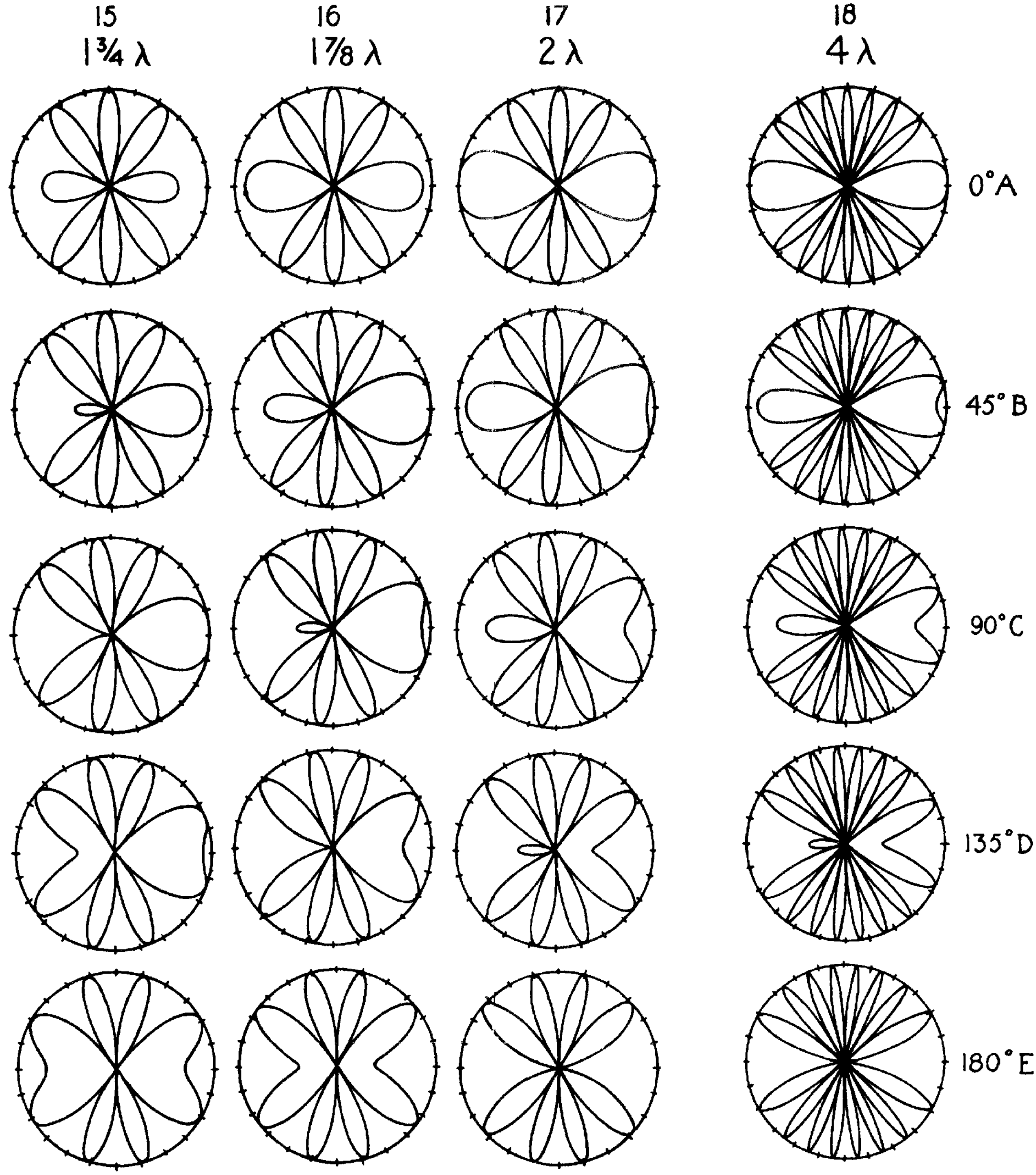
GRATING FACTOR (SHEET 2)

FIG. 8,
CHAP. XV



GRATING FACTOR (SHEET 3)

FIG. 8,
CHAP. XV



GRATING FACTOR (SHEET 4)

FIG. 8.
CHAP. XV

equivalent to a single dipole carrying a current of two units, hence the diagram corresponds with the limiting circle. For other values of $\frac{d}{\lambda}$, upon the line joining the two aerials, the field strength varies with the spacing. As d is increased toward the value $\frac{\lambda}{2}$, the field strength gradually decreases, and when $d = \frac{\lambda}{2}$ the radiation from the two aerials is in anti-phase at all points along this line, so that complete interference, i.e. cancellation, results. For values of d greater than $\frac{\lambda}{2}$, multiple lobes appear. In the directions $\theta = 0$ and $\theta = 180°$, the Grating Factor is 2, if n is even, and zero, if n is odd, whenever $d = \frac{n\lambda}{2}$. Upon a line perpendicular to that joining the two aerials, the Grating Factor is equal to 2 for all values of d, because the radiation from both aerials reaches all points simultaneously.

17. Now consider the field produced in the equatorial plane by two dipoles in which the currents are of equal magnitude but differ in phase. Let the aerials be A, carrying a current $i_A = \mathcal{I}_0 \cos \omega t$ and B with the current $i_B = \mathcal{I}_0 \cos (\omega t + \beta)$. Then

$$\gamma_A = K \mathcal{I}_0 \cos \left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r - \frac{\pi d}{\lambda} \cos \theta \right)$$

$$\gamma_B = K \mathcal{I}_0 \cos \left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r + \frac{\pi d}{\lambda} \cos \theta + \beta \right).$$

If $\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r \right) = \omega t_0$

$$\gamma_A + \gamma_B = K \mathcal{I}_0 \left\{ \cos \left(\omega t_0 - \frac{\pi d}{\lambda} \cos \theta \right) + \cos \left(\omega t_0 + \frac{\pi d}{\lambda} \cos \theta + \beta \right) \right\}$$

and the total field becomes

$$\gamma_0 = K \mathcal{I}_0 \left[\cos \left(\omega t_0 - \frac{\pi d}{\lambda} \cos \theta \right) + \cos \left(\omega t_0 + \frac{\pi d}{\lambda} \cos \theta + \beta \right) \right]. \qquad (11)$$

To simplify, put $\omega t_0 = X$, $\frac{\pi d}{\gamma} \cos \theta = Y$, $P = X - Y$, $R = X + Y$, $Q = R + \beta$; then

$$\gamma_0 = K \mathcal{I}_0 \{\cos (X - Y) + \cos (X + Y + \beta)\}$$
$$= K \mathcal{I}_0 \{\cos P + \cos Q\}.$$

By Chapter V, $\cos P + \cos Q = 2 \frac{P + Q}{2} \cos \frac{P - Q}{2}$.

Whence

$$\gamma_0 = 2K \mathcal{I}_0 \cos \frac{X - Y + (X + Y + \beta)}{2} \cos \frac{(X - Y) - (X + Y) - \beta}{2}$$

$$= 2K \mathcal{I}_0 \left\{ \cos \left(X + \frac{\beta}{2} \right) \cos - \left(Y + \frac{\beta}{2} \right\} \right)$$

$$= 2K \mathcal{I}_0 \left[\cos \left(\omega t_0 + \frac{\beta}{2} \right) \right] \cos \left(\frac{\pi d}{\lambda} \cos \theta + \frac{\beta}{2} \right). \qquad \ldots \quad \ldots \quad (12)$$

The Grating Factor is therefore $2 \cos \left(\frac{\pi d}{\lambda} \cos \theta + \frac{\beta}{2} \right)$ and is plotted in polar co-ordinates in fig. 8, lines B to E, for various values of β and $\frac{d}{\lambda}$. Of particular interest is the bottom row, which shows the fields due to two aerials carrying currents in anti-phase. Obviously, the radiation from the two cancels out along a line perpendicular to that joining the two aerials, while along that line, the Grating Factor is zero if d is an even number of half wavelengths, and equal to 2, if d is an odd number of half wavelengths. Line C will again be referred to in connection with reflector aerials, while line E is of importance in the study of loop aerials, both for reception and transmission. We see then that fig. 8 has many important applications and will repay a very careful study. To facilitate the enlarged reproduction of any particular diagram, each limiting circle has been divided at 15° intervals, and a series of concentric circles of various radii inserted within the limiting circle of fig. 8, A 1. When adding these diagrams, it must be noted that the radius vector changes sign on passing through a zero. An example is given in paragraph 41.

Combinations of pairs of dipoles

18. Consider an array of four parallel dipoles spaced one-half wavelength apart and fed with equal, syn-phased currents as in fig. 9a. The polar diagram in the equatorial plane may be obtained in the following manner. Divide the array into two pairs of aerials; the polar diagram of each of these pairs is, by the previous paragraph, the elongated figure-of-eight shown in diagram A 5 of fig. 8, and repeated in fig. 9b. For the four aerials spaced $\frac{\lambda}{2}$ apart, we may now substitute two aerials, each having the above polar diagram, but spaced one wavelength apart as in fig. 9c. According to diagram A 9 of fig. 8, two dipoles with this spacing, and syn-phased currents, have a polar diagram with four lobes; this diagram is repeated in fig. 9d.

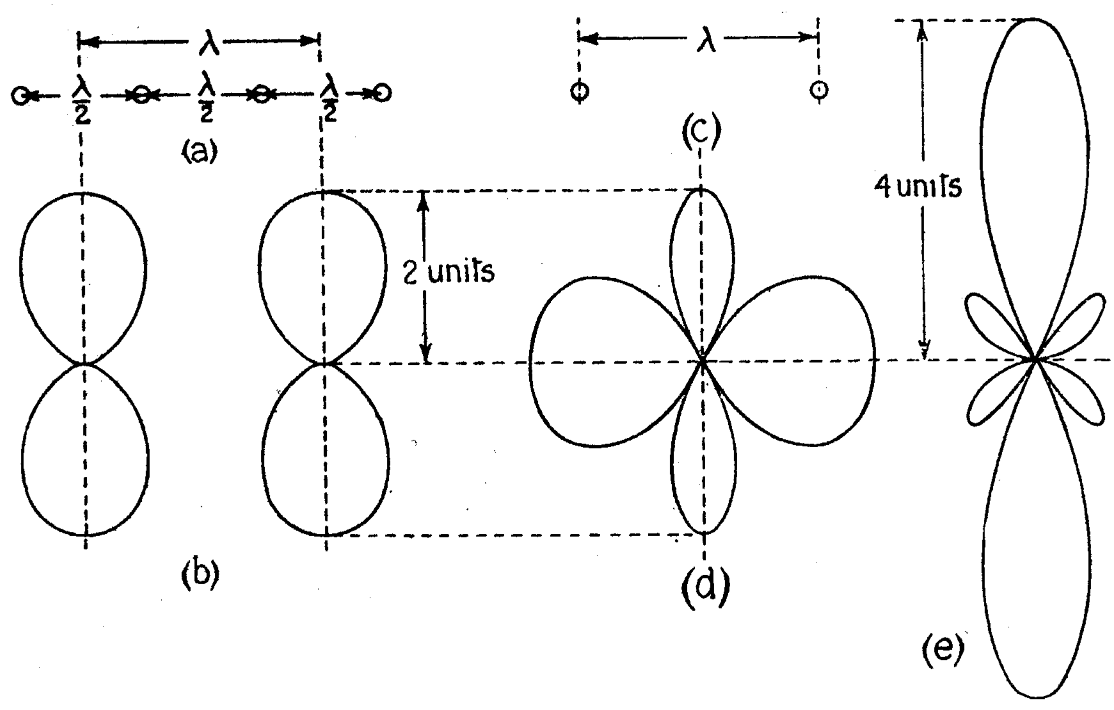


FIG. 9, CHAP. XV.—Development of polar diagram for four parallel dipoles carrying syn-phased current.

The field distribution of the combination of two radiators which, individually, give the figure-of-eight diagram, fig. 9b, may now be obtained by multiplying together the corresponding polar radii of figs. 9b and 9d, giving the result shown in fig. 9e. The principle of combining parallel aerials carrying syn-phased currents is the basis of what are called broadside arrays.

19. The above process may obviously be extended to obtain the field distribution in the equatorial plane for any number of dipoles irrespective of the spacing and the phase of current in the respective aerials. Thus, suppose we have an array of four parallel dipoles A B C D, fig. 10a, spaced $\frac{\lambda}{4}$ apart, each carrying a current of I amperes. Let the current in B lead by 90° on that in A, the current in C lead by 90° on that in B and so on. The polar diagram for the pair A and B, or for the pair C and D, is given in fig. 8, diagram C 3, which is reproduced in fig. 10b; it

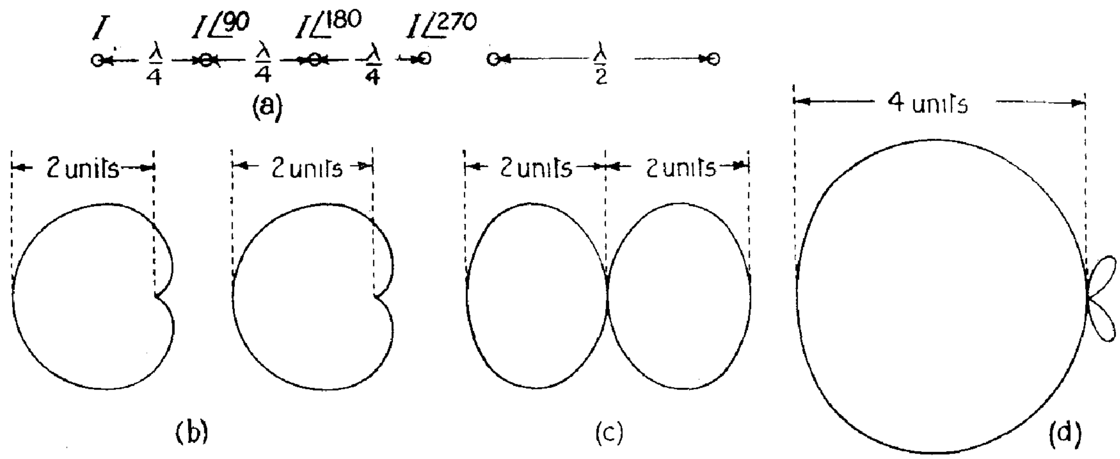


FIG. 10, CHAP. XV.—Development of polar diagram for four parallel dipoles with currents in progressive phase difference of 90°.

is a cardioid or heart-shape. For the four dipoles, we may now substitute two radiators each having this polar diagram, spaced $\frac{\lambda}{2}$ apart, and with currents in anti-phase. Two dipoles with this spacing and current phase give the polar diagram E 5, fig. 8, reproduced in fig. 10c. The polar diagram of the four-element array is found by multiplying together the corresponding polar radii of diagrams C 3 and E 5, resulting in the diagram shown in fig. 10d. It will be observed that the array is substantially uni-directional, maximum radiation being directed in the direction of the aerial in which the phase of the current is lagging. The principle of combining parallel aerials carrying currents differing by a constant angle, which in turn is related to the spacing of the elements, is the basis of what are called end-fire aerial arrays.

Radiation in the plane of the aerials.

20. We may now investigate the shape of the polar diagram in the plane containing the aerials. In fig. 11 let A and B be two parallel dipoles each carrying a loop current $i = \mathcal{I}_0 \cos \omega t$. At a point P at an angle φ above the equatorial plane, situated at a distance r from the origin O, where $r >> \lambda$, the fields will be

$$\gamma_A = K \mathcal{I}_0 \frac{\cos\left(\frac{\pi}{2} \sin \varphi\right)}{\cos \varphi} \cos\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r + \frac{d}{2}\cos \varphi\right)\right]$$

$$\gamma_B = K \mathcal{I}_0 \frac{\cos\left(\frac{\pi}{2} \sin \varphi\right)}{\cos \varphi} \cos\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r - \frac{d}{2}\cos \varphi\right)\right].$$

The Current Distribution Factor $\dfrac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}$

has previously been introduced to account for the fact that the dipole does not radiate uniformly in any plane perpendicular to the equatorial. The combined field is $\gamma_\varphi = \gamma_A + \gamma_B$ and

$$\gamma_\varphi = \left[2K\,\mathcal{I}_0\,\frac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}\cos\left(\frac{\pi d}{\lambda}\cos\varphi\right)\right]\cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}r\right). \quad (13)$$

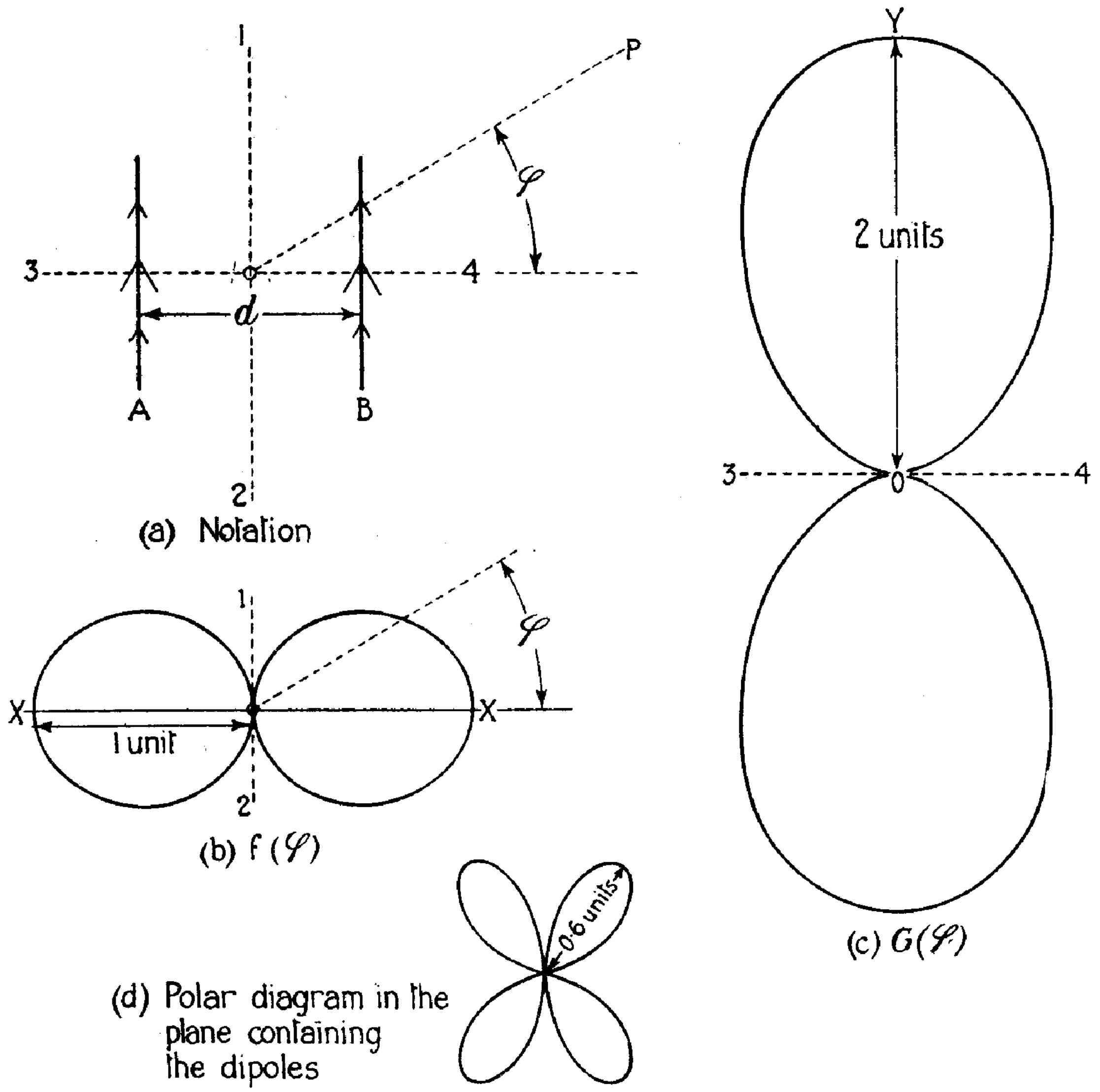


FIG. 11, CHAP. XV.—Radiation in plane containing two parallel syn-phased dipoles.

The portion of the right-hand member which is enclosed in square brackets is the amplitude of the field in the direction O P. It is the product of three factors

(i) $K \mathcal{I}_0 = \frac{60}{r} \mathcal{I}_0$.

(ii) $\frac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}$, i.e. the Current Distribution Factor of a $\frac{\lambda}{2}$ dipole.

(iii) $2\cos\left(\frac{\pi d}{\lambda}\cos\varphi\right)$, i.e. the Grating Factor for a pair of syn-phased dipoles in the plane containing them. This is of exactly the same form as the equatorial plane Grating Factor, but is a function of the angle φ instead of being a function of the angle θ.

The Current Distribution Factor is given by fig. 6, and the Grating Factor by the upper row of diagrams in fig. 8. Thus the resultant amplitude in any particular case may be obtained by multiplying the constant $\frac{60}{r}\mathcal{I}_0$ by two polar radii obtainable from the diagrams. As an example, take $d = \frac{\lambda}{2}$. Fig. 11b is the Current Distribution Factor, fig. 11c the Grating Factor, and the diagram obtained from the polar products is shown in fig. 11d. This product has a maximum value of 0·6, at an angle of approximately 55°. If the Grating Factor in this plane is denoted by G (φ), the R.M.S. field in the plane containing the aerials is

$$\Gamma_\varphi = \frac{60}{r} I_0 \times F \times f(\varphi) \times G(\varphi)$$

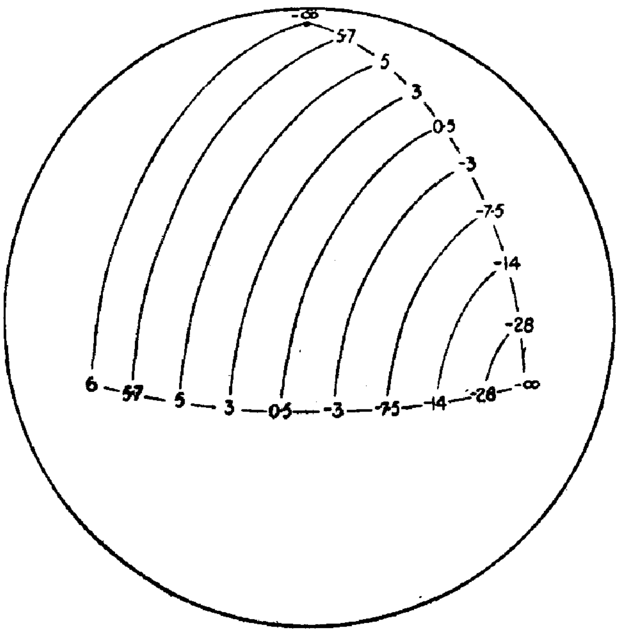
where F and $f(\varphi)$ are the Form and Current Distribution Factors as before. It will be seen later that if the co-ordinates of the point P are r, θ, φ, the Grating Factor becomes

$$G(\theta, \varphi) = 2\cos\left(\frac{\pi d}{\lambda}\cos\theta\cos\varphi\right).$$

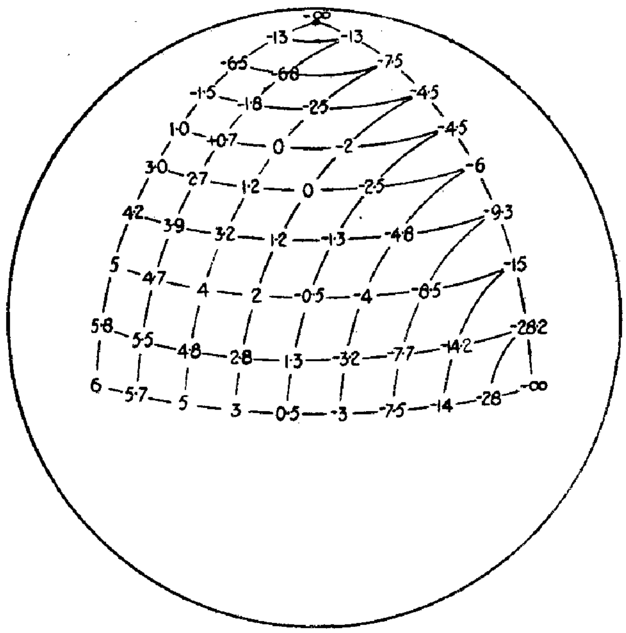
Three dimensional polar diagram

21. We have now shown how to obtain the polar diagrams for a pair of spaced aerials in the equatorial plane and in the perpendicular plane containing the aerials. While it is possible to calculate the polar diagram of any combination of aerials in all directions in space, the process becomes very tedious when more than two or three aerials are involved. The solid polar diagram may however be obtained by combining the diagrams for the equatorial plane and that containing the aerials. The process will be illustrated by taking the two parallel dipoles, spaced $\frac{\lambda}{2}$ apart in free space as before (fig. 11a) and carrying equal, syn-phased currents. The R.M.S. field strength at an angle φ with respect to the equatorial plane is proportional to the Current Distribution Factor $f(\varphi)$; in this particular case $f(\varphi) = \frac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}$ and has already been plotted in fig. 6. If this diagram is rotated about the axis 1, 2, fig. 11a, the resulting solid figure (when multiplied by $\frac{60}{r} I_0$) gives the three-dimensional diagram of a single dipole. The combination of two such dipoles introduces a Grating Factor which has been shown to be $2\cos\left(\frac{\pi d}{\lambda}\cos\theta\right)$ in the equatorial plane and $2\cos\left(\frac{\pi d}{\lambda}\cos\varphi\right)$ in the plane containing the aerials.

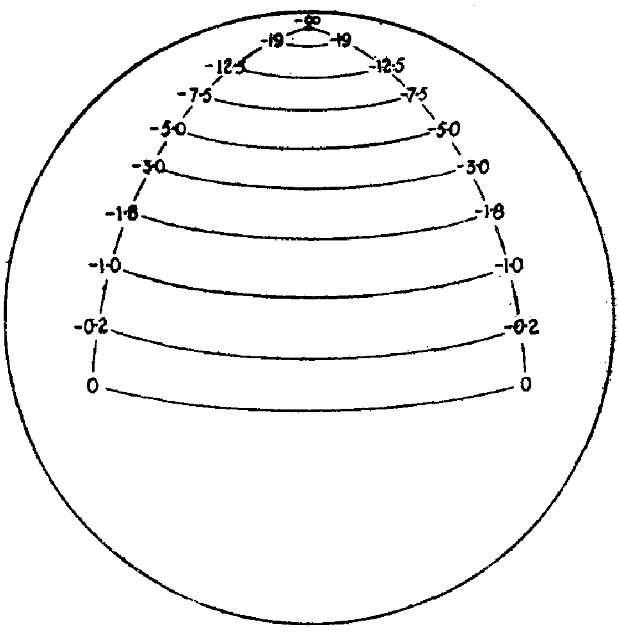
The Grating Factor is plotted in fig. 11c ; if it is rotated about the axis 3, 4, the result is another solid figure. The polar radii of the latter, for any direction in space, gives a factor by which the quantity $\frac{60}{r} I_0 f(\varphi)$ must be multiplied, in order to give the R.M.S. field at any particular point. The radius O X in fig. 11b is equal to unity, and the radius O Y in fig. 11c is equal to two units. Thus, in the equatorial plane, the field strength in a direction perpendicular to the line upon which the dipoles are situated is twice that of a single dipole.



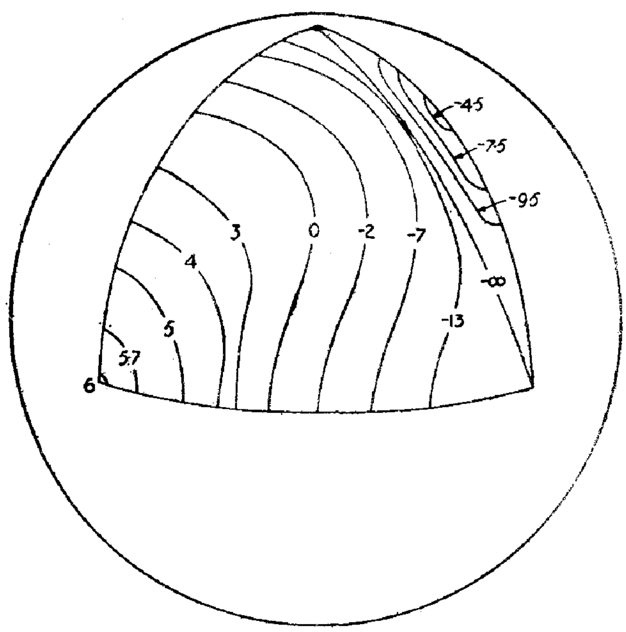
(a) Sphere with parallels corresponding to grating factor G (φ) in db



(c) Sum of G (φ) and f(φ) in db



(b) Sphere with parallels corresponding to C.D factor f (φ) in db



(d) Contour lines derived from (c)

FIG. 12, CHAP. XV.—Gain contours on spherical surface.

22. Instead of in units of length, the radii may be expressed in decibels above or below unity. The field in any direction is then given, in decibels above or below the equatorial field of a single dipole, merely by adding the decibels corresponding to the respective radii of figs. 11b and 11c. If we take a spherical surface and mark off a number of equal zones parallel to the equatorial plane, each of the boundary lines between adjacent zones may be marked to show the number of decibels below the field strength in the equatorial plane as in fig. 12b, the figures being derived from the Current Distribution Factor. Similarly, if we draw a number of equal zones in a plane perpendicular to the equatorial plane and to the plane containing the aerials, the boundary lines between these zones may also be marked in decibels above or below unity as in fig. 12a, the figures being obtained from the Grating Factor. At the points of intersection of any two lines, the field strength is above or below the equatorial field of a single aerial by the sum of the decibels appropriate to the two intersecting lines. One quadrant of a spherical surface, with both sets of zones superimposed, is shown in fig. 12c. It must be particularly noted that although the boundary lines in fig. 12b correspond to parallels of latitude, the boundary lines of fig. 12a do not pass through the pole and are not analogous to meridians of longitude.

23. If now we insert, at the intersection of all boundary lines in the spherical surface, a number equal to the algebraic sum of the decibels appertaining to the two intersecting lines, we obtain the gain or loss in decibels compared with the standard at different points on the sphere. These figures have been inserted in fig. 12c. By joining all points of equal gain, we obtain a field strength contour diagram as in fig. 12d. A close examination of this figure shows that in each quadrant of the surface there are two maxima. One, corresponding to the main lobe, is 6 db. above the standard while the other is at an elevation of about 55°, in the plane containing the aerials, and its maximum is about 4 db. below the standard. This lobe has already been found to exist (paragraph 20). From this date we may make a solid model of the polar diagram in plasticine, as shown in fig. 13. To do this, the gain in db. above or below the standard must be converted back to absolute field strength.

Field strength map—the sinusoidal projection

24. When the principles involved in the production of the solid diagram are thoroughly appreciated, it will be found easy to construct a map showing the gain or loss in different directions. We may consider the aerial array to be situated at the centre of a sphere and to illuminate

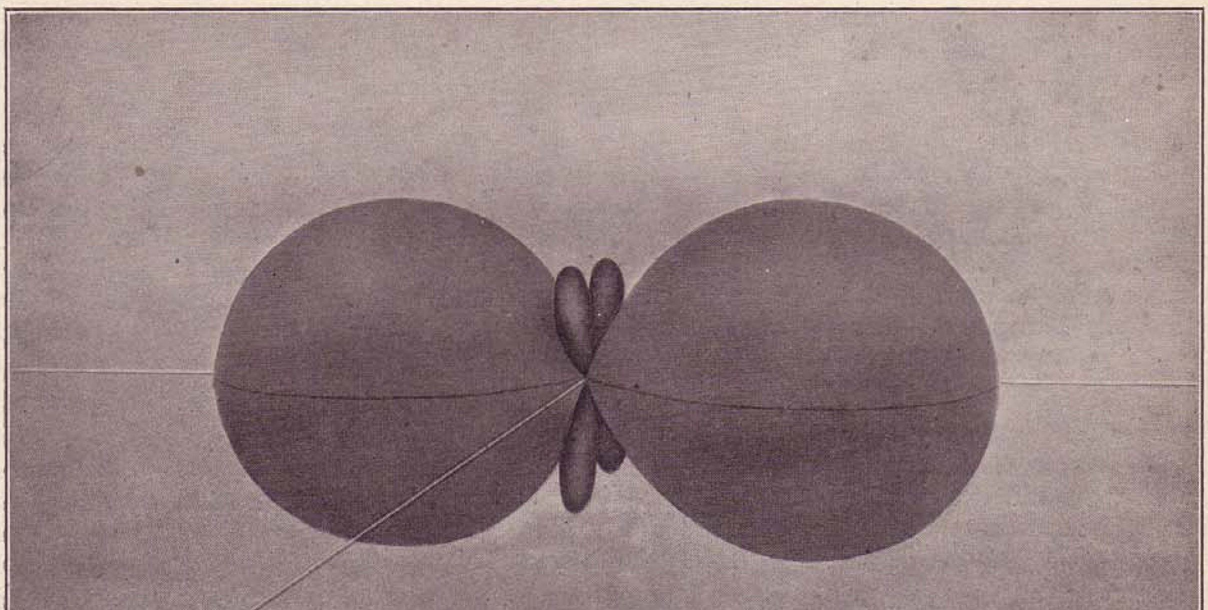


FIG. 13, CHAP. XV.—Solid polar diagram—parallel dipoles in free space.

different regions with greater or less intensity. The delineation of a spherical surface upon a plane is most familiar in the form of Mercator's projection of the earth. This projection is unsuitable for general use in aerial array theory because the high latitudes cannot be shown with accuracy, and it may be necessary to show the field strength vertically over the aerial. A suitable projection is that known as sinusoidal, in which the length of a parallel of latitude is proportional to the cosine of the latitude. This is shown in fig. 14. In the original drawing, the length of one quadrant of a parallel of latitude in the equatorial plane, i.e. latitude 0°, is 9 inches. The length of the corresponding quadrant in latitude 10° is 9 *cos* 10° = 8·85 inches, in latitude 20° is 8·45 inches and so on. The meridian corresponding to longitude 0° (with the convention of fig. 10) is a line through the points given above and is a cosine curve. Longitudes 10°, 20°, etc., are also cosine curves obtained by the division of each 90° into nine equal parts. Once the sinusoidal graticule has been prepared, the

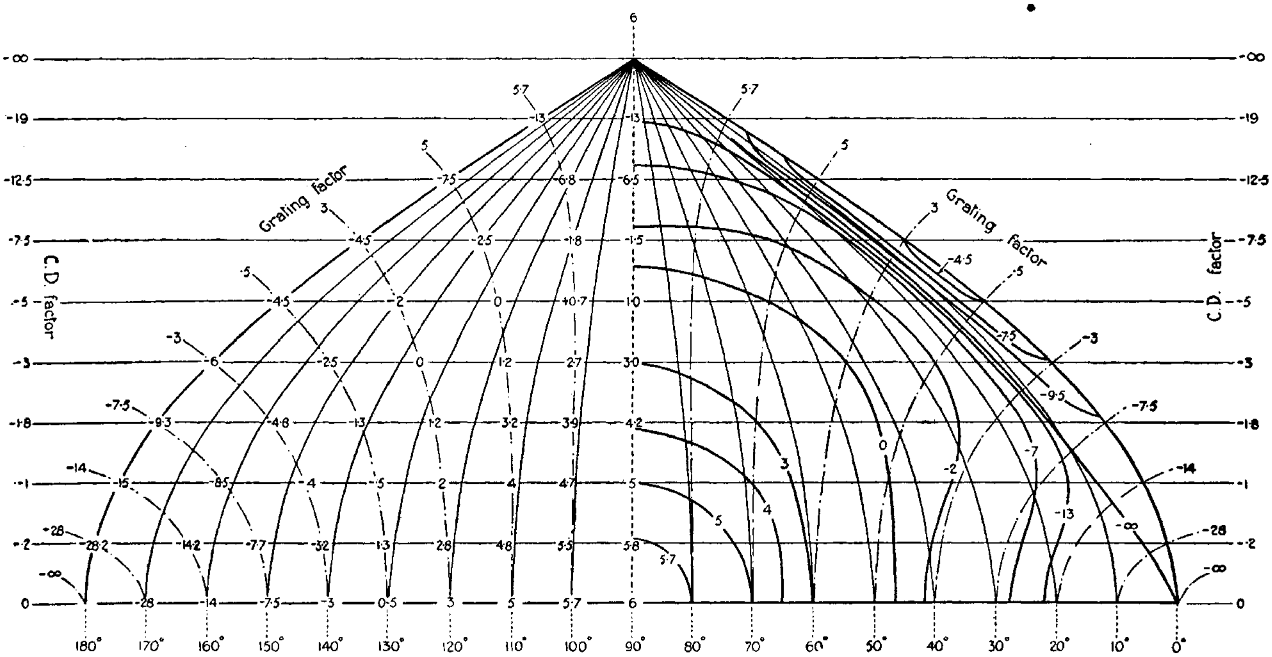


FIG. 14, CHAP. XV.—Gain contours shown on sinusoidal projection.

parallels of latitude may be allotted the appropriate Current Distribution Factors (in decibels) and the zones perpendicular to these may be inserted by freehand drawing with sufficient accuracy for most purposes. In fig. 14 these are denoted by chain-dotted lines. The latter are allotted their appropriate values of Grating Factor (in decibels). The total gain is then inserted at the intersecting points, and the gain contours drawn. Alternatively, the gain in decibels may be reconverted to absolute field strength. There is of course no objection to working in absolute field strength from the beginning, but this would necessitate finding the product of two numbers for each intersecting point.

Use of vector algebra

25. When it is necessary to calculate the field at a point P in space, having the co-ordinates r, θ, φ, the method now to be described will be found more convenient for algebraic purposes than the purely trigonometric methods previously adopted. Instead of considering the sinusoidal current as the product of a constant, i.e. the amplitude $\mathcal{I}$, and a trigonometrical function of time, e.g. $\cos \omega t$, it is considered as the product of a vector $\mathbf{I}$ and a vector operator $\epsilon^{j\omega t}$. Now $\epsilon^{j\omega t} = \cos \omega t + j \sin \omega t$ so that $\mathbf{I}\epsilon^{j\omega t} = \mathbf{I} \cos \omega t + j\, \mathbf{I} \sin \omega t$. Since in operations involving complex quantities of this kind, the real and imaginary parts are entirely independent, we may

deal with a current $i = \mathscr{I} \cos \omega t$ by saying "let i be the real part of $\mathbf{I}\, \varepsilon^{j\omega t}$." The magnitude of the vector **I** is of course equal to the amplitude $\mathscr{I}$ of the current, in fact **I** may be regarded as the product of the scalar $\mathscr{I}$ and a unit vector. In practice, it is usual to write merely "let $i = \mathbf{I}\, \varepsilon^{j\omega t}$" the real part only of the final result being taken. For example, consider the field due to a $\frac{\lambda}{2}$ dipole in free space, carrying a loop current $i_0 = \mathscr{I}_0 \cos \omega t$. Let this current be $\mathbf{I}_0\, \varepsilon^{j\omega t}$. Then the field γ_φ at the point $\mathrm{P} = r, \varphi$, will be

$$\gamma_\varphi = \frac{60}{r} f(\varphi)\, \mathscr{I}_0 \cos\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right) \qquad \ldots \quad \ldots \quad \ldots \quad (14a)$$

in the notation of previous paragraphs. In the present notation,

$$\gamma_\varphi = \frac{60}{r} f(\varphi)\; I_0\, \varepsilon^{j\omega t}\, \varepsilon^{j\frac{\pi}{2} - j\frac{2\pi}{\lambda} r} \qquad \ldots \quad \ldots \quad \ldots \quad (14b)$$

or more economically

$$\gamma_r = \frac{60}{r} f(\varphi)\, \mathbf{I}_0\, \varepsilon^{\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)}. \qquad \ldots \quad \ldots \quad \ldots \quad (41c)$$

26. The advantage of this notation lies in the ease with which the fields due to two or more radiators can be combined, even if they differ both in magnitude and phase. Referring to Chapter V, an impedance of magnitude $Z = \sqrt{R^2 + X^2}$ ohms may be represented both in magnitude and in its effect on the phase angle of a current, in any of the following ways,

$$Z \underline{/\theta} = R + jX = Z\, \varepsilon^{j\theta}$$

or

$$Z \,/\theta = R - jX = Z \varepsilon^{-\theta}$$

$$\left(\theta = \tan^{-1} \frac{X}{R}\right)$$

depending upon whether the reactance is positive (inductive) or negative (capacitive). Obviously $Z \underline{/\theta} = Z \setminus -\theta$ and vice versa. For example, an E.M.F. $\mathbf{E}\, \varepsilon^{j\omega t}$ acting in a circuit of $Z \underline{/\theta}$ ohms, will produce an instantaneous current,

$$i = \frac{\mathbf{E}\, \varepsilon^{j\omega t}}{Z \,/\theta} = \frac{\mathbf{E}\, \varepsilon^{j\omega t}}{Z} \overline{\setminus \theta}$$

$$= \frac{\mathbf{E}}{Z} \varepsilon^{j\omega t}\, \varepsilon^{-j\theta}$$

$$= \frac{\mathbf{E}}{Z} \varepsilon^{j(\omega t - \theta)} \text{ (real part only)}$$

$$= \frac{\mathscr{E}}{Z} \cos(\omega t - \theta),$$

i.e. a positive reactance produces a lagging current. Similarly, an impedance $Z \; \overline{\backslash \theta}$ ohms, acting under the same conditions, produces an instantaneous current

$$i = \frac{\mathbf{E}\,\varepsilon^{j\omega t}}{Z\,\overline{\backslash \theta}} = \frac{\mathbf{E}\,\varepsilon^{j\omega t}}{Z}\,\underline{/\,\theta}$$

$$= \frac{\mathbf{E}}{Z}\,\varepsilon^{j(\omega t + \theta)} \text{ (real part only)}$$

$$= \frac{\mathscr{E}}{Z}\cos(\omega t + \theta),$$

i.e. a negative reactance produces a leading current. It is also convenient to adopt a distinctive type of symbol for any vector operator, for use where it is unnecessary or impossible to define its properties completely. In the following text such operators will be denoted by lower-case (i.e. "small") Clarendon type, thus naturally associating with vector quantities. The latter are printed in Clarendon capitals, except where the symbol is a Greek character, when a bar superior is used thus $\bar{\Gamma}$. Vector operators are often used to denote the vector ratio between two quantities as follows:—Suppose we have two currents, $i_1 = \mathscr{I}_1 \cos(\omega t - \theta)$ and $i_2 = \mathscr{I}_2 \cos(\omega t + \varphi)$. The ratio $\frac{\mathscr{I}_2}{\mathscr{I}_1}$ of the amplitudes is a mere number and may be denoted by M. We also require to know their relative phase, and in the vector notation

$$i_1 = \mathbf{I}_1\,\varepsilon^{j(\omega t - \theta)} = I_1\,\underline{/\,\omega t - \theta}$$

$$i_2 = \mathbf{I}_2\,\varepsilon^{j(\omega t + \varphi)} = I_2\,\underline{/\,\omega t + \varphi}$$

$$\frac{i_2}{i_1} = \frac{I_2\,\underline{/\,\omega t + \varphi}}{I_1\,\underline{/\,\omega t - \theta}}$$

$$= \frac{I_2}{I_1}\,\underline{/\,\omega t + \varphi - \omega t + \theta}$$

$$= M\,\underline{/\,\varphi - \theta}$$
$$= \mathbf{m}\,.$$

The above method of treatment leads to the simple algebraic solution of problems which would otherwise be comparatively difficult and much more tedious.

General case of two parallel radiators

27. Referring to fig. 15, let A and B be two parallel but not necessarily identical aerials. Their midpoints are, however, equally spaced on either side of an origin O in the equatorial phase, the distance apart being d. Consider the field at the point $P = r, \varphi, \theta$, where $r = OP$ is very much greater than d. Let the angle $XOP = \alpha$. Then $XAP = XBP = \alpha$ also. We may therefore write

$$AP = r_A = r + \frac{d}{2}\cos\alpha$$

$$BP = r_B = r - \frac{d}{2}\cos\alpha$$

with negligible error. Now suppose the currents at the midpoints to be $\mathbf{I'}_A = \mathbf{I}_A\, \varepsilon^{j\omega t}$ and $\mathbf{I'}_B = \mathbf{I}_B\, \varepsilon^{j(\omega t + \beta)}$. If F_A, F_B are the Form Factors and $f_A(\varphi), f_B(\varphi)$, the Current Distribution Factors of the respective aerials with regard to their midpoints, the individual fields due to the two aerials will be

$$\gamma_A = j\frac{60}{r} F_A . f_A(\varphi)\, \mathbf{I'}_A\, \varepsilon^{-j\frac{2\pi}{\lambda} r_A}$$

$$\gamma_B = j\frac{60}{\gamma} F_B . f_B(\varphi)\, \mathbf{I'}_B\, \varepsilon^{-j\frac{2\pi}{\gamma} r_B}$$

or

$$\gamma_A = j\frac{60}{r} F_A . f_A(\varphi)\, \mathbf{I'}_A\, \varepsilon^{-j\frac{2\pi}{\lambda} r}\, \varepsilon^{-j\frac{\pi d}{\lambda} \cos}$$

$$\gamma_B = j\frac{60}{r} F_B f_B(\varphi)\, \mathbf{I'}_B\, \varepsilon^{-j\frac{2\pi}{\lambda} r}\, \varepsilon^{+j\frac{\pi d}{\lambda} \cos a}$$

Thus the combined field is

$$\gamma = j\frac{60}{r} \varepsilon^{-j\frac{2\pi}{\lambda} r} \left[F_A . f_A(\varphi)\, \mathbf{I'}_A\, \varepsilon^{-j\frac{\pi d}{\lambda} \cos a} + F_B . f_B(\varphi)\, \mathbf{I'}_B\, \varepsilon^{+j\frac{\pi d}{\lambda} \cos a} \right] \dots \quad \dots \quad \dots \quad (15)$$

The above process may obviously be applied to an array consisting of any number of elements.

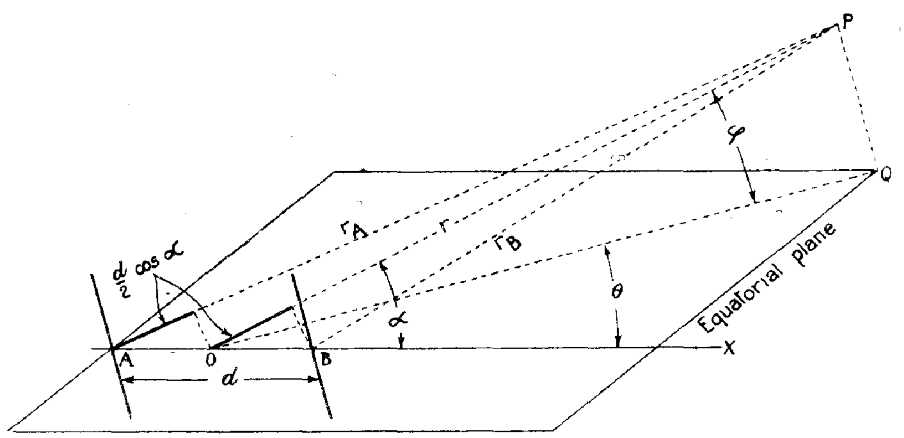


FIG. 15, CHAP. XV.—Notation, general case of parallel radiators.

28. Let us now take the specific case where the aerials are identical. Then $F_A . f_A(\varphi) = F_B . f_B(\varphi) = F . f(\varphi)$.

For brevity we may write $K = j\frac{60}{r} F.\, f(\varphi)\, \varepsilon^{-j\frac{2\pi}{\lambda} r}$, and proceed to allow for the difference in magnitude and phase of the currents. Let the amplitudes be $\mathcal{I}_A$, $\mathcal{I}_B$, and $\mathcal{I}_B = M\, \mathcal{I}_A$. The current in the aerial B leads on the current in the aerial A by an angle β, or $\mathbf{I'}_B = M \underline{/ \beta}\, \mathbf{I'}_A$.

The total field is therefore

$$\begin{aligned}\gamma &= K\left[\mathbf{I'}_A\,\varepsilon^{-j\frac{\pi d}{\lambda}\cos\alpha} + M\,\underline{/\beta}\,\mathbf{I'}_A\,\varepsilon^{+j\frac{\pi d}{\lambda}\cos\alpha}\right] \\ &= K\,\mathbf{I'}_A\left[\varepsilon^{-\frac{\pi d}{\lambda}\cos\alpha} + M\,\varepsilon^{j(\beta+\frac{\pi d}{\lambda}\cos\alpha)}\right] \\ &= K\,\mathbf{I'}_A\,\varepsilon^{-j\frac{\pi d}{\lambda}\cos\alpha}\left[1 + M\,\varepsilon^{j(\beta+\frac{2\pi d}{\lambda}\cos\alpha)}\right]. \qquad \ldots\ \ldots\ (16)\end{aligned}$$

Now $K\,\mathbf{I'}_A\,\varepsilon^{-j\frac{\pi d}{\lambda}\cos\alpha}$ is the field due to the aerial A alone and may be denoted by γ_A. Then

$$\begin{aligned}\gamma &= \gamma_A\left[1 + M\,\varepsilon^{j(\beta+\frac{2\pi d}{\lambda}\cos\alpha)}\right] \\ &= \gamma_A\left[1 + M\,\underline{/\beta + \frac{2\pi d}{\lambda}\cos\alpha}\right].\end{aligned}$$

It now only remains to find the scalar value of the quantity enclosed in brackets. To do this let $\beta + \frac{2\pi d}{\lambda}\cos\alpha = \psi$.

$$M\,\underline{/\psi} = M\cos\psi + j\,M\sin\psi.$$

The required scalar is that of $1 + M\,\underline{/\psi}$ or $1 + M\cos\psi + j\,M\sin\psi$, and from Chapter V this is known to be $\sqrt{(1 + M\cos\psi)^2 + M^2\sin^2\psi}$. This easily reduces to $\sqrt{1 + 2M\cos\psi + M^2}$ and therefore

$$\gamma = \gamma_A\sqrt{1 + 2\,M\cos\psi + M^2}. \qquad \ldots\ \ldots\ \ldots(17)$$

The R.M.S. field will be

$$\Gamma = \frac{60}{r}\,F.f(\varphi)\,I_A\sqrt{1 + 2M\cos\psi + M^2}. \qquad \ldots\ \ldots\ \ldots\ (18)$$

29. Before proceeding further, let us examine the angle α, which is more conveniently expressed in terms of the angles θ, φ. An examination of fig. 15 shows that the projection O Q of O P upon the equatorial plane is O P $\cos\varphi$, and that the projection of O Q upon the datum line O X is O Q $\cos\theta$. Since O P $= r$, O Q $= r\cos\varphi$ and the projection of O P upon O X is $r\cos\varphi\cos\theta$. The direct projection of O P upon O X is obviously O P $\cos\alpha$ or $r\cos\alpha$, i.e.

$$r\cos\alpha = r\cos\varphi\cos\theta$$

$$\cos\alpha = \cos\varphi\cos\theta.$$

30. We are now in a position to discuss the polar diagram. With unequal currents in the aerials, the only satisfactory basis of comparison with a single aerial is for equal power. From paragraph 14 we know that the powers radiated by the aerials A and B will be $I_A^2 R_A$ and $I_B^2 R_B$ respectively, where R_A and R_B are the radiation resistances. With identical aerials $R_A = R_B$ and the total radiated power is

$$P_T = I_A^2 R_A + (M\,I_A)^2 R_A. \qquad \ldots\ \ldots\ \ldots(19)$$

For a given power, then, the current I_A must be equal to $\sqrt{\frac{P_T}{R_A(1 + M^2)}}$.

Inserting this value for the current, we have, for the R.M.S. field

$$\Gamma = \frac{60}{r} F.f(\varphi)\sqrt{\frac{P_T}{R_A(1+M^2)}}\sqrt{1+M^2+2M\cos\left(\beta+\frac{2\pi d}{\lambda}\cos\varphi\cos\theta\right)} \qquad \text{(20)}$$

The polar diagram of a single aerial of the same kind, situated at the origin O, is given by the expression

$$\Gamma = \frac{60}{r} F.f(\varphi)\sqrt{\frac{P_T}{R_A}} \qquad \text{(21)}$$

and is a circle about the origin. The ratio of the field produced by the two aerials, to that produced by a single aerial radiating an equal power, is

$$G(\varphi, \theta) = \sqrt{\frac{1+M^2+2M\cos\left(\beta+\frac{2\pi d}{\lambda}\cos\varphi\cos\theta\right)}{1+M^2}}.$$

31. From this expression it is possible to calculate diagrams similar to those of fig. 8, for any value of M/β and at any angle φ with respect to the equatorial plane. It is obviously impossible to portray all the possibilities here; equatorial plane diagrams corresponding with fig. 8 are derived by putting $M = 1$, $\varphi = 0$, and letting β take any required value. With these substitutions

$$G(\theta) = \frac{\sqrt{2+2\cos\left(\beta+\frac{2\pi d}{\lambda}\cos\theta\right)}}{\sqrt{2}}$$

$$= \sqrt{1+\cos\psi}.$$

By trigonometry $1 + \cos\psi = 2\cos^2\frac{\psi}{2}$ so that

$$G(\theta) = \sqrt{2}\cos\frac{\psi}{2}$$

$$= \sqrt{2}\cos\left(\frac{\beta}{2}+\frac{\pi d}{\lambda}\cos\theta\right).$$

This expression is the same as that developed by a different method in paragraph 17 except that the factor $\sqrt{2}$ appears instead of 2. This is because we have obtained the Grating Factor for equal power in single aerial and array respectively, whereas in paragraph 17 the power in the array was four times that in the single aerial.

Co-linear dipoles

32. Instead of being placed parallel to each other, single wire radiators, particularly $\frac{\lambda}{2}$ dipoles, are sometimes placed end to end as shown in fig. 16, and are then said to be co-linear. The polar diagram of a simple array consisting of two co-linear dipoles can be found as follows. In fig. 16 let all measurements be made from an origin O lying between the two dipoles and on their common axis, and let their current loops be separated by a distance d. For simplicity let

the loop current in each dipole be $\mathbf{I}\,\varepsilon^{j\omega t}$. Then at a point P having the co-ordinates r, φ, the field due to the aerial A will be

$$\gamma_A = \frac{60}{r} f(\varphi)\, \mathbf{I}\, \varepsilon^{\left(j\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r_A\right)} \quad \ldots \quad (22a)$$

$$\gamma_B = \frac{60}{r} f(\varphi)\, \mathbf{I}\, \varepsilon^{\left(j\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r_B\right)} \quad \ldots \quad (22b)$$

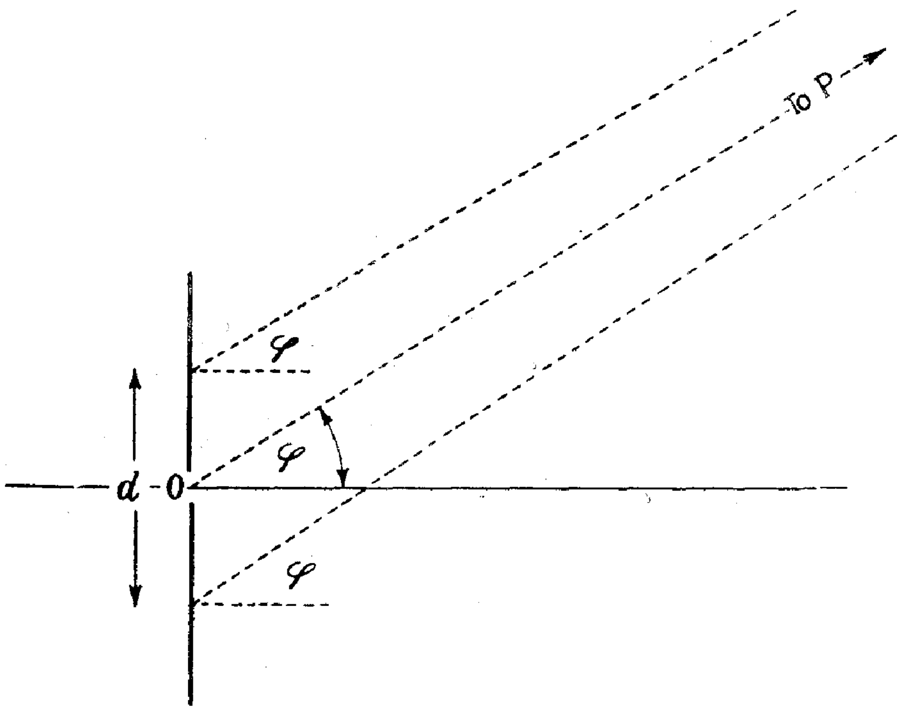
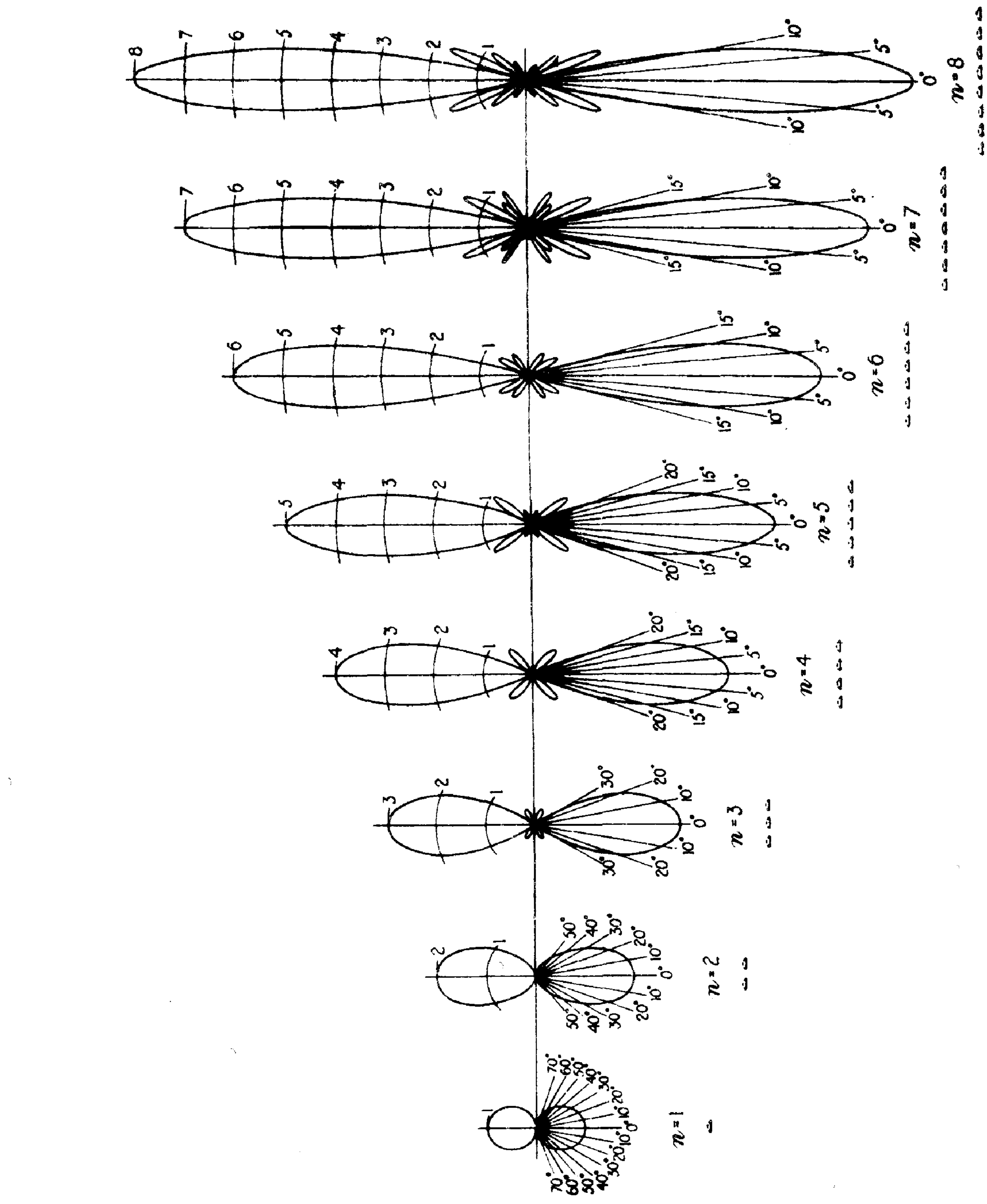


FIG. 16, CHAP. XV.—Co-linear dipoles.

These expressions are of the same form as in the case of parallel dipoles, and the total R.M.S. field strength is easily found to be

$$\begin{aligned} \Gamma_t &= \frac{60}{r} f(\varphi)\, I \left\{ \varepsilon^{j\frac{\pi d}{\lambda}\cos\varphi} + \varepsilon^{-j\frac{\pi d}{\lambda}\cos\varphi} \right\} \\ &= \frac{60}{r} f(\varphi)\, I \times 2\cos\left(\frac{\pi d}{\lambda}\cos\varphi\right) \\ &= \frac{60}{r} I\, f(\varphi)\, G(\varphi), \quad \ldots \quad (23) \end{aligned}$$

hence the field strength is the product of $\frac{60}{r} I$ and two factors which are obtainable from figs. 6 and 8 respectively. With respect to $G(\varphi)$, given by the latter, only columns 5 and above are applicable for obvious reasons, and due regard must be paid to the direction from which φ is measured. From physical considerations it is obvious that the dipoles radiate most strongly in a direction perpendicular to their common axis and this axis is a horizontal line in the diagrams of fig. 8. The polar diagrams of arrays consisting of combinations of more than two co-linear dipoles



CO-LINEAR DIPOLES IN FREE SPACE

FIG. 17
CHAP. XV.

CO-LINEAR DIPOLES WITH REFLECTORS

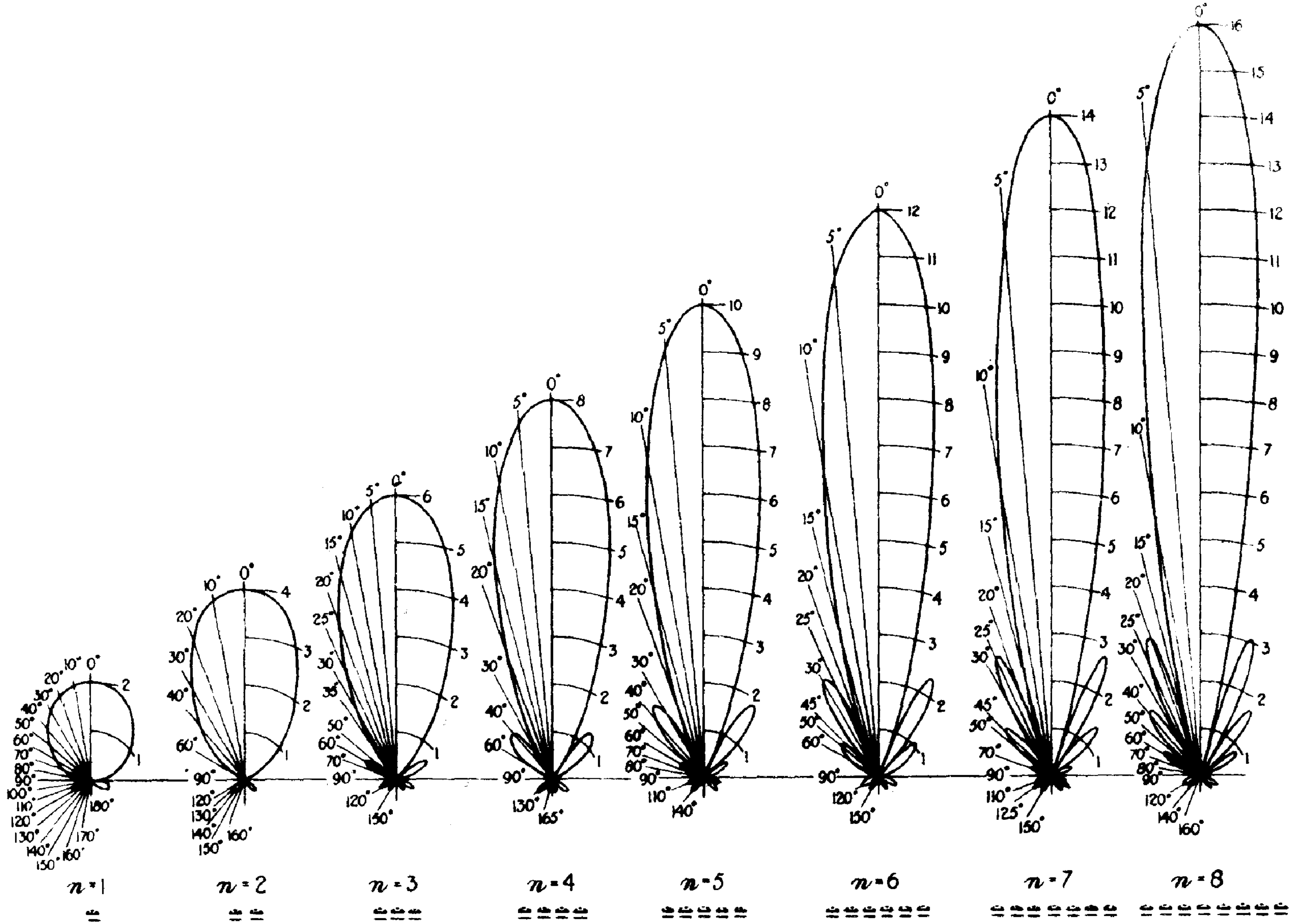


FIG. 18
CHAP. XV

can be found by the methods explained with reference to parallel dipoles. Fig. 17 shows the product $f(\varphi)\ G(\varphi)$ for all numbers up to 8 co-linear dipoles. This figure has an important bearing on the radiation from arrays of horizontal dipoles.

Reflector aerial

33. Hitherto, in considering the radiation from two parallel aerials, we have ignored the effect of one aerial upon the other. It is obvious that when both are supplied with energy each will receive energy from the other. Of the received energy, a portion is converted into heat and the remainder radiated into space. Now let us consider two parallel dipoles, A and B, $\frac{\lambda}{4}$ apart in free space, and consider their radiation in the equatorial plane, when A is supplied with a current $i = \vartheta_A \cos \omega t$, and B is unenergized. Then the field due to the aerial A will induce an E.M.F. in B, and consequently an oscillatory current of the same frequency. This " induction " is really caused by both the induction and the radiation fields of A, but for the present we shall neglect the former. The magnitude ϑ_B of the induced current in B, under these conditions, will be equal to that of the current in A, but ϑ_B will lead on ϑ_A by 90°. The aerials A and B therefore radiate an equal amount of energy per second, and the field at any point can be calculated as in previous paragraphs, putting the angle β equal to 90° ; the polar diagram in the equatorial plane is given by fig. 8, C 3. The effect in the plane containing the aerials is somewhat similar, the only modification being due to the Current Distribution Factor. For comparison with fig. 17, fig. 18 gives the polar diagram in this plane of co-linear dipoles with reflectors, calculated on the above assumptions.

34. The exact manner in which a reflector aerial will function depends upon three factors ; first, its distance from the energized aerial ; second, the ratio of its induced current to that in the energized aerial ; third, whether it is reactive or non-reactive, i.e. tuned or untuned to the frequency of the energized aerial. The second factor is obviously not independent of the third. Reference to fig. 8 shows that if the currents in the energized and reflector aerials are equal, the polar diagram is more or less uni-directional whenever the phase difference between the two currents is greater than 0° and less than 180°, provided that the spacing is less than $\frac{\lambda}{2}$. Particular attention is directed to the diagrams B 2, B 3, B 4, C 2, C 3, C 4, D 2, D 3, D 4, of fig. 8, which show the theoretical possibilities which may arise. The effect of the induction field cannot be entirely ignored, and will receive further consideration.

Effect of the ground

35. In practice, the field produced at a given point by a transmitting aerial is always affected by the presence of the earth's surface, but a complete treatment allowing for the curvature of the earth, and its finite conductivity and permittivity, is extremely complex. For many purposes, however, the earth may be considered as a flat, perfectly conductive surface, and this simplification is of great help in visualizing the nature of the effect. A perfectly conducting earth would act as a perfect reflector of electro-magnetic waves, and a flat earth as a plane reflector. If then we consider the earth's surface in the immediate vicinity of the aerial to be both perfectly conductive and plane, we may treat certain problems by a method analogous to that used in geometrical optics, i.e. by supposing the reflector to give a virtual image of the actual radiator. The virtual image is defined as a point from which rays appear to diverge after reflection, although no rays actually pass through the point. This conception is illustrated in fig. 19a which shows a hertzian doublet A B situated above a perfectly reflecting earth. At the instant depicted, the current is flowing from A to B, and consequently a positive charge is accumulating at B.

36. Now take a point B′ situated at a distance OB′ = OB on the other side of the reflecting surface, so that B O B′ is straight and perpendicular to the surface. B′ is then the geometrical virtual image of B. Similarly, we may locate the geometrical virtual image A′ of the point A. Now considering B to be a small sphere, it must possess capacitance with respect to the perfectly conducting surface and if it carries a positive charge, its electrostatic field would be distributed somewhat as shown in fig. 19b. The field due to a similar charge of opposite polarity, situated at the point B′, is also shown, and it is seen, in conjunction with fig. 19a, that a positive charge

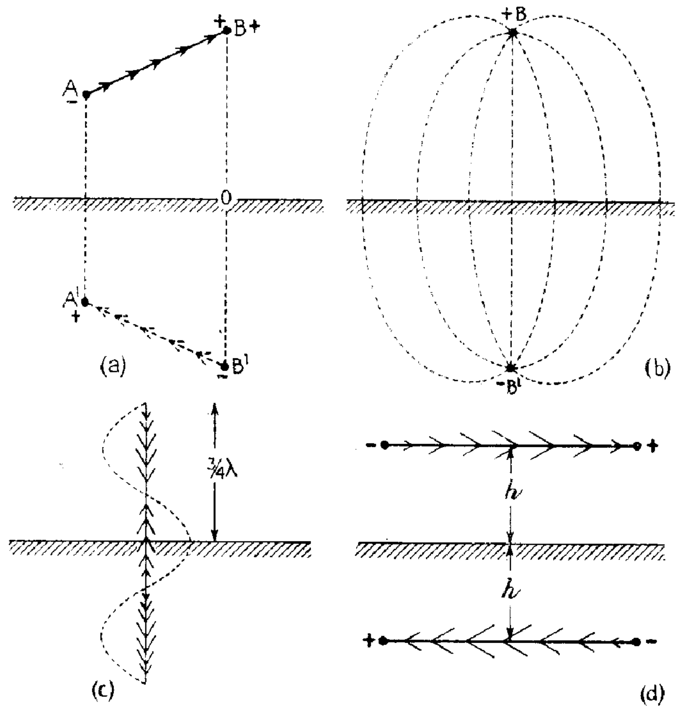


FIG. 19, CHAP. XV.—Illustrations of image theorem.

at B, above a perfect reflector, will have the same field as would be set up if the reflecting surface were removed and an equal and opposite charge placed at the point B′. Similar considerations apply to the points A and A′ and therefore, when a current is flowing from A to B, an equal current must be considered to flow from B′ to A′. The field strength at any point above the reflector is found by algebraic addition of the fields due to the aerial itself and to its virtual image, with due regard to the direction of current in the latter, as determined by the above considerations. Fig. 19c shows the distribution of current in a vertical $\frac{3}{4}\lambda$ aerial, and the current in a horizontal dipole is shown in fig. 19d together with those of the images. In certain circumstances the image theorem lends itself to comparatively simple application, and it will now be applied to find the radiation characteristics of the horizontal dipole.

Horizontal dipole

37. For purposes of notation, the dipole is shown in figs. 20a and 20b. We shall consider the vertical polar diagram in the plane of the latter figure. By the preceding paragraph the image A′ B′ of the dipole A B is as shown, and therefore we require to find the polar diagram of two parallel dipoles, $d = 2h$ apart, carrying currents in phase opposition. If γ_A is the field due to the aerial at the point P, and γ_R the field due to the image, the total field is $\gamma = \gamma_A + \gamma_R$ where

$$\gamma_A = \frac{60}{r} \mathbf{I} \, \varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}(r - h \sin\varphi)\right]} \quad \ldots \quad \ldots \quad \ldots \quad (24a)$$

$$\gamma_R = -\frac{60}{r} \mathbf{I} \, \varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}(r + h \sin\varphi)\right]} \quad \ldots \quad \ldots \quad \ldots \quad (24b)$$

$$\gamma = \frac{60}{r} \mathbf{I} \, \varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)} \left(\varepsilon^{j\frac{2\pi}{\lambda} h \sin\varphi} - \varepsilon^{-j\frac{2\pi}{\lambda} h \sin\varphi}\right).$$

$$\text{Now} \left(\varepsilon^{j\frac{2\pi}{\lambda} h \sin\varphi} - \varepsilon^{-j\frac{2\pi}{\lambda} h \sin\varphi}\right) = 2j \sin\left(\frac{2\pi}{\lambda} h \sin\varphi\right)$$

$$\therefore \gamma = \frac{60}{r} \mathbf{I} \, \varepsilon^{j\left(\omega t + \pi - \frac{2\pi}{\lambda} r\right)} \times 2 \sin\left(\frac{2\pi}{\lambda} h \sin\varphi\right). \quad \ldots \quad \ldots \quad (24c)$$

The R.M.S. field being

$$F = \frac{60}{r} I \times 2 \sin\left(\frac{2\pi}{\lambda} h \sin\varphi\right). \qquad (25)$$

The factor $2 \sin\left(\frac{2\pi}{\lambda} h \sin\varphi\right)$ may be called the Vertical Distribution Factor and denoted by $D(\varphi)$; it is obviously analogous to the Grating Factor previously used in the case of aerials in free space, the change from " cosine " to " sine " being due merely to the choice of a different datum. The Vertical Distribution Factor is in fact given by the series of polar diagrams in fig. 8,

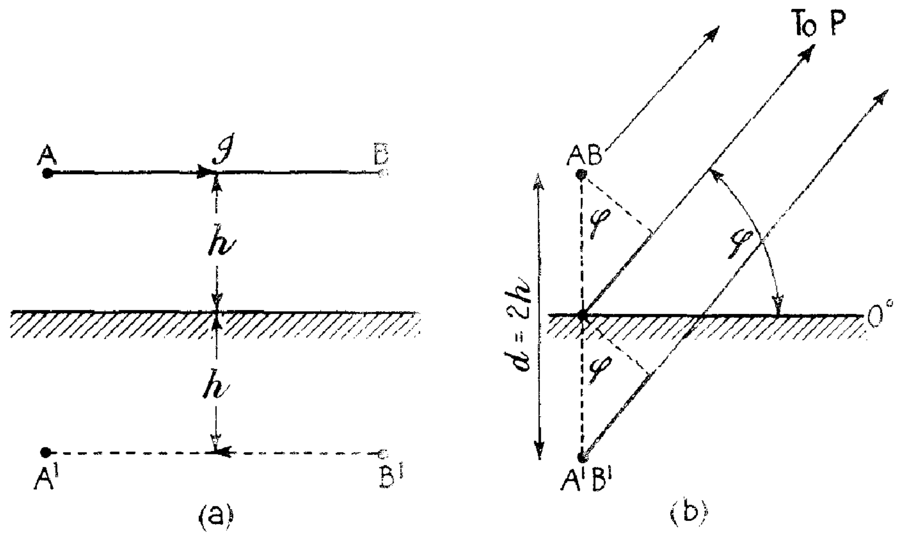


FIG. 20, CHAP. XV—Notation, horizontal dipole.

line E, except that they must be turned through 90°. In the first few diagrams, one half of the limiting circle has been shaded to represent the ground, so serving as a reminder to perform the necessary rotation. The relation $d = 2h$ must not be forgotten, e.g. the diagram E 18, $d = 4\lambda$, gives $D(\varphi)$ for a height of 2λ.

Effect of height of dipole

38. Several additional vertical polar diagrams are given in fig. 21 (Sheets 1 and 2), and from these it is apparent that no matter what the height may be, there is no radiation along the surface of the earth, even if the latter is perfectly conductive. It follows that with a horizontal aerial no communication can be performed by means of a true ground ray. If h is less than $0 \cdot 3\lambda$ the greater part of the energy is radiated vertically; if h is increased to about $0 \cdot 4\lambda$, the diagram shows signs of breaking into two lobes. This kind of diagram is very suitable for a marker beacon in a blind approach system, but for very little else. As h is increased to $0 \cdot 5\lambda$ the two lobes become fully developed, the vertical radiation falling to zero and the maximum field being developed at an angle of 30° to the horizontal. A further increase of height leads to the development of additional lobes, and whenever h is an integral multiple of $\frac{\lambda}{2}$, e.g. λ, $\frac{3}{2}\lambda$, 2λ, etc., the number of lobes is equal to $\frac{4h}{\lambda}$, thus a height of $\frac{5}{4}\lambda$ gives five lobes, and so on. It follows that one lobe is vertical whenever h is an odd multiple of $\frac{\lambda}{4}$ and that no vertical radiation occurs when h is an even multiple of $\frac{\lambda}{4}$. The angles at which successive maxima occur can be determined only

approximately from the polar diagrams, but are given to a higher degree of accuracy in fig. 22. The practical use of figs. 21 and 22 is to determine the most effective type of aerial for any particular service. As already stated, heights less than about $0 \cdot 3\lambda$ are useless except for marker beacons and the like. If $\frac{h}{\lambda}$ is increased to say $0 \cdot 45$ or $0 \cdot 5$, the aerial may be suitable for short-distance transmission, e.g. up to about 500 miles, for a projection angle of 45° will give a signal at about that distance by reflection from the F region, assuming the height of the latter to be about 200 miles. For distances of 1,000 miles or more, a projection angle of about 12° to 15° is required; from fig. 21 it is seen that to get maximum radiation at this angle a height of from λ to $\frac{5}{4}\lambda$ is necessary. Thus if $\lambda = 80$ metres, the radiator must be raised to a height of some 250 to 300 feet.

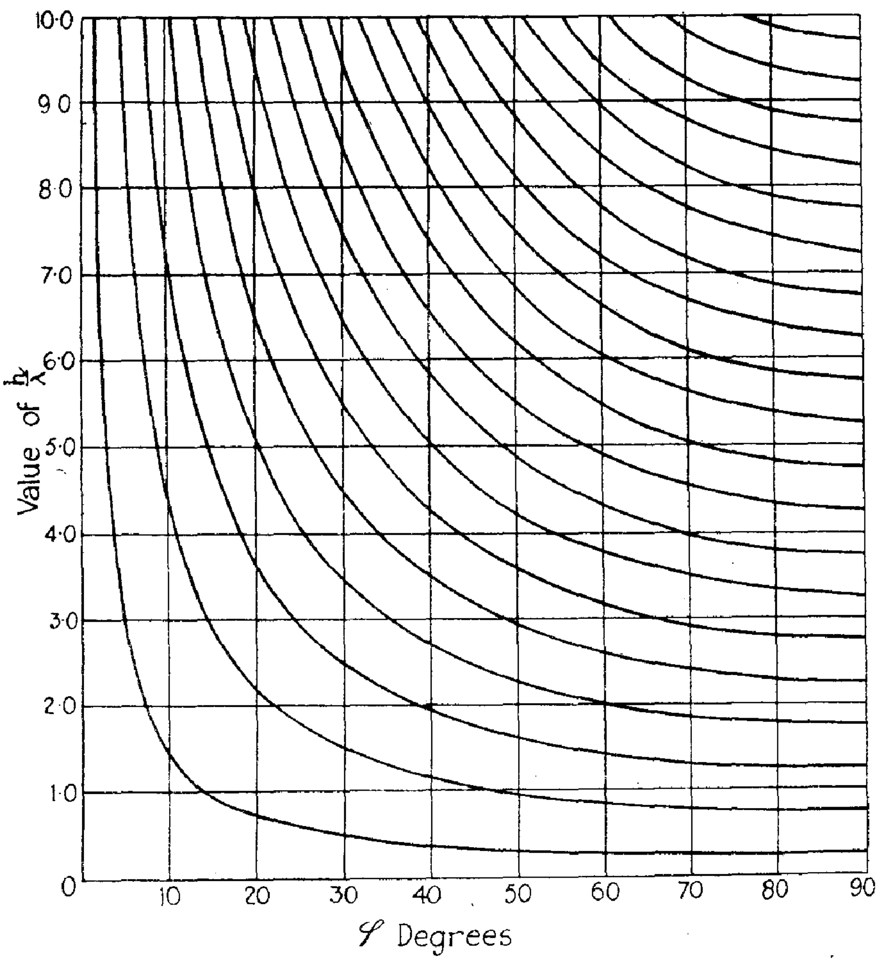
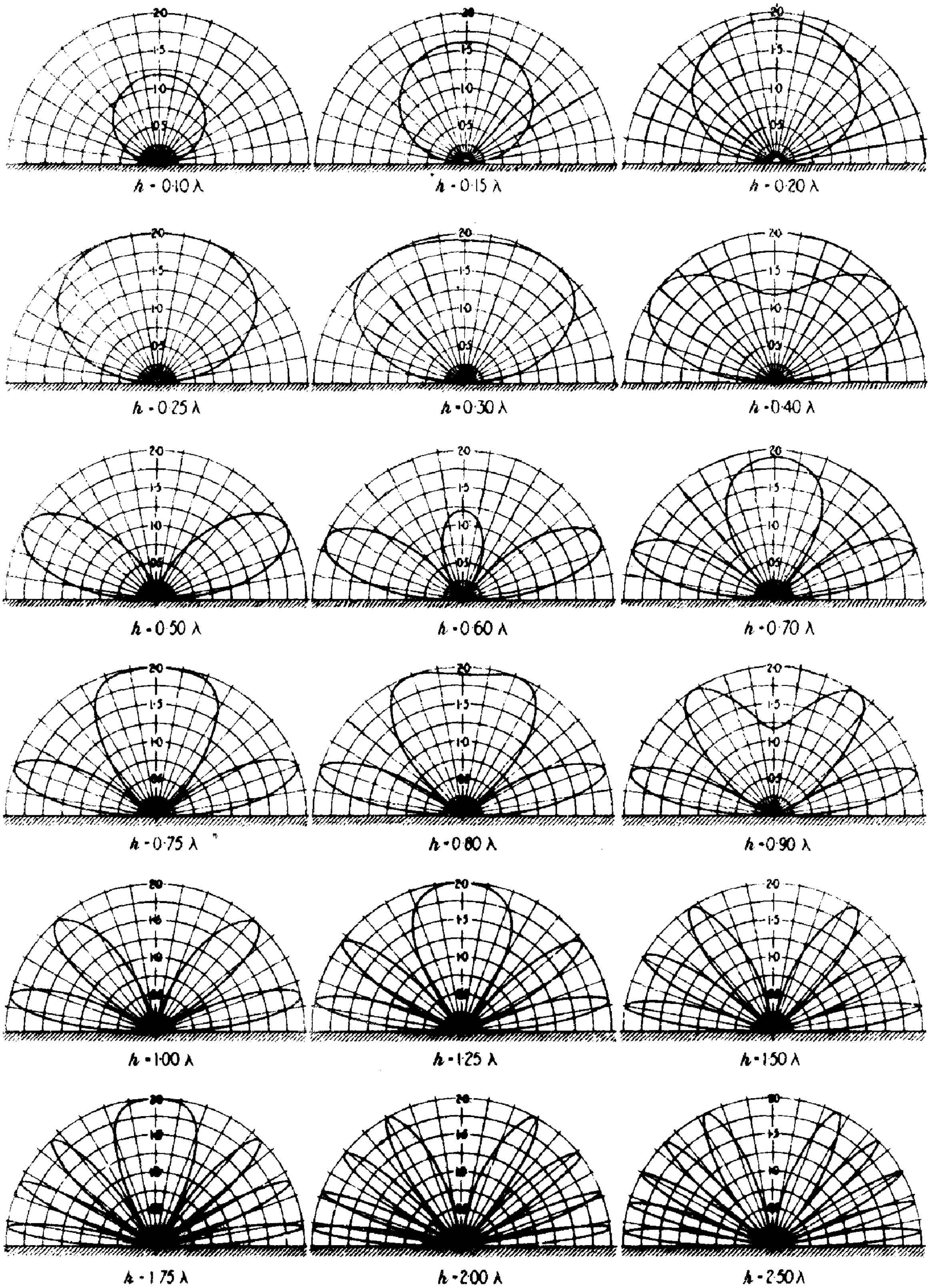


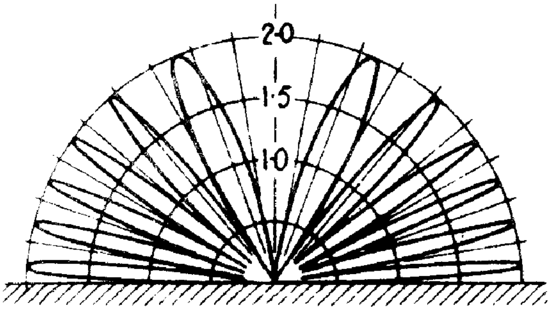
FIG. 22, CHAP. XV.—Horizontal dipole: angles at which field maxima occur.

39. The projection angle of 12° to 15° is suggested for transmission *via* the ionosphere, but where direct-ray communication is necessary, as in ground to air service, a much greater height would be desirable. An aircraft at 10,000 feet and a range of 50 miles is only about 2° above the earth, and if the maximum of the first lobe is to be brought down to 2°, the height of the aerial must be about 7λ. This is of course quite impracticable with a wavelength of 80 metres, and is not very easily or cheaply achieved with so short a wavelength as 3 metres. Nevertheless, it must be regarded as axiomatic that for efficient ground to air communication on high and very

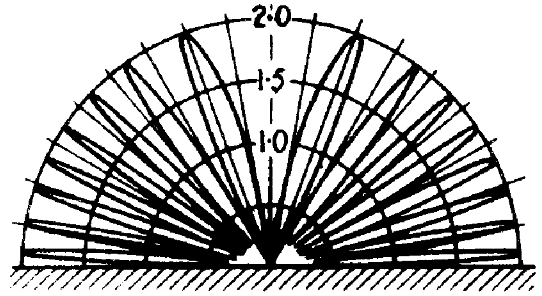


VERTICAL POLAR DIAGRAMS OF HORIZONTAL DIPOLES (SHEET I)

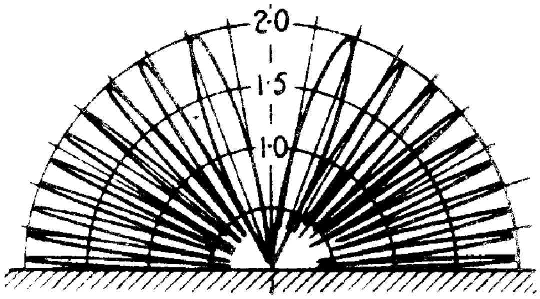
FIG. 21.
CHAP. XV.



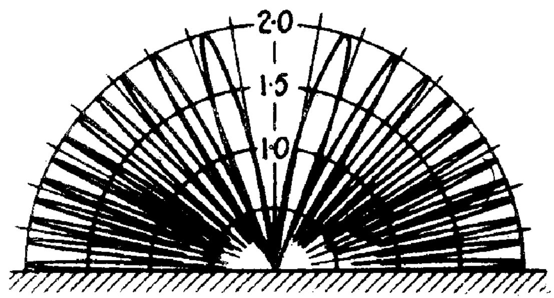
$h = 3{\cdot}00\,\lambda$



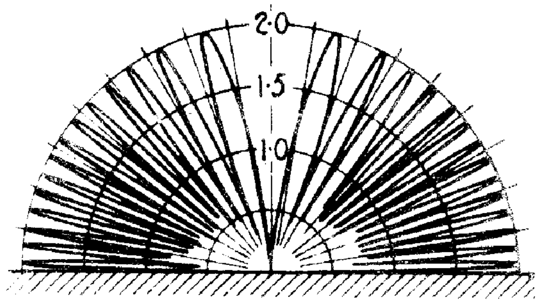
$h = 4{\cdot}00\,\lambda$



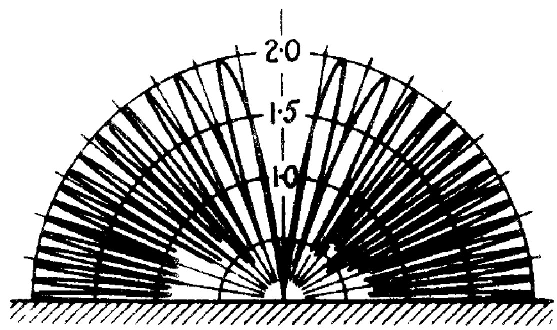
$h = 5{\cdot}00\,\lambda$



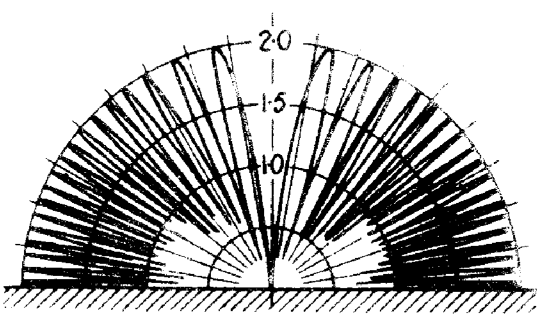
$h = 6{\cdot}00\,\lambda$



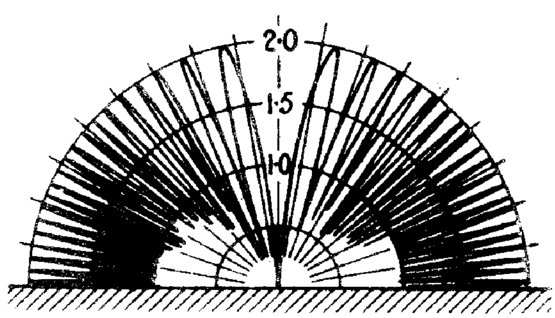
$h = 7{\cdot}00\,\lambda$



$h = 8{\cdot}00\,\lambda$



$h = 9{\cdot}00\,\lambda$



$h = 10{\cdot}00\,\lambda$

VERTICAL POLAR DIAGRAMS OF HORIZONTAL DIPOLES
(SHEET 2)

FIG. 21
CHAP. XV.

high frequencies, the highest possible masts must be used. Even at comparatively short ranges, the energy received at ground level will generally arrive by reflection from the ionosphere, and the signal will generally be subject to severe fading. When dealing with low angle radiation, i.e. up to a few degrees, the height h, which gives the first maximum at an angle φ_1 radians, is found thus,

$$h\,\varphi_1 = \frac{\lambda}{4}$$

$$h = \frac{\lambda}{4\varphi_1}.$$

If φ_1 is in degrees,

$$h = \frac{14{\cdot}3\lambda}{\varphi_1}.$$

Vertical aerials

40. The vertical polar diagrams of vertical aerials situated on or near the surface of a perfectly conducting earth are obtained by summing the effects of all the elementary hertzian doublets which comprise the aerial, together with those constituting its image. The elevation of the centre point of the aerial above the ground level is of considerable importance; examples of this, in the case of a $\frac{\lambda}{2}$ dipole, are given qualitatively in Chapter VII. A single example will be given

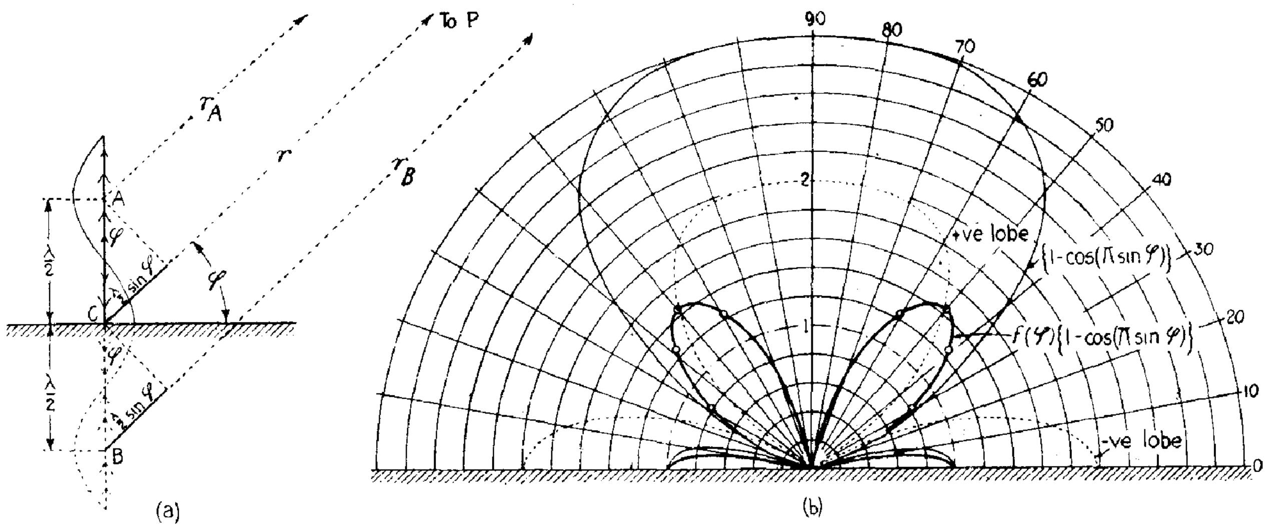


FIG. 23, CHAP. XV.—Calculation of vertical polar diagram of $\frac{3}{4}\lambda$ aerial on perfect earth.

to illustrate the method of calculation. Referring to fig. 23a which shows a $\frac{3}{4}\lambda$ aerial with its lower end at ground level, together with the assumed current distribution in the aerial and its image, it is seen that the length of wire conveniently divides into three $\frac{\lambda}{2}$ sections, A, B, C, each of which may be considered to give rise to a field at the point P. The notation is given in the diagram. The section C gives rise to a field

$$\gamma_C = \frac{60}{r} f(\varphi)\, \mathrm{I}\, \varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)}. \qquad \ldots \quad \ldots \quad \ldots \quad (26)$$

Note that $f(\varphi)$ is the appropriate Current Distribution Factor and geometrically is identical with fig. 6, although the angle (φ) is in the present example measured from the ground. The distance of the point P from the current loop of section A is $\left(r - \frac{\lambda}{2} \sin \varphi\right)$, and from the current loop of section B $\left(r + \frac{\lambda}{2} \sin \varphi\right)$. The currents in these sections are in anti-phase to that in section C. The fields due to these sections are therefore

$$\gamma_A = -\frac{60}{r} f(\varphi)\, \mathrm{I}\, \varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r_A\right)}$$

$$= -\frac{60}{r} f(\varphi)\, \mathrm{I}\, \varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r - \frac{\lambda}{2}\sin\varphi\right)\right]} \quad \ldots \quad (27a)$$

$$\gamma_B = -\frac{60}{r} f(\varphi)\, \mathrm{I}\, \varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}\left(r + \frac{\lambda}{2}\sin\varphi\right)\right]} \quad \ldots \quad (27b)$$

and the total field is

$$\gamma = \gamma_A + \gamma_B + \gamma_C = \frac{60}{r} f(\varphi)\, \mathrm{I}\, \varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)} \left[1 - \left(\varepsilon^{+j\pi \sin\varphi} + \varepsilon^{-j\pi\sin\varphi}\right)\right] \quad \ldots \quad (27c)$$

the R.M.S. field being

$$\Gamma = \frac{60}{r} f(\varphi)\, I \left[1 - \left(\varepsilon^{+j\pi\sin\varphi} + \varepsilon^{-j\pi\sin\varphi}\right)\right]$$

$$= \frac{60}{r} f(\varphi)\, I \left[1 - 2\cos(\pi \sin\varphi)\right] \quad \ldots \quad (27d)$$

Vertical polar diagrams

41. The above expression is easily plotted with the aid of fig. 8. Ignoring the term $\frac{60}{r} I$, the shape of the polar diagram can be found thus. Setting aside the factor $f(\varphi)$ for a time, we have to plot $[1 - 2\cos(\pi\sin\varphi)]$. Now $[2\cos(\pi\sin\varphi)]$ is given by fig. 8, A 9, turned through 90°, and is shown in dotted line in fig. 23b. Before proceeding further, it is necessary to note that in any diagram of fig. 8 which possesses more than two lobes, the latter are alternatively of positive and negative sign, the lobe extending in the direction 0° being positive. In the dotted line diagram of fig. 23b the signs have been reversed, so that what is shown is $-[2\cos(\pi\sin\varphi)]$. To this, a circle of radius + 1 unit (shown in chain line) must be added, giving the result shown in thin solid line. The latter diagram represents $[1 - 2\cos(\pi\sin\varphi)]$, and to obtain the polar diagram, it must be multiplied by the appropriate Current Distribution Factor $f(\varphi)$ (fig. 6) giving the final result shown in heavy line. Proceeding in the above manner the vertical diagrams for vertical aerials of various heights up to 2λ have been calculated, and are shown in fig. 24. It will be observed that all lobes have a common tangent, and further that if the number of quarter wavelengths in the actual aerial is odd, there is always some radiation along the ground, bearing in mind that the latter is assumed to be a perfect conductor. The effect of the finite conductivity will be dealt with later.

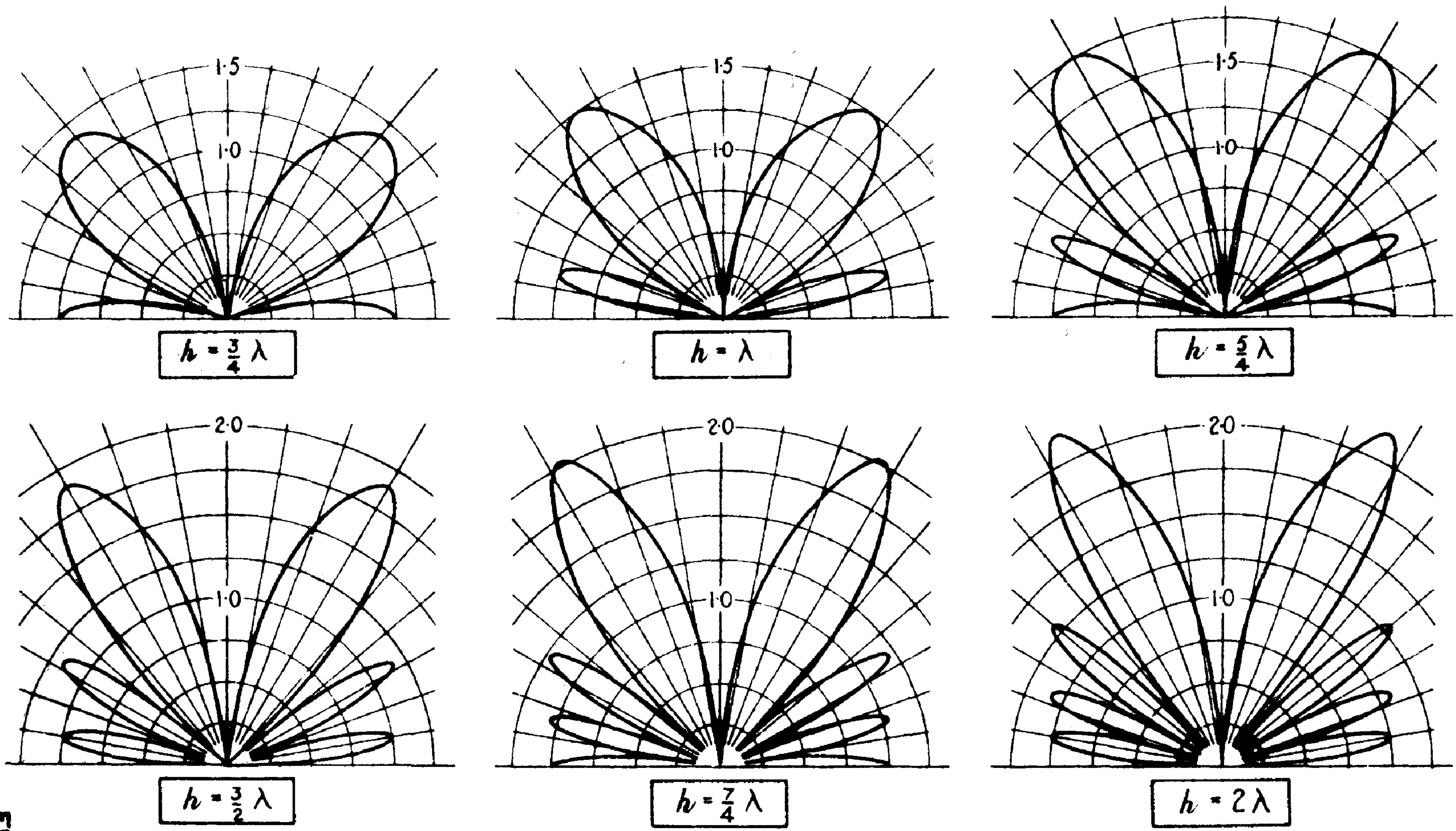


FIG. 24
CHAP. XV.

VERTICAL POLAR DIAGRAMS; VERTICAL AERIALS ON PERFECTLY CONDUCTING EARTH

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Mutual impedance between adjacent radiators

42. Although the elementary theory indicates that certain results will be obtained when given conductors are of particular lengths, or spaced at a particular distance from earth or another conductor, it is found in practice that optimum results are often obtained with slightly different dimensions. In many cases, this is due to the assumption that the mutual impedance between conductors, or between conductor and earth, is zero, whereas it may in fact be of the same order as the impedance of the conductor itself. The magnitude of the mutual impedance between two radiators in free space is defined as the ratio of the induced voltage at the current loop of a second radiator, to the loop current of the first radiator; when dealing with earthed aerials it is convenient to refer the mutual impedance to the base current. The sign of the mutual impedance, when defined in this way, is negative. Thus, if $\mathbf{z}_{AB}$ and $\mathbf{z}_{BA}$ denote the mutual impedances of a radiator A with respect to a radiator B, and *vice versa*, $\mathbf{V}_{BA}$ denotes the induced voltage in B due to the current in A. Let $\mathbf{I}_A$ be the loop current in A, then

$$\mathbf{z}_{AB} = \mathbf{z}_{BA} = -\frac{\mathbf{V}_{BA}}{\mathbf{I}_A}.$$

In the following discussion the notation will be as under :—

$\mathbf{I}_A$ = vector current in aerial A ; R.M.S. value I_A

$\mathbf{I}_B$ = vector current in aerial B ; R.M.S. value I_B

$\mathbf{V}_A$ = vector voltage at terminals of aerial A ; R.M.S. value V_A

$\mathbf{V}_B$ = vector voltage at terminals of aerial B ; R.M.S. value V_B

$Z_A = \sqrt{R_A^2 + X_A^2}$ = magnitude of self-impedance of aerial A

$\theta_A = tan^{-1}\frac{X_A}{R_A}$

$\mathbf{z}_A = R_A + jX_A = Z_A \underline{/\theta_A}$

$Z_B = \sqrt{R_B^2 + X_B^2}$ = magnitude of self-impedance of aerial B

$\theta_B = tan^{-1}\frac{X_B}{R_B}$

$\mathbf{z}_B = R_B + jX_B = Z_B \underline{/\theta_B}$

β = phase difference between $\mathbf{I}_B$ and $\mathbf{I}_A$

$M = \frac{I_B}{I_A}$, i.e. $\mathbf{I}_B = M\mathbf{I}_A \underline{/\beta}$

$Z_M = \sqrt{R_M^2 + X_M^2}$ = magnitude of mutual impedance between radiators A and B

$\theta_M = tan^{-1}\frac{X_M}{R_M}$

$\mathbf{z}_M = R_M + jX_M = Z_M \underline{/\theta_M}$

R_A' = sum of self and mutual resistance of aerial A

X_A' = sum of self and mutual reactance of aerial A

R_B' = sum of self and mutual resistance of aerial B

X_B' = sum of self and mutual reactance of aerial B

$\mathbf{z}_A' = R_A' + jX_A'$

$\mathbf{z}_B' = R_B' + jX_B'$.

43. The evaluation of the mutual impedance between two radiators is very tedious and there are very few data available. For identical radiators, the curves of figs. 25 and 26 may be used. These diagrams give the magnitude Z_M and phase angle θ_M for various values of $\frac{d}{\lambda}$. The first-named diagram is applicable to vertical aerials located directly above a perfectly conductive earth, the mutual impedance being referred to the current at the base of the aerial in all cases except for the $\frac{\lambda}{2}$ aerial which is referred to the loop current as usual. Fig. 26a gives Z_M and θ_M for parallel $\frac{\lambda}{2}$ dipoles in free space. It may be noted that the above curves give the self-impedance of the aerial also, i.e. $Z_A \underline{/\theta_A} = Z_M \underline{/\theta_M}$ when $\frac{d}{\lambda} = 0$. Fig. 26b may be used for co-linear dipoles. In this case the ordinate, Z_M or θ_M, is plotted against the separation of the adjacent ends. The impedance $Z_M \underline{/\theta_M}$ is easily resolved into its resistive and reactive components by methods previously explained, as in the following.

Example

Find the mutual impedance and reactance between two parallel $\frac{\lambda}{2}$ dipoles A, B, $\frac{\lambda}{2}$ apart in free space.

From fig. 26a, for $\frac{d}{\lambda} = 0{\cdot}5$

$$Z_M \underline{/\theta_M} = 33 \underline{/-117}.$$

$$\therefore R_M = 33 \cos 117 = -33 \cos 63 = -15 \text{ ohms}$$

$$X_M = -33 \sin 117 - 33 \sin 63 - 29{\cdot}4 \text{ ohms}.$$

From the above example it follows that if the members of such an array are energized by equal, syn-phased currents, the impedance of each is $\mathbf{z} + \mathbf{z}_M = 73{\cdot}3 + j\,42{\cdot}5 - (15 + j\,29{\cdot}4)$ or $58{\cdot}3 + j\,13{\cdot}1$ ohms. The reactance of each member will be annulled by suitable tuning arrangements, while the whole array will have a radiation resistance of 116·6 ohms.

44. Now suppose the two parallel aerials to be energized by voltages $\mathbf{V}_A, \mathbf{V}_B$, in such a manner that $\mathbf{I}_B = \mathbf{m}\mathbf{I}_A = M \underline{/\beta}\, \mathbf{I}_A$. By Kirchoff's laws,

$$\mathbf{V}_A = \mathbf{I}_A \mathbf{z}_A + \mathbf{I}_B \mathbf{z}_M \quad \ldots \quad (28a)$$

$$\mathbf{V}_B = \mathbf{I}_A \mathbf{z}_M + \mathbf{I}_B \mathbf{z}_B \quad \ldots \quad (28b)$$

where $\mathbf{z}_B$ may or may not be equal to $\mathbf{z}_A$. Substituting for $\mathbf{I}_B$ in equation (28a)

$$\mathbf{V}_A = \mathbf{I}_A \{R_A + jX_A + \mathbf{m}\,\mathbf{z}_M\}$$

$$= \mathbf{I}_A \{R_A + jX_A + MZ_M \underline{/\theta_M + \beta}\}$$

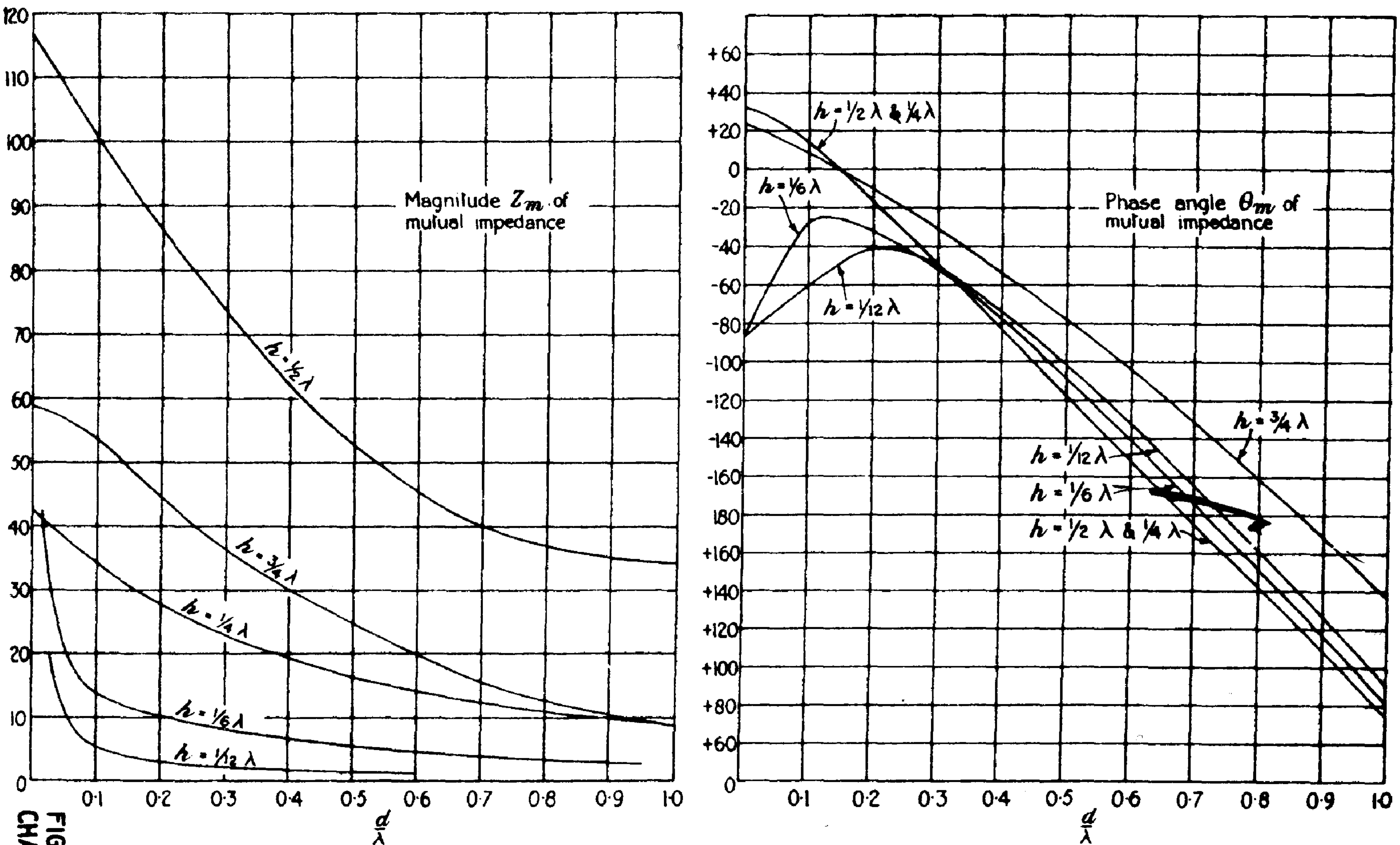
or

$$\mathbf{z}_A' = R_A + jX_A + MZ_M \cos(\theta_M + \beta) + jMZ_M \sin(\theta_M + \beta),$$

i.e.

$$R_A' = R_A + MZ_M \cos(\theta_M + \beta) \quad \ldots \quad (29a)$$

$$X_A' = X_A + MZ_M \sin(\theta_M + \beta). \quad \ldots \quad (29b)$$



MUTUAL IMPEDANCE $Z_m \angle \theta_m$ BETWEEN IDENTICAL VERTICAL AERIALS

FIG. 25
CHAP. XV

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Similarly

$$R_B' = R_B + \frac{Z_M}{M} \cos (\theta_M - \beta) \qquad \text{(29c)}$$

$$X_B' = X_B + \frac{Z_M}{M} \sin (\theta_M - \beta). \qquad \text{(29d)}$$

It is now seen that the radiation resistances of the two aerials are only equal when $M = 1$ and β is either 0° or 180°. It is of interest to note that in any other circumstances the reactances are also unequal, so that the tuning reactances of the two aerials will differ.

Effect upon power distribution

45. As an example of the effects of the value of **m**, let us consider two $\frac{\lambda}{4}$ aerials, A and B, $\frac{\lambda}{4}$ apart upon a perfect earth, and energized with currents I_A, I_B, where $I_B = 0{\cdot}8\ I_A\ \underline{/90^\circ}$. The self impedance of each aerial will be $36{\cdot}6 + 21{\cdot}5$ ohms. From fig. 25, $\mathbf{z}_M = 25\ \underline{/-36}$ ohms. Then

$$\begin{aligned} R_A' &= R_A + MZ_M \cos (90 - 36) \\ &= 36{\cdot}6 + 0{\cdot}8 \times 25 \cos 54 \\ &= 36{\cdot}6 + 0{\cdot}8 \times 25 \times 0{\cdot}588 \\ &= 36{\cdot}6 + 11{\cdot}76 \\ &= 48{\cdot}36 \text{ ohms} \\ R_B' &= R_B + \frac{Z_M}{M} \cos (-90 - 36) \\ &= 36{\cdot}6 + \frac{25}{0{\cdot}8} \cos (-126) \\ &= 36{\cdot}6 - \frac{25}{0{\cdot}8} \times 0{\cdot}588 \\ &= 36{\cdot}6 - 18{\cdot}4 \\ &= 17{\cdot}2 \text{ ohms.} \end{aligned}$$

Let us now find how these aerials would share a power of 500 watts.

$$\begin{aligned} I_A^2 R_A' + I_B^2 R_B' &= 500 \\ I_A^2 (48{\cdot}36 + {\cdot}8^2 \times 17{\cdot}2) &= 500 \\ I_A^2 \times 59{\cdot}36 &= 500 \\ I_A &= \sqrt{\frac{500}{59{\cdot}36}} \\ &= 2{\cdot}9 \text{ amperes} \\ I_B &= 0{\cdot}8 I_A = 2{\cdot}32 \text{ amperes} \\ P_A = I_A^2 R_A' &= 2{\cdot}9^2 \times 48{\cdot}36 \\ &= 405 \text{ watts} \\ P_B = I_B^2 R_B' &= 2{\cdot}32^2 \times 17{\cdot}2 \\ &= 93 \text{ watts} \\ P_T &= 498 \text{ watts,} \end{aligned}$$

the slight discrepancy being due to arithmetical approximations. In order to energize the aerials in the manner specified, the applied voltages $\mathbf{V}_A$, $\mathbf{V}_B$ must differ both in phase and magnitude. The vector ratio $\frac{\mathbf{V}_A}{\mathbf{V}_B}$ is easily found

$$\mathbf{V}_A = \mathbf{I}_A\,\mathbf{z}_A + \mathbf{m}\,\mathbf{I}_A\,\mathbf{z}_M$$

$$\mathbf{V}_B = \mathbf{I}_A\,\mathbf{z}_M + \mathbf{m}\,\mathbf{I}_A\,\mathbf{z}_B$$

$$\therefore \frac{\mathbf{V}_A}{\mathbf{V}_B} = \frac{\mathbf{z}_A + \mathbf{m}\,\mathbf{z}_M}{\mathbf{z}_M + \mathbf{m}\,\mathbf{z}_B}. \qquad \text{(30)}$$

This equation determines the nature of the network which must separate the feeding points of A and B, if they are to be supplied from the same feeder line.

Effect upon polar diagram

46. It is also of importance to appreciate the extent to which the polar diagrams of an array are affected by the mutual impedance. Taking the two aerials A and B of the previous paragraph, but separated by an unspecified distance d, the total power radiated will be $I_A{}^2R_A' + I_B{}^2R_B'$.

Putting $I_B = M/\underline{\beta}\,I_A$ as before, the total power radiated is

$$P_T = I_A{}^2\,(R_A + M^2R_A + 2MR_M \cos\beta).$$

Thus, for a given power P_T the current in A is

$$I_A = \sqrt{\frac{P_T}{R_A\left(1 + M^2 + 2M\,\frac{R_M}{R_A}\cos\beta\right)}}$$

In paragraph 29 the polar diagram of two parallel radiators is derived on the assumption that $R_M = 0$. It is then shown that at any point P having the co-ordinates r, θ, φ, the R.M.S. field is

$$\Gamma = \frac{60}{r}\,F.f(\varphi)\sqrt{\frac{P_T}{R_A(1+M^2)}}\sqrt{1 + M^2 + 2M\cos\left(\beta + \frac{2\pi}{\lambda}\,d\cos\varphi\cos\theta\right)} \qquad \text{(31)}$$

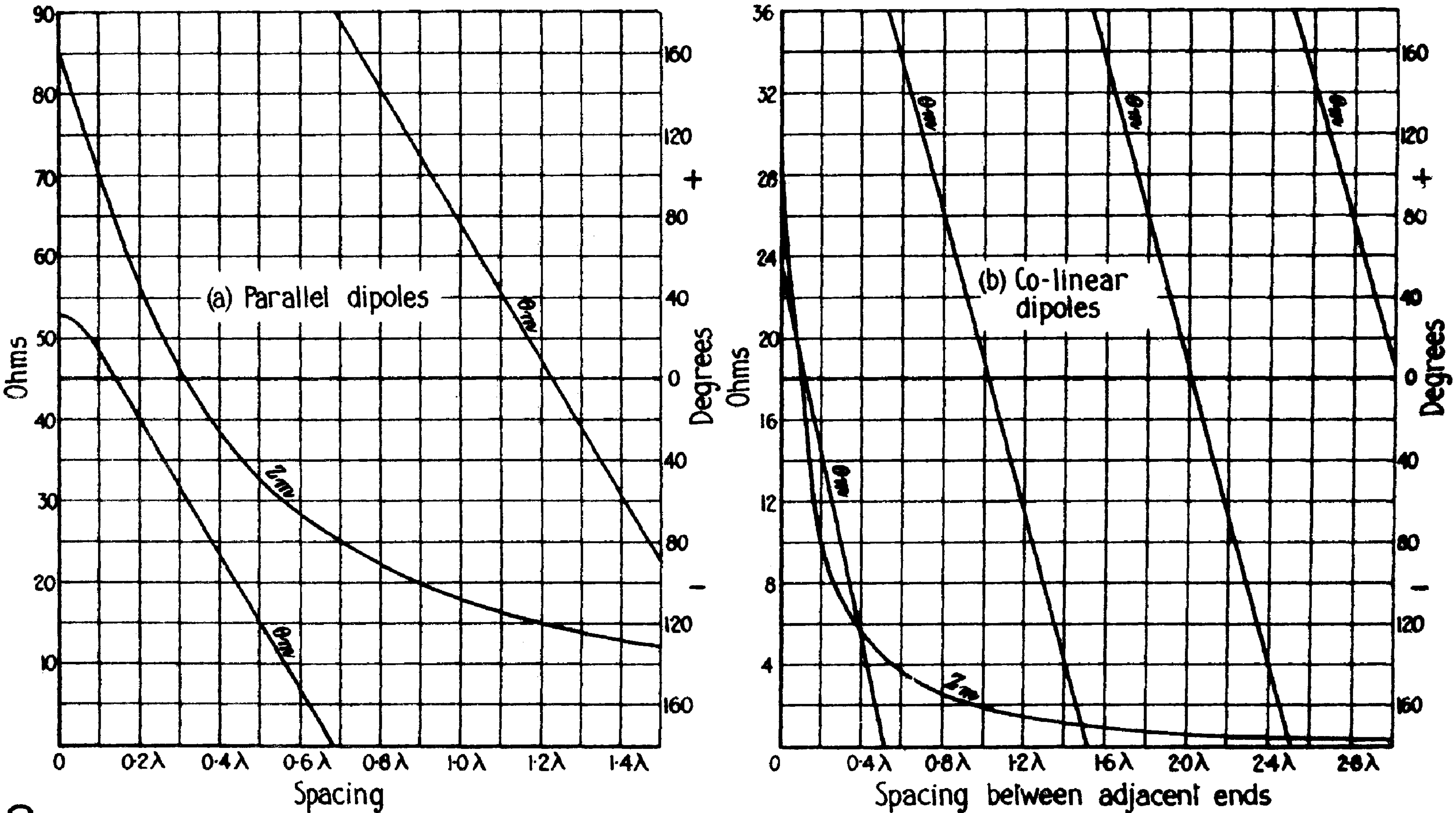
where $\sqrt{\frac{P_T}{R_A\,(1+M^2)}}$ is the current in aerial A.

To allow for the mutual impedance, then, we have only to substitute the value of I_A as modified by the mutual resistance R_M, giving

$$\Gamma = \frac{60}{r}\,F.f(\varphi)\sqrt{\frac{P_T\left[1 + M^2 + 2M\cos\left(\beta + \frac{2\pi d}{\lambda}\cos\varphi\cos\theta\right)\right]}{R_A\left(1 + M^2 + 2M\,\frac{R_M}{R_A}\cos\beta\right)}} \qquad \text{(32)}$$

The horizontal polar diagram is obtained by putting $\varphi = 0$, $\cos\varphi = 1$. The field at ground level is then

$$\Gamma_0 = \frac{60}{r}\,F\sqrt{\frac{P_T\left[1 + M^2 + 2M\cos\left(\beta + \frac{2\pi d}{\lambda}\cos\theta\right)\right]}{R_A\left(1 + M^2 + 2M\,\frac{R_M}{R_A}\cos\beta\right)}} \qquad \text{(33)}$$



MUTUAL IMPEDANCE OF DIPOLES IN FREE SPACE

FIG. 26
CHAP. XV.

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If the same power were supplied to the aerial A alone, B being entirely removed, the polar diagram would be a circle, the field strength being given by

$$\Gamma_{0(A)} = \frac{60}{r} F \sqrt{\frac{P_T}{R_A}},$$

from paragraph 14. The ratio of Γ_0 to $\Gamma_{0(A)}$ is

$$\frac{\Gamma_0}{\Gamma_{0(A)}} = \sqrt{\frac{1 + M^2 + 2M \cos\left(\beta + \frac{2\pi}{\lambda} d \cos\theta\right)}{1 + M^2 + 2M \frac{R_M}{R_A} \cos\beta}}. \quad .. \quad .. \quad .. \text{ (34)}$$

47. From this ratio it is easy to plot polar diagrams corresponding to those of fig. 8, but it is obviously impossible to portray all the possibilities. For the particular case when $M = 1$, $R_M \neq 0$, we have a further simplification

$$\frac{\Gamma_0}{\Gamma_{0(A)}} = \sqrt{\frac{1 + \cos\left(\beta + \frac{2\pi}{\lambda} d \cos\theta\right)}{1 + \frac{R_M}{R_A} \cos\beta}}$$

$$= \frac{\sqrt{2} \cos\left(\frac{\beta}{2} + \frac{\pi d}{\lambda} \cos\theta\right)}{\sqrt{1 + \frac{R_M}{R_A} \cos\beta}}$$

The numerator of this expression obviously gives the ratio $\frac{\Gamma_0}{\Gamma_{0(A)}}$ if $R_M = 0$, as in paragraph 31. Since neither $\frac{R_M}{R_A}$ nor $\cos\beta$ can exceed unity, the product $\frac{R_M}{R_A} \cos\beta$ cannot exceed unity and may be very much less. Thus, with equal currents in both aerials, the shape of the polar diagram is very little affected by the mutual impedance, so that fig. 8 may be used for practical purposes even though the mutual impedance was not taken into account in calculating the diagrams.

Effect of $\mathbf{Z}_M$ upon current in radiating members

48. (i) In an array consisting of more than two radiating members, the mutual impedance between the radiators may exercise a considerable influence upon the radiation characteristics. As an illustration the array shown in fig. 27 will be considered briefly. Here A, B, and C are parallel $\frac{\lambda}{2}$ aerials on a perfectly conductive earth, and are spaced $\frac{\lambda}{2}$ apart. Each aerial has a self-impedance $\mathbf{z} = R + jX$ ohms; tuning reactances X_1, X_2, X_3, may or may not be included. Let $\mathbf{z}_A = \mathbf{z} + jX_1$, $\mathbf{z}_B = \mathbf{z} + jX_2$, $\mathbf{z}_C = \mathbf{z} + jX_3$. Let us first assume that the tuning reactances are absent, and that the aerials are fed from a common source of voltage $\mathbf{V}$. We then have

$$\mathbf{V} = \mathbf{z}\mathbf{I}_A + \mathbf{z}_P\mathbf{I}_B + \mathbf{z}_Q\mathbf{I}_C \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \text{ (35a)}$$

$$\mathbf{V} = \mathbf{z}_P\mathbf{I}_A + \mathbf{z}\mathbf{I}_B + \mathbf{z}_P\mathbf{I}_C \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \text{ (35b)}$$

$$\mathbf{V} = \mathbf{z}_Q\mathbf{I}_A + \mathbf{z}_P\mathbf{I}_B + \mathbf{z}\mathbf{I}_C. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \text{ (35c)}$$

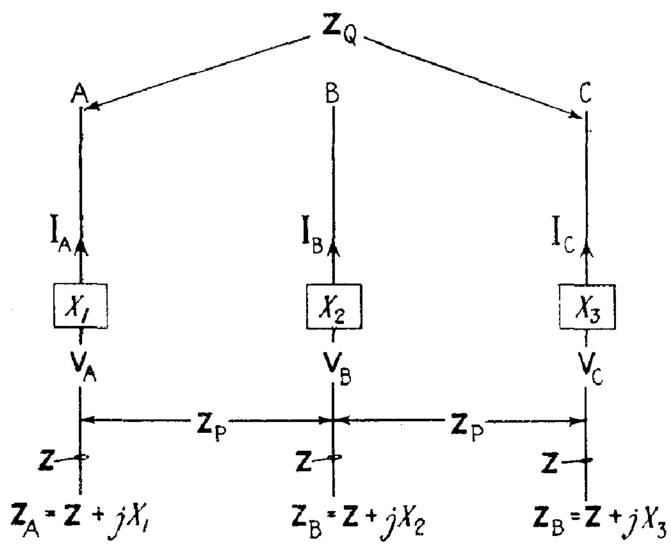


FIG. 27, CHAP. XV.—Notation—three parallel aerials.

From the symmetry of the arrangement it is obvious that $\mathbf{I}_A = \mathbf{I}_C$, although $\mathbf{I}_B$ is not necessarily equal to $\mathbf{I}_A$. Let $\mathbf{I}_B = \mathbf{m}\mathbf{I}_A$. Then

$$\mathbf{V} = (\mathbf{z} + \mathbf{z}_Q + \mathbf{m}\mathbf{z}_P)\,\mathbf{I}_A$$

$$\mathbf{V} = (2\mathbf{z}_P + \mathbf{m}\mathbf{z})\,\mathbf{I}_A$$

$$\mathbf{z} + \mathbf{z}_Q + \mathbf{m}\mathbf{z}_P = 2\mathbf{z}_P + \mathbf{m}\mathbf{z}$$

$$\text{and } \mathbf{m} = \frac{\mathbf{z} + \mathbf{z}_Q - 2\mathbf{z}_P.}{\mathbf{z} - \mathbf{z}_P} \qquad \text{(36)}$$

This ratio is easily evaluated. From fig. 25 the various impedances are found to be (to the nearest integers)

$$\mathbf{z} = 100 + j58$$

$$\mathbf{z}_P = -\,(24 + j47)$$

$$\mathbf{z}_Q = 10 + j32$$

$$\therefore \mathbf{m} = \frac{100 + j58 + 10 + j32 + 48 + j94}{100 + j58 + 24 + j47}$$

$$\doteq \frac{158 + j184}{124 + j105}$$

$$= 1\cdot43 + j0\cdot277$$

$$= 1\cdot46 \ \underline{/\,11^\circ} \text{ approximately.}$$

Thus the current in the centre aerial is nearly 50 per cent. greater than in the outer ones and is slightly out of phase. For practical purposes the horizontal polar diagram may be obtained by adding a circle of radius 1·46 units to diagram A 9 of fig. 8, taking the vertical lobes to be of positive and the horizontal lobes to be of negative sign, and ignoring the effect of the slight phase difference.

(ii) Next we shall suppose the aerials to be individually resonant, so that $\mathbf{z}_A = \mathbf{z}_B = \mathbf{z}_C = R = 100$ ohms. From the previous example it is easily seen that in this case

$$\begin{aligned}\mathbf{m} &= \frac{R + \mathbf{z}_Q - 2\mathbf{z}_P}{R - \mathbf{z}_P} \\ &= \frac{100 + 10 + j32 + 48 + j94}{100 + 24 + j47} \\ &= \frac{158 + j126}{124 + j47} \\ &= 1{\cdot}49 \underline{/18^\circ}\end{aligned}$$

Thus the current in the centre aerial is still nearly fifty per cent. greater than in the outer ones, and is out of phase by a greater angle than before.

49. Finally, let us find the conditions under which $\mathbf{m} = 1 \underline{/0^\circ}$, the current in each aerial to be in phase with its supply voltage. To achieve this it will be necessary to feed the centre aerial in such a manner that $\mathbf{V}_B$ is not equal to $\mathbf{V}_A$.

$$\mathbf{V}_A = \mathbf{z}_A\mathbf{I}_A + \mathbf{z}_P\mathbf{I}_B + \mathbf{z}_Q\mathbf{I}_C \qquad \text{(37a)}$$

$$\mathbf{V}_B = \mathbf{z}_P\mathbf{I}_A + \mathbf{z}_B\mathbf{I}_B + \mathbf{z}_P\mathbf{I}_C. \qquad \text{(37b)}$$

But

$$\mathbf{I}_A = \mathbf{I}_B = \mathbf{I}_C;$$

$$\mathbf{V}_A = (\mathbf{z} + jX_1 + \mathbf{z}_P + \mathbf{z}_Q)\,\mathbf{I}_A \qquad \text{(38a)}$$

$$\mathbf{V}_B = (2\mathbf{z}_P + \mathbf{z} + jX_2)\,\mathbf{I}_B, \qquad \text{(38b)}$$

i.e.

$$\begin{aligned}\mathbf{z}_A' &= R + jX + jX_1 + R_P + jX_P + R_Q + jX_Q \\ R_A' &= R + R_P + R_Q \\ &= 100 - 24 + 10 \\ &= 86 \text{ ohms} \\ X_A' &= X + X_1 + X_P + X_Q = 0 \\ X_1 &= -(X + X_P + X_Q) \\ &= -(58 - 47 + 32) \\ &= -43 \text{ ohms} \\ \mathbf{z}_B' &= 2R_P + 2jX_P + \mathrm{R} + jX + jX_2 \\ R_B' &= R + 2R_P \\ &= 100 - 48 \\ &= 52 \text{ ohms.} \\ X_B' &= X + X_2 + 2X_P = 0 \\ X_2 &= -(X + 2X_P) \\ &= -(58 - 94) \\ &= 36 \text{ ohms.}\end{aligned}$$

Thus in order to tune the array correctly, it is necessary to insert capacitive reactances in the outer members and an inductive reactance in the centre member. In practice this tuning may be achieved by suitable adjustment of the length of the radiator, or by the addition of a susceptance in parallel with the aerial instead of a series reactance. Such susceptances may take the form of short lengths of non-radiating feeder line. It is obviously necessary to supply the outer aerials at a higher voltage than the centre one, i.e. $\mathbf{V}_A = \mathbf{V}_C = 1{\cdot}65\ \mathbf{V}_B$. Since the aerials are carrying equal currents, we may refer to the radiation resistance of the whole aerial without ambiguity. This is equal to $R_A' + R_C' + R_B' = 208$ ohms, the average resistance per radiator being 69 ohms. It is found that as the number of parallel syn-phased dipoles is increased, the average resistance per aerial falls slightly, approaching about 56 ohms for an infinite number of aerials. On the other hand, if $\frac{\lambda}{2}$ aerials, spaced $\frac{\lambda}{2}$ apart, are fed with equal currents having a progressive phase difference of 180°, in order to obtain an " end-fire " diagram, a repetition of the above calculation gives the total radiation resistance of these aerials as 416 ohms, an average of 139 ohms per radiator.

Effect of $\mathbf{Z}_M$ between radiator and reflector

50. Let us now consider the action of a reflector aerial more fully than in paragraphs 33 and 34. In the notation previously used let A and B be two parallel vertical aerials separated by a distance d, A being energized by the application of an oscillatory voltage $\mathbf{V}$, and B being un-energized. Let their self-impedance operators be $\mathbf{z}_A$, $\mathbf{z}_B$ and their mutual impedance operator $\mathbf{z}_M$. By Kirchoff's law

$$\mathbf{V} = \mathbf{z}_A\mathbf{I}_A + \mathbf{z}_M\mathbf{I}_B \qquad \text{(39a)}$$

$$0 = \mathbf{z}_M\mathbf{I}_A + \mathbf{z}_B\mathbf{I}_B. \qquad \text{(39b)}$$

The current in the reflector B is

$$\mathbf{I}_B = -\frac{\mathbf{z}_M}{\mathbf{z}_B}\mathbf{I}_A$$

$$= \mathbf{m}\mathbf{I}_A.$$

Thus

$$\mathbf{m} = -\frac{\mathbf{z}_M}{\mathbf{z}_B} = -\frac{Z_M \,\underline{/\,\theta_M}}{Z_B \,\underline{/\,\theta_B}}$$

$$= \frac{Z_M}{Z_B}\,\underline{/\,\pi + \theta_M - \theta_B}$$

$$\mathbf{V} = \left\{\mathbf{z}_A - \frac{(\mathbf{z}_M)^2}{\mathbf{z}_B}\right\}\mathbf{I}_A$$

$$\frac{\mathbf{V}}{\mathbf{I}_A} = \mathbf{z}_A' = \mathbf{z}_A - \frac{(\mathbf{z}_M)^2}{\mathbf{z}_B}$$

$$= R_A + jX_A - \frac{(Z_M)^2}{Z_B}\,\underline{/\,2\pi + 2\theta_M - \theta_B}$$

$$= R_A + jX_A - \frac{(Z_M)^2}{Z_B}\,\underline{/\,2\theta_M - \theta_B}$$

so that

$$R_A' = R_A - \frac{(Z_M)^2}{Z_B}\cos\,(2\theta_M - \theta_B) \qquad \text{(40a)}$$

$$X_A' = X_A - \frac{(Z_M)^2}{Z_B}\sin\,(2\theta_M - \theta_B). \qquad \text{(40b)}$$

51. As would be expected, then, the presence of the reflector modifies both the resistance and the reactance of the energized aerial. The polar diagram is calculated by methods already explained. If Γ_0 is the field strength in the horizontal plane in the direction θ when the reflector is absent, and Γ_1 the field in the same direction with the reflector present, for the same input power,

$$\Gamma_1 = \Gamma_0 \sqrt{\frac{R_A}{R_A'}\left\{1 + M^2 + 2M \cos\left(\beta - \frac{2\pi}{\lambda} d \cos\theta\right)\right\}}$$

$$= \Gamma_0 \sqrt{\frac{1 + M^2 + 2M \cos\left(\beta - \frac{2\pi}{\lambda} d \cos\theta\right)}{1 - M \frac{Z_M}{R_A} \cos(2\theta_M - \theta_B)}}. \quad \ldots \quad \ldots \quad (41)$$

A study of the above results will show that it is almost if not quite impossible to fulfil the conditions required to give a horizontal polar diagram corresponding exactly with fig. 8 C 3. To obtain the latter diagram it is necessary to have $\mathbf{m} = 1\underline{/\frac{\pi}{2}}$. Now $M = \frac{Z_M}{Z_B}$ and Z_B cannot be less than R_B. If the aerials are $\frac{\lambda}{2}$ dipoles on a perfectly conductive earth, the minimum value of R_B is 100 ohms. The current in the reflector aerial leads on that in the energized aerial by an angle $\beta = 180° + \theta_M - \theta_B$, and if $Z_B = R_B$, $\theta_B = 0$. Thus the mutual impedance should have a phase angle of $-90°$. Reference to fig. 25 shows that $\theta_M = -90°$ when the spacing is approximately $0{\cdot}4\lambda$ and Z_M is then only 60 ohms, so that $\mathbf{m} = 0{\cdot}6\underline{/\frac{\pi}{2}}$ instead of $1\underline{/\frac{\pi}{2}}$.

52. The above position may be summarized by the statement that it is impossible simultaneously to fulfil the conditions that the forward radiation shall be double that of the single energized aerial, and the backward radiation absolutely annulled, by the use of a single reflector aerial. So far as it is possible to generalize, it may be said that in the case of a single aerial with reflector, both tuned to the same frequency, the optimum spacing for maximum forward radiation is approximately $\frac{\lambda}{2\pi}$, and for minimum backward radiation, $\frac{\lambda}{\pi}$. For the optimum ratio of forward to backward radiation, the separation should be about $0{\cdot}28\lambda$. These results are only of practical importance when a single energized member and a single reflector are used. When reflector aerials are used in conjunction with arrays consisting of several radiating members, the spacing is not critical, and it is found that a spacing of $\frac{\lambda}{4}$ is as effective as any, the reflector being usually slightly mistuned as explained below. In certain designs, particularly on the higher frequencies, a spacing of $\frac{3}{4}\lambda$ is sometimes adopted.

Mistuning of reflector

53. It is possible to obtain a near approach to the desired cardioid diagram by mistuning the reflector aerial, the degree of mistuning being dependent upon the spacing ; when this expedient is adopted the $\frac{\lambda}{4}$ spacing is in most circumstances as effective as any other. For any given set of conditions, the horizontal polar diagram is easily calculated from the expressions given above, particularly since, if only the shape of the diagram is required, it is sufficient to plot the portion

$\sqrt{1 + M^2 + 2\,M \cos\,(\beta - \frac{2\pi}{\lambda}\,d \cos\,\theta)}$. The method may be seen from the following example. If A and B are vertical $\frac{\lambda}{2}$ aerials, $\frac{\lambda}{4}$ apart upon a conductive earth, A only being energized, the radiation resistance of each wire will be 73 ohms and the dead-loss resistance may be only 2 ohms, so that the total resistance of each is 75 ohms. It is not suggested that the dead-loss resistance can be kept within so low a figure in practice, but it will be seen that unless the dead-loss resistance is very low the desired diagram cannot be obtained.

54. For $\frac{d}{\lambda} = 0{\cdot}25$, fig. 25 gives $Z_M \underline{/\theta_M}$ as $80 \underline{/-35}$. Suppose the reflector to be mistuned, having a positive, i.e. (inductive) reactance, its impedance being $Z_B \underline{/45^\circ}$.

Then $$Z_B = \frac{R_B}{\cos\,\theta_B} = \frac{75}{0{\cdot}707} = 106 \text{ ohms}$$

and $$M = \frac{Z_M}{Z_B} = \frac{80}{106} = 0{\cdot}755$$
$$M^2 = 0{\cdot}57$$

The angle β by which the current I_B leads on I_A is given by
$$\beta = \pi + \theta_M - \theta_B = (180 - 35 - 45) \text{ degrees}$$
$$= 100^\circ$$

Substituting these values of M and β in the expression $1 + M^2 + 2M \cos\,(\beta - \frac{2\pi}{\lambda}\,d \cos\,\theta)$ we obtain $1{\cdot}57 + 1{\cdot}51 \cos\,(100 - 90 \cos\,\theta)$; when $\theta = 0$, $\cos\,\theta = 1$, $(100 - 90 \cos\,\theta) = 10$.
$$1{\cdot}51 \cos 10 = 1{\cdot}486$$
$$1{\cdot}57 + 1{\cdot}486 = 3{\cdot}056.$$
Hence the field in this direction is
$$\Gamma_0 = \Gamma_A \sqrt{\frac{R_A}{R_A'}} \sqrt{3{\cdot}056}.$$

Ignoring the terms Γ_A, $\sqrt{\frac{R_A}{R_A'}}$, which give the scale of the diagram
$$\Gamma_0 = \sqrt{3{\cdot}056}$$
$$= 1{\cdot}75.$$

When $$\theta = 180,\ \cos\,\theta = -1,\ (100 - 90 \cos\,\theta) = 190$$
$$1{\cdot}51 \cos 190 = -1{\cdot}51 \cos 10 = -1{\cdot}486$$
$$1{\cdot}57 - 1{\cdot}486 = 0{\cdot}084$$
$$\Gamma_{180} = \sqrt{0{\cdot}084}$$
$$= 0{\cdot}29$$

The field in other directions is found in the same manner and so the shape of the horizontal polar diagram is determined. Actually a good approximation may be found by calculating Γ_0 and Γ_{180} as above, and in addition, Γ_{90} and the minimum field. The field Γ_{90} is obviously

$\sqrt{1 + M^2 + 2M \cos \beta}$; in the given example this becomes $\sqrt{1 \cdot 57 + 1 \cdot 51 \cos 100°}$ $= \sqrt{1 \cdot 57 - 1 \cdot 51 \times 0 \cdot 1736} = 1 \cdot 15$. The minimum field obviously occurs when $\cos\left(\beta - \frac{2\pi}{\lambda} d \cos \theta\right) = -1$, i.e. when $\left(\beta - \frac{360d}{\lambda} \cos \theta\right) = 180$. In the present instance we have

$$100 - 90 \cos \theta = 180$$

$$\cos \theta = -\frac{8}{9}$$

$$\theta = 153°$$

The field Γ_{153} is equal to $\sqrt{1 + M^2 - 2M} = \sqrt{1 \cdot 57 - 1 \cdot 51} = 0 \cdot 245$.

55. With regard to the scale, we have to find Γ_A, the field which would be set up by the aerial A alone. This is equal to $\frac{60}{r} I_A f(\varphi)$. Geometrically $f(\varphi)$ is identical with fig. 6 and its magnitude in the horizontal plane ($\varphi = 0$) is unity. Next the expression $\sqrt{\frac{R_A}{R_A'}}$, must be evaluated. From equation 40a,

$$\frac{R_A'}{R_A} = 1 - \frac{Z_M^2}{Z_B R_A} \cos (2\theta_M - \theta_B)$$

$$= 1 - M \frac{Z_M}{R_A} \cos (2\theta_M - \theta_B).$$

In the present example this becomes

$$\frac{R_A'}{R_A} = 1 - \frac{0 \cdot 755 \times 80}{73} \cos (-70 - 45)$$

$$= 1 \cdot 35$$

$$\frac{R_A}{R_A'} = 0 \cdot 74$$

$$\sqrt{\frac{R_A}{R_A'}} = 0 \cdot 86.$$

Finally, allowance must be made for the radiation contributed by the virtual image of the array. This entails the introduction of a Vertical Distribution Factor $D(\varphi)$; since the centre point of the array is $\frac{\lambda}{4}$ above the earth, the appropriate factor is geometrically identical with fig. 8 A 5, but must be turned through 90° so that it has the value 2 along the ground ($\varphi = 0$, $\theta = 0$). Thus the R.M.S. field in the direction $\theta = 0$, along a perfectly conductive ground will be $2 \times 0 \cdot 86 \times 1 \cdot 75 \doteq 3 \times \frac{60}{r} I$ and in the direction $\theta = 180$, $2 \times 0 \cdot 86 \times 0 \cdot 29 = 0 \cdot 465 \times \frac{60}{r} I$. The complete horizontal polar diagram is therefore that shown in fig. 28b. From the manner in which it is obtained it is obvious that if the diagram is rotated through 90° about the axis XX, and then multiplied by the appropriate values of $f(\varphi)$ and $D(\varphi)$ as defined above, the vertical polar diagram (fig. 28c) is obtained. A few points so calculated will give a sufficiently close approximation.

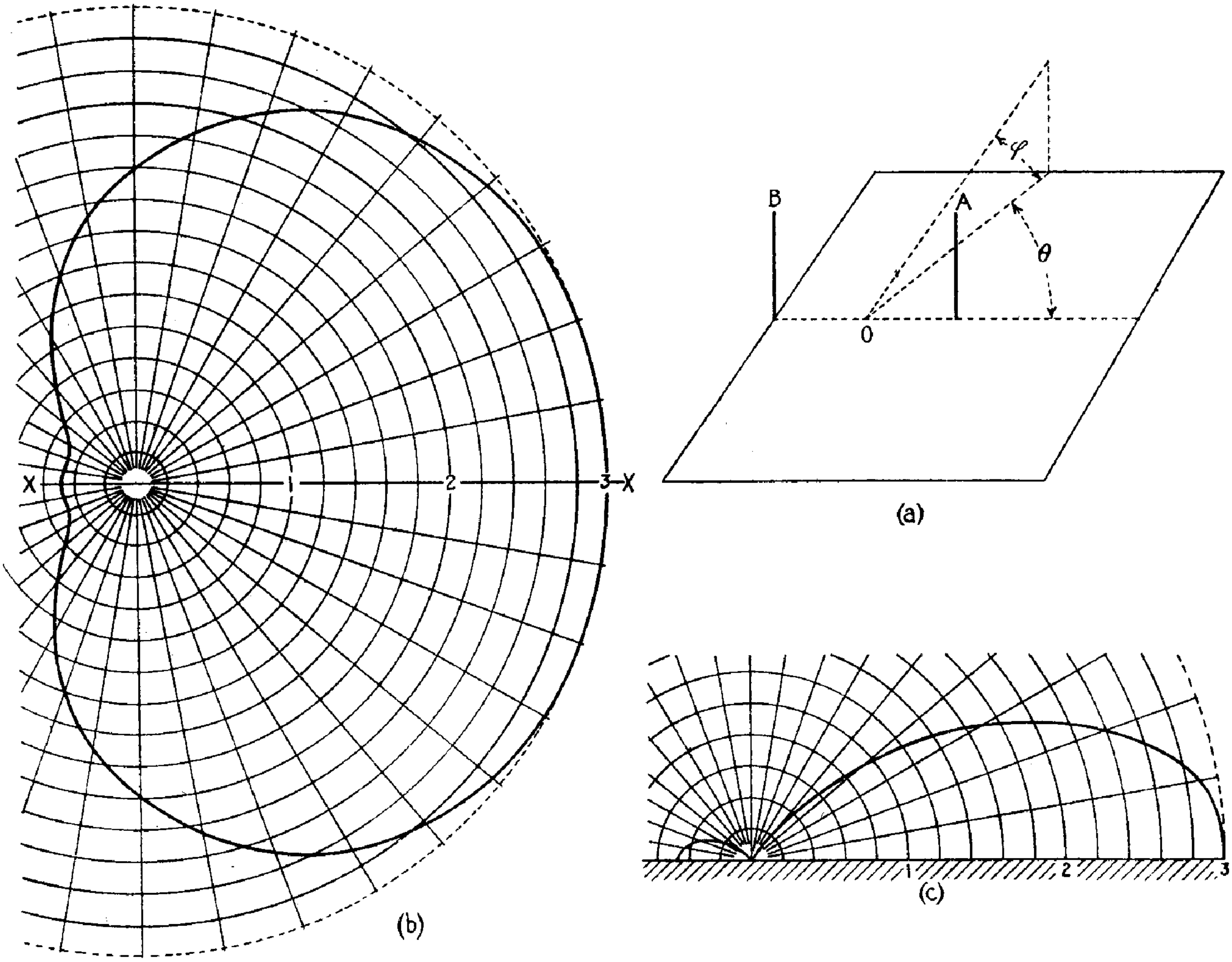


FIG. 28, CHAP. XV.—Example of calculation of polar diagrams.

Effect of dead-loss resistance

56. The importance of low dead-loss resistance in the reflector aerial can easily be appreciated. Again referring to fig. 8, which, it will be remembered, is constructed on the basis of equal currents in the two aerials, it is seen that if this condition is fulfilled, an approach to the desired unidirectional diagram is attainable even if the respective currents are not in quadrature. For example, compare diagrams B 3, B 4, D 2, D 3, with diagram C 3. If the two currents are not equal, however, the attainment is much more difficult, and it is therefore desirable to make M approach unity as closely as possible. Since $M = \frac{Z_M}{Z_B}$ and Z_M is constant for any particular spacing, M can only approach unity if Z_B is kept small. Even with zero dead-loss resistance and zero resistance, Z_B is equal to the radiation resistance, e.g. for a $\frac{\lambda}{2}$ dipole, Z_B must be more than 73 ohms. This considerably narrows the range of $\frac{d}{\lambda}$ from which a suitable value of Z_M can be chosen.

Influence of finite conductivity and permittivity of ground

57. We may now briefly discuss the errors involved in the assumption that the surface of the earth is a perfect conductor, so far as its properties as a reflector are concerned. Since we are only concerned with the earth in the vicinity of the aerial, we shall consider the surface to be plane as before. Fresnel's equations governing the reflection of a plane electro-magnetic wave at a plane surface are given in the previous chapter, in terms of the angle of incidence as defined for physical purposes. For the present purpose, however, it is more convenient to state them in terms of the ground angle as previously used in this Chapter. The ratio of the field strength on reflection, Γ_r, to the incident field strength Γ_i, is then a complex number. Two different solutions occur according to the plane of polarization of the incident wave. For a wave polarized in the plane of incidence, i.e. vertical polarization, we have

$$\frac{\Gamma_r}{\Gamma_i} = K_v \underline{/ \theta_v} = K_v \, \varepsilon^{j\theta_v} = \mathbf{k}_v$$

while if the wave is polarized perpendicularly to the plane of incidence, i.e. horizontal polarization,

$$\frac{\Gamma_r}{\Gamma_i} = K_h \underline{/ \theta_h} = K_h \, \varepsilon^{j\theta_h} = \mathbf{k}_h.$$

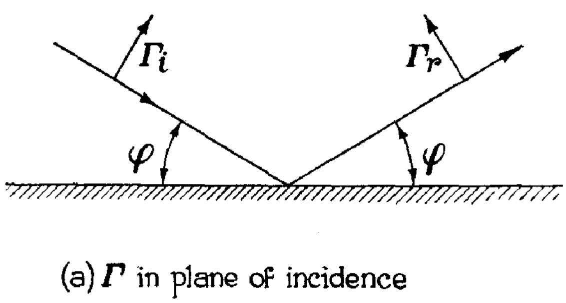
In terms of the ground angle φ, Fresnel's equations become

$$\mathbf{k}_v = \frac{\left(\varkappa - j\,\frac{2\sigma}{f}\right) \sin\varphi - \sqrt{\varkappa - \cos^2\varphi - j\,\frac{2\sigma}{f}}}{\left(\varkappa - j\,\frac{2\sigma}{f}\right) \sin\varphi + \sqrt{\varkappa - \cos^2\varphi - j\,\frac{2\sigma}{f}}} \quad \ldots \quad (42)$$

$$\mathbf{k}_h = \frac{\sqrt{\varkappa - \cos^2\varphi - j\,\frac{2\sigma}{f}} - \sin\varphi}{\sqrt{\varkappa - \cos^2\varphi - j\,\frac{2\sigma}{f}} + \sin\varphi} \quad \ldots \quad (43)$$

where $\varkappa$ is the permittivity of the ground.
σ is the conductivity of the ground in E.S.U.
φ is the ground angle, i.e. the complement of the angle of incidence.
f is the frequency in cycles per second.

58. The expressions $\mathbf{k}_v$, $\mathbf{k}_h$ are referred to as the complex coefficients of reflection for the respective cases. Their moduli, K_v and K_h, are always less than unity. The angle θ_v or θ_h must be added to the phase of the incident wave to obtain the phase of the reflected wave. This angle is always negative, and lies between 0 and — 180°. In using these equations it is most important to observe the conventions which have been adopted in obtaining them. These are shown diagrammatically in fig. 29. Taking the vertical polarization case first, Γ_i and Γ_r are both considered to be positive in the upward direction, along the plane of the paper. In the case of



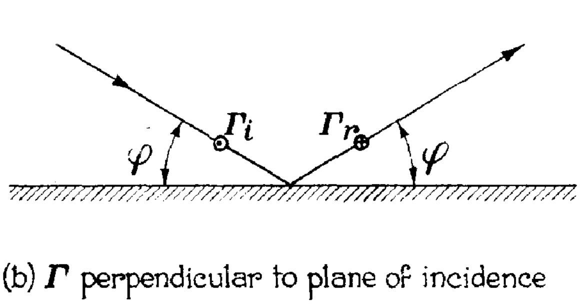


FIG. 29, CHAP. XV.—Conventional positive directions of electric field vector.

horizontally polarized waves Γ_i is considered to be positive when it is in a direction upward from the surface of the paper, and the positive direction of Γ_r is considered to be downward, i.e. below the surface of the paper. The importance of these conventions becomes apparent when the sum of the incident and reflected waves is to be found.

K / θ curves

59. As the computation of the reflection coefficient is very tedious, curves of K_v, θ_v and K_h, θ_h, for the different states of polarization and for several different kinds of ground surface, are given in fig. 30 (Sheets 1 to 4). Once these are known the total field at a distance from an aerial may be found by the methods already given. For example, take a horizontal dipole operated at a frequency f, which is situated at a height h over ground for which σ and κ are known, and consider the total field at a point P at an angle φ to the horizon and at a distance r ($>>h$) from the aerial. Let the aerial current be $\mathbf{I}\,\varepsilon^{j\omega t}$. Then, due to the aerial alone, we have at P a field

$$\gamma_A = \frac{60}{r}\,\mathbf{I}\,\varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)}.$$

Also, due to the wave reflected at the ground, a field

$$\gamma_B = \frac{60}{r}\,\mathbf{k}_h\,\mathbf{I}\,\varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}(r + 2\,h \sin \varphi)\right]}.$$

Paying due regard to the conventional positive directions of γ_A and γ_B, therefore,

$$\gamma_T = \frac{60}{r}\,\mathbf{I}\,\varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)}\left[1 - K_h\,\varepsilon^{j\theta_h}\,\varepsilon^{-j\frac{4\pi}{\lambda} h \sin \varphi}\right]$$

$$= \gamma_A\left[1 - K_h \,\big/\, \theta_h - \frac{4\pi}{\lambda} h \sin \varphi\right]. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

60. (i) By methods already explained, the amplitude of the total field is found to be

$$\hat{\Gamma}_T = \hat{\Gamma}_A \sqrt{1 + (K_h)^2 - 2K_h \cos\left(\theta_h - \frac{4\pi}{\lambda} h \sin \varphi\right)} \quad \ldots \quad \ldots \quad (45)$$

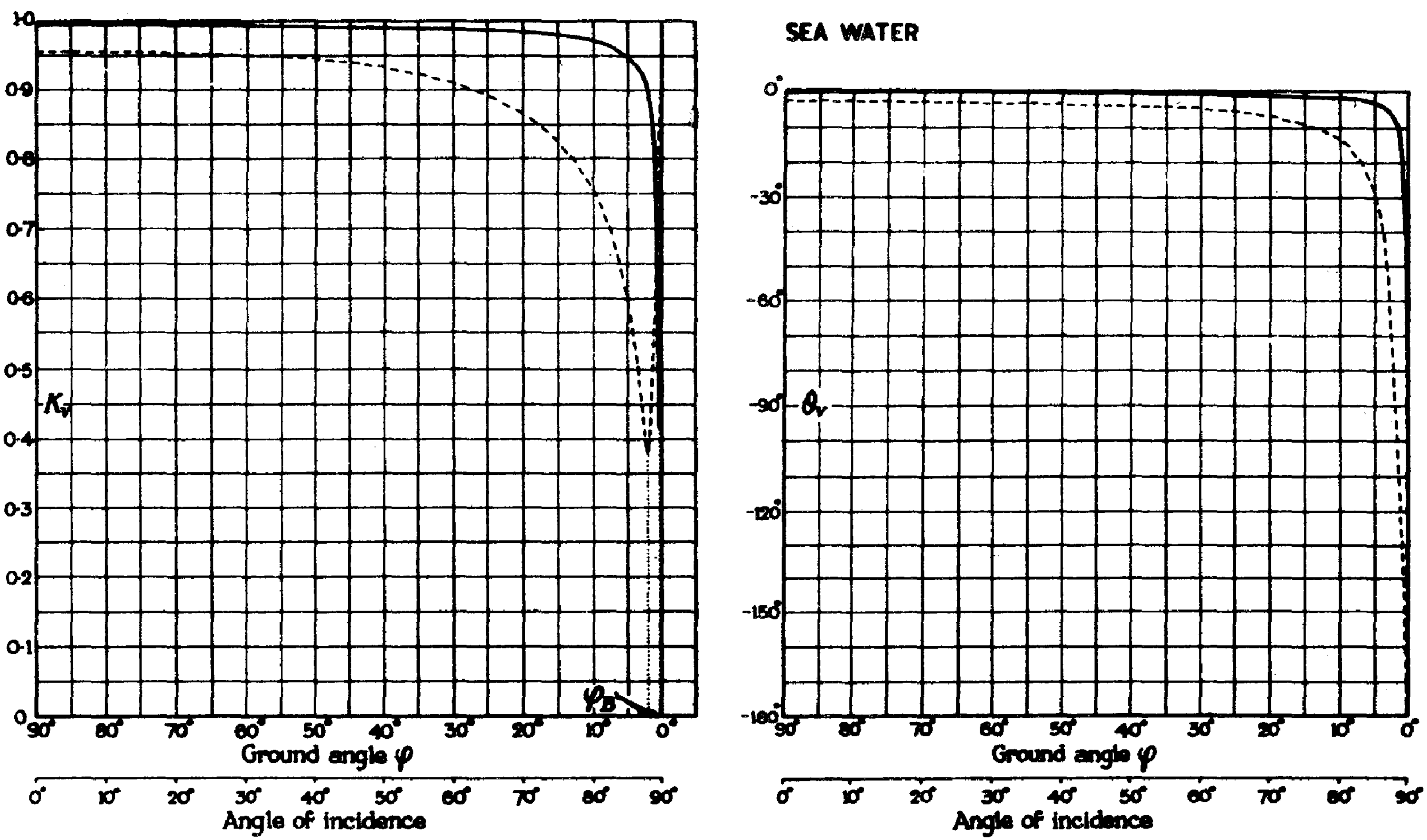
and we are not further concerned with its phase. The expression under the square root sign is therefore a factor by which the field strength $\hat{\Gamma}_A$, due to the aerial alone, must be multiplied, in order to allow for the effect of the earth. It is seen to be of the same form as that which takes into account the mutual impedance between an aerial and a reflector, but θ_h is a function of the angle φ instead of being constant for a given set of conditions.

(ii) If the above calculation is repeated for the case of a vertical dipole, with due regard to the sign convention, the amplitude of the total field is found to be

$$\hat{\Gamma}_T = \hat{\Gamma}_A \sqrt{1 + (K_v)^2 + 2K_v \cos\left(\theta_v - \frac{4\pi}{\lambda} h \sin \varphi\right)} \quad \ldots \quad \ldots \quad (46)$$

which is of a similar form to that obtained for horizontal polarization. The quantities under the square root signs are the Ground Reflection Factors and may be denoted by $\varrho_v(\varphi)$ and $\varrho_h(\varphi)$ respectively. Once these factors have been calculated, they may of course be applied to any array which in free space would radiate equally well above and below the equatorial plane, provided that h is taken as the height above earth of the electrical centre of the array. The R.M.S. field at any point having co-ordinates r, θ, φ, may therefore be written

$$\Gamma = \frac{60}{r}.\ F.f(\varphi).\ \varrho(\varphi).\ I_A.$$

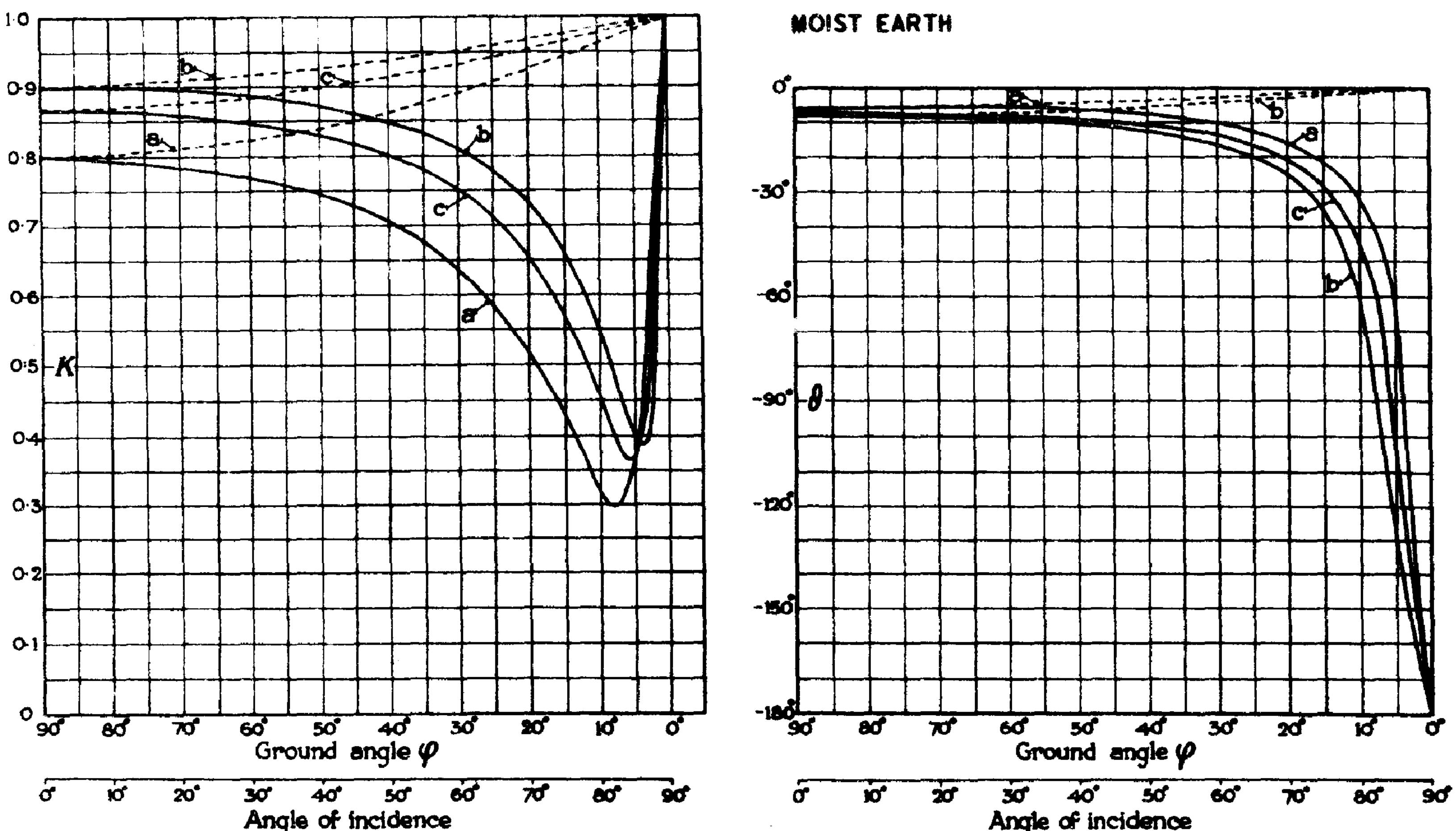


Γ IN PLANE OF PROPAGATION

$$\left.\begin{array}{l} \text{------ } f = 10^8 \sim/\text{sec.} \quad \sigma = 5 \times 10^{10}\ \text{E.S.U. at } 0^\circ\text{C} \\ \text{——— } f = 10^6 \sim/\text{sec.} \quad \sigma = 5 \times 10^{10}\ \text{E.S.U. at } 25^\circ\text{C} \end{array}\right\} \varkappa = 80$$

REFLECTION COEFFICIENTS

FIG. 30
SHEET I
CHAP. XV

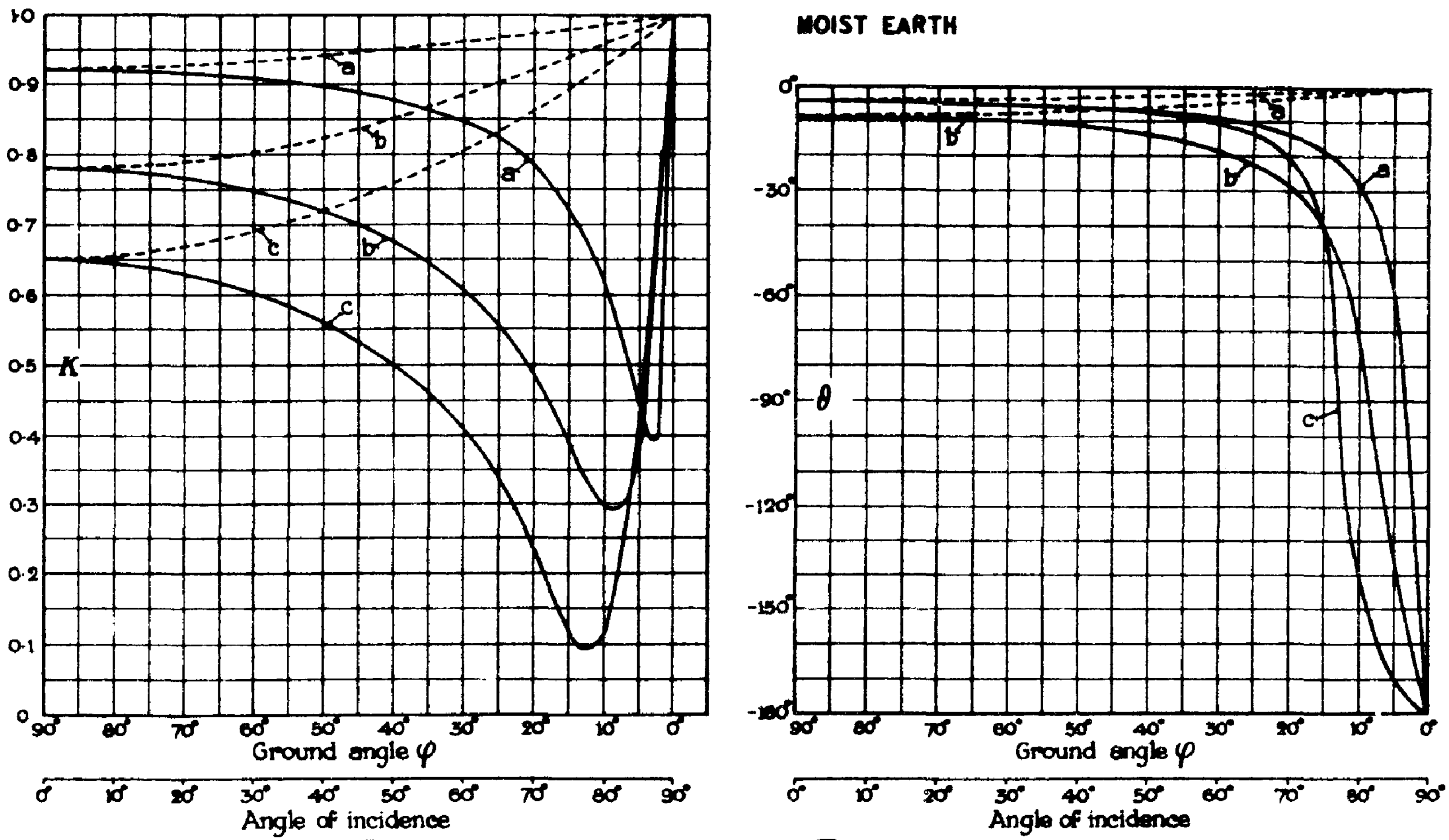


—Γ IN PLANE OF PROPAGATION. ----Γ PERPENDICULAR TO PLANE OF PROPAGATION

(a) $f = 10^6 \sim/\text{sec.}$ $\sigma = 10^8$ E.S.U.
(b) $f = 10^7 \sim/\text{sec.}$ $\sigma = 2{\cdot}5 \times 10^8$ E.S.U.
(c) $f = 10^8 \sim/\text{sec.}$ $\sigma = 55 \times 10^8$ E.S.U.
$\kappa = 20$

FIG. 30
SHEET 2
CHAP. XV

REFLECTION COEFFICIENTS

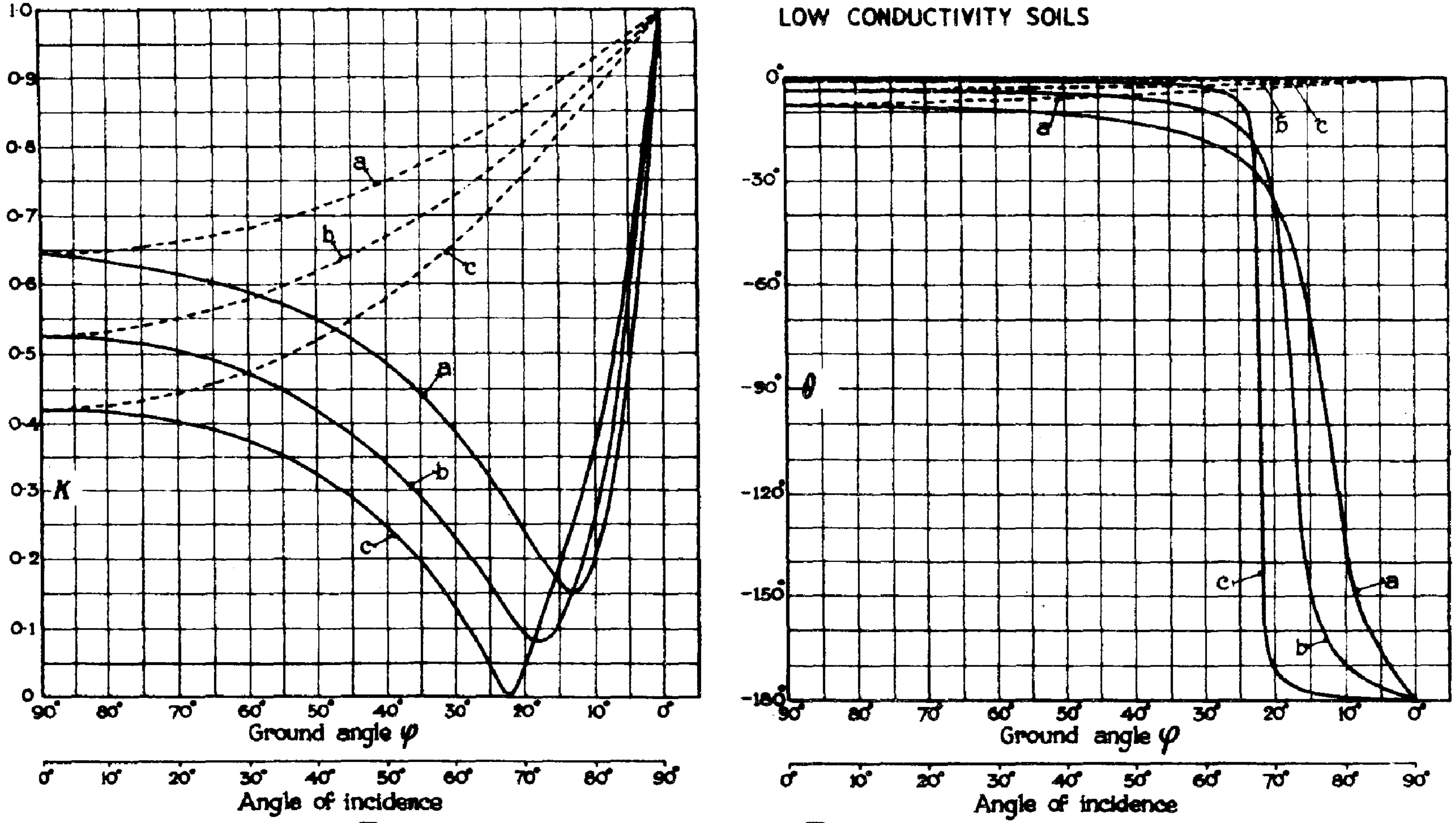


—Γ IN PLANE OF PROPAGATION ---Γ PERPENDICULAR TO PLANE OF PROPAGATION

(a) $f = 10^6 \sim/\text{sec.}$ $\sigma = 1{\cdot}5 \times 10^8$ E.S.U.
(b) $f = 10^7 \sim/\text{sec.}$ $\sigma = 2 \times 10^8$ E.S.U.
(c) $f = 10^8 \sim/\text{sec.}$ $\sigma = 4 \times 10^8$ E.S.U.
$\Big\}$ $\kappa = 20$

FIG. 30
SHEET 3
CHAP. XV

REFLECTION COEFFICIENTS



—Γ IN PLANE OF PROPAGATION ----Γ PERPENDICULAR TO PLANE OF PROPAGATION

(a) Fine sands and gravels	$\varkappa = 15$	$\sigma = 0{\cdot}5 \times 10^8$ E.S.U.
(b) Shingles and grits	$\varkappa = 10$	$\sigma = 0{\cdot}1 \times 10^8$ E.S.U.
(c) Hard rocks	$\varkappa = 6$	$\sigma = 0{\cdot}01 \times 10^8$ E.S.U.

FIG. 30
SHEET 4
CHAP. XIV

REFLECTION COEFFICIENTS

61. Referring now to the curves showing the variation of $\mathbf{k}_h$ with φ (figs. 30, Sheets 2 to 4) it is seen that for ground of moderately high conductivity, K_h may be of the order of $0 \cdot 7$ or more (for $\varphi = 90°$), gradually increasing to unity when the radiation reaches the ground at a very small angle to the horizon. For very high values of σ, e.g. for sea water, K_h is rarely less than $\cdot 96$ (for $\varphi = 90°$) and may therefore be taken as unity for practical purposes. For a surface of hard rock, on the other hand, K_h (for $\varphi = 90°$) may be as low as $0 \cdot 4$ or even less. Given the values of $\varkappa$, σ and f, it is not difficult to calculate the reflection coefficient for vertical incidence (i.e. $\varphi = 90°$), as in the following example.

Example.—If $\sigma = 10^6$ E.S.U. $\varkappa = 6$, $f = 10^7$ cycles per second, find the reflection coefficient for vertical incidence.

When $\varphi = 90°$ $\cos\varphi = 0$, $\sin\varphi = 1$

$$\mathbf{k}_h = \frac{\sqrt{\varkappa - j\dfrac{2\sigma}{f}} - 1}{\sqrt{\varkappa - j\dfrac{2\sigma}{f}} + 1}$$

$$\frac{2\sigma}{f} = 0 \cdot 2$$

$$\mathbf{k}_h = \frac{\sqrt{6 - j\,0 \cdot 2} - 1}{\sqrt{6 - j\,0 \cdot 2} + 1}$$

$$\begin{aligned}
\sqrt{6 - j\,0 \cdot 2} &= \nu - j\alpha \\
6 - j\,0 \cdot 2 &= \nu^2 - 2j\nu\alpha - \alpha^2 \\
\therefore\ \nu^2 - \alpha^2 &= 6. \\
2\nu\alpha &= 0 \cdot 2 \\
\nu^4 - 2\nu^2\alpha^2 + \alpha^4 &= 36 \\
4\nu^2\alpha^2 &= 0 \cdot 04 \\
\nu^4 + 2\nu^2\alpha^2 + \alpha^4 &= 36 \cdot 04 \\
\nu^2 + \alpha^2 &= \sqrt{36 \cdot 04} \\
&= 6 \cdot 0033 \\
(\nu^2 + \alpha^2) + (\nu^2 - \alpha^2) &= 12 \cdot 0033 \\
\nu^2 &= 6 \cdot 00165 \\
\nu &= \sqrt{6 \cdot 00165} \\
&= 2 \cdot 45
\end{aligned}$$

and

$$\begin{aligned}
\alpha^2 &= 0 \cdot 00165 \\
\alpha &= 0 \cdot 0406 \\
\mathbf{k}_h &= \frac{\nu - j\alpha - 1}{\nu - j\alpha + 1} \\
&= \frac{2 \cdot 45 - 1 - j\,0 \cdot 0406}{2 \cdot 45 + 1 - j\,0 \cdot 0406} \\
&= \frac{1 \cdot 45 - j\,0 \cdot 0406}{3 \cdot 45 - j\,0 \cdot 0406} \\
K_h &= \sqrt{\frac{1 \cdot 45^2 + 0 \cdot 0406^2}{3 \cdot 45^2 + 0 \cdot 0406^2}} \\
&\doteq 0 \cdot 42
\end{aligned}$$

When an approximate value for K_h ($\varphi = 90°$) has been obtained, the approximate curve for other values of φ may be sketched in by noting that it closely resembles one quarter of the negative portion of a sine curve. The error in drawing the curve in this manner is greatest at about 30°, but even then is probably not greater than that occasioned by our imperfect knowledge of the electrical properties of the particular ground.

62. Turning now to the curves showing the value of K_v, it is at once evident that the phenomenon is more complicated than in the case of horizontal polarization. It is on this account that it is difficult to draw general conclusions as to the radiating properties of vertical aerials. The magnitude $K_{v(max.)}$ of the reflection coefficient for $\varphi = 90°$ is the same as for horizontal polarization. This is obvious from physical considerations, for strictly, "vertical" and "horizontal" polarization have no significance for a wave perpendicularly incident. As the ground angle decreases, K_v also decreases and passes through a minimum value at some angle φ_B, afterwards increasing fairly rapidly, and reaching unity when $\varphi = 0$. The angle φ_B is known as the pseudo-Brewster angle from its relation to certain phenomena in optics. If the minimum value of K_v and the pseudo-Brewster angle φ_B are known for any particular kind of ground surface, the curve may be sketched in with sufficient accuracy for most purposes by observing the general trend of the calculated curves given. To facilitate this procedure, the curves shown in fig. 31 may be used.

63. The phase angles θ_v and θ_h are also plotted in fig. 30 for conditions corresponding to those for which K_v and K_h are given. Again it is obvious that for $\varphi = 90°$, $\theta_v = \theta_h$ and is rarely more than a few degrees. In the case of horizontal polarization, the angle θ_h gradually decreases with an increase of φ and is zero when $\varphi = 0$. If its maximum value is known, the curve may be sketched in with fair accuracy by noting its general resemblance to a sine curve as in paragraph 62 above. As an example of the calculation we may find θ_h for the conditions previously discussed.

In paragraph 61 we found, for $\varphi = 90°$

$$\begin{aligned}
\mathrm{k}_h &= \frac{1\cdot45 - j\,0\cdot0406}{3\cdot45 - j\,0\cdot0406} \\
&= \frac{(1\cdot45 - j\,0\cdot0406)\,(3\cdot45 + j\,0\cdot0406)}{3\cdot45^2 + 0\cdot0406^2} \\
&= \frac{5 + \cdot00165 - j\,0\cdot14 + j\,0\cdot06}{11\cdot9} \\
&\doteqdot \frac{5 - j\,0\cdot08}{11\cdot9} \\
\therefore\ \theta_h &= -\tan^{-1}\frac{0\cdot08}{5} \\
&= -0°\,55'
\end{aligned}$$

The above example has been worked at some length, but the following "short-cut" should be noted. When $\sqrt{\varkappa - j\,\dfrac{2\sigma}{f}} = \nu - j\alpha$ has been evaluated

$$\begin{aligned}
\underset{(\varphi = 90°)}{K_h/\theta_h} &= \frac{\nu - 1 - j\alpha}{\nu + 1 - j\alpha} \\
&= \frac{\{(\nu - 1) - j\alpha\}\,\{(\nu + 1) + j\alpha\}}{(\nu + 1)^2 + \alpha^2} \\
&= \frac{\nu^2 + \alpha^2 - 1 - j\,2\alpha}{(\nu + 1)^2 + \alpha^2}
\end{aligned}$$

and when, as is generally the case in practice, $\nu^2 >> \alpha^2$

$$K_{\mathrm{h}\,(\varphi = 90^\circ)} \doteqdot \frac{\nu - 1}{\nu + 1}$$

$$\theta_{\mathrm{h}} \doteqdot -\tan^{-1}\frac{2\alpha}{\nu^2 - 1}.$$

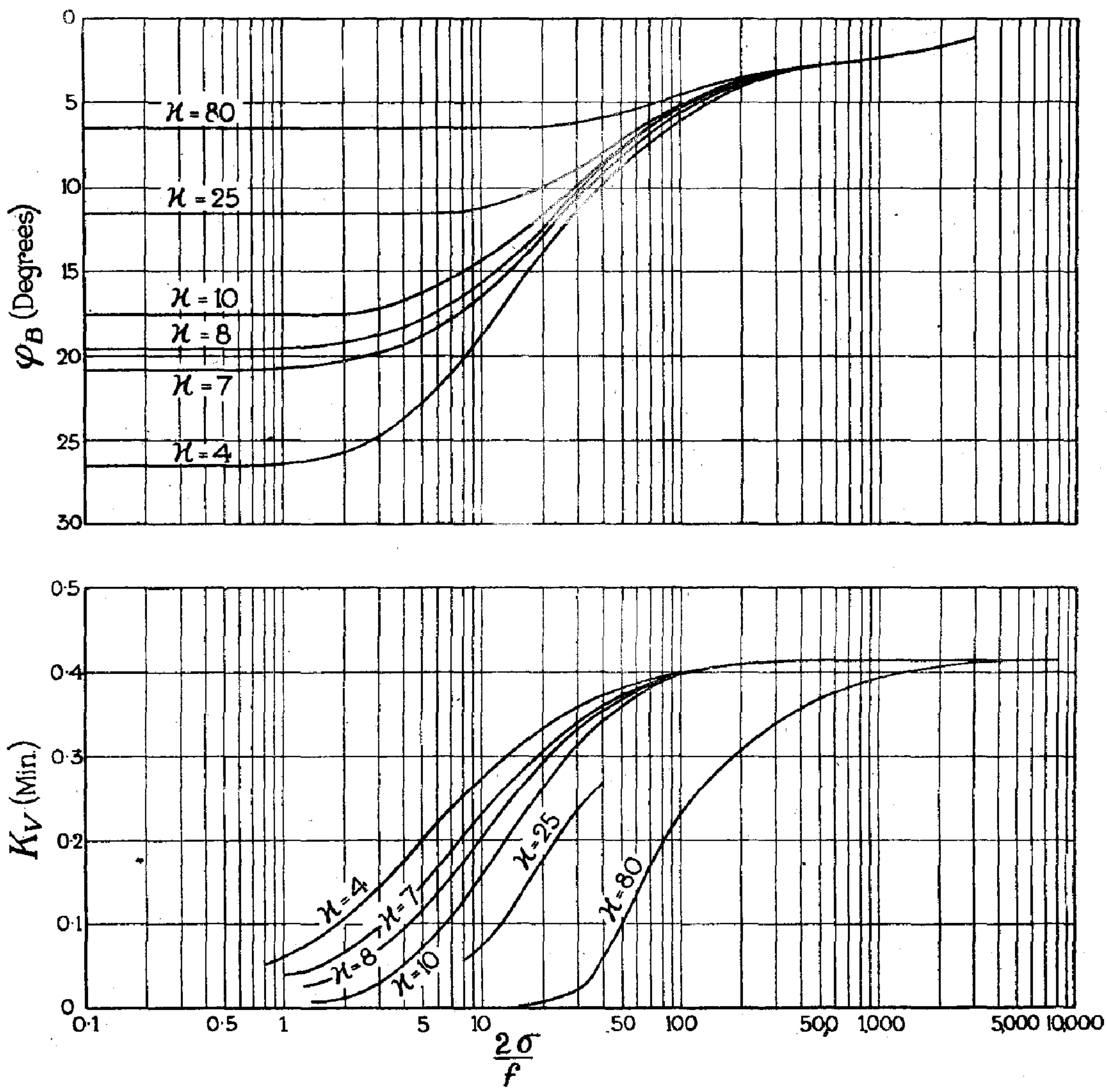


FIG. 31, CHAP. XV.—Pseudo-Brewster angle φ_{B} and corresponding reflection coefficient K_{v} (min.) for various kinds of ground.

64. The variation of θ_{v} with φ is very different from that of θ_{h}. Commencing with a small negative value equal to θ_{h}, when $\varphi = 90^\circ$, it is seen to increase in size very gradually until φ approaches the value φ_{B}, when a very rapid variation takes place; when $\varphi = \varphi_{\mathrm{B}}$, $\theta_{\mathrm{v}} = -90^\circ$, and for angles smaller than φ_{B}, θ_{v} continues to increase in size, becoming -180° when $\varphi = 0$. The minimum value of K_{v} and the corresponding angle φ_{B} are given in fig. 31 for various values of $\varkappa$ and $\frac{2\sigma}{f}$ in order to facilitate the construction of approximate curves of K_{v} and θ_{v}.

65. (i) When the appropriate values of K and θ have been obtained and tabulated for any given conditions, the corresponding Reflection Factor can be plotted for various values of φ, and the resulting polar curve used as a correction factor for the free space diagram of any aerial or aerial array, taking the place of the Vertical Distribution Factor. As an example, the expression $\sqrt{1 + K_V^2 + 2K_V \cos\left(\theta_V - \frac{4\pi}{\lambda} h \sin \varphi\right)}$ which is appropriate to vertically polarized radiation, has been plotted in thin solid line in fig. 32 for the following conditions, viz., $\kappa = 20$, $\sigma = 4 \times 10^8$, $f = 10^8$, $h = \frac{\lambda}{4}$. No great accuracy has been attempted as the intention is merely to indicate the kind of curve to be expected. The curve shown in dotted line represents the Current Distribution Factor for some unspecified form of aerial. The curve shown in heavy line is the polar product of the two former curves; and gives the shape of the vertical polar diagram of the aerial or array. Absolute values of R.M.S. field strength are of course obtained by multiplying by the factor $\frac{60}{r} F.I.$

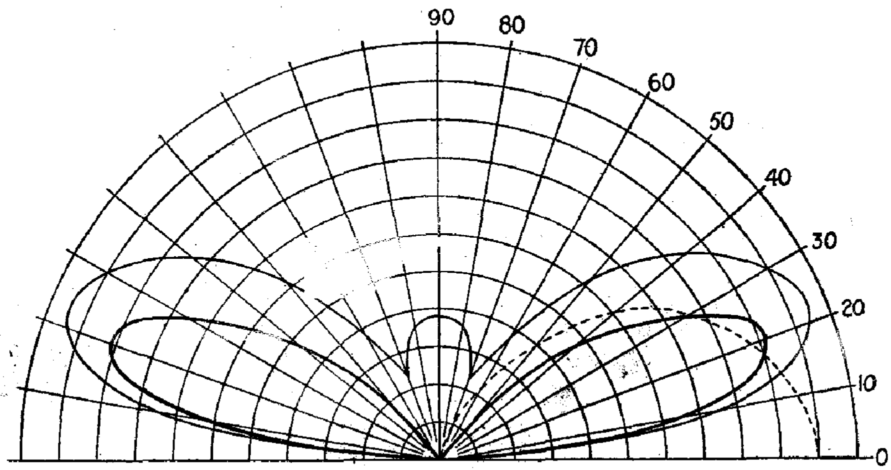


FIG. 32, CHAP. XV.—Vertical polar diagram of aerial over ground, $\sigma = 4 \times 10^8$, $\kappa = 20$, ($f = 10^8$, $h = \frac{\lambda}{4}$).

(ii) The foregoing theory assumes that at the boundary between the air and ground, the wave front is a plane surface. This incorrect assumption does not lead to significant error in the case of horizontal polarization, but with respect to vertically polarized waves, the field radiated along the surface of the earth is not absolutely zero as the simplified theory indicates. According to certain physicists, a vertical aerial at ground level gives rise to a surface wave additional to that derived from the simple radiation theory, but this view is not unreservedly accepted. Its protagonists agree that if this surface wave does exist, it suffers very heavy attenuation within a few wavelengths from the source, and need not be taken into account in long distance H/F and V. H/F communication.

TRANSMISSION LINES

Theory of transmission line

66. In Chapter VII reference is made to the propagation of electro-magnetic waves along a conductor such as a transmitting aerial. It is now necessary to enter somewhat more thoroughly into the theory of electro-magnetic waves on transmission lines such as the radio-frequency feeder lines used for supplying power from a transmitter to an aerial array, or from an aerial array to a radio receiver. The complete theory is also applicable to telephone and voice-frequency L/T lines. Before dealing with the mathematical theory, the physical aspect will be discussed.

67. Consider a transmission line consisting of a pair of parallel wires of high conductivity, perfectly insulated from and at a considerable height above the earth. Let these be connected to a battery by means of a reversing switch S as shown in fig. 33. A rapid reversal of the switch S is then equivalent to the application of an alternating E.M.F. having a perfectly flat-topped waveform. If the switch is closed at a given instant, so that the point A is at a positive potential with respect to the point B, an electric field will be set up between these points. The field does not however appear instantaneously at all points along the line, we may in fact consider the battery continuously to generate lines of electric force. Thus, if a single line of force appears between A and B when the switch is closed, and new lines are constantly being generated, the second line repels the first, causing the latter to travel along between the wires. As lines of electric force (unless closed upon themselves) must terminate upon electric charges, the movement of the electric lines implies the existence of moving electric charges, i.e. an electric current in the wires themselves. Thus, associated with the moving lines of electric force, we have a magnetic field consisting of a number of closed magnetic lines forming concentric circles round each conductor. The direction of the magnetic field relative to the direction of the electric field and the current is found by the first law of electro-dynamics.

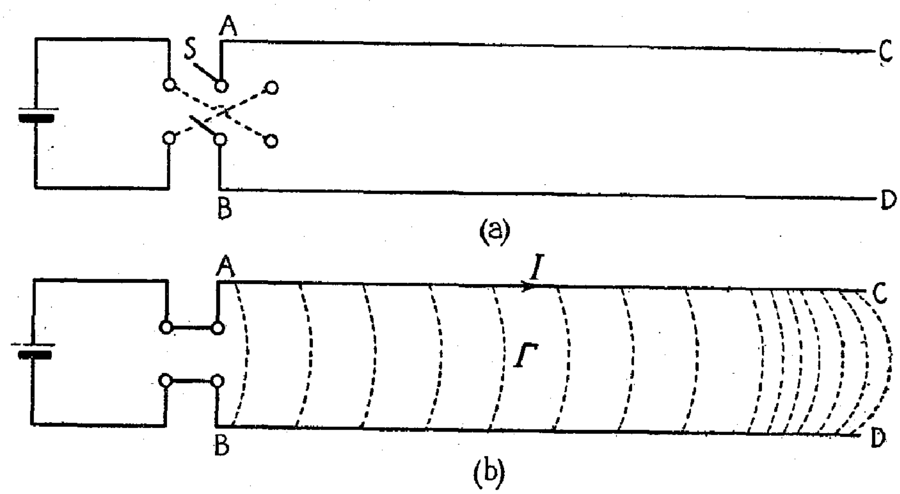


FIG. 33, CHAP. XV.—Electric field between parallel wires.

68. Now consider what happens when the travelling electric flux reaches the end of the line remote from the battery. If the wires are on open circuit, the lines of electric force can travel no further, and must tend to "pile up" at the points C D. In being brought to rest, however, they set up a magnetic field of opposite polarity to the original, and the growth of this field in turn recreates new lines of electric force. These lines now travel back towards the battery. This phenomenon may be summarized by the statement that on arrival at the open-circuited end of a transmission line, the electric field is reflected without change of phase, while the magnetic field is reflected with a phase change of 180°. At the moment of reversal, the magnetic field strength must fall to zero, and the electric field strength is doubled. If the remote end of the line is closed upon itself, forming what is called a short-circuited line, the reflection process is somewhat different. Instead of tending to pile up at the end, the electric lines must gradually collapse. In collapsing, however, they give rise to an additional magnetic field which travels on round the short-circuited end of the conductor. This magnetic field in turn recreates the electric field as before but with reverse polarity. At the exact instant at which the electric field is zero, the magnetic field strength is doubled. If the remote end of the line is connected to an impedance, partial reflection will occur, unless the terminal impedance has a particular nature and magnitude which will be dealt with later.

General equations for line current and voltage

69. Although the foregoing physical aspect enables one to form a crude mental picture of the process of reflection it is necessary to enter somewhat more deeply into the effects of the line constants upon the mechanism of propagation. The line constants are first, the resistance, R (ohms per unit length), second, the inductance, L (henries per unit length), third, the capacitance, C (farads per unit length), between the two lines (or between line and earth in an " earth return " circuit), and fourth, the leakage conductance G (siemens, or mhos, per unit length). The resistance and inductance are measured per unit length of line, not per unit length of wire. It will easily be seen that in an element of line of length dx, the resistance will be $R.dx$, the inductance $L.dx$, the capacitance $C.dx$, and the leakage conductance $G.dx$ (fig. 34). Suppose, then, that an E.M.F. is applied to one end of the line, which will be called the input, or sending end, setting up at a

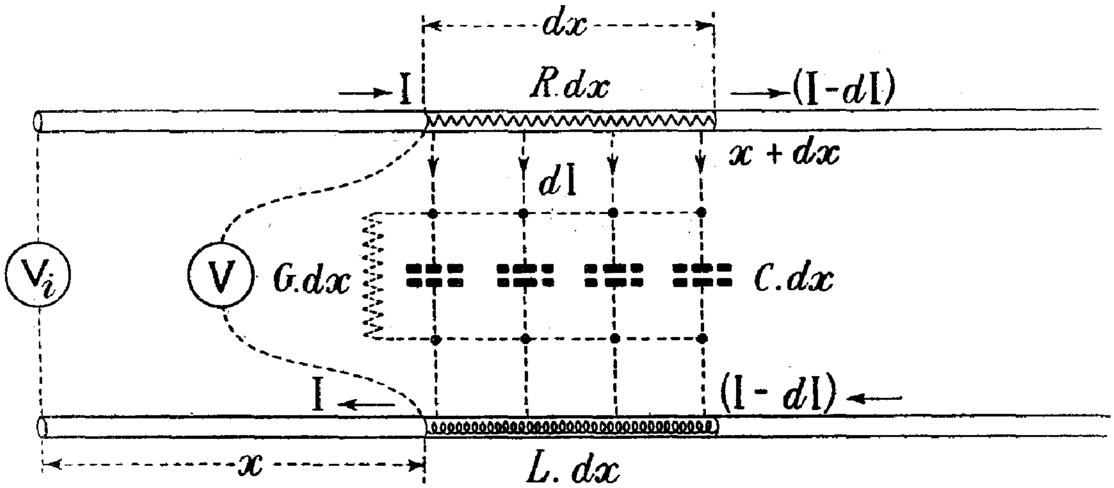


FIG. 34, CHAP. XV.—Notation used in transmission line theory.

point distant x centimetres along the line a P.D., $\mathbf{V}$. If we now take an elementary length of line dx extending from x to $x + dx$, the P.D. between x and $x + dx$ will be $- d\mathbf{V}$, where

$$- d\mathbf{V} = (R.dx)\, \mathbf{I} + (L.dx)\, \frac{d\mathbf{I}}{dt}.$$

If the current in the line at the point x is $\mathbf{I}$, it will be $\mathbf{I} - d\mathbf{I}$ at $x + dx$, owing to the element of current $d\mathbf{I}$ which flows in the capacitance $C.\ dx$ and leakage conductance $G.\ dx$. It is easily seen that

$$- d\mathbf{I} = (G.dx)\, \mathbf{V} + (C.dx)\, \frac{d\mathbf{V}}{dt}.$$

Hence the rate of change of $\mathbf{V}$ and $\mathbf{I}$, with respect to the distance from the sending end, is

$$-\frac{d\mathbf{V}}{dx} = R\mathbf{I} + L\frac{d\mathbf{I}}{dt}$$

$$-\frac{d\mathbf{I}}{dx} = G\mathbf{V} + C\frac{d\mathbf{V}}{dt}.$$

Solution for sinusoidal conditions

70. The above expressions are perfectly general, and subsequent work will be considerably simplified if the applied E.M.F. is considered to be sinusoidal. Under these conditions instead of the above equations we may write

$$-\frac{d\mathbf{V}}{dx} = (R + j\omega L)\,\mathbf{I} \qquad \text{(1)}$$

$$-\frac{d\mathbf{I}}{dx} = (G + j\omega C)\,\mathbf{V} \qquad \text{(2)}$$

because to a sinusoidal E.M.F., each unit length of line offers a vector impedance $R + j\omega L$, while shunted across each unit length we have a vector admittance $G + j\omega C$. Equations 1 and 2 are the fundamental basis of the theory of the transmission line. In developing the latter, we must first separate the variables **V** and **I**; to do this differentiate equation 1 with respect to x :—

$$-\frac{d^2\mathbf{V}}{dx^2} = (R + j\omega L)\,\frac{d\mathbf{I}}{dx}.$$

Substituting for $\frac{d\mathbf{I}}{dx}$ from equation (2)

$$\frac{d^2\mathbf{V}}{dx^2} = (R + j\omega L)\,(G + j\omega C)\,\mathbf{V}$$

$$= P^2\mathbf{V} \qquad \text{(3)}$$

In a similar manner we obtain

$$\frac{d^2\mathbf{I}}{dx^2} = P^2\mathbf{I} \qquad \text{(4)}$$

71. These equations define $P = \sqrt{(R + j\omega L)\,(G + j\omega C)} = \alpha + j\beta$. It will be observed that P is complex and possesses the dimensions $\sqrt{\frac{\text{ohms}}{\text{length}} \times \frac{\text{siemens}}{\text{length}}} = \frac{1}{\text{length}}$, so that quantities like Pl, Px, etc., are mere numbers. The complex quantity P is called the transfer constant of the line, and consists of a real part α called the attenuation constant, and an imaginary portion β called the wavelength constant, or latterly, the phase constant. Equations 3 and 4 are standard forms and the solutions are known to be

$$\mathbf{V} = \mathbf{M}_1\,\varepsilon^{-Px} + \mathbf{N}_1\,\varepsilon^{Px} \qquad \text{(5)}$$

$$\mathbf{I} = \mathbf{M}_2\,\varepsilon^{-Px} + \mathbf{N}_2\,\varepsilon^{Px} \qquad \text{(6)}$$

where $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{N}_1$, $\mathbf{N}_2$, are quantities which depend upon the terminal conditions of the line, and are not entirely independent of each other. Since ε^n is a mere number, $\mathbf{M}_1$, $\mathbf{M}_2$, etc., must be vectors and are therefore printed in Clarendon. It will be shown that $\mathbf{M}_1 = \mathbf{M}_2\,\mathbf{z}_0$, $\mathbf{N}_1 = -\,\mathbf{M}_2\,\mathbf{z}_0$ where

$$\mathbf{z}_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}.$$

Relation between $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{N}_1$, $\mathbf{N}_2$

72. If the values of **V** and **I** given in equations (5) and (6) are inserted in equation (1) we obtain

$$P\left(\mathbf{M}_1\,\varepsilon^{-Px} - \mathbf{N}_1\,\varepsilon^{Px}\right) = (R + j\omega L)\left(\mathbf{M}_2\,\varepsilon^{-Px} + \mathbf{N}_2\,\varepsilon^{Px}\right) \qquad \text{(7a)}$$

and if inserted in equation (2)

$$P\left(\mathbf{M}_2\,\varepsilon^{-Px} - \mathbf{N}_2\,\varepsilon^{Px}\right) = (G + j\omega C)\left(\mathbf{M}_1\,\varepsilon^{-Px} + \mathbf{N}_1\,\varepsilon^{Px}\right). \quad \ldots \quad (7b)$$

On multiplying across by $\dfrac{P}{G + j\omega C}$

$$P\left(\mathbf{M}_1\,\varepsilon^{-Px} + \mathbf{N}_1\,\varepsilon^{Px}\right) = \frac{P^2}{G + j\omega C}\left(\mathbf{M}_2\,\varepsilon^{-Px} - \mathbf{N}_2\,\varepsilon^{Px}\right)$$

$$= (R + j\omega L)\left(\mathbf{M}_2\,\varepsilon^{-Px} - \mathbf{N}_2\,\varepsilon^{Px}\right). \quad \ldots \quad (8)$$

Adding equations (7a) and (8)

$$2P\,\mathbf{M}_1\,\varepsilon^{-Px} = (R + j\omega L) \times 2\,\mathbf{M}_2\,\varepsilon^{-Px}$$

$$\therefore \mathbf{M}_1 = \mathbf{M}_2\,\frac{R + j\omega L}{P}$$

$$= \mathbf{M}_2\sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \mathbf{M}_2\,\mathbf{z}_0.$$

Subtracting (8) from (7a)

$$-\,2P\,\mathbf{N}_1\,\varepsilon^{Px} = +\,(R + j\omega L) \times 2\,\mathbf{N}_2\,\varepsilon^{Px}$$

$$\therefore \mathbf{N}_1 = -\,\mathbf{N}_2\,\frac{R + j\omega L}{P}$$

$$= -\,\mathbf{N}_2\,\mathbf{z}_0.$$

The quantity $\mathbf{z}_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$ is of great importance; it is termed the characteristic impedance or surge impedance of the line.

Introduction of hyperbolic functions

73. Equations (5) and (6) may now be written

$$\mathbf{V} = \mathbf{M}_1\,\varepsilon^{-Px} + \mathbf{N}_1\,\varepsilon^{Px} \quad \ldots \quad (9)$$

$$\mathbf{I} = \frac{\mathbf{M}_1}{\mathbf{z}_0}\,\varepsilon^{-Px} - \frac{\mathbf{N}_1}{\mathbf{z}_0}\,\varepsilon^{Px}. \quad \ldots \quad (10)$$

It is now convenient to introduce hyperbolic functions, writing

$$\cosh Px + \sinh Px = \varepsilon^{Px}$$

$$\cosh Px - \sinh Px = \varepsilon^{-Px}$$

so that (9) and (10) become

$$\mathbf{V} = \mathbf{M}_1\,(\cosh Px - \sinh Px) + \mathbf{N}_1\,(\cosh Px + \sinh Px)$$

$$= (\mathbf{M}_1 + \mathbf{N}_1)\cosh Px - (\mathbf{M}_1 - \mathbf{N}_1)\sinh Px, \quad \ldots \quad (11)$$

$$\mathbf{I} = \frac{1}{\mathbf{z}_0}\left[(\mathbf{M}_1 - \mathbf{N}_1)\cosh Px - (\mathbf{M}_1 + \mathbf{N}_1)\sinh Px\right] \quad \ldots \quad (12)$$

Note that we have reduced the number of quantities depending upon the terminal conditions to two. Provided $\mathbf{M}_1$ and $\mathbf{N}_1$ can be determined we are able to obtain complete information regarding the current and voltage distribution in the line.

Equations for infinite line

74. The simplest problem to consider, and one of great importance because it brings out the physical signification of $\mathbf{z}_0$ and P, is a line of infinite length to which a known voltage $\mathbf{V}_i$ (i for "input") is applied to the sending end. At the point $x = 0$ we have $\mathbf{V} = \mathbf{V}_i$; inserting known quantities in equation (11)

$$\mathbf{V}_i = (\mathbf{M}_1 + \mathbf{N}_1)\, cosh\, 0 - (\mathbf{M}_1 - \mathbf{N}_1)\, sinh\, 0$$
$$(cosh\, 0 = 1,\ sinh\, 0 = 0)$$
$$\mathbf{V}_i = \mathbf{M}_1 + \mathbf{N}_1$$

On the other hand, it is obvious from physical reasons that as we go further from the sending end both $\mathbf{V}$ and $\mathbf{I}$ become smaller, and ultimately when $x \to \infty$, $\mathbf{V} \to 0$, $\mathbf{I} \to 0$. But when $x \to \infty$, both $cosh\, Px$ and $sinh\, Px$ approach the value $\frac{1}{2}\varepsilon^{Px}$, and therefore

$$\mathbf{V}_{(x \to \infty)} = \Big[(\mathbf{M}_1 + \mathbf{N}_1) - (\mathbf{M}_1 - \mathbf{N}_1)\Big]\frac{\varepsilon^{Px}}{2} = 0.$$

Now $\frac{1}{2}\varepsilon^{Px}$ is not equal to zero, therefore

$$\mathbf{M}_1 + \mathbf{N}_1 - (\mathbf{M}_1 - \mathbf{N}_1) = 0$$
$$\therefore \mathbf{N}_1 = 0.$$

But $\qquad \mathbf{M}_1 + \mathbf{N}_1 = \mathbf{V}_i$

$$\therefore \mathbf{V}_i = \mathbf{M}_1.$$

Inserting in (11)

$$\begin{aligned}\mathbf{V} &= \mathbf{M}_1\, cosh\, Px - \mathbf{M}_1\, sinh\, Px \\ &= \mathbf{V}_i\,(cosh\, Px - sinh\, Px) \\ &= \mathbf{V}_i\,\varepsilon^{-Px}, \end{aligned} \qquad (13)$$

and in (12),

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}_i}{\mathbf{z}_0}(cosh\, Px - sinh\, Px) \\ &= \frac{\mathbf{V}_i}{\mathbf{z}_0}\varepsilon^{-Px}. \end{aligned} \qquad (14)$$

Magnitudes of α and β

75. As already stated $P\sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$. It follows that

$$RG + j\omega CR + j\omega LG - \omega^2 LC = \alpha^2 + 2j\alpha\beta - \beta^2$$
$$\alpha^2 - \beta^2 = RG - \omega^2 LC \qquad (15a)$$
$$2\alpha\beta = \omega\,(CR + LG)$$
$$(RG - \omega^2 LC)^2 = \alpha^4 - 2\alpha^2\beta^2 + \beta^4$$
$$\omega^2\,(CR + LG)^2 = 4\alpha^2\beta^2$$
$$\alpha^2 + \beta^2 = \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(CR + LG)^2} \qquad (15b)$$

From equations 15a and 15b we obtain

$$\alpha = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2 + \omega^2L^2)\,(G^2 + \omega^2C^2)} + (GR - \omega^2LC)\right\}} \quad .. \quad (15c)$$

$$\beta = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2 + \omega^2L^2)\,(G^2 + \omega^2C^2)} - (GR - \omega^2LC)\right\}}. \quad .. \quad (15d)$$

Physical significance of α and β

76. Equation 13 may be written

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_i\, \varepsilon^{-(\alpha + j\beta)x} \\ &= \mathbf{V}_i\, \varepsilon^{-\alpha x}\, \varepsilon^{-j\beta x} \\ &= \left(\mathbf{V}_i\, \varepsilon^{-\alpha x}\right)(\cos \beta x - j \sin \beta x). \end{aligned}$$

The portion within the first pair of brackets may be called the amplitude factor. It indicates that the amplitude $\mathscr{V}$ of the voltage at x is equal to the amplitude $\mathscr{V}_i$ of the input voltage divided by $\varepsilon^{\alpha x}$. The factor within the second pair of brackets is a vector operator of unit magnitude. Its presence signifies that the phase of $\mathbf{V}$ lags behind that of $\mathbf{V}_i$ by an angle βx. Thus if

$$v_i = \mathscr{V}_i \cos(\omega t + \varphi)$$

$$v_x = \frac{\mathscr{V}_i}{\varepsilon^{\alpha x}} \cos(\omega t + \varphi - \beta x).$$

In a very long or infinite line, therefore, the voltage amplitude at a distance x from the input end decreases exponentially by the factor $\varepsilon^{-\alpha x}$, while the phase angle lags behind the phase at the sending end by an angle βx. At a distance such that $\beta x = 2\pi$ the line voltage is in phase with the supply voltage. Similarly if $\beta x = 4\pi$, 6π, etc., in fact, the line voltage is in phase with that at the sending ends at all points where $\beta x = 2\pi n$, and n is an integer.

Physical significance of $\mathbf{z}_0$.

77. We may now consider the current in an infinite line. At the sending end, where $x = 0$, let it be $\mathbf{I}_i$. From equation (14)

$$\mathbf{I}_i = \frac{\mathbf{V}_i}{\mathbf{z}_0}\varepsilon^0 = \frac{\mathbf{V}_i}{\mathbf{z}_0} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16a)$$

Hence $\mathbf{z}_0$ is the quotient of voltage and current at the input end of an infinite line, and is therefore the input impedance of such a line. It also follows that equation 14 may be written

$$\mathbf{I}_x = \mathbf{I}_i\, \varepsilon^{-Px} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (17)$$

This equation is of exactly the same form as equation 13 and may be interpreted in the same way, i.e. in passing along the line the current is attenuated and its phase delayed just as in the case of the voltage. Further, at any point in the line

$$\frac{\mathbf{V}_x}{\mathbf{I}_x} = \mathbf{z}_0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16b)$$

thus $\mathbf{z}_0$ is the ratio of voltage to current at any point in the line.

Short-circuited lines.

78. We have now shown the physical meanings of the quantities α, β and $\mathbf{z}_0$, and may apply these to more practical cases, e.g. a line of finite length terminated by an impedance of some kind. Consider a length of line l which is short-circuited at the output or receiving end. Let a voltage $\mathbf{V}_i$ be applied at the point $x = 0$.

Then the following data are known :—

$$\text{at } x = 0, \mathbf{V} = \mathbf{V}_0 = \mathbf{V}_i$$
$$\text{at } x = l, \mathbf{V} = \mathrm{V}_l = 0.$$

Inserting these conditions in equation (11)

$$\mathbf{V}_i = \mathbf{M}_1 + \mathbf{N}_1$$
$$\mathbf{V}_l = 0 = (\mathbf{M}_1 + \mathbf{N}_1)\, cosh\ Pl - (\mathbf{M}_1 - \mathbf{N}_1)\, sinh\ Pl$$
$$= \mathbf{V}_i\, cosh\ Pl - (\mathbf{M}_1 - \mathbf{N}_1)\, sinh\ Pl$$
$$\therefore \mathbf{M}_1 - \mathbf{N}_1 = \frac{\mathbf{V}_i\, cosh\ Pl}{sinh\ Pl}$$
$$= \mathbf{V}_i\, coth\ Pl \quad \ldots \quad (18)$$

so that equation 11 may be written

$$\mathbf{V} = \mathbf{V}_i\,(cosh\ Px - coth\ Pl\ sinh\ Px)$$
$$= \mathbf{V}_i \frac{sinh\ P\,(l - x)}{sinh\ Pl}$$

and equation 12 becomes

$$\mathbf{I} = \frac{\mathbf{V}_i}{\mathbf{z}_0}(coth\ Pl\ cosh\ Px - sinh\ Px)$$
$$= \frac{\mathbf{V}_i}{\mathbf{z}_0} \cdot \frac{cosh\ P\,(l - x)}{sinh\ Pl}.$$

Now at the input end, $x = 0$. The current entering the line is therefore

$$\mathbf{I}_i = \frac{\mathbf{V}_i}{\mathbf{z}_0} \cdot \frac{cosh\ Pl}{sinh\ Pl}$$
$$= \frac{\mathbf{V}_i}{\mathbf{z}_0\, tanh\ Pl} \quad \ldots \quad (19)$$

Thus we have a most important result, namely that the input impedance of a length l of line short-circuited at the end remote from the input terminals, is $\mathbf{z}_0\ tanh\ Pl$.

Open lines

79. Another case of interest is that of a line of length l, with the output end on open circuit. At the input end $x = 0$, $\mathbf{V} = \mathbf{V}_i$, while at the output end $x = l$, $\mathbf{I} = 0$.

From equation (11)

$$\mathbf{V}_i = \mathbf{M}_1 + \mathbf{N}_1$$
$$0 = \frac{1}{\mathbf{z}_0}\{(\mathbf{M}_1 - \mathbf{N}_1)\, cosh\ Pl - (\mathbf{M}_1 + \mathbf{N}_1)\, sinh\ Pl$$
$$\mathbf{M}_1 - \mathbf{N}_1 = (\mathbf{M}_1 + \mathbf{N}_1)\frac{sinh\ Pl}{cosh\ Pl}$$
$$= \mathbf{V}_i\, tanh\ Pl \quad \ldots \quad (20)$$

and the line equations become

$$\mathbf{V}_x = \mathbf{V}_i \cosh Px - \mathbf{V}_i \tanh Pl \sinh Px$$

$$\mathbf{I}_x = \frac{\mathbf{V}_i}{\mathbf{z}_0}\{\tanh Pl \cosh Px - \sinh Px\}$$

or

$$\mathbf{V}_x = \mathbf{V}_i \frac{\cosh P\,(l-x)}{\cosh Pl}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_i}{\mathbf{z}_0}\ \frac{\sinh P\,(l-x)}{\cosh Pl}$$

The input current is obtained by putting $x = 0$, hence

$$\mathbf{I}_i = \frac{\mathbf{V}_i}{\mathbf{z}_0}\ \frac{\sinh Pl}{\cosh Pl}$$

$$= \frac{\mathbf{V}_i}{\mathbf{z}_0 \coth Pl} \qquad \text{(21)}$$

Thus the input impedance of a length l of open-circuited line is $\mathbf{z}_0 \coth Pl$.

Line terminated by finite impedance

80. Having cleared the air by these preliminary investigations we arrive at the most important practical case, namely, a line of finite length l, terminated by a finite impedance of $\mathbf{z}_r$ ohms, its nature being unspecified. As before it is known that $\mathbf{V} = \mathbf{V}_i$ at $x = 0$. At the other end, where $x = l$, the current and voltage will be denoted by $\mathbf{I}_r$ and $\mathbf{V}_r$ (r for " receiving "). Then $\mathbf{I}_r$ is the current through $\mathbf{z}_r$ due to the P.D. $\mathbf{V}_r$, and

$$\mathbf{I}_r = \frac{\mathbf{V}_r}{\mathbf{z}_r}.$$

Putting $\mathbf{V} = \mathbf{V}_i$ when $x = 0$,

$$\mathbf{M}_1 + \mathbf{N}_1 = \mathbf{V}_i$$

Since $\mathbf{V}_r = \mathbf{z}_r\mathbf{I}_r$ it follows that when $x = l$, equations (11) and (12) become

$$\mathbf{V}_i \cosh Pl - (\mathbf{M}_1 - \mathbf{N}_1) \sinh Pl = \frac{\mathbf{z}_r}{\mathbf{z}_0}\left\{(\mathbf{M}_1 - \mathbf{N}_1) \cosh Pl - \mathbf{V}_i \sinh Pl\right\}$$

$$\mathbf{V}_i \left\{\cosh Pl + \frac{\mathbf{z}_r}{\mathbf{z}_0} \sinh Pl\right\} = (\mathbf{M}_1 - \mathbf{N}_1)\left\{\sinh Pl + \frac{\mathbf{z}_r}{\mathbf{z}_0} \cosh Pl\right\}$$

$$\mathbf{M}_1 - \mathbf{N}_1 = \frac{\mathbf{V}_i \left\{\cosh Pl + \frac{\mathbf{z}_r}{\mathbf{z}_0} \sinh Pl\right\}}{\sinh Pl + \frac{\mathbf{z}_r}{\mathbf{z}_0} \cosh Pl} \qquad \text{(22)}$$

hence for these conditions, equations (13) and (14) become

$$\mathbf{V} = \mathbf{V}_i \frac{\mathbf{z}_r \cosh P\,(l-x) + \mathbf{z}_0 \sinh P\,(l-x)}{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl} \qquad \text{(23)}$$

and

$$\mathbf{I} = \frac{\mathbf{V}i}{\mathbf{z}_0}\ \frac{\mathbf{z}_r \sinh P\,(l-x) + \mathbf{z}_0 \cosh P\,(l-x)}{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl} \qquad \text{(24)}$$

Equations (23) and (24) give the voltage and current at any distance x along the line. For many purposes we require to know only those at the input and output ends respectively. Putting $x = l$ in (23) and (24,)

$$\mathbf{V}_r = \mathbf{V}_i \frac{\mathbf{z}_r}{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl} \quad \ldots \quad (25)$$

$$\mathbf{I}_r = \mathbf{V}_i \frac{1}{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl} \quad \ldots \quad (26)$$

while putting $x = 0$ in (24) gives

$$\mathbf{I}_i = \frac{\mathbf{V}_i}{\mathbf{z}_0} \frac{\mathbf{z}_r \sinh Pl + \mathbf{z}_0 \cosh Pl}{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl} \quad \ldots \quad (27a)$$

The input impedance is therefore

$$\mathbf{z}_i = \mathbf{z}_0 \frac{\mathbf{z}_r \cosh Pl + \mathbf{z}_0 \sinh Pl}{\mathbf{z}_r \sinh Pl + \mathbf{z}_0 \cosh Pl} \quad \ldots \quad (27b)$$

Correctly terminated line

81. A very important case in practice is that which occurs when the terminating load $\mathbf{z}_r$ is equal to the surge impedance $\mathbf{z}_0$ of the line. When this is so, equation (24) becomes

$$\mathbf{I}_i = \frac{\mathbf{V}_i}{\mathbf{z}_0} \frac{\mathbf{z}_0 (\sinh Pl + \cosh Pl)}{\mathbf{z}_0 (\sinh Pl + \cosh Pl)}$$

$$= \frac{\mathbf{V}_i}{\mathbf{z}_0} \quad \ldots \quad (28a)$$

while

$$\mathbf{I}_r = \frac{\mathbf{V}_i}{\mathbf{z}_0 (\cosh Pl + \sinh Pl)}$$

$$= \frac{\mathbf{V}_i}{\mathbf{z}_0 \, \varepsilon^{Pl}}$$

$$= \frac{\mathbf{V}_i}{\mathbf{z}_0} \varepsilon^{-Pl}. \quad \ldots \quad (28b)$$

This is exactly the same expression as was found in paragraph 77 for the current $\mathbf{I}_x$ at a distance x from the input end of an infinitely long line. It follows then that if a line of finite length is terminated by an impedance equal to its surge impedance, all the energy reaching the output terminals of the line passes into the load impedance, which is usually the desired object. When the operating conditions are such that $\mathbf{z}_r = \mathbf{z}_0$ the line is said to be correctly terminated. When incorrectly terminated the whole of the received energy does not pass the output terminals, a portion being reflected back towards the input end. The importance of avoiding reflection in a transmission line may perhaps be emphasized by comparing it with an aerial. With a few exceptions, aerials are built up of conductors with free ends, so that reflection occurs, and the length (including the image in certain cases) is made electrically equal to a multiple of $\frac{\lambda}{2}$ so that stationary waves are set up in the aerial. By this means we obtain syn-phased currents over each half-wavelength of wire (approximately) as explained in the early paragraphs of this chapter. The energy supplied to the aerial is then partly radiated and partly degraded into heat. In a transmission line, however, the object is to convey as much energy as possible from one point (the transmitter) to another point (the load impedance), avoiding all unnecessary dissipation en route. For this conveyance to be highly efficient, then, the load impedance must be equal to the surge impedance of the line.

82. Returning now to the expression for the current in the load impedance in the case of a correctly terminated line, i.e.

$$\mathbf{I_r} = \frac{\mathbf{V_i}}{\mathbf{Z_0}} \varepsilon^{-Pl}$$

putting

$$P = \alpha + j\beta$$

$$\mathbf{I_r} = \frac{\mathbf{V_i}}{\mathbf{Z_0}} \varepsilon^{-\alpha l} \varepsilon^{-j\beta l}.$$

We have already seen that the magnitude of the attenuation constant α depends upon the leakage conductance G and resistance R, per unit length. If G, R and l are very small $\varepsilon^{-\alpha l}$ is very nearly unity and the received current becomes

$$\mathbf{I_r} = \frac{\mathbf{V_i}}{\mathbf{Z_0}} \varepsilon^{-j\beta}. \qquad (29)$$

At radio frequencies, α is given by the following approximate formula which is derived from equation (15c);

$$\alpha = \frac{R}{2Z_0} + \frac{GZ_0}{2},$$

and is always a very small quantity. Suppose the line to have a resistance of 20 ohms per mile and an insulation resistance of 5 megohms per mile. One mile is roughly $1{\cdot}6 \times 10^5$ centimetres, i.e. $R = 1{\cdot}2\ 10^{-4}$ ohms, $\frac{1}{G} = 5 = 10^6 \times 1{\cdot}6 \times 10^5$ ohms per centimetre, and $G = 1{\cdot}25 = 10^{-12}$ siemens per centimetre. If the surge impedance of the line is 500 ohms, the attenuation constant is

$$\alpha = \frac{1{\cdot}2}{2 \times 10^4 \times 500} + \frac{1{\cdot}25 \times 500}{10^{12} \times 2}$$

$$= 1{\cdot}203 \times 10^{-7}.$$

Radio-frequency feeders

83. In connecting an aerial array to its transmitting or receiving equipment, it is necessary to utilize a transmission line consisting of either a twin wire line or a concentric line. In either instance the length rarely exceeds a few hundred feet, and the line may be designed to have a very low attenuation constant. The theory may then be considerably simplified by assuming the attenuation to be negligible, i.e. that the line itself has negligible resistance and perfect insulation so that in the equation $P = \sqrt{(R + j\omega L)\ (G + j\omega C)}$, $R = 0$ and $G = 0$. Then $P = \alpha + j\beta = \sqrt{j\omega L \times j\omega C} = j\omega\sqrt{LC}$, and therefore $\alpha = 0$, $\beta = \omega\sqrt{LC}$. Similarly the equation $\mathbf{Z_0} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$ becomes $\mathbf{Z_0} = \sqrt{\frac{L}{C}}$. In these circumstances $\mathbf{Z_0}$ is not complex and therefore possesses no reactive component, i.e. the surge impedance is purely resistive, and may be denoted by Z_0. When its non-reactive nature is to be particularly stressed it will be denoted by R_0.

Twin wire feeders

84. The inductance of a pair of parallel wires of radius r, separated by a distance D, is

$$L = \frac{9{\cdot}2104}{10^9} \log_{10} \frac{D}{r} \text{ henries per centimetre,}$$

and the capacitance

$$C = \frac{1 \cdot 208}{10^{13} \log_{10} \frac{D}{r}} \text{ farads per centimetre,}$$

so that

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{9 \cdot 2104}{10^9} \log_{10} \frac{D}{r} \times \frac{10^{13}}{1 \cdot 208} \log_{10} \frac{D}{r}}$$

$$= 276 \log_{10} \frac{D}{r} \text{ ohms.}$$

The phase constant of the line is easily found :—

$$\beta = \omega\sqrt{LC}$$

$$= 2\pi f\sqrt{\frac{9 \cdot 2104}{10^9} \times \frac{1 \cdot 208}{10^{13}}}.$$

$$= \frac{2\pi f}{3 \times 10^{10}}$$

$$= \frac{2\pi f}{c}$$

where c is the natural constant equal to the velocity of electro-magnetic waves in free space. Since $\frac{f}{c} = \lambda$, where λ is the wavelength in free space

$$\beta = \frac{2\pi}{\lambda}.$$

Concentric feeders

85. A concentric feeder consists of an outer tubular conductor containing an inner conductor which may be either solid or tubular. If D is the internal diameter of the outer tube and d the external diameter of the inner conductor, its inductance and capacitance are given by the formulae

$$L = \frac{4 \cdot 605}{10^9} \log_{10} \frac{D}{d} \text{ henries per centimetre}$$

$$C = \frac{2 \cdot 416}{10^{13} \log_{10} \frac{D}{d}} \text{ farads per centimetre}$$

and, therefore,

$$Z_0 = 138 \log_{10} \frac{D}{d} \text{ ohms.}$$

It is easily shown that, as for the twin wire feeder,

$$\beta = \frac{2\pi}{\lambda}.$$

As stated in Chapter VII, where twin wire feeders are used, it is usual to arrange, if possible, that the surge impedance is 600 ohms. This implies that the ratio $\frac{D}{r} = 150$. For example, 18 s.w.g. wire has a diameter of ·048 inch, and gives a surge impedance of 600 ohms if spaced 3·6 inches apart. The surge impedance of a concentric feeder is usually about 60 to 100 ohms, and the copper losses are a minimum when $\frac{D}{d} = 3{\cdot}6$, i.e. when the surge impedance is 75 ohms. For ratios smaller than 2 the copper losses are very heavy, but they are not seriously increased by an increase of $\frac{D}{d}$ up to about 8.

Properties of lines of various lengths

86. (i) The input impedance of a length l of line, short-circuited at the output end, is $\mathbf{z}_i = \mathbf{z}_0 \, tanh\, Pl$. If the attenuation is negligible, $P = j\beta = j\frac{2\pi}{\lambda}$,

$$\mathbf{z}_i = \mathbf{z}_0 \, tanh\, j\frac{2\pi}{\lambda}l.$$

Since, however, $\mathbf{z}_0$ is a purely ohmic resistance of $\sqrt{\frac{L}{C}}$ ohms,

$$\mathbf{z}_i = \sqrt{\frac{L}{C}}\, tanh\, j\frac{2\pi}{\lambda}l$$

$$= j\sqrt{\frac{L}{C}}\, tan\frac{2\pi}{\lambda}l, \quad \ldots \quad (30)$$

i.e. $\mathbf{z}_i$ is purely reactive and may be either positive or negative. The graph of the magnitude Z_i of the input reactance, against the length l, is plotted in fig. 35. It is seen that at the point $l = 0$, $Z_i = 0$. As l increases Z_i assumes positive values, e.g. at $l = \frac{\lambda}{8}$, $Z_i = Z_0$, and increases until at $l = \frac{\lambda}{4}$, Z_i becomes infinite. In the range $l = 0$ to $l = \frac{\lambda}{4}$ then, the line behaves as an inductance, the value of which may lie anywhere between zero and infinity. Consequently, a

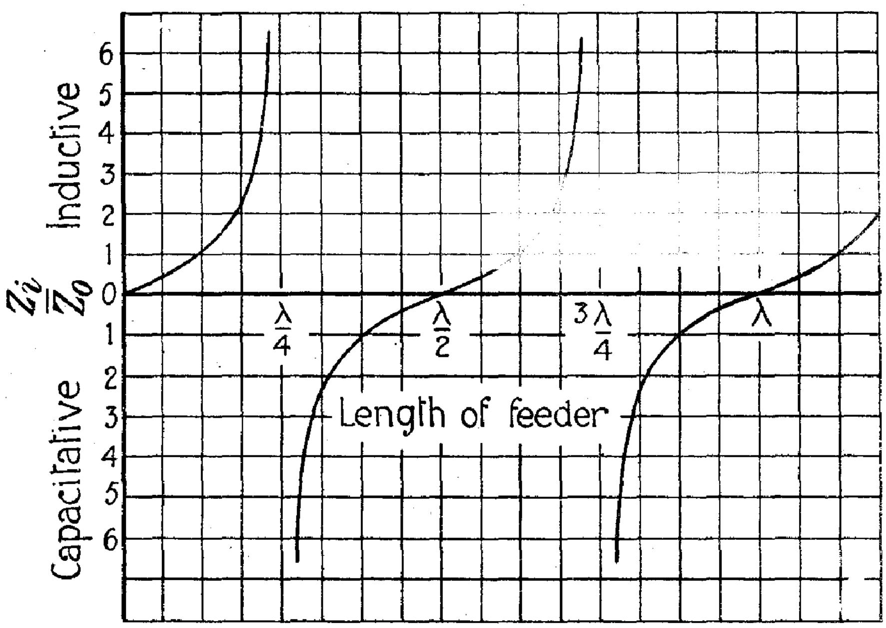


FIG. 35, CHAP. XV.—Reactance of short-circuited line.

length of line may be determined which will act as an inductance of any desired value for a given frequency. Suppose we desire a line to have an inductance L',

$$j\omega L' = j\,Z_0\,tan\,\frac{2\pi}{\lambda}l$$

$$tan\,\frac{2\pi}{\lambda}l = \frac{\omega L'}{Z_0}$$

$$\frac{2\pi}{\lambda}l = tan^{-1}\,\frac{\omega L'}{Z_0}$$

$$l = \frac{\lambda}{2\pi}\,tan^{-1}\,\frac{\omega L'}{Z_0}.$$

Example

Calculate the length of 600 ohms line which will act as an inductance of $5\mu H$ at a frequency of 5 M/cs.

$$\lambda = \frac{3 \times 10^8}{5 \times 10^6} = 60 \text{ metres}$$

$$l = \frac{60}{2\pi}tan^{-1}\,\frac{2\pi \times 5 \times 10^6 \times 5 \times 10^{-6}}{600}$$

$$= 9{\cdot}56\,tan^{-1}\,\frac{\pi}{12}$$

$$tan^{-1}\,\frac{\pi}{12} = 14^\circ\,41' \text{ or } 0{\cdot}25 \text{ radians}$$

$$l = 9{\cdot}56 \times 0{\cdot}25$$

$$= 2{\cdot}4 \text{ metres.}$$

In the range $\frac{\lambda}{4}$ to $\frac{\lambda}{2}$, $tan\,\frac{2\pi}{\lambda}l$ is negative and the reactance of the short-circuited line is capacitive, varying from infinity to zero. By a suitable choice of l, its reactance may be of any value whatever. If it is required to obtain a line of capacitance C' farads,

$$\frac{1}{j\omega C} = j\,Z_0\,tan\frac{2\pi}{\lambda}\,l$$

$$tan\,\frac{2\pi}{\lambda}l = -\,\frac{1}{\omega C' Z_0}$$

$$l = \frac{\lambda}{2\pi}tan^{-1}\left(-\,\frac{1}{\omega C' Z_0}\right)$$

Over the range $\frac{\lambda}{2}$ to λ the curve repeats the values in tne range 0 to $\frac{\lambda}{2}$ and so on.

(ii) The input impedance of a length of line having its output end free, is $Z_i = Z_0\,coth\,Pl$. For negligible attenuation this becomes

$$Z_i = -\,j\,Z_0\,cot\,\frac{2\pi}{\lambda}\,l. \qquad (31)$$

As before, then, the impedance is purely reactive; Z_i is plotted against l in fig. 36. By using a length of line less than $\frac{\lambda}{4}$ we may obtain a negative (capacitive) reactance of any value between $-\infty$ and 0 while lengths between $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$ behave inductively.

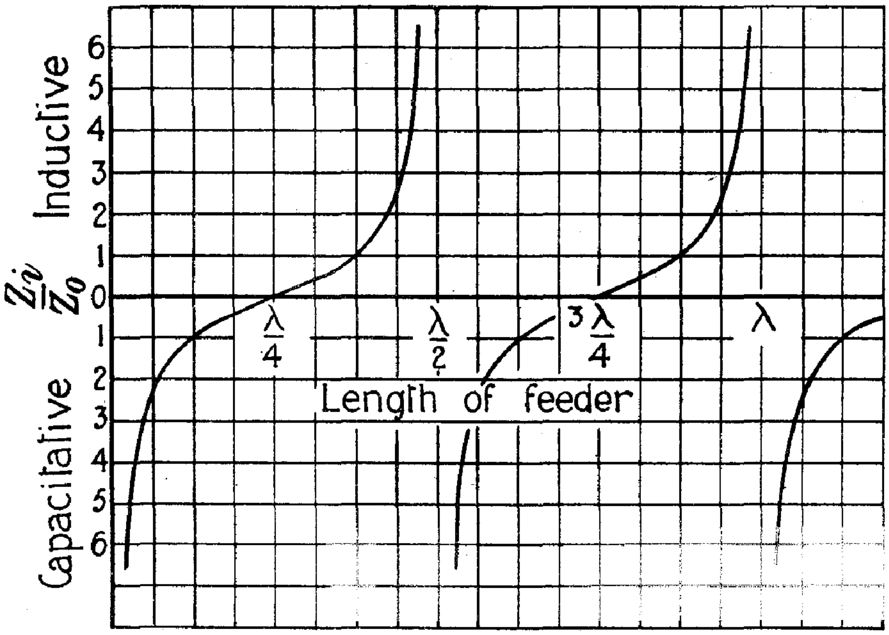


FIG. 36, CHAP. XV.—Reactance of open-circuited line.

(iii) Lengths of line are often used in this manner in matching an aerial array to a transmission line. It is obviously desirable, as a rule, to use the shortest possible lengths of wire, so that in practice a length less than $\frac{\lambda}{4}$ is generally employed. It must be short-circuited if required to act inductively, and on open circuit if required to act capacitively.

Properties of quarter-wave line

87. The properties of a line exactly $\frac{\lambda}{4}$ in length are of particular importance. The input impedance of a loss-free $\frac{\lambda}{4}$ line, terminated by a non-reactive impedance Z_r, is given by

$$Z_i = Z_0 \frac{Z_r \cos\frac{\pi}{2} + j Z_0 \sin\frac{\pi}{2}}{Z_0 \cos\frac{\pi}{2} + j Z_r \sin\frac{\pi}{2}}$$

$$= Z_0 \frac{j Z_0}{j Z_r}, \text{ because } \cos\frac{\pi}{2} = 0,\ \sin\frac{\pi}{2} = 1.$$

$$\therefore Z_i = \frac{Z_0^2}{Z_r}. \qquad \qquad (32)$$

This property of the $\frac{\lambda}{4}$ line is used for matching purposes. Suppose we have a 100 ohm load, fed from a 600 ohm line. Then $Z_r = 100$, $Z_0 = 600$, and if they are directly connected, reflection will occur at the termination. To avoid this, we may interpose a $\frac{\lambda}{4}$ length of feeder of such a spacing that its surge impedance Z_m is equal to $\sqrt{Z_0 Z_r}$, i.e. to $\sqrt{100 \times 600} = 245$ ohms. The 600 ohm line will then be correctly terminated, for the input impedance of the $\frac{\lambda}{4}$ line, terminated by 100 ohms, is $\frac{Z_m^2}{Z_r} = \frac{245^2}{100} = 600$ ohms.

Example

In the instance cited above, calculate the spacing of the $\frac{\lambda}{4}$ line in order that $Z_m = 245$ ohms, if the wire is 18 s.w.g.

Diameter of 18 s.w.g. wire is ·048 inch, i.e. $r = \cdot 024$.

$$Z_0 = 276 \log_{10} \frac{D}{r}$$

$$\log_{10} \frac{D}{r} = \frac{245}{276} = 0 \cdot 888$$

$$\text{Antilog } 0 \cdot 888 = 7 \cdot 727$$

$$\therefore \frac{D}{r} = 7 \cdot 727$$

$$D = 7 \cdot 727 \times \cdot 024$$

$$= 0 \cdot 185 \text{ inch.}$$

It is not practicable to space wires as closely as this, except possibly in the case of feeders connected to receiving aerials. A possible solution is a multiple-wire transformation feeder.

Properties of lines of length $\frac{n\lambda}{2}$.

88. We will now consider the input impedance of a length of line equal to some integral multiple of $\frac{\lambda}{2}$, terminated by a non-reactive impedance Z_r. Then

$$Z_i = Z_0 \frac{Z_r \cos \frac{2\pi}{\lambda} l + j Z_0 \sin \frac{2\pi}{\lambda} l}{Z_0 \cos \frac{2\pi}{\lambda} l + j Z_r \sin \frac{2\pi}{\lambda} l} \quad \ldots \quad (33)$$

Putting $l = n\frac{\lambda}{2}$, we see that Z_i depends upon whether n is even or odd. If n is odd, $\frac{2\pi}{\lambda} l = n\pi = 3\pi$, 5π, etc., and $\sin n\pi = 0$, $\cos n\pi = -1$. Hence $Z_i = Z_0 \times \frac{Z_r}{Z_0} = Z_r$. The magnitude of the voltage at the output terminals is

$$V_r = V_i \frac{Z_r}{j Z_0 \sin \frac{2\pi}{\lambda} l + Z_r \cos \frac{2\pi}{\lambda} l}$$

$$= V_i \frac{Z_r}{-Z_r}.$$

$$= -V_i.$$

Thus we have the important result that a length of line equal to an odd multiple of $\frac{\lambda}{2}$ is a perfect unity-ratio transformer, the output P.D. being equal in magnitude to the input voltage, with a phase difference of 180°. On the other hand if n is even we have $\sin n\pi = 0$, $\cos n\pi = +1$. Hence

$$V_r = V_i \frac{Z_r}{j Z_0 \sin n\pi + Z_r \cos n\pi}$$

$$= V_i \frac{Z_r}{Z_r}$$

$$= V_i.$$

Also $$Z_i = Z_0 \frac{Z_r + 0}{Z_0 + 0}$$

$$= Z_r$$

Thus, a length of line equal to an even multiple of $\frac{\lambda}{2}$ is a perfect unity-ratio transformer, the input and output P.D.'s being in phase.

Voltage distribution along a feeder

89. The voltage distribution along a feeder may be calculated from the formulae given in previous paragraphs. Taking a length of feeder terminated by a non-reactive impedance $Z_r = Z_0$, we may apply equation (13) of paragraph 74.

$$\mathbf{V}_x = \mathbf{V}_i\, \varepsilon^{-Px} = \mathbf{V}_i\, \varepsilon^{-j\frac{2\pi}{\lambda}x}.$$

Thus if $$V_i = \mathscr{V} \cos(\omega t + \varphi)$$

$$V_x = \mathscr{V} \cos\left(\omega t + \varphi - \frac{2\pi}{\lambda}x\right),$$

i.e. the amplitude of the voltage is the same all along the feeder because we have assumed the attenuation to be zero. The phase changes continuously, so that points $\frac{\lambda}{2}$ apart are in opposite phase. The input current is $\frac{\mathscr{V}}{Z_0} \cos(\omega t + \varphi)$ but at a distance x from the input end the current is $\frac{\mathscr{V}}{Z_0} \cos\left(\omega t + \varphi - \frac{2\pi}{\lambda}x\right)$. Since ammeters and voltmeters do not measure phase difference, such a meter will indicate the same R.M.S. current (or voltage) at all points along the line.

90. We will now consider a general case, in which the feeder is terminated by an impedance $m Z_0$ where m may have any positive finite value, either integral or fractional. Applying equations (23) to (27) of paragraphs 80 *et seq.*

$$\mathbf{V}_x = \mathbf{V}_i \left\{ \frac{m Z_0 \cos\frac{2\pi}{\lambda}(l - x) + j Z_0 \sin\frac{2\pi}{\lambda}(l - x)}{m Z_0 \cos\frac{2\pi}{\lambda}l + j Z_0 \sin\frac{2\pi}{\lambda}l} \right\}$$

$$= \mathbf{V}_i \left\{ \frac{m \cos\frac{2\pi}{\lambda}(l - x) + j \sin\frac{2\pi}{\lambda}(l - x)}{m \cos\frac{2\pi}{\lambda}l + j \sin\frac{2\pi}{\lambda}l} \right\} \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

We wish to find how $\mathbf{V}_x$ varies with x, and therefore need consider only the numerator of the bracketed portion. This is complex, and its modulus is $\sqrt{m^2\cos^2\theta + \sin^2\theta} = N(V_x)$, and $\theta = \frac{2\pi}{\lambda}(l - x)$.

91. The nature of the variation of V_x, at different points in the line, can therefore be obtained by plotting $N(V_x)$ against x. Its maxima and minima may also be obtained by the differential calculus. Differentiating $N(V_x)$ with respect to θ, and equating to zero, we find that maxima or minima are given by $(1 - m^2)\sin\theta\cos\theta = 0$.

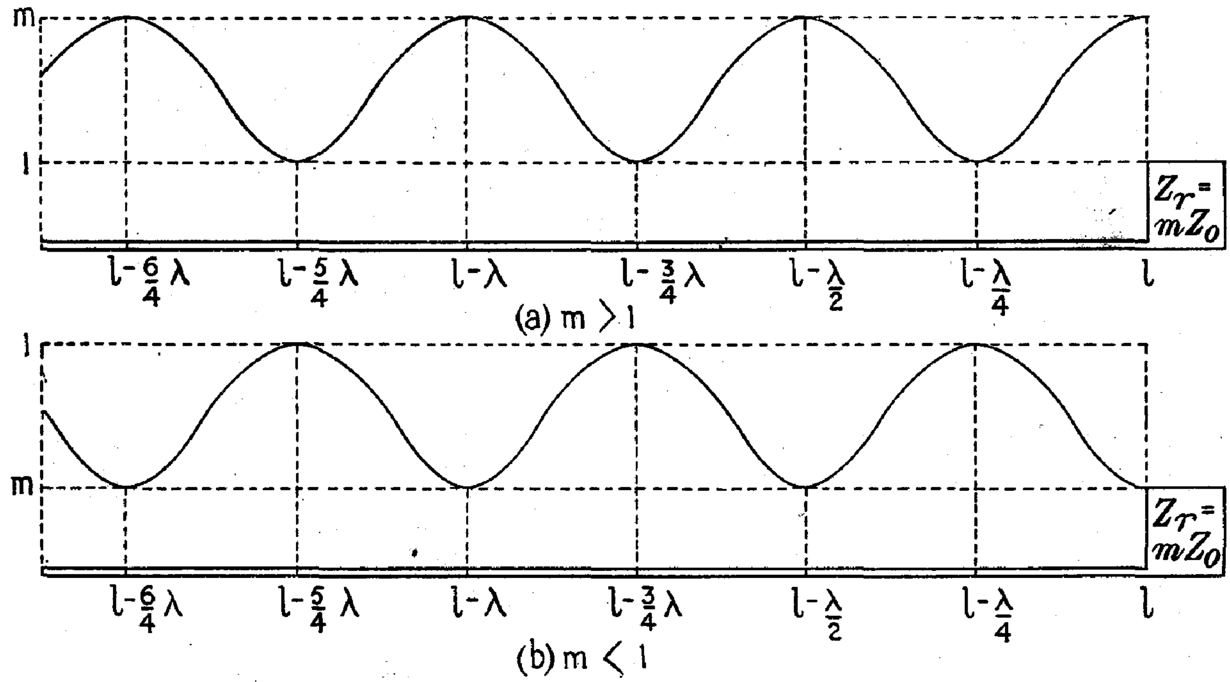


FIG. 37, CHAP. XV.—Location of voltage maxima for values of m greater and less than unity.

Unless $m = 1$, $\sin\theta\cos\theta$ must therefore equal zero, which is the case if θ is any multiple of $\frac{\pi}{2}$ radians. Hence the maxima or minima occur when $\theta = \frac{n}{2}\pi$ where n is any positive integer or zero. That is, when $\frac{2\pi}{\lambda}(l - x) = \frac{n}{2}\pi$, or $(l - x) = n\frac{\lambda}{4}$, or zero. It follows, therefore, that either a maximum or a minimum of voltage will occur at the termination, i.e. where $l = x$. If m is greater than unity it will be a maximum, if m is less than unity, a minimum. At a distance of $\frac{\lambda}{4}$ from the termination, there will be another turning point so that the voltage distribution will be either as in fig. 37a or fig. 37b, depending on whether $m > 1$ or $m < 1$. It will be seen that m is the ratio of the maximum to the minimum P.D. or *vice versa*. The current distribution along the feeder may be calculated in a similar manner. The resulting curves are very nearly sinusoidal but not exactly so, except in the case of short-circuited or free lines, because a terminal load will necessarily call for a feed current. The calculated current distribution for a 600 ohm line, for ratios $\frac{I_{min}}{I_{max}}$ from 0·1 to 0·9, are given in fig. 38.

Measurement of surge impedance

92. (i) If the operating frequency is sufficiently low, it is possible to determine the surge impedance of a line by actual measurement. Let the length of the line be l. The input impedance is first measured, with the receiving end on open circuit; let this be Z_f (f for "free"). The input impedance Z_c (c for "closed") with the receiving end on short-circuit is also found. Then

$$Z_f = Z_0 \, coth \, Pl$$

$$Z_c = Z_0 \, tanh \, Pl$$

from paragraphs 78 and 79.

It follows that

$$Z_f Z_c = Z_0 \, coth \, Pl \times Z_0 \, tanh \, Pl$$

$$= Z_0^2$$

$$\therefore Z_0 = \sqrt{Z_f Z_c}.$$

(ii) The following method has also been proposed. On erection, the line is extended for rather more than $\frac{\lambda}{8}$ past the proposed terminating point, and is then energized at the intended frequency, or a closely adjacent one, in such a manner that stationary waves are set up along the line. The wavelength on the line itself is obtained by observation of the minimum current at adjacent current nodes by means of an ammeter and transformer (*see* paragraphs 4 and 128). A $\frac{\lambda}{8}$ length of line is then removed from the free end and a calibrated variable condenser joined across the ends of the line in its place. The capacitance is varied as necessary until the current minima appear at the same points as before; when this is achieved, the capacitance is exactly equivalent to the length of line which was removed. Referring to paragraph 79, the impedance of a $\frac{\lambda}{8}$ length of open loss-free line is

$$Z_{\frac{\lambda}{8}} = -jZ_0 \, cot \, \frac{2\pi}{\lambda} l$$

$$= -jZ_0 \, cot \, \frac{2\pi}{\lambda} \frac{\lambda}{8}$$

$$= -jZ_0 \, cot \, \frac{\pi}{4}$$

$$= -jZ_0.$$

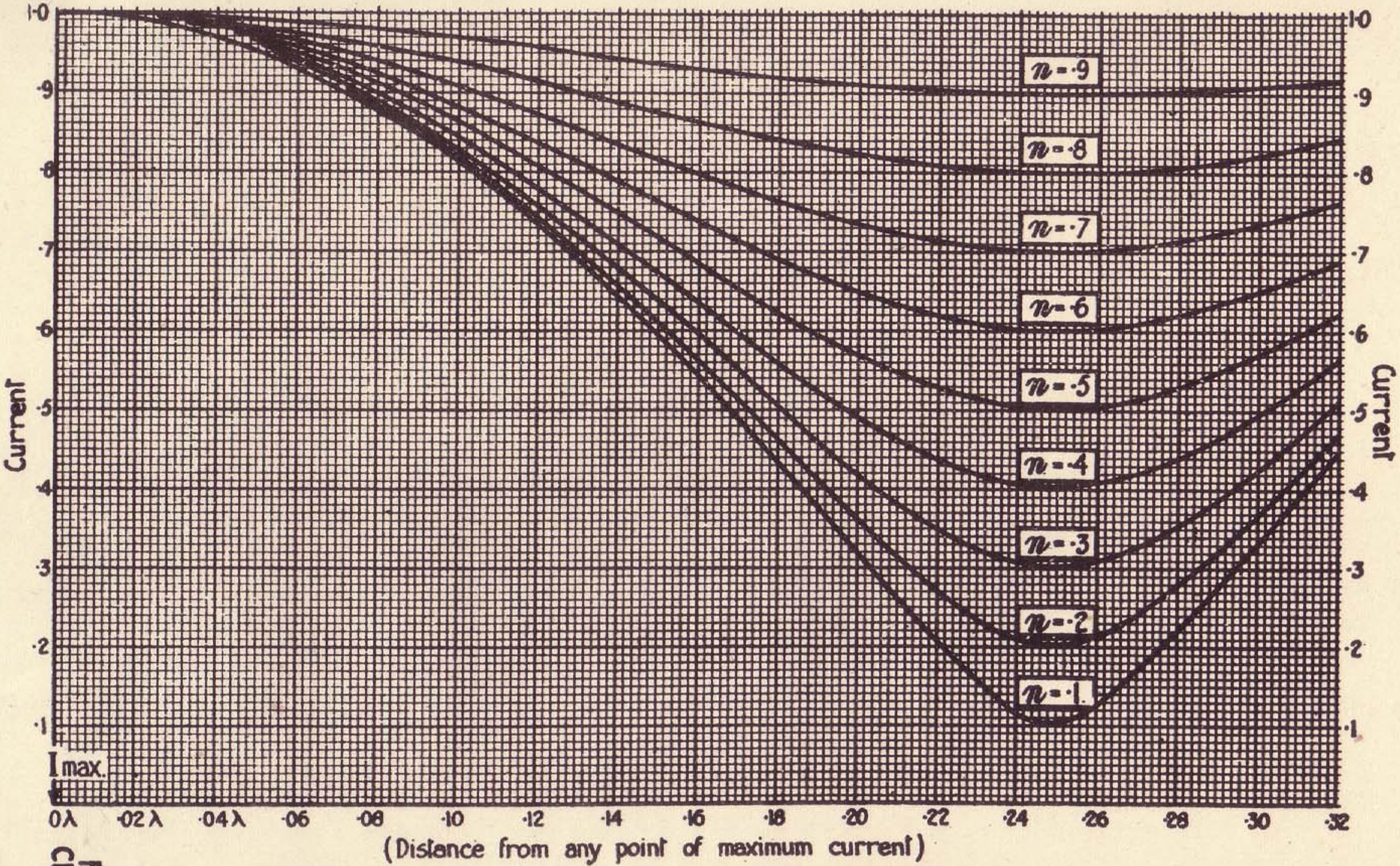
Since the capacitance C has exactly the same effect on the line as the impedance $Z_{\frac{\lambda}{8}}$ it follows that

$$\frac{1}{j\omega C} = Z_{\frac{\lambda}{8}}$$

$$\frac{1}{j\omega C} = -jZ_0$$

or

$$Z_0 = \frac{1}{\omega C}.$$



CURRENT DISTRIBUTION ALONG 600 OHM FEEDER

FIG. 38
CHAP. XV

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Radiation due to travelling wave

93. Although in most forms of aerial the arrangement is such that stationary waves are established along the wires, it must not be thought that this is an essential requirement for radiation to occur. The fact that radiation can and does occur from wires carrying travelling waves is of importance from two points of view. First, in the case of properly terminated feeder lines, considerable radiation will occur unless the lines are very close together, i.e. less than about $0 \cdot 05\lambda$. Second, it is possible to design aerial arrays for directional transmission and reception, the action depending entirely upon the radiating properties of a long, properly terminated wire. The directivity of such an aerial for receiving purposes depends upon the reciprocal properties mentioned in paragraph 3.

94. If a long straight wire is situated in free space and carries a travelling current wave, the radiation field set up by the current is easily calculated. Referring to fig. 39 consider a wire of length l and let the current at an origin O at the mid-point of the length be $\mathbf{I}_0 = \mathbf{I}\,\varepsilon^{j\omega t}$. At a

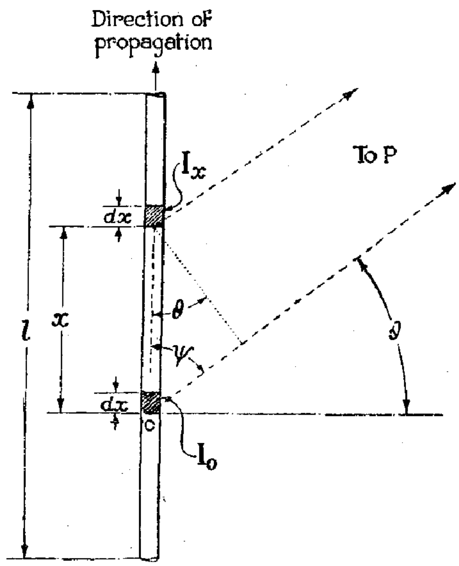


FIG. 39, CHAP. XV.—Wire carrying travelling wave.

point P, at a distance r ($>>l$) from the origin, and at an angle θ to the perpendicular through O, the field $d\gamma_0$ due to the current in a short length dx of conductor closely adjacent to O, may be found by treating the length dx as a hertzian doublet, giving

$$d\gamma_0 = \frac{60}{r}\,\frac{\pi}{\lambda}\cos\theta . dx \times \mathbf{I}\,\varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)} . \qquad \ldots \quad \ldots \quad \ldots \quad (35)$$

Consider another element of length dx at a distance x from the origin. Since the wire carries a travelling wave, the current $\mathbf{I}_x$ in this element will have the same amplitude as at the origin, but will be out of phase with it. If x is measured in the direction of propagation along the wire, $\mathbf{I}_x$ lags on $\mathbf{I}_0$ by $2\frac{\pi}{\lambda}x$ radians. Also, the point P is distant $(r - x\sin\theta)$ from the element, and the field due to the latter will be

$$d\gamma = \frac{60}{r}\,\frac{\pi}{\lambda}\cos\theta . dx \times \mathbf{I}_x\,\varepsilon^{j\left[\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}(r - x\sin\theta)\right]}$$

$$= \frac{60}{r}\,\frac{\pi}{\lambda}\cos\theta . dx \times \mathbf{I}_x\,\varepsilon^{j\left(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} r\right)}\,\varepsilon^{j\frac{2\pi}{\lambda}x\sin\theta} \qquad \ldots \quad \ldots \quad (36)$$

But $$\mathbf{I}_x = \mathbf{I}_0\, \varepsilon^{-j\frac{2\pi}{\lambda}x}$$

$$\therefore\ d\gamma = \frac{60}{r}\frac{\pi}{\lambda}\cos\theta\, \mathbf{I}_0\, \varepsilon^{j\left(\frac{\pi}{2}-\frac{2\pi}{\lambda}r\right)}\varepsilon^{j\frac{2\pi}{\lambda}x(\sin\theta - 1)}.dx,$$

$$= \mathbf{A}\cos\theta\, \varepsilon^{j\frac{2\pi}{\lambda}x(\sin\theta-1)}.dx \qquad \ldots \quad \ldots \quad \ldots \quad (37)$$

where, for brevity, $\mathbf{A} = \frac{60}{r}\frac{\pi}{\lambda}\mathbf{I}\,\varepsilon^{j\left(\omega t+\frac{\pi}{2}-\frac{2\pi}{\lambda}r\right)}$.

The total field set up by the whole length of conductor is obtained by integrating between the limits $x = +\frac{l}{2}$ and $x = -\frac{l}{2}$, giving

$$\gamma = \mathbf{A}\cos\theta \int_{x=-\frac{l}{2}}^{x=+\frac{l}{2}} \varepsilon^{j\frac{2\pi}{\lambda}x(\sin\theta-1)}.dx$$

$$= \frac{\mathbf{A}\cos\theta}{j\frac{2\pi}{\lambda}(\sin\theta-1)}\left[\varepsilon^{j\frac{2\pi}{\lambda}l(\sin\theta-1)} - \varepsilon^{-j\frac{2\pi}{\lambda}l(\sin\theta-1)}\right]$$

$$= -j\frac{\lambda\,\mathbf{A}\cos\theta}{2\pi(\sin\theta-1)} \times 2\sin\frac{\pi l}{\lambda}(\sin\theta-1). \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Polar diagrams

95. Neglecting the factors $-j$ and $\mathbf{A}$ for the present, the field varies with the angle θ in accordance with the equation

$$f(\theta) = \frac{\cos\theta}{\sin\theta - 1}\sin\left[\frac{\pi l}{\lambda}(\sin\theta-1)\right],$$

and this is more compactly expressed in terms of the angle $\psi = \frac{\pi}{2} - \theta$ which is the angle of the direction r with reference to the axis of the wire. Then

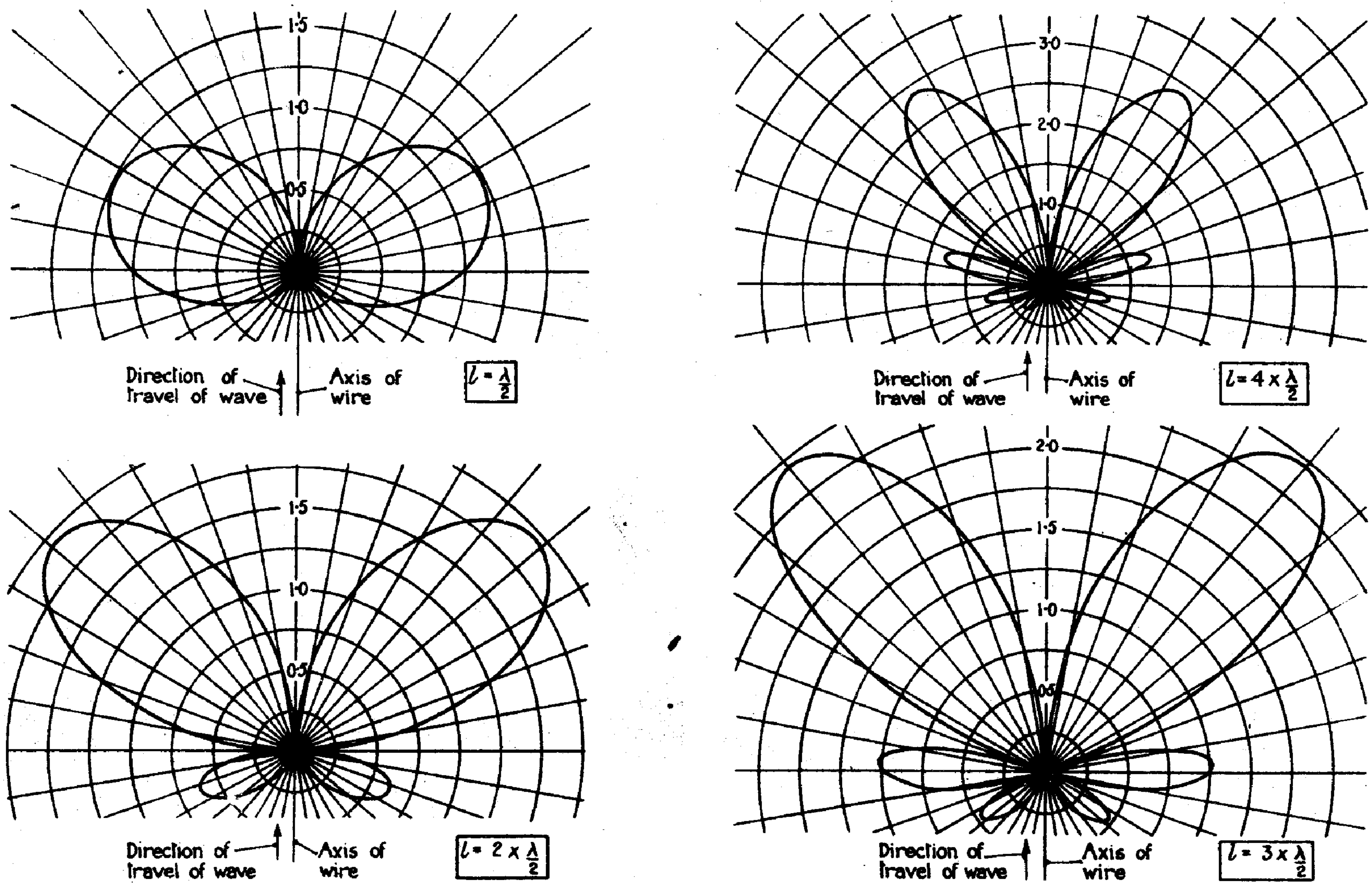
$$f(\psi) = \cot\frac{\psi}{2}\sin\left[\frac{2\pi l}{\lambda}\sin^2\frac{\psi}{2}\right].$$

The total instantaneous field is therefore

$$\gamma = \frac{60}{r}\frac{\pi}{\lambda}\mathbf{I}\,\varepsilon^{j\left(\omega t+\frac{\pi}{2}-\frac{2\pi}{\lambda}r\right)}\left\{-j\frac{\lambda}{\pi}\cot\frac{\psi}{2}\sin\left[\frac{2\pi l}{\lambda}\sin^2\frac{\psi}{2}\right]\right\}$$

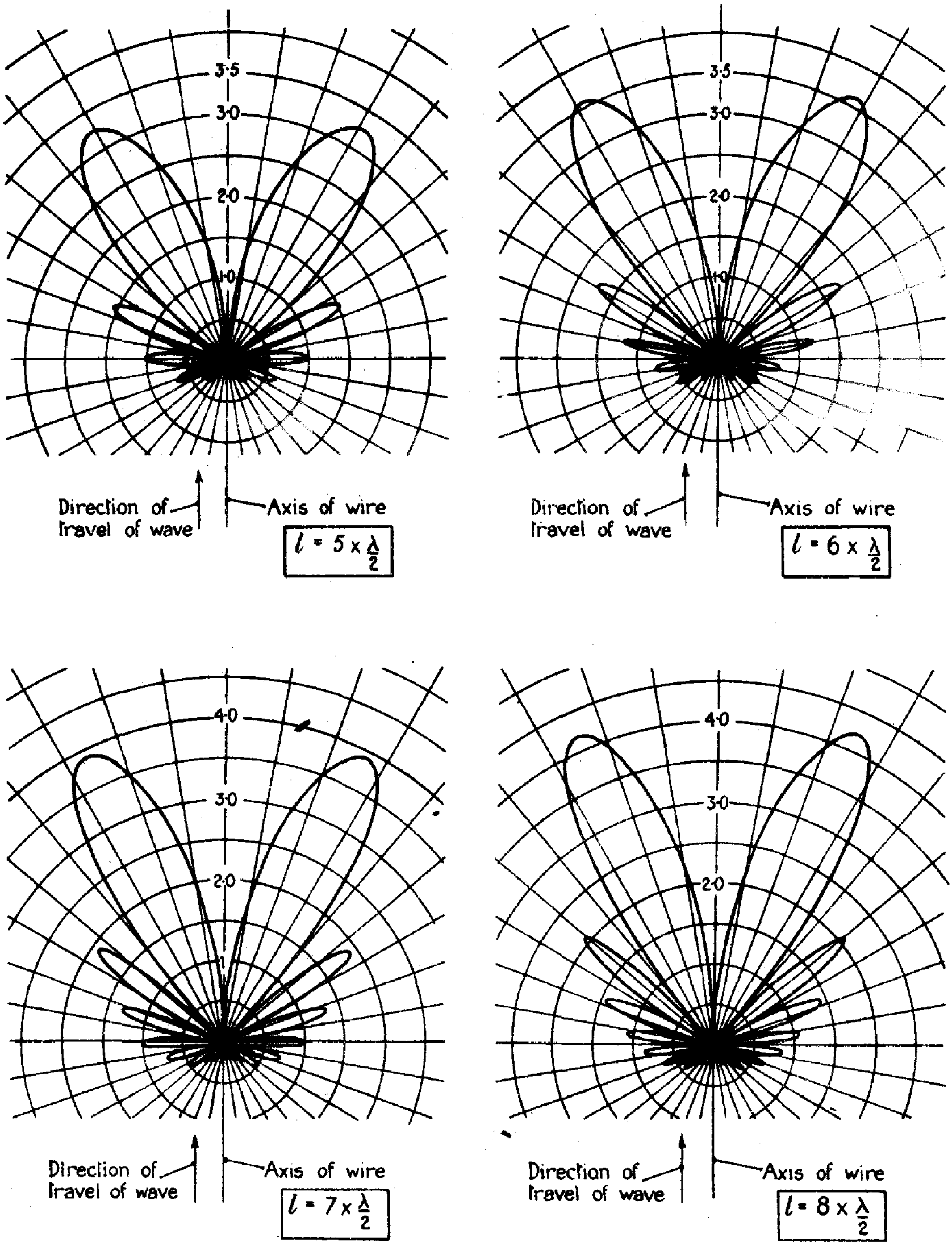
$$= \frac{60}{r}\mathbf{I}\,\varepsilon^{j\left(\omega t-\frac{2\pi}{\lambda}r\right)}\left\{\cot\frac{\psi}{2}\sin\left[\frac{2\pi l}{\lambda}\sin^2\frac{\psi}{2}\right]\right\} \quad \ldots \quad \ldots \quad (39)$$

It is interesting to note that this field lags by $\frac{\pi}{2}$ radians on that which would be produced by a stationary wave in the same wire. The portion enclosed in curved brackets, that is, $f(\psi)$, is plotted



FIELD STRENGTH DUE TO STRAIGHT WIRE CARRYING TRAVELLING WAVE

FIG. 40
SHEET I
CHAP. XV



FIELD STRENGTH DUE TO STRAIGHT WIRE
CARRYING TRAVELLING WAVE

FIG. 40
SHEET 2
CHAP. XV

in fig. 40, sheets 1 and 2, for various values of l up to 4λ. It is seen that the length of wire has an influence upon the magnitude of the maximum radius vector and also upon the angle of the main lobe with reference to the axis of the wire. These are collected and shown graphically in fig. 41.

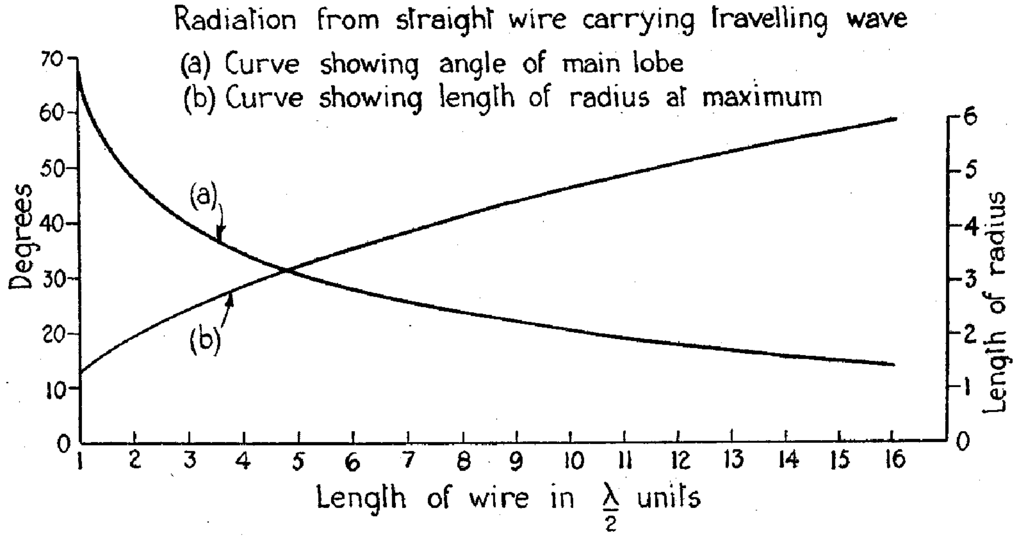


FIG. 41, CHAP. XV.—Radiation due to travelling wave.

96. It is of interest to refer to the particular case when $l = \frac{\lambda}{2}$. The R.M.S. field is then

$$\Gamma_{(l=\lambda/2)} = \frac{60}{r} I \cot\frac{\varphi}{2} \sin\left(\pi \sin^2\frac{\varphi}{2}\right)$$

As previously shown, a dipole gives a field

$$\Gamma\text{ (dipole)} = \frac{60}{r} I \frac{\cos\left(\frac{\pi}{2}\sin\varphi\right)}{\cos\varphi}$$

which is equal to $\frac{60}{r} I$ when $\varphi = 0$. For a length $l = \frac{\lambda}{2}$ the maximum value of $f(\varphi)$ occurs when $\varphi \doteqdot 65°$. This value of $f(\varphi)$ is nearly $1 \cdot 25$, so that a $\frac{\lambda}{2}$ length of conductor carrying a travelling wave sets up a field 25 per cent. greater than a dipole carrying the same current. In the former case the current is of course the same at all points in the line, whereas in the dipole the current referred to is that at the mid-point.

Application to radiation from transmission line

97. Care must be taken when applying the above results to transmission lines. Let us suppose a twin wire transmission line to have such small separation that the two wires may be regarded as coincident in space. Each wire has a current I and carries a travelling wave; the polar diagram will thus be the algebraic sum of those corresponding to the respective waves. The currents in the two wires are in opposite directions but the wave direction is the same. Hence the two diagrams lie upon each other but are of opposite sign, and the feeder is shown to be non-radiative. If, however, the wires are energized in such a manner that both the wave direction and the

instantaneous current are reversed in direction, the polar diagram is the sum of two diagrams appropriate to the length of conductor, one of which is turned upside down and so placed that the two origins coincide. This is easily seen by tracing the diagram for $l = \frac{\lambda}{2}$ (fig. 40) on tracing paper, turning it about the origin through 90°, and then adding the polar radii of the tracing and the original diagram. The result is found to be identical in shape with fig. 6, but has a maximum radius of two units. This is not surprising since fig. 6 is by hypothesis the polar diagram of a conductor carrying one half of a stationary wave formed by reflection at its open ends, the loop current being $2I$. By suitable manipulation, then, the travelling wave diagrams may be used to determine the polar diagrams of conductors carrying stationary waves.

98. Reverting to the case of a single wire carrying a travelling wave it is obvious that, in free space, the polar diagram in the plane perpendicular to the axis of the wire is a circle, i.e. the wire radiates uniformly in all directions. The solid polar diagram is therefore obtained by rotating the axial diagram about the wire. It follows that if we have two parallel wires carrying travelling waves, the polar diagram of the two is obtained by multiplying the axial diagram by the appropriate Grating Factor. In a twin wire transmission line carrying equal and opposite currents, the grating factor is given by row E of fig. 8. These diagrams are however of little practical use, for the present purpose, because transmission line spacing is usually much less than $\frac{\lambda}{8}$, the smallest spacing given. For closer spacing, however, the grating factor diagram closely approximates to two circles in contact at the origin; as the Grating Factor is $2 \cos \left(90 - \frac{180d}{\lambda} \cos \theta\right)$, the diameter of this circle is easily seen to be $2 \sin \frac{180d}{\lambda}$. For example, if $\frac{d}{\lambda} = 0{\cdot}05$, $\frac{180d}{\lambda} = 9°$, $2 \sin 9° = 0{\cdot}3128$ which is the diameter of each circle. It will be seen that for this spacing there is quite appreciable radiation, but this is greatly reduced as the spacing is decreased.

Effect of unbalanced currents in transmission line

99. When the spacing is very small, i.e. of the order of $\frac{\lambda}{50}$, the effect of unbalanced currents is of more importance than the actual spacing. By methods already used it is easily shown that if the currents are I_A, I_B, and $I_B = M/\underline{\beta}\, I_A$, the field at a radius r and angle ψ is approximately

$$\Gamma = \frac{60}{r} f(\psi)\, I_A \sqrt{1 + M^2 + 2M \cos \beta}.$$

Thus, if $\frac{d}{\lambda}$ is very small, and $\beta = 180°$, the Grating Factor diagram becomes a circle of diameter $1 - M$.

Effect of proximity of ground

100. So far the presence of the ground below the feeder line has been neglected. If perfect conductivity is assumed, the image of the feeder must be considered to carry a wave travelling in the same direction as in the feeder, but with the instantaneous current in the opposite direction at all points. The field due to the feeder must be therefore multiplied by the Vertical Distribution Factor appropriate to a horizontal dipole at a distance above ground equal to that of the feeder.

Properties of twin wire and concentric feeders

101. In order that the transmission line theory may hold, it is necessary that the current at each point in one of a pair of twin wires shall be equal in magnitude to the current at the corresponding point in the other. Now each wire has a capacitance with respect to earth, and there is also a capacitance between the two wires. The line currents at corresponding points can only be of equal magnitude if all corresponding points have equal and opposite voltages with respect to earth, and the currents, although equal in magnitude, are then exactly 180° out of phase. Under these conditions the line is said to be balanced with respect to earth. In a properly terminated and balanced line, the power losses are almost entirely due to the ohmic resistance, and the efficiency of transmission fairly high. As an approximation, the efficiency may be taken as $\left(100 - \frac{2l}{\lambda}\right)$ per cent., l being the length of the line. A little reflection will show that although the line itself may be balanced, the circuit as a whole cannot be so, unless the input and output impedances are also symmetrical with respect to earth. Thus, a horizontal dipole (fig. 42a) is a suitable load for a twin wire feeder, but an earthed aerial (fig. 42b) is not. If it is necessary to feed the latter by means of a twin wire feeder, a coupled circuit may be used, as in fig. 42c. It may be necessary to place an electro-static screen between the two coils as shown. As an alternative the circuit of fig. 42d is suggested. Here the electrical centre of the

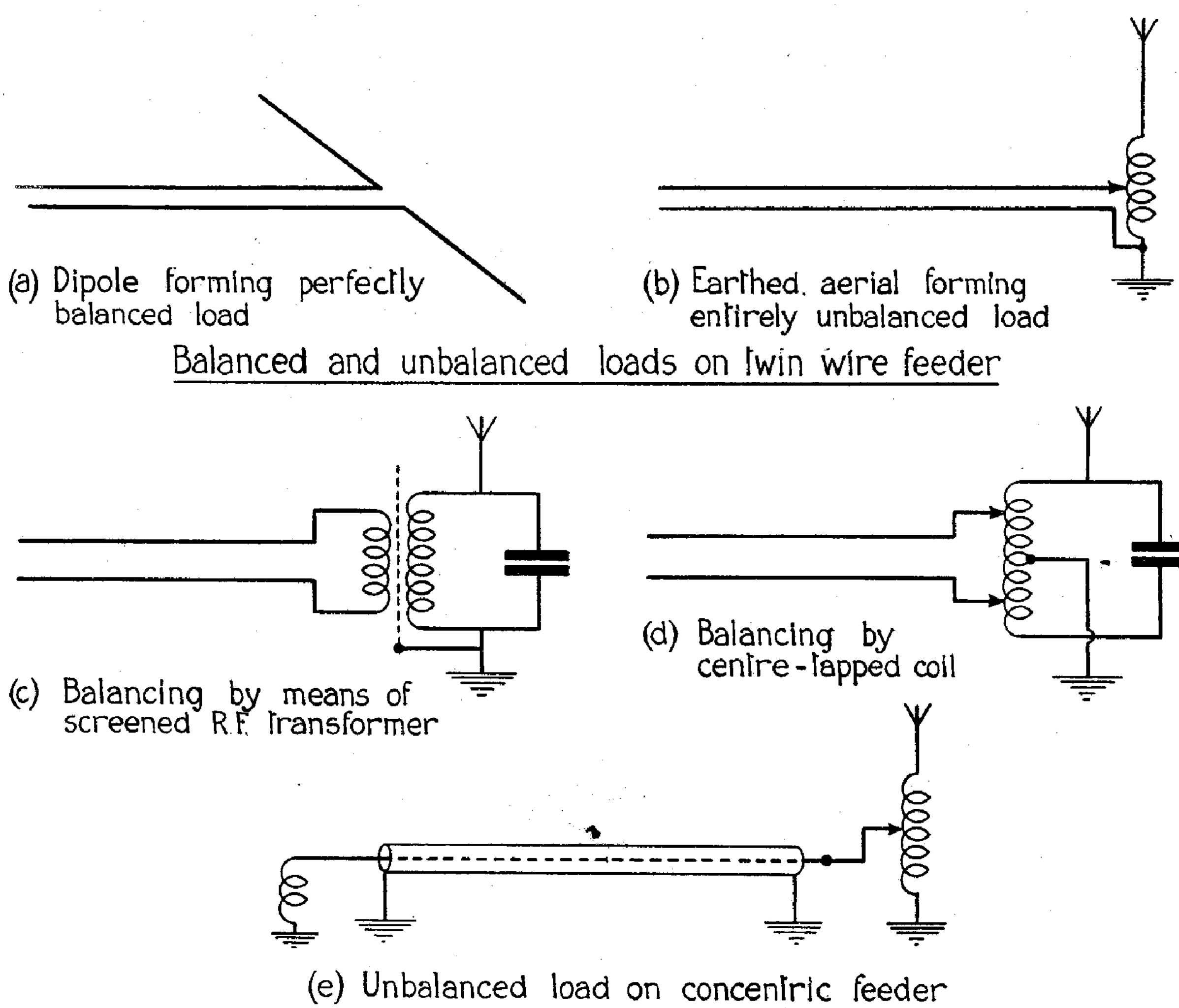


FIG. 42, CHAP. XV.—Balanced and unbalanced loads.

coil is earthed, and the feeders are tapped in at electrically equidistant points on each side of earth. It must be noted that since the aerial is connected to one end of the coil, the capacitance to earth of its two ends may be very different, and the electrical and geometrical centres are not usually coincident. Similar considerations apply to the input end of the feeder. Although it is possible to transmit power along an unbalanced line (for example, as in fig. 42b) it is found that standing waves are set up, with maxima and minima of different values and at non-corresponding points in the two wires. The efficiency of transmission is then very low and, in addition, it is impossible to predict, even approximately, the behaviour of an unbalanced line.

102. In a concentric feeder there is practically no external field, because the currents in the outer conductor are confined to a very thin layer on the inner surface, and the outer portions act merely as a screen. The outer conductor may, therefore, be earthed without affecting the electrical characteristics as a transmission line. It follows, therefore, that an earthed aerial may be fed by means of a concentric feeder as in fig. 42e. On the other hand, if it is required to feed a balanced load from a concentric feeder, some form of coupling device must be employed. This is exactly opposite to the conditions governing the use of twin wire feeders.

Methods of balancing the concentric feeder

103. As it is often necessary to feed a balanced aerial or aerial array by means of a concentric feeder, two methods of doing so will be described. The first is very simple and depends upon the fact that a length $\frac{\lambda}{2}$ of transmission line acts as a perfect 1/1 transformer (with a phase reversal of 180°). Referring to fig. 43a, suppose T_1, T_2 to be the input terminals and T_3, T_4 the output terminals of a section of feeder $\frac{\lambda}{2}$ in length. If an impedance Z_r is connected across T_3, T_4, and a P.D. V_i exists at T_1, T_2, the voltage across Z_r at T_3, T_4, will be $-V_i$. In addition, the impedance of the line, as measured at the terminals T_1, T_2, with the load Z_r so connected, will be Z_r ohms. The load so connected is, however, completely unbalanced. Now suppose that at the end of a transmission line, we measure backwards towards the input end a distance equal to $\frac{\lambda}{2}$, and bring out a suitably insulated connection from the inner conductor, as shown in fig. 43b. The terminals T_3 and T_4 are then at equal and opposite potentials with respect to earth, and are suitable for

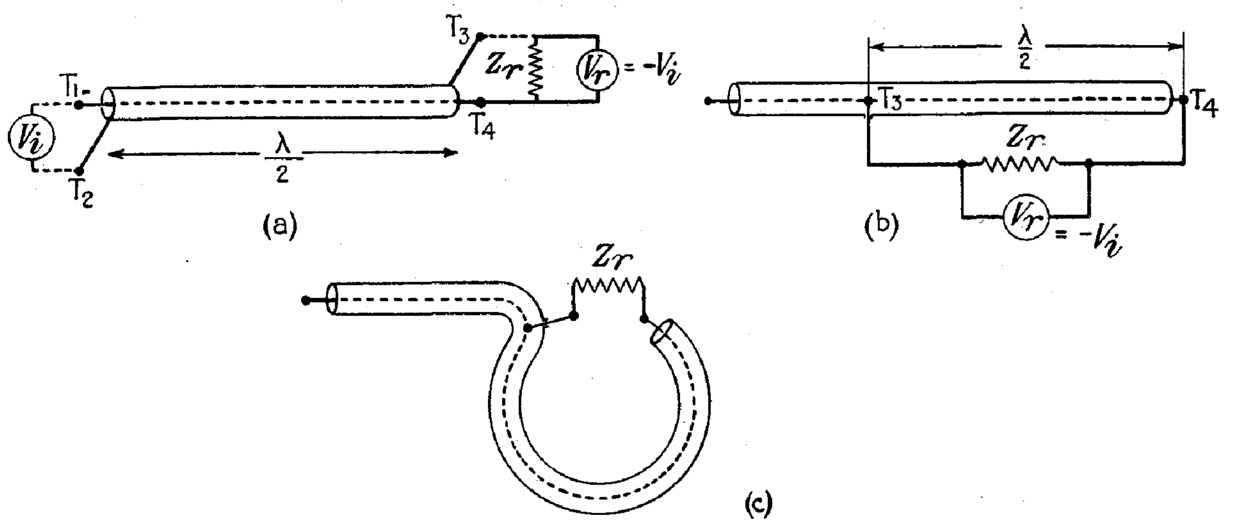


FIG. 43, CHAP. XV.—Balanced output from concentric feeder—first method.

feeding a balanced load. Since, however, T_3 and T_4 are usually required to be in proximity it is convenient to fold the $\frac{\lambda}{2}$ length of cable as in fig. 43c. This will probably affect the velocity of the wave along this portion and it may be necessary to determine the exact length by trial and error.

104. The effective impedance of the load, when connected to the output terminals, is only $\frac{Z_r}{4}$. This may be seen from the following considerations. Since the actual impedance between T_3 and T_4 is Z_r, and it is balanced with respect to earth, its centre point is at or near earth potential. The outer conductor is also at or near earth potential, so that between T_3 and earth we have an impedance of $\frac{Z_r}{2}$ ohms and an impedance of $\frac{Z_r}{2}$ ohms between T_4 and earth. But the latter impedance may equally be considered to be connected between T_3 and earth, from the argument in the preceding paragraph, and therefore the transmission line must be considered as being terminated at T_3, by two impedances in parallel, each of $\frac{Z_r}{2}$ ohms, i.e. by $\frac{Z_r}{4}$ ohms.

105. The second method is as follows. A length l of copper tube A B of the same external diameter as the outer conductor of the feeder, is placed parallel to the end of the transmission line, and is electrically connected to the latter at A as in fig. 44a. The inner conductor of the

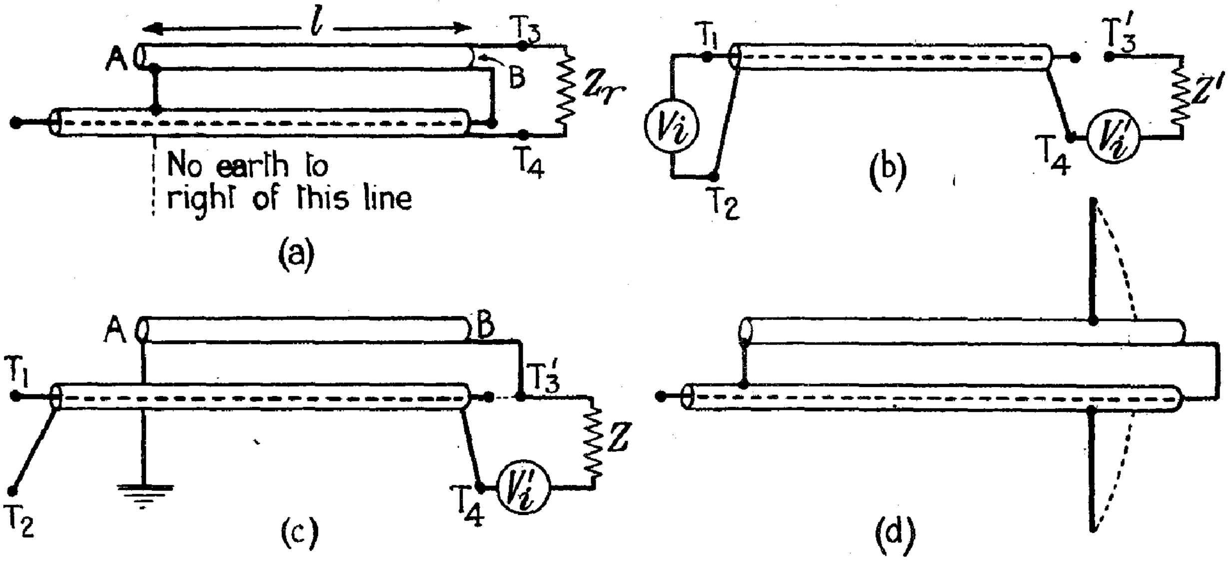


FIG. 44, CHAP. XV.—Balanced output from concentric feeder—second method.

transmission line is connected to the added tube at B and the balanced output is then taken from the terminals T_3 T_4, which are connected to the external conductor of the line and the added conductor respectively. It must be noted that the length l of both these conductors must be insulated from and preferably symmetrically disposed with respect to earth. The explanation of the operation of this device is simple, but rather more difficult to visualize than that previously described. Suppose that at the input end we apply a voltage V_i, it is possible to find an equivalent voltage V_i' and impedance Z' which, when connected between the terminals T_3', T_4, will cause the same P.D. at these terminals and will deliver the same current to the load. Now the point T_3' has (practically) zero capacitance to earth because it is entirely shielded by the outer conductor. It may therefore, as a preliminary, be considered as an entirely isolated point connected to T_4 by a generator of voltage V_i' and an impedance Z' in series, neither of which possess capacitance with respect to earth (fig. 44b). If now the extra length of conductor is added as in fig. 44c, and the terminal T_3' connected to the point B, it is obvious that the output impedance is symmetrical with respect to earth. The inner conductor may therefore be connected to T_3' as in the dotted line. We have, however, added, at the points T_3' T_4, an additional parallel impedance due to the short transmission line formed by the two parallel tubes. Denoting this by Z_2, it has already been shown that $Z_2 = j\,Z_0'\,tan\,\frac{2\pi}{\lambda}l$, where Z_0' is the surge impedance of the length l of parallel tube. If $l = \frac{\lambda}{4}$, Z_2 becomes infinite, while if $l = \frac{\lambda}{2}$, Z_2 is zero. On either side of $\frac{\lambda}{4}$, Z_2 is either capacitive or inductive, hence by suitable choice of l an effective reactance of any desired value may be placed in parallel with the actual load. The resistance of the load may be matched to the surge impedance of the feeder by choosing suitable locations for the terminals T_3', T_4; in fig. 44d, a $\frac{\lambda}{2}$ dipole is fed in this way, the output terminals being located a short distance from the ends of the parallel tubes.

Comparison of twin-wire and concentric lines

106. In practice, both twin-wires and concentric feeders are used according to local conditions. It is not possible to give any definite rules which will govern the adoption of either type. The following summary of their relative advantages and disadvantages should be taken into account in any decision.

(i) *Type of load*

(*a*) Twin feeders are inherently balanced and are suitable for any type of load consisting of an arrangement of dipoles. If the load is not symmetrical with respect to earth, some form of coupling device must be adopted.

(*b*) Concentric feeders are inherently unbalanced and are suitable for unbalanced loads such as earthed aerials. If the load is symmetrical with respect to earth some form of coupling device must be adopted.

(ii) *Constructional*

(*a*) Twin feeders are cheap to construct and repair. In the field it is even possible to erect a workable line from field telegraph poles and improvised insulators such as glass bottles, although of course a high transmission efficiency cannot be expected. On the other hand, since a twin-wire line should be several feet above the ground, it is difficult to adopt this type where the transmitter or receiver is installed underground.

(*b*) Concentric feeders are expensive to construct, difficult to repair in the event of a mechanical failure, and are impossible to improvise. They may, however, be buried and led to an underground station.

(iii) *Convenience*

(*a*) Twin-wire feeders occupy a considerable space, particularly where a large number of aerials are energized from transmitters in the same building, because they have an external field, and unless different lines are well apart they will affect each other.

(*b*) Concentric feeders are very compact and may be placed in proximity without mutual interaction.

(iv) *Breakdown voltage*

(*a*) The breakdown voltage between twin wires of suitable spacing is very high. Flash over is not likely to occur with properly matched and balanced loads, at any rate with the power required in service transmitters.

(*b*) With concentric feeders, the spacing between conductors is comparatively small and they are more likely to flash over. It is therefore essential, from this point of view alone, to ensure that no standing waves exist in the feeder.

(v) *Transmission losses*

For the powers used in the service, i.e. up to a few kilowatts, there is probably little to choose between the two types, although accurate figures are not available. For very high power (e.g. 500 kW.) concentric feeders may be better.

(vi) *Radiation and pick-up*

(*a*) The power radiated by a correctly matched twin-wire feeder is not large. In reception the pick-up is correspondingly small, but is often difficult to eliminate completely.

(*b*) With concentric feeders, radiation and pick-up are practically non-existent. For special purposes where freedom from pick-up is absolutely essential, e.g. a remote D/F system, concentric feeders must be employed. Again, if it is proposed to use a very high frequency transmitter in an aeroplane, and a feeder line is necessary, the concentric type is almost compulsory.

MATCHING DEVICES

Necessity for matching

107. (i) It will be appreciated from the foregoing that in order to convey the greatest possible amount of power from a transmitter to an aerial system by means of a feeder line, the input impedance of the feeder must be matched to the output impedance of the transmitter, and the input impedance of the aerial must be matched to the surge impedance of the feeder. At the

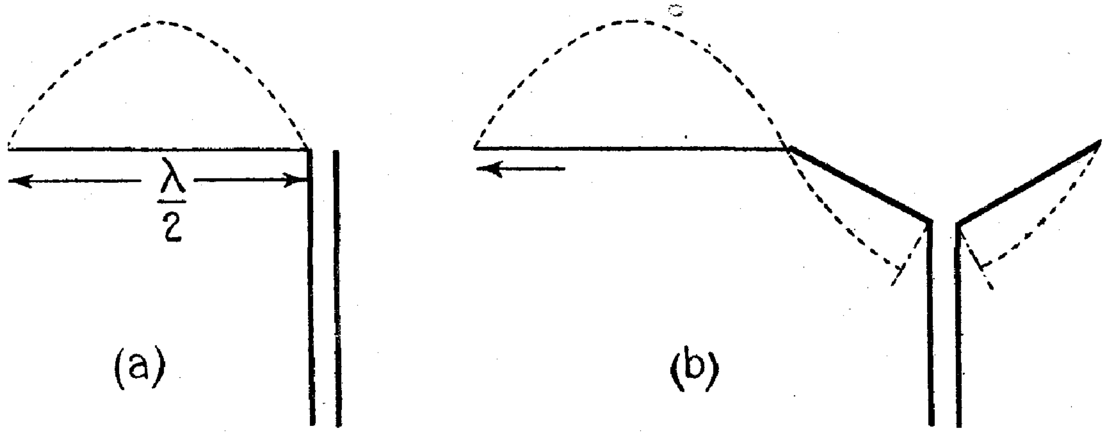


FIG. 45, CHAP. XV.—Example of mis-matching.

transmitter end, the suitable matching devices are usually incorporated in the design of the transmitter and need no further comment. The matching of aerial to feeder must often be dealt with by the personnel responsible for bringing the station into operation, especially on active service. Unless this matching is fairly close, the efficiency of the station may be very low. As an example of what must be avoided if possible, take the arrangement shown in fig. 45a. This is fairly satisfactory if the feeders are, electrically, $\frac{\lambda}{4}$ in length, as explained in Chapter VII. If, however, a feeder of indefinite length is used, a purely physical consideration will show that the arrangement is far from efficient. If it is considered as a $\frac{\lambda}{2}$ aerial connected to one side of a transmission line, the aerial has an input resistance of some 3,000 ohms, while the feeder will have a surge impedance of the order of 600 ohms. On the other hand, we may assume that the feeder is effectively terminated at a point $\frac{\lambda}{4}$ from the aerial connection. In fig. 45b the final $\frac{\lambda}{4}$ length of feeder has been splayed out in order to show the current distribution on this assumption. It is now seen that the arrangement is equivalent to a λ aerial fed at a current loop, and as the aerial is so folded that a length of $\frac{\lambda}{2}$ is non-radiative, its input resistance is of the order of 80 to 100 ohms only. Regarding the arrangement in either of these ways it is seen that the ratio $\frac{Z_r}{Z_0}\left(\text{or } \frac{Z_0}{Z_r}\right)$ is of the order of 5. Although it is rarely possible to obtain a perfect match $\left(\frac{Z_r}{Z_0} = 1\right)$, the aim should be to attain this within 25 per cent.

(ii) Since a well-designed radio-frequency feeder is practically loss-free, and its surge impedance is to all intents and purposes purely resistive, stationary waves will only be suppressed if the termination is also purely resistive. If the input impedance of the aerial has a reactive component, this must be balanced out by incorporating an equal and opposite reactance in the termination.

Limitations of R.F. transformer as matching device

108. At first sight the problem of achieving an approximate match between the feeder and the aerial system would appear to be comparatively simple, merely involving the design of a suitable radio-frequency transformer. In practice, however, it is very difficult to obtain a practical solution by this method. This would present little difficulty if it were practicable in a transformer of this kind to achieve a coupling factor approaching unity, but actually it is rarely possible for it to exceed 0·5. This is due to the necessity for well spacing the coils, in order to avoid capacitance coupling and to permit the development of high voltage across the input and output terminals without insulation breakdown. It is highly desirable that, looking into the input terminals of the matching device, the load shall be non-reactive. If this is not so the power factor of the load will be less than unity and, for a given input to the aerial, the line current must be greater than with a non-reactive load. Since the line cannot have zero resistance, this must lead to power loss in the feeder and to low efficiency. Further, the surge impedance of a radio-frequency line is practically non-reactive and should, therefore, be terminated by a purely resistive load. It is possible to bring the power factor to unity by the introduction of a suitable condenser or condensers in addition to the transformer, but the design of such condensers is again beset with difficulty. They must be located near the aerial and, therefore, in weatherproof casing, and yet must be very highly insulated from earth. In some instances, the capacitance may be only about $100\mu\mu F$ and yet the plate area must be sufficient to carry the full feed current without overheating. In addition, consideration of breakdown voltage may necessitate a large spacing although the external field must be negligible. The two latter requirements lead to a very bulky and extremely expensive condenser. Unless a transformer is absolutely essential it is customary to perform the matching by means of an electrical network consisting of arrangements of impedances. These impedances often take the form of suitable lengths of transmission line.

Principle of matching network

109. In dealing with matching by means of electrical networks, we shall assume that the aerial itself, at its input terminals, offers resistance only. This resistance will be denoted by R_A. The line will have a surge impedance Z_0, which may be taken to be purely resistive and denoted by R_0. The resistances R_A and R_0 being unequal, we require to insert some matching device between R_0 and R_A. This will be some arrangement of reactances which must be of the lowest possible resistance. Before dealing with some of the various possible arrangements consider the matching unit shown in fig. 46, in which the exact arrangement of the apparatus is unknown.

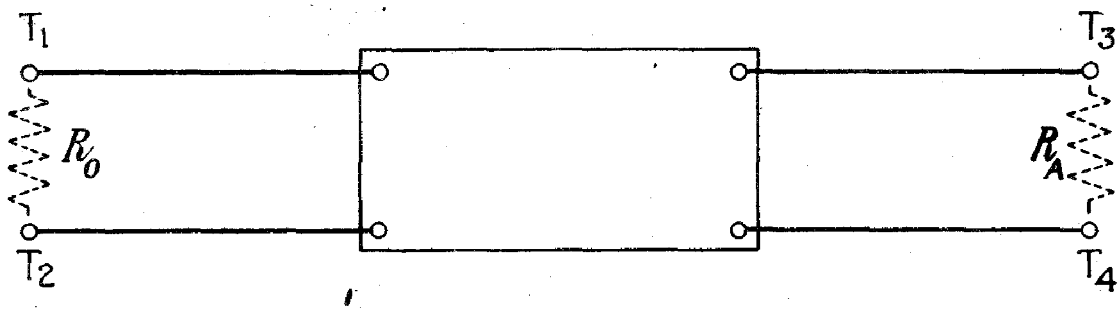


FIG. 46, CHAP. XV.—Insertion of matching network.

The requirements are that if the resistance R_A is connected across the terminals T_3, T_4, the impedance, measured at the terminals T_1, T_2, is R_0. On the other hand, if a resistance R_0 is connected to T_1, T_2, and the impedance measured at T_3, T_4, it must be equal to R_A. This reciprocal relation is the essential property of any matching network, and is true if the latter consists of reactances only.

L unit

110. The simplest possible arrangement is the "L unit" which consists of two reactances jA and jB ohms. If R_A is greater than R_0, say $R_A = nR_0$, $n > 1$, the arrangement is as shown in fig. 47a, whereas if $R_0 > R_A$, say $R_0 = nR_A$, the arrangement will be as shown in fig. 47b. Taking the former case, the input impedance will be

$$Z_i = jA + \frac{jBR_A}{R_A + jB} \qquad \text{.. .. ..} \quad (40a)$$

$$= jA + \frac{jBR_A\,(R_A - jB)}{R_A{}^2 + B^2} \qquad \text{.. .. ..} \quad (40b)$$

$$= \frac{B^2R_A}{R_A{}^2 + B^2} + j\left(\frac{BR_A{}^2}{R_A{}^2 + B^2} + A\right) \qquad \text{.. .. ..} \quad (40c)$$

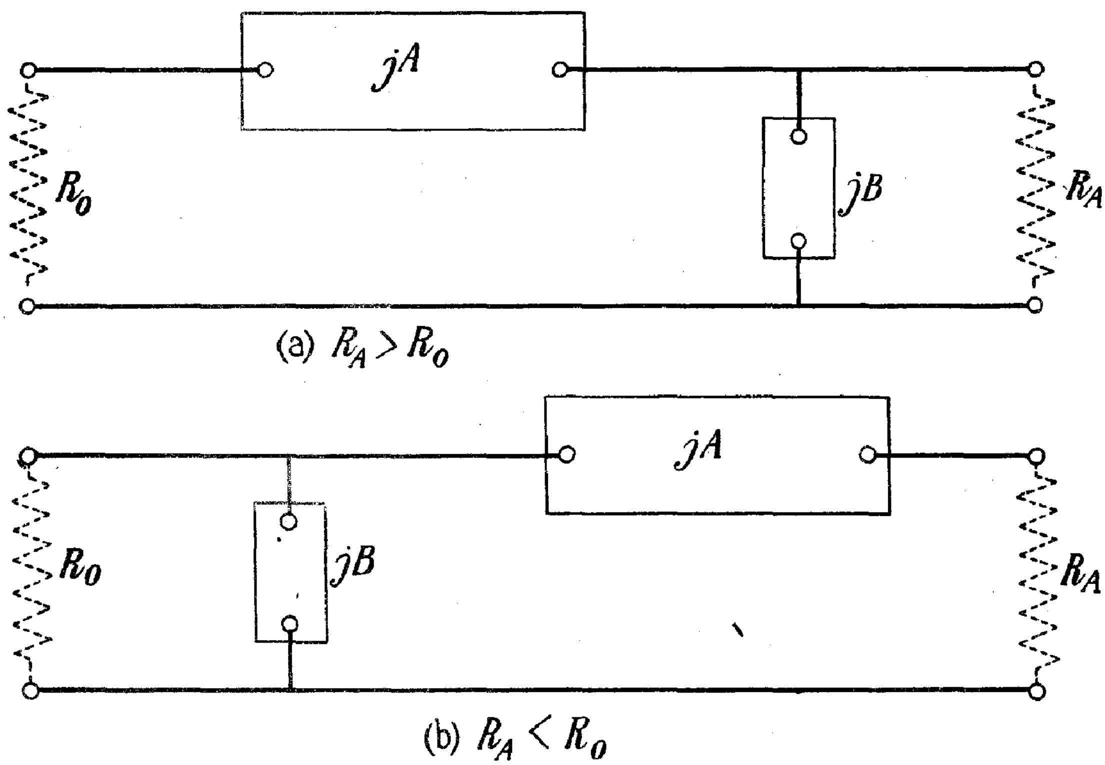


FIG. 47, CHAP. XV.—L-type matching units.

We require Z_i to be equal to R_0, i.e. purely resistive, and the imaginary part must vanish, i.e.

$$A + \frac{BR_A^2}{R_A^2 + B^2} = 0 \qquad \text{(40d)}$$

$$R_0 = \frac{B^2 R_A}{R_A^2 + B^2} \qquad \text{(40e)}$$

from the above equations

$$R_0 (R_A^2 + B^2) = B^2 R_A$$

$$B^2 (R_0 - R_A) = R_0 R_A^2$$

$$B^2 = \frac{R_0 R_A^2}{R_0 - R_A}$$

$$B = \mp \frac{n}{\sqrt{n-1}} R_0, \qquad \text{(40f)}$$

because $R_A = nR_0$.

Also

$$A = -\frac{BR_A^2}{R_A^2 + B^2}$$

Inserting the above value of B,

$$A = \pm \frac{\dfrac{n}{\sqrt{n-1}} R_0 \times n^2 R_0^2}{n^2 R_0^2 + \dfrac{n^2}{n-1} R_0^2}$$

$$= \pm \sqrt{n-1}\, R_0. \qquad \text{(40g)}$$

Thus the series and shunt reactances must be of opposite sign; if A is inductive B must be capacitive and *vice versa*. It will also be observed that $AB = nR_0^2 = R_A R_0$.

Example.—If $R_A = 3{,}000$ ohms, and $R_0 = 600$ ohms, $n = 5$, $\sqrt{n-1} = 2$.

$$\therefore A = \pm 2R_0 = \pm 1{,}200 \text{ ohms.}$$

$$AB = R_A R_0 = 3{,}000 \times 600$$

$$B = \frac{3{,}000 \times 600}{\pm 1{,}200}$$

$$= \pm 1{,}500 \text{ ohms.}$$

If $f = 6$ Mc/s, and we decide that A is to be inductive, say ωL ohms,

$$\omega L = +1{,}200$$

$$L = \frac{1{,}200}{2\pi \times 6 \times 10^6} \text{ henries}$$

$$= 31 \cdot 8\, \mu H$$

and B will be capacitive, say $-\dfrac{1}{\omega C}$ ohms.

$$-\frac{1}{\omega C} = -1{,}500$$

$$C = \frac{1}{2\pi \times 6 \times 10^6 \times 1{,}500} \text{ farad,}$$

$$= 16 \cdot 67\, \mu\mu F.$$

111. In the second case, $R_0 > R_A$ or $R_0 = nR_A$ (fig. 47b), and the input impedance will be

$$\frac{1}{Z_i} = \frac{1}{jB} + \frac{1}{R_A + jA} \quad \text{.. .. ..} \quad (41a)$$

$$= \frac{1}{jB} + \frac{R_A - jA}{R_A^2 + A^2} \quad \text{.. .. ..} \quad (41b)$$

Again, $\frac{1}{Z_i}$ must be equal to $\frac{1}{R_0}$, and its imaginary part must vanish, i.e.

$$\left.\begin{aligned} \frac{1}{R_0} &= \frac{R_A}{R_A^2 + A^2} \\ \text{and} \quad \frac{1}{jB} - j\frac{A}{R_A^2 + A^2} &= 0 \end{aligned}\right\} \quad \text{.. .. ..} \quad (41c)$$

Hence

$$R_0 = \frac{R_A^2 + A^2}{R_A} \quad \text{.. .. ..} \quad (41d)$$

$$A^2 = R_0R_A + R_A^2$$

Substituting $\qquad R_A = \frac{R_0}{n}$

$$A^2 = \frac{n-1}{n^2} R_0^2$$

$$A = \pm \frac{\sqrt{n-1}}{n} R_0 \quad \text{.. .. ..} \quad (41e)$$

and

$$\frac{1}{jB} = j\frac{A}{R_A^2 + A^2}$$

$$B = -\frac{R_A^2 + A^2}{A}$$

Inserting the above value of A,

$$B = \mp \frac{R_0}{\sqrt{n-1}}. \quad \text{.. .. ..} \quad (41f)$$

As before, A and B are of opposite sign and

$$AB = R_AR_0$$

Example.—If $R_A = 120$ ohms and $R_0 = 600$ ohms, $n = 5$

$$B = \mp\frac{R_0}{2} = \mp 300 \text{ ohms}$$

$$A = \pm R_0 \times \frac{2}{5} = \pm 240 \text{ ohms.}$$

Let $A = \omega L$, $B = -\frac{1}{\omega C}$, $\omega = 2\pi \times 6 \times 10^6$.

$$L = \frac{240}{2\pi \times 6}\,\mu H$$
$$= 6{\cdot}37\ \mu H.$$
$$-\frac{1}{\omega C} = -300$$
$$C = \frac{10^6}{300 \times 2\pi \times 6}\,\mu\mu F$$
$$= 88{\cdot}5\ \mu\mu F.$$

Symmetrical T and Π units

112. Having shown the method of deriving the matching conditions in two of the simplest cases, we may now describe briefly certain other arrangements. Fig. 48a shows the symmetrical T, and fig. 48b the symmetrical Π networks. In each case we have three reactances each having a magnitude of A ohms. The sign of each of the series reactances is the same, but opposite to the

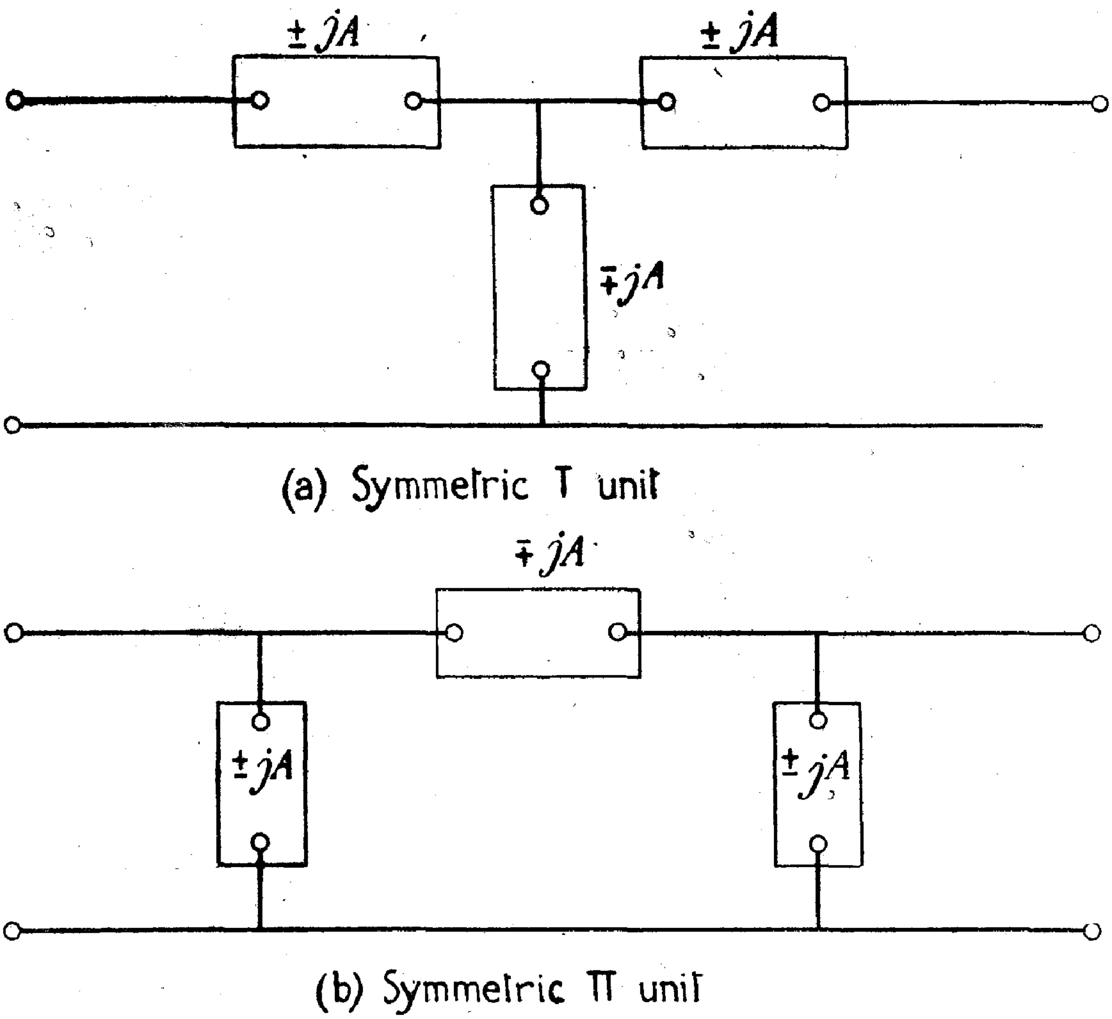


FIG. 48, CHAP. XV.—Symmetric matching units.

sign of the corresponding parallel reactance. Such a network is equivalent electrically to a $\frac{\lambda}{4}$ length of transmission line of surge impedance A ohms. Hence the required reactance is immediately found by the relation $A = \sqrt{R_0 R_A}$.

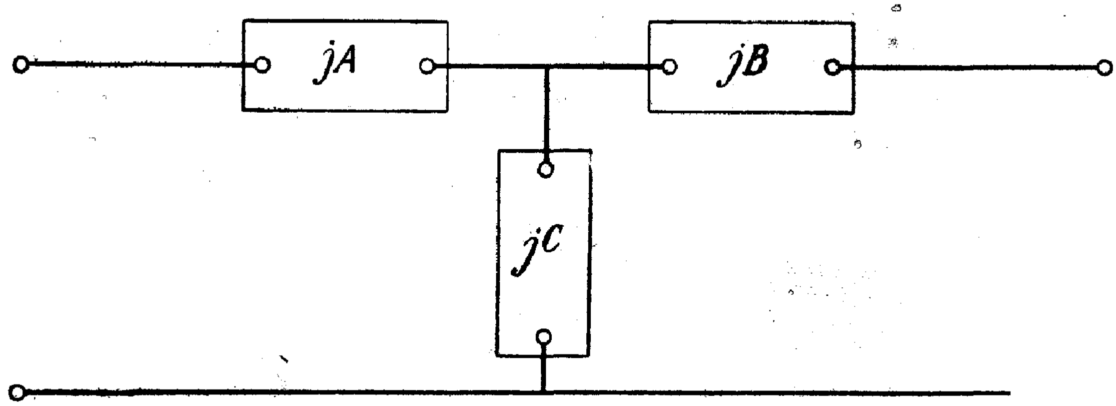
Unsymmetrical T and Π units

113. These are shown in fig. 49a and fig. 49b respectively. In each case we have three reactances A, B, C. Let $R_0 = nR_A$, where n may be greater or less than unity. Then in the T unit if $A = aR_A$, $B = bR_A$, $C = cR_A$, the numerics a, b and c are interdependent. If a suitable value is selected for C and therefore for c.

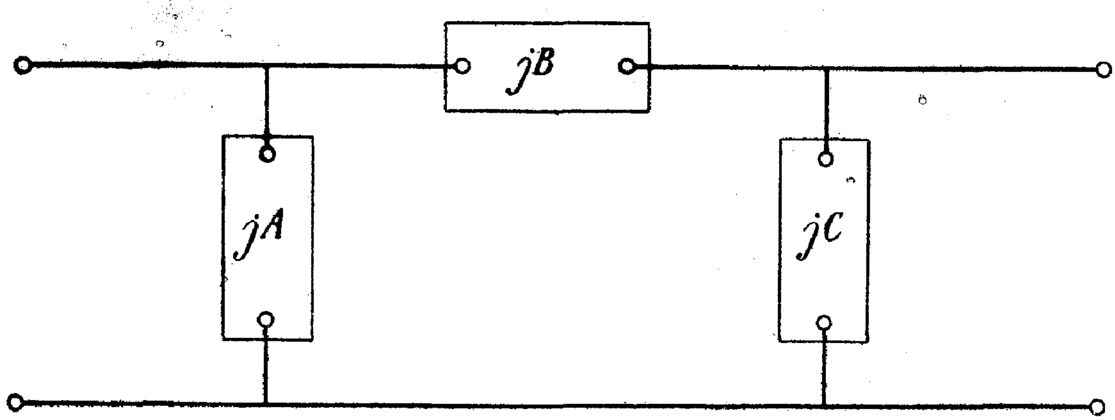
$$a = -c \pm n\sqrt{\frac{c^2}{n} - 1}$$

$$b = -c \pm \sqrt{\frac{c^2}{n} - 1}$$

In choosing the value for c, therefore, it is essential to make it greater than $\sqrt{n}$. When $c = \sqrt{n}$ the circuit becomes symmetrical. In the unsymmetric Π unit, we have $A = aR_A$, $B = bR_A$,



(a) Unsymmetric T unit



(b) Unsymmetric Π unit

FIG. 49, CHAP. XV.—Unsymmetric matching units.

$C = cR_A$ as before. If the value of B is suitably selected, then, the values of a and c are related to b as follows.

$$a = -\frac{bn}{n \pm \sqrt{n - b^2}}$$

$$c = -\frac{b}{1 \pm \sqrt{n - b^2}},$$

i.e. if $b = \sqrt{n}$ the network becomes symmetrical. Hence b must be less than $\sqrt{n}$.

T – Π network

114. This is shown in fig. 50. It possesses the following important property. If the reactances A, B, C, D are so chosen that $A + B + C = 0$, and $\frac{D}{A} = \frac{B}{C} = -n$, so that D is of opposite sign to A, and B to C, the network is equivalent to an ideal transformer of turns ratio n. Thus if an

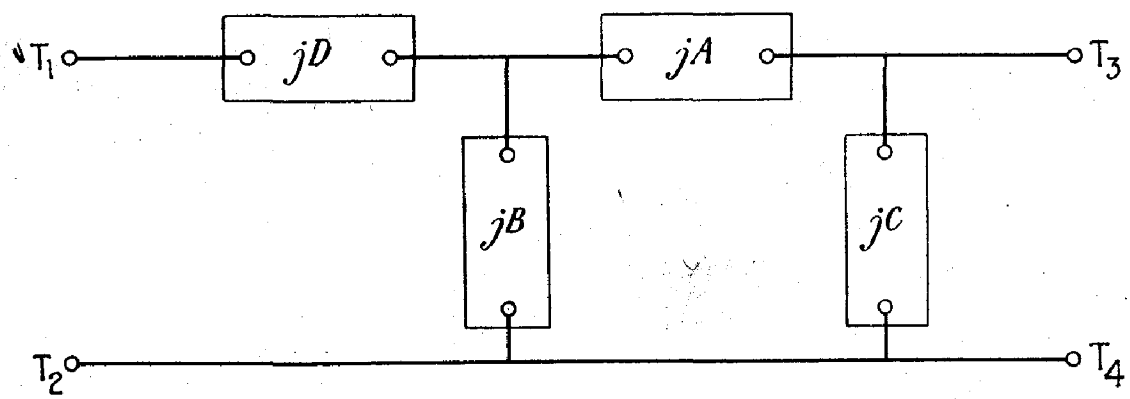


FIG. 50, CHAP. XV.—T – Π network.

aerial of resistance R_A is connected to the terminals T_3, T_4, the input impedance at T_1 T_2 is n^2R_A. To match the aerial to the line therefore, we must make $n^2R_A = R_0$, or $n = \sqrt{\frac{R_0}{R_A}}$.

Annulment of capacitive reactance of load

115. If an aerial is connected to a feeder line in such a manner that a current node exists at a point in the aerial within a distance of $\frac{\lambda}{4}$ from the junction of line and aerial, the latter offers capacitive reactance as well as resistance and can be represented by an impedance $R_A + \frac{1}{j\omega C}$ ohms, or by an admittance $G + jB_c$ ohms, where

$$G = \frac{\omega^2 C^2 R_A}{1 + \omega^2 C^2 R_A^2}$$

$$B_c = \frac{\omega C}{1 + \omega^2 C^2 R_A^2}$$

$$= \omega C',$$

where C' is the effective shunt capacitance. Thus we may annul the reactance of the aerial by connecting, in series, an inductance $L = \frac{1}{\omega^2 C}$, or, in parallel, an inductive susceptance $\frac{1}{\omega L'}$, where

$$\frac{1}{\omega L'} = \omega C' = \frac{\omega C}{1 + \omega^2 C^2 R_A{}^2}$$

or

$$L' = \frac{1 + \omega^2 C^2 R_A{}^2}{\omega^2 C}$$

$$= \frac{1}{\omega^2 C} + C R_A{}^2$$

116. If the matching is performed by means of an L, Π or T network, there is no necessity to add a physical inductance in this manner, for the actual capacitance C may be considered to form part of the matching network, the constants of the latter being adjusted accordingly.

Example.—Suppose the aerial to be terminated at a point such that at 6 Mc/s its impedance is $100 - j\,25$ ohms.

$$\frac{1}{\omega C} = 25$$

$$C = \frac{1}{2\pi \times 6 \times 10^6 \times 25} \text{ farad}$$

$$= 106 \mu\mu\text{F}.$$

To annul this we may use a series inductance

$$L = \frac{25}{2\pi \times 6 \times 10^6} \text{ henries}$$

$$= 0{\cdot}66 \mu\text{H}.$$

The equivalent shunt capacitance is $\frac{C}{1 + \omega^2 C^2 R_A{}^2} = C'$

$$C' = \frac{106 = 10^{-12}}{1 + (2\pi \times 6 \times 10^6 \times 10^6 \times 10^{-12} \times 100)^2}$$

$$= \frac{106}{1{\cdot}16}$$

$$= 91{\cdot}5 \mu\mu\text{F}.$$

and this may be annulled by a shunt inductance

$$L' = \frac{1{\cdot}16}{(2\pi \times 6 \times 10^6)^2 \times 106 \times 10^{-12}}$$

$$= 7 \mu\text{H}.$$

Annulment of inductive reactance of load

117. If the aerial is so connected that a current loop exists in it, within a distance of $\frac{\lambda}{4}$ from the feeding point, the aerial offers inductive reactance and its impedance is $R_A + j\omega L$ ohms. Its admittance is $G - jB_L$ where

$$G = \frac{R_A}{R_A{}^2 + \omega^2 L^2}$$

$$B_L = \frac{\omega L}{R_A{}^2 + \omega^2 L^2}.$$

Thus by connecting a capacitance C in series with the aerial, its inductive reactance may be annulled. The value of C is obviously $\frac{1}{\omega^2 L}$. Alternatively a capacitance C' may be connected in parallel, its value being given by

$$C' = \frac{L}{R_A^2 + \omega^2 L^2} \text{ farads.}$$

Again, instead of adding a physical component to the aerial itself it is possible to insert the required reactance in the last member of the matching network. Thus, no matter at what point an aerial is terminated it is always possible to ensure that its input impedance is purely resistive and suitable for matching to a non-reactive line.

Quarter-wave matching

118. We may now explain the theory of quarter-wave matching more thoroughly. Suppose a transmission line to have a surge impedance of Z_0 ohms and to be ultimately terminated by an impedance $Z_r = nZ_0$. It is therefore necessary to insert some matching device between the line and the load. A section of line $\frac{\lambda}{4}$ long, of surge impedance Z_0', terminated by an impedance Z_r, has an input impedance $Z_1 = \frac{(Z_0')^2}{Z_r}$. If $Z_r = nZ_0$, $Z_1 = \frac{(Z_0')^2}{nZ_0}$, and if $Z_0' = \sqrt{n}\, Z_0$,

$$Z_1 = \frac{(Z_0')^2}{nZ_0} = \frac{n(Z_0)^2}{nZ_0} = Z_0.$$

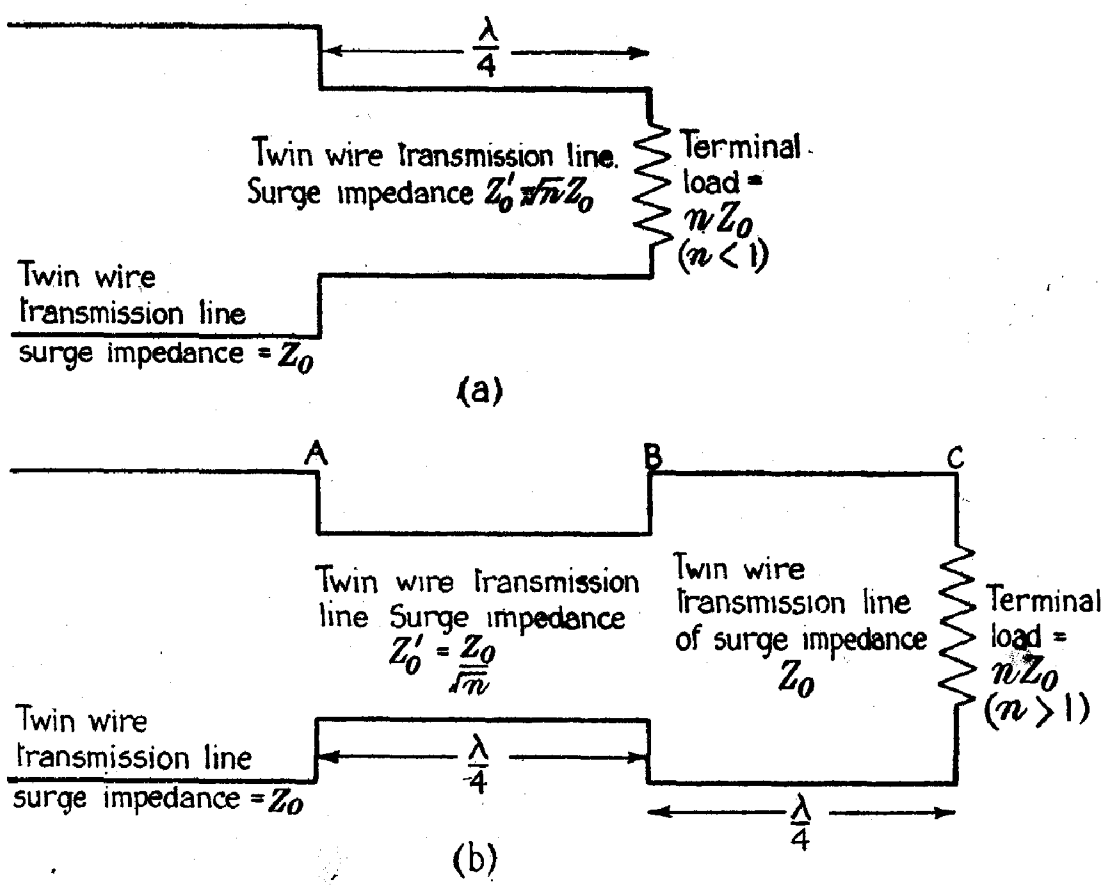


FIG. 51, CHAP. XV.—Quarter-wave matching.

Thus, if a $\frac{\lambda}{4}$ length of line, of surge impedance $Z_0' = \sqrt{n}\, Z_0$, is inserted between the actual line and the load proper, the line is terminated by an impedance equal to its surge impedance, which is what is required.

119. If n is less than unity, Z_0' must be less than Z_0, and this may be achieved simply by reducing the spacing of the line over the final $\frac{\lambda}{4}$ length, as shown in fig. 51a. In effect this last section is a part of the aerial system in that it carries a stationary wave, whereas in the line proper stationary waves should be entirely suppressed. If n is greater than unity, this method of matching would entail an increase in the spacing, and consequently to increased radiation from the line in transmission, and greater pick-up in reception. It is then necessary to adopt an artifice, and arrange the feeder as in fig. 51b. The input impedance of the section B C is $\frac{Z_0^2}{n\,Z_0} = \frac{Z_0}{n}$, and we are back to the original problem ($n<1$). If the surge impedance of the section A B is Z_0' the input impedance of this section is $\frac{(Z_0')^2\, n}{Z_0}$, and we require this to be equal to Z_0, i.e.

$$\frac{n(Z_0')^2}{Z_0} = Z_0$$

$$Z_0' = \frac{Z_0}{\sqrt{n}}.$$

If these conditions are achieved, the line will be matched at the point A, and the portion A C becomes in effect a part of the aerial, carrying a $\frac{\lambda}{2}$ portion of a standing wave.

Example.—A load of (*a*) 120 ohms, (*b*) 3,000 ohms is to be matched to a 600 ohm line. Find the required conditions

(*a*) Here $Z_r = 120, = nZ_0, Z_0 = 600, n = \frac{1}{5}$. We therefore require to insert a $\frac{\lambda}{4}$ section, of surge impedance $Z_0' = \sqrt{\frac{1}{5}}Z_0 = \frac{600}{2\cdot 24} = 268$ ohms.

(*b*) Here $Z_r = 3{,}000$, $Z_0 = 600$, $n = 5$, and the line is arranged as in fig. 51b. The surge impedance of the part B C is equal to that of the line proper, namely, 600 ohms. The portion A B will, however, be of such a spacing that its surge impedance is 268 ohms.

Loop matching

120. If a pair of transmission lines is incorrectly terminated, the standing waves in the line may be as shown in fig. 52. Then at a current minimum, e.g. at A, the input impedance, looking towards Z_r is purely resistive, say r ohms, and r is less than Z_0. At a current maximum, e.g. at B,

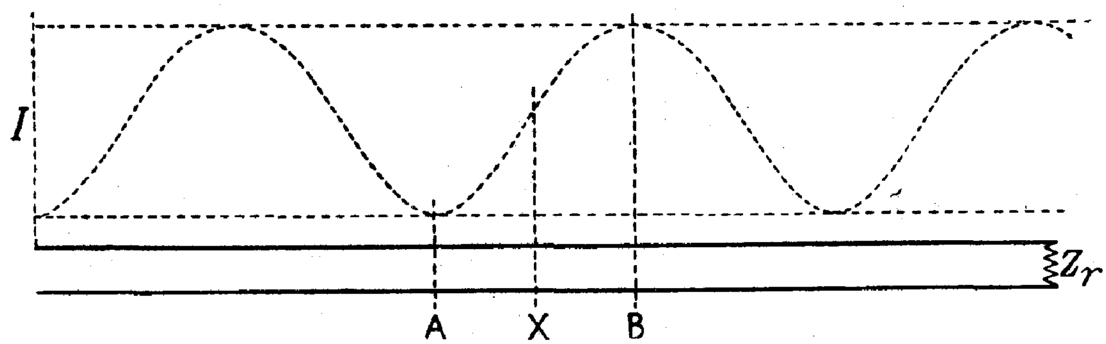


FIG. 52, CHAP. XV.—Impedance at various points in transmission line.

the input impedance is purely resistive, say R ohms, where R is greater than Z_0, R and r being related by the equation $rR = Z_0^2$. At any intermediate point, e.g. at X, the input admittance is complex, say $G + jB = \dfrac{1}{r' + jx'}$, and it is possible to match the line up to the point X (approximately) by connecting a susceptance of $-jB$ ohms so that the line becomes non-reactive at the point X.

121. It has already been shown that the input impedance of the section of line of length l, between X and Z_r, is

$$Z_x = Z_0 \frac{Z_r \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_r \sin \beta l}$$

and its admittance is

$$\frac{1}{Z_x} = Y_x = \frac{Z_0 \cos \beta l + jZ_r \sin \beta l}{Z_0 (Z_r \cos \beta l + jZ_0 \sin \beta l)}.$$

Rationalizing the denominator

$$Y_x = \frac{1}{Z_0} \frac{Z_r Z_0 (\cos^2 \beta l + \sin^2 \beta l) + j (Z_r^2 - Z_0^2) \sin \beta l \cos \beta l}{Z_r^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}. \quad \dots \quad (42)$$

If the matching is to be achieved by the addition of a purely susceptive device, the real part of the admittance must be equal to $\dfrac{1}{Z_0}$,

i.e.

$$\frac{1}{Z_0} = \frac{1}{Z_0} \frac{Z_r Z_0}{Z_r^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}$$

or

$$\frac{1}{Z_0} = \frac{Z_r}{Z_r^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}. \quad \dots \quad \dots \quad \dots \quad \dots \quad (43)$$

If $Z_r = nZ_0$ we have

$$\frac{1}{Z_0} = \frac{nZ_0}{n^2 Z_0^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}$$

$$\frac{1}{n} = \frac{1}{n^2 \cos^2 \beta l + \sin^2 \beta l}. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (44)$$

To solve this we observe that if $\tan^2\theta = m$, $\sin^2\theta = m \cos^2\theta$ and $m \sin^2\theta + \sin^2\theta = m^2 \cos^2\theta + \sin^2\theta$.

So that if $n^2 \cos^2 \beta l + \sin^2 \beta l = n$

$$n \sin^2 \beta l + \sin^2 \beta l = n$$

$$\left.\begin{aligned} \therefore \sin^2 \beta l &= \frac{n}{n+1} \\ \cos^2 \beta l &= \frac{1}{n+1} \\ \tan^2 \beta l &= n \end{aligned}\right\} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (45)$$

It follows that there is a value of βl in every quadrant of $\dfrac{\pi}{2}$ radians which will meet the required condition. If l' is the lowest value of l which will do this, the above equation may be written

$$l = \frac{m\lambda}{2} \pm l', \text{ where } m \text{ is a positive integer.}$$

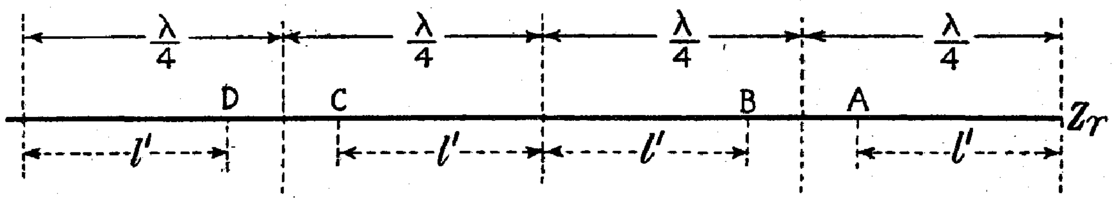
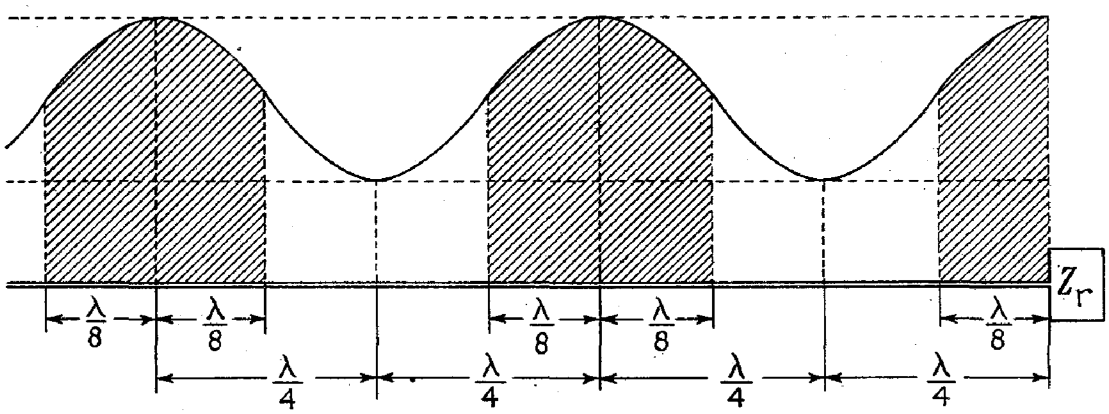


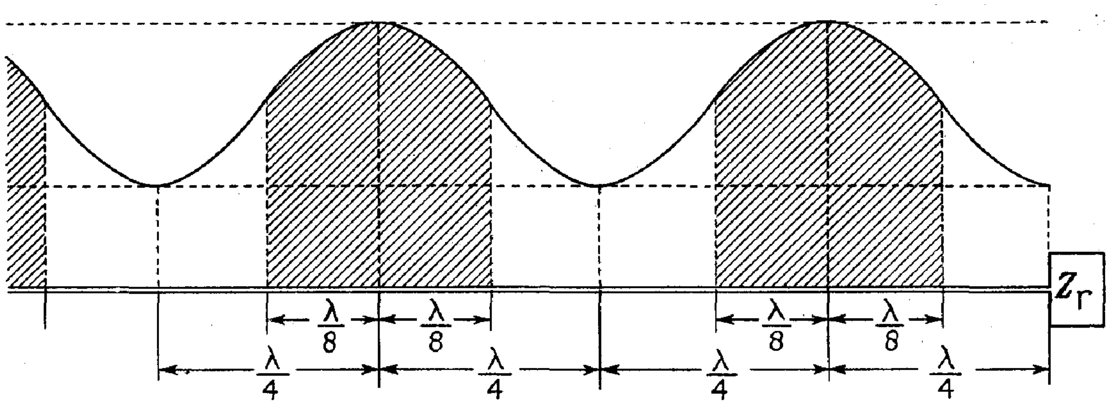
FIG. 53, CHAP. XV.—Physical meaning of $l = \frac{m\lambda}{2} + l'$.

This result may be translated into a physical picture of the feeder, fig. 53, in which A, B, C, D, etc., are possible positions for the matching impedance.

122. We must now consider the effect of the value of n. If n is less than unity, $\sin^2 \beta l'$ $\left(= \frac{n}{n+1}\right)$ must be less than $\frac{1}{2}$. Then, $\sin \beta l'$ is less than ·707 and $\beta l'$ less than $\frac{\pi}{4}$. Since $\beta = \frac{2\pi}{\lambda}$, $\frac{2\pi}{\lambda} l' < \frac{\pi}{4}$ means that l' must be less than $\frac{\lambda}{8}$. There will be a current maximum at the end of the line, and l' must be within the shaded areas of fig. 54a, i.e. if n is less than unity, the added susceptance must be applied within a distance of $\frac{\lambda}{8}$ from a point of maximum current.



(a) $n < 1$, current maximum at Z_r



(b) $n > 1$, current minimum at Z_r

FIG. 54, CHAP. XV.—Positions of matching susceptance

If n is greater than unity, $\sin \beta l'$ must be greater than $\cdot 707$ and the value of $\beta l'$ must be between $\frac{\pi}{4}$ and $\frac{\pi}{2}$. Hence l' must be between $\frac{\lambda}{8}$ and $\frac{\lambda}{4}$. The end of the line is a current minimum and l' must be within the shaded areas of fig. 54b, i.e. if n is greater than unity the matching susceptance must be applied within $\frac{\lambda}{8}$ of a current maximum as before. Thus we do not need to know whether Z_r is greater or less than Z_0, provided that we measure l' from a current maximum. It has already been shown that the ratio $\frac{I_{min}}{I_{max}}$ is equal either to n or $\frac{1}{n}$, and as it does not matter whether n is greater or less than unity, the practical method is to measure $\frac{I_{min}}{I_{max}}$ and call this ratio n. Substituting this value in equation (45) will give a value for $\sin^2 \beta l$ and therefore for l, fixing the possible positions at which a matching susceptance must be applied. It will be seen that there is a choice of positions. Before dealing with these, we may find the value of the matching susceptance B_M. This must be equal in magnitude but of opposite sign to the imaginary part of equation 42,

i.e.
$$jB_M = -\frac{j}{Z_0}\,\frac{(Z_r^2 - Z_0^2)\sin \beta l \cos \beta l}{Z_r^2 \cos^2 \beta l + Z_0^2 \sin^2 \beta l}$$

$$B_M = -\frac{1}{Z_0}\,\frac{\left[\left(\frac{Z_r}{Z_0}\right)^2 - 1\right]\sin \beta l \cos \beta l}{\left(\frac{Z_r}{Z_0}\right)^2 \cos^2 \beta l + \sin^2 \beta l}$$

$$= -\frac{1}{Z_0}\,\frac{(n^2 - 1)\sin \beta l \cos \beta l}{n^2 \cos^2 \beta l + \sin^2 \beta l}$$

$$= \frac{1}{Z_0}\,\frac{(1 - n^2)\sin \beta l \cos \beta l}{n^2 \cos^2 \beta l + \sin^2 \beta l}, \qquad (46)$$

since we need only consider the case when $n < 1$. Now $\sin \beta l \cos \beta l = \frac{\sin 2\beta l}{2}$ and $\sin 2\beta l$ may be either positive or negative. If $l = l'$, $2\beta l$ cannot exceed $\frac{\pi}{4}$ and the sign of $\sin 2\beta l$ will be positive. This also applies if $l = m\pi + l'$. Under these circumstances B_M will be a positive, i.e. capacitive susceptance. If, however, $l = m\pi - l'$, B_M becomes negative, corresponding to an inductive susceptance.

123. If the matching reactance is placed on the input side of a current maximum, therefore, it must be capacitive, while if placed on the opposite side it must be inductive. This is shown

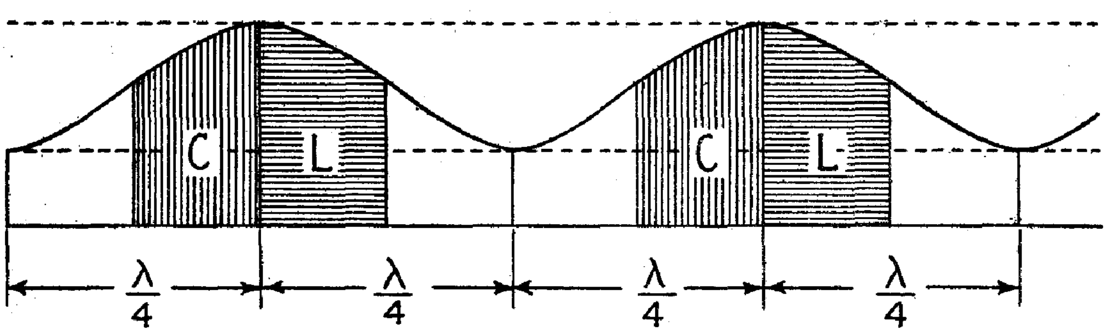


FIG. 55, CHAP. XV.—Nature of matching susceptance.

diagrammatically in fig. 55. The magnitude of the matching susceptance will now be found. From equation (46),

$$B_M = \frac{1}{Z_0} \frac{(1 - n^2) \sin \beta l \cos \beta l}{n^2 \cos^2 \beta l + \sin^2 \beta l}$$

and from equation (45), $\sin^2 \beta l = \dfrac{n}{n+1}$, $\cos^2 \beta l = \dfrac{1}{n+1}$

$$B_M = \frac{(1 - n^2) \dfrac{\sqrt{n}}{\sqrt{n+1}} \times \dfrac{1}{\sqrt{n+1}}}{Z_0 \left(n^2 \dfrac{1}{n+1} + \dfrac{n}{n+1} \right)}$$

which simplifies to

$$B_M = \frac{1 - n}{\sqrt{n}\, Z_0} \qquad \ldots \qquad (47)$$

For example, if $n = 0 \cdot 16$, $Z_0 = 600$ ohms, $B_M = \dfrac{1 - 0 \cdot 16}{\sqrt{0 \cdot 16} \times 600} = 0 \cdot 0035$ siemens (mho). At a frequency of 2 Mc/s ($\omega = 4\pi \times 10^6$), $B_M = \omega C$ or $\dfrac{1}{\omega L}$.

Suppose we decide to add capacitance,

$$C = \frac{0 \cdot 0035}{\omega} = \frac{0 \cdot 0035}{4\pi \times 10^6} \text{ farads}$$
$$= 278\ \mu\mu\text{F}.$$

If we decide to add inductance,

$$L = \frac{1}{0 \cdot 00035\, \omega}$$
$$= \frac{1}{0 \cdot 00035 \times 4\pi \times 10^6} \text{ henries}$$
$$= 22 \cdot 8\ \mu\text{H}.$$

124. In practice the matching inductance or capacitance is usually added by means of a section of line, either on open or short circuit, as explained in paragraphs 86, 115 *et seq.* It will be found that by a judicious choice of the side of the current maximum upon which this line is connected, it is always possible for these additional "matching lines", as they are called, to be less than $\dfrac{\lambda}{8}$ in length. Thus, continuing the above example, we will calculate the position of the added susceptance. From equation (45)

$$\tan \beta l = \sqrt{n} = \sqrt{0 \cdot 16} = 0 \cdot 4$$

and from tables we find $0 \cdot 4 = \tan^{-1} 21° 48'$.

Converting to radians,

$$21 \cdot 48 \text{ degrees} = \frac{21 \cdot 8 \times \pi}{180} \text{ radians}$$

$$\beta l' = \frac{2\pi}{\lambda} l' = \frac{21 \cdot 8\pi}{180}$$

$$l' = \frac{21 \cdot 8\pi}{180} \times \frac{\lambda}{2\pi}$$
$$= 0 \cdot 0605\lambda.$$

Now suppose we decide to connect the matching line on the output side of a current maximum. Then B_M must be inductive. From paragraph 86 we find that the susceptance of a short-circuited line of length l_1 is

$$B_1 = -\frac{1}{Z_0} \cot \beta l_1$$

But
$$B_M = -\frac{1 - n}{Z_0 \quad \sqrt{n}}$$

and the length l_1 must be such that

$$\cot \beta l_1 = \frac{1 - n}{\sqrt{n}}$$

Continuing the example, $n = 0{\cdot}16$, $\sqrt{n} = 0{\cdot}4$,

$$\cot \beta l_1 = \frac{0{\cdot}84}{0{\cdot}4} = 2{\cdot}1.$$

From tables, $\beta l_1 \doteqdot 25{\cdot}45°$

$$\frac{2\pi}{\lambda} l_1 = \frac{25{\cdot}45 \times \pi}{180}$$

$$l_1 = \frac{25{\cdot}45 \times \pi}{180} \times \frac{\lambda}{2\pi}$$

$$= 0{\cdot}071\lambda$$

Note that l' gives the distance, measured from a current maximum towards the output end, at which the loop must be placed, while l_1 gives the length of the short-circuited loop of matching line which must be added.

125. Let us now find what must be done to achieve capacitive matching. The distance l' will be the same as before but must be measured from a current maximum towards the transmitter. The susceptance B_2 of a short length l_2 of open-circuited line is, by equation (31), paragraph 86,

$$B_2 = \frac{1}{Z_0} \tan \beta l_2$$

Equating as before, since $B_M = \frac{1 - n}{Z_0 \quad \sqrt{n}}$,

$$\tan \beta l_2 = \frac{1 - n}{\sqrt{n}}$$

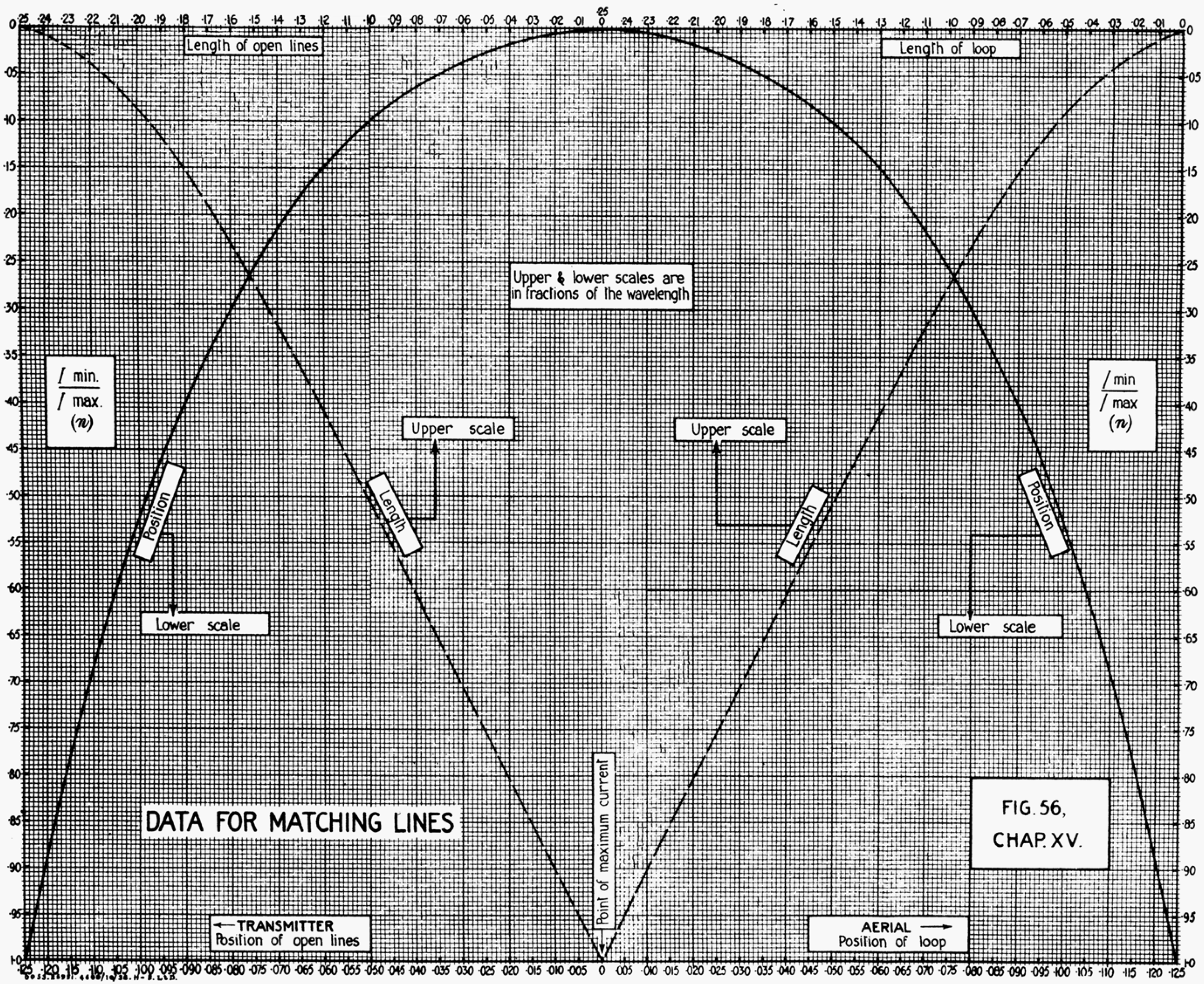
$$= 2{\cdot}1$$

From tables,
$$\beta l_2 = 64{\cdot}5°$$

$$\frac{2\pi}{\lambda} l_2 = \frac{64{\cdot}5 \times \pi}{180}$$

$$l_2 = 0{\cdot}179\lambda.$$

To avoid the necessity for these computations, however, fig. 56 has been developed. For any given value of n, we may read off the necessary length, either of closed (l_1) or open (l_2) matching line, from the dotted curve and top scale, and the distance l' from the current maximum by means of the full-line curve and the lower scale.


Length of open lines
Length of loop
Upper & lower scales are in fractions of the wavelength
/ min.
/ max.
(n)
Upper scale
Lower scale
Position
Length
Point of maximum current
DATA FOR MATCHING LINES
FIG. 56, CHAP. XV.
← TRANSMITTER
Position of open lines
AERIAL →
Position of loop

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126. The foregoing theory assumes that the feeder line is terminated by a purely resistive impedance. In practice this may not be the case, but it can be shown that provided the datum point for measurement is a current maximum, the actual calculations are exactly as for a resistive termination. This is because if the line is otherwise terminated, all the current maxima and minima are shifted equally along the line.

Practical application of loop matching

127. The application of the above theory to the matching of an aerial array is as follows. The array itself will usually consist either of half-wave or quarter-wave (electrical) elements, although their actual length may not exceed $\cdot 46\lambda$ and $\cdot 23\lambda$ respectively. The transmission line may be a pair of conductors, supported upon poles as high as practicable above the ground, and clear of all irregularities of terrain. It is essential that the insulation at the points of support shall be maintained at a very high value, for a lumped leakage conductance at any point constitutes a change in the electrical character of the line and gives rise to reflection, with a consequent production of quasi-stationary waves, thus leading to both heat losses and undesired radiation from the feeder. For the same reason, the conductors should be symmetrical with respect to earth, the transmitter and the aerial array, and sharp bends must be avoided.

128. The first step in matching the array to the line is to energize the line, and observe the stationary waves in the latter. A suitable arrangement for this purpose is shown in fig. 57. It consists of a thermo-ammeter reading 0–120 milliamperes, which is mounted in a loop circuit; this loop may be suspended from one of the conductors forming the transmission line. The size of loop shown is suitable for an input into the line of the order of 1 kilowatt. The line is energized at a reduced input and the loop is drawn along it, and the current reading observed, field glasses being of assistance in this process. The current maximum at the point nearest the array is then selected for particular observation, and the power increased until the ammeter gives nearly a full scale deflection. The exact position on the line of the current maximum should be marked, and the actual scale reading, I_{max}, noted. The loop is then drawn along the line to the adjacent minimum, the current, I_{min}, being noted and its position marked. Especial care must be taken

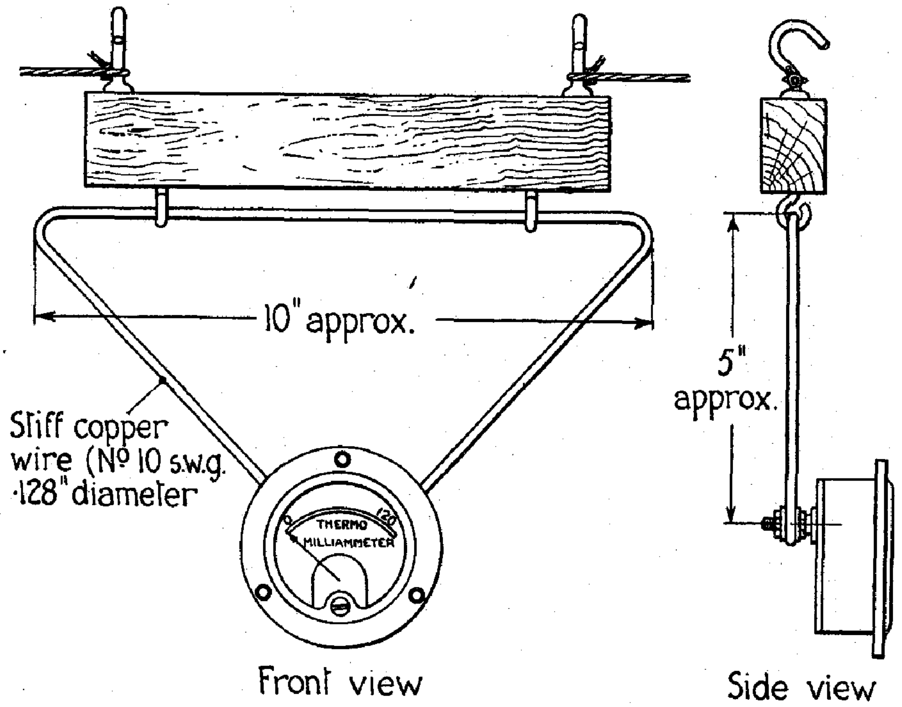


FIG. 57, CHAP. XV.—Ammeter and transformer for matching purposes.

in reading the minimum current since the lower part of the scale is very cramped. At this point it is advisable to change the ammeter over to the other conductor and verify that the currents along both lines are equal and that the maxima and minima are in the same positions in each line. If this is not so, the line and the input termination should be examined with a view to the elimination of any out-of-balance effects, as it is hopeless to attempt to match an unbalanced line. The distance between adjacent maximum and minimum positions should also be checked. This should be between $\cdot 23\lambda$ and $\cdot 25\lambda$. When all is satisfactory a final check of I_{max} and I_{min} will give the ratio $\frac{I_{min}}{I_{max}} = n$.

129. Referring to fig. 56, we now locate the position of the matching line by means of the curve marked "Position", e.g. if $n = 0 \cdot 3$, reference to the left hand half of the diagram shows that matching may be achieved by means of an open line, distant $\cdot 080\lambda$ from the current maximum, on the side nearer to the transmitter, or by a closed loop $\cdot 080\lambda$ from the current maximum on the side nearer the aerial. Note that the bottom scale is to be read. Reference to the "Length" curve will now give the length of the matching line; in the example given, $n = 0 \cdot 3$, and reading from the top scale of the diagram, we find that an open line of $\cdot 144\lambda$ or a closed loop of $\cdot 108\lambda$ will produce the desired effect, the respective positions being of course on different sides of the current maximum.

130. The choice of open line or closed loop must be governed by local circumstances. For instance if $n = 0 \cdot 8$, we obtain from the closed loop curves a value of $0 \cdot 116\lambda$ for the position and $0 \cdot 21\lambda$ for the length, while from the open line curves we obtain $0 \cdot 116\lambda$ for the position and $0 \cdot 04\lambda$ for the length. If the wavelength is great compared with the height of the line above the ground, it may not be convenient to attach a loop of $0 \cdot 21\lambda$ to the line, whereas an open line of $0 \cdot 04\lambda$ may be only a few feet in length and easily suspended from the line. Whether open or closed matching lines are used, they must be perpendicular to the transmission line, otherwise interaction will occur between the transmission and matching lines and will give rise to losses.

131. (i) When the matching line has been attached to the transmission line the ammeter should be drawn along the latter and the maximum and minimum readings again taken; n should now approach unity. If n is less than $0 \cdot 833$ $\left(\frac{1}{n} > 1 \cdot 2\right)$ a slight adjustment of the position of the matching line, or of its length, may improve matters, but an alteration of only an inch or so at a time should be made. If the position of the maxima and minima have interchanged, over-correction is indicated. Typical readings for an aerial consisting of a single dipole are given below.

Maximum current $= 0 \cdot 114$ amperes

Minimum current $= 0 \cdot 031$ amperes

$n = \cdot 272$

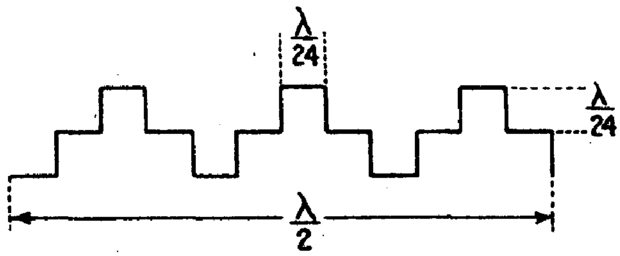
Position of matching impedance $= 0 \cdot 078\lambda$

Length of loop $= 0 \cdot 1\lambda$, or

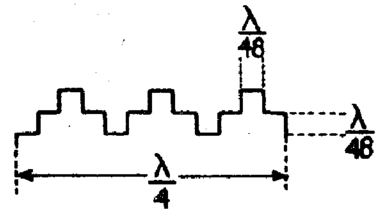
Length of open line $= 0 \cdot 15\lambda$

The ratio of maximum to minimum current after matching was $1 \cdot 14$.

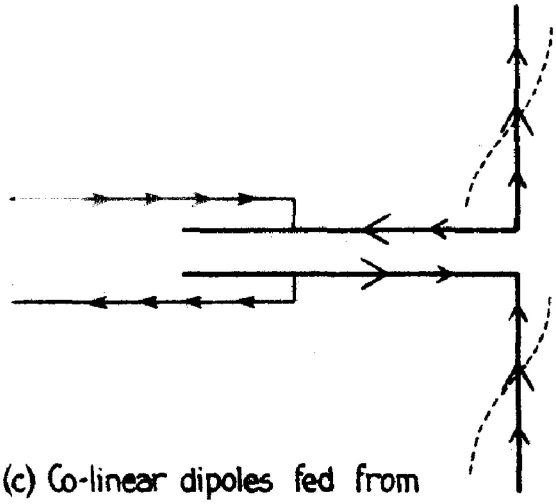
(ii) Even where matching at the aerial termination is performed by some other method, loop matching lends itself to the compensation for the lack of uniform capacitance per unit length of line. This lack of uniformity always exists to some extent because the line must in practice be supported by insulators having a permittivity greater than that of air. At frequencies of the order of 3 Mc/s the effect may be insignificant, but it will probably be appreciable at frequencies above 10 Mc/s, particularly on long lines. The symptom of such lack of uniformity is that whereas the ratio of maximum to minimum current over the first few half-wavelengths from the aerial



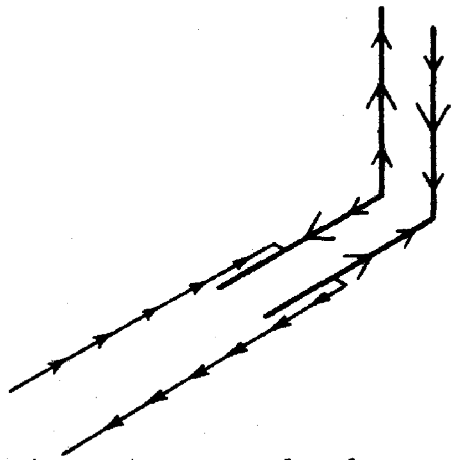
(a) One wavelength of wire acting as 1:1 transformer



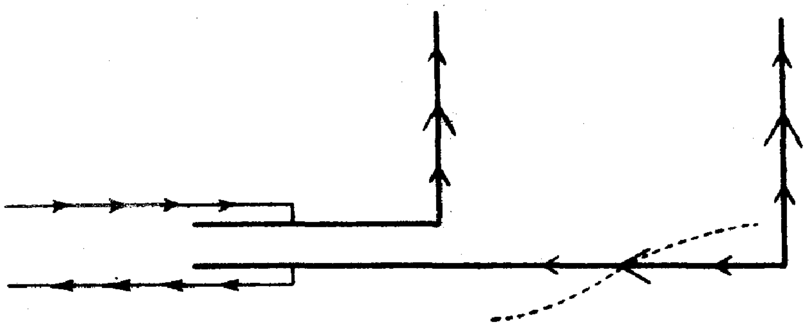
(b) One half wavelength of wire acting as -1:1 transformer



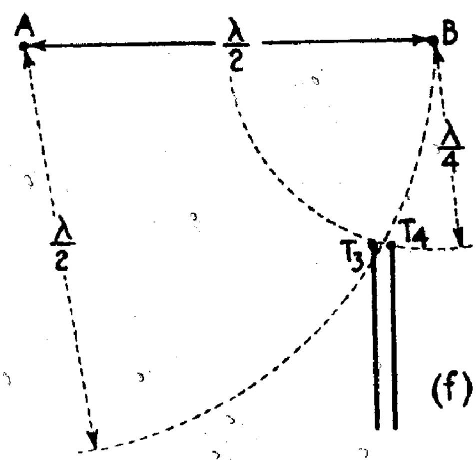
(c) Co-linear dipoles fed from transmission line.
Fields assist each other

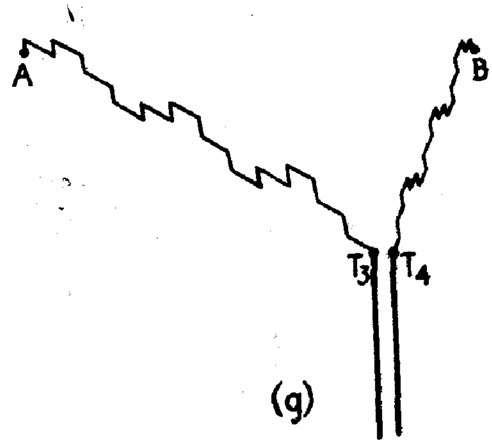


(d) Parallel dipoles fed from transmission line.
Fields cancel each other



(e) Synphased parallel dipoles fed from opposite sides of feeder line by half wave conductor acting as -1:1 transformer





FEEDERS FOR PARALLEL $\frac{\lambda}{2}$ AERIALS

FIG. 58
CHAP. XV

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end may be quite near to unity, quasi-stationary waves develop at more remote points, the mis-matching becoming more serious as the transmitter is approached. The remedy for this state of affairs is to add additional matching loops at intervals in order to maintain the $\frac{I_{min}}{I_{max}}$ ratio as near as possible to unity along the whole length of the feeder.

Suppression unit

132. It has been stated that a length λ of loss-free conductor acts as a perfect 1 : 1 ratio transformer while a loss-free length of $\frac{\lambda}{2}$ acts as a perfect transformer of ratio — 1 : 1. Although a conductor cannot be entirely loss-free, if it is arranged in such a manner as to be practically non-radiating, its loss will be almost entirely due to joulean heat and may be very small. It is possible to approach the desired non-radiating property by folding the conductor symmetrically as shown in fig. 58a and fig. 58b, in which it will be observed that the total length of wire is double the actual distance between the input and output terminals. A feeder arranged in this manner is sometimes called a " suppression unit ". In fig. 58a, the feeder is required to act as a 1 : 1 transformer and the total length of wire is one λ. It is bent rectangularly at intervals of $\frac{\lambda}{24}$, forming a kind of chequer pattern in space. Similarly in fig. 58b, we have a length $\frac{\lambda}{2}$ of conductor bent into 24 rectangular loops at intervals of $\frac{\lambda}{48}$, the whole acting as a — 1 : 1 transformer. It is recommended that where this expedient is adopted the number of bent portions shall never be less than 24. The velocity of the wave along the wire is probably at least 10 per cent. less than in free space and the theoretical length of conductor should be reduced by this factor.

133. As an example of the use of such a feeder, let us consider the problem of feeding two parallel vertical dipoles, with syn-phased current from a twin-wire transmission line. If instead of being parallel, they are arranged co-linearly as in fig. 58c, they could be fed directly from the line through a suitable matching device. If however the lower aerial of the two is turned upwards so that the dipoles are parallel instead of co-linear (fig. 58d), it is seen that the currents in the two aerials are in opposite directions, and the desired polar diagram will not be obtained. Some form of — 1 : 1 transformer must therefore be inserted in one of the aerials, and the only question is the form it shall take. If the parallel dipoles are $\frac{\lambda}{2}$ apart the most obvious method is to use a single $\frac{\lambda}{2}$ length of horizontal conductor as in fig. 58e. This conductor will however possess radiating properties and consequently the power to be supplied from one side of the line will be considerably greater than from the other, i.e. the aerial system is unbalanced. Nor is this all; unbalanced currents must flow in the transmission line and consequently this will also radiate. The final result may well be that the polar characteristics are very different from those aimed at.

134. An alternative method of feeding is shown in fig. 58f, in which the output terminals T_3 T_4 of the matching network are located as follows. From the aerial A, draw an arc of radius $\frac{\lambda}{2}$, and from B an arc of radius $\frac{\lambda}{4}$. Then T_3, T_4 are to be located at the intersection of these arcs. From the output terminal T_3 to the aerial A we may now connect a one-λ section of non-radiative

feeder as described above, which will behave as a 1 : 1 transformer, and from the output terminal T_4 to the aerial B a $\frac{\lambda}{2}$ section giving a transformation ratio of $-1:1$. The arrangement is then as shown in fig. 58g. The two aerials will then be energized in syn-phase and will give the required polar diagram, i.e. A 5 of fig. 8. The same method may be adopted for any aerial spacing up to $\frac{3}{\lambda}\lambda$, for which the terminals T_3 T_4 will lie upon the line joining the two aerials. This form of non-radiating feeder is often used to feed the vertical radiators in the Franklin uniform array, and also in conjunction with multiple unit series phase arrays.

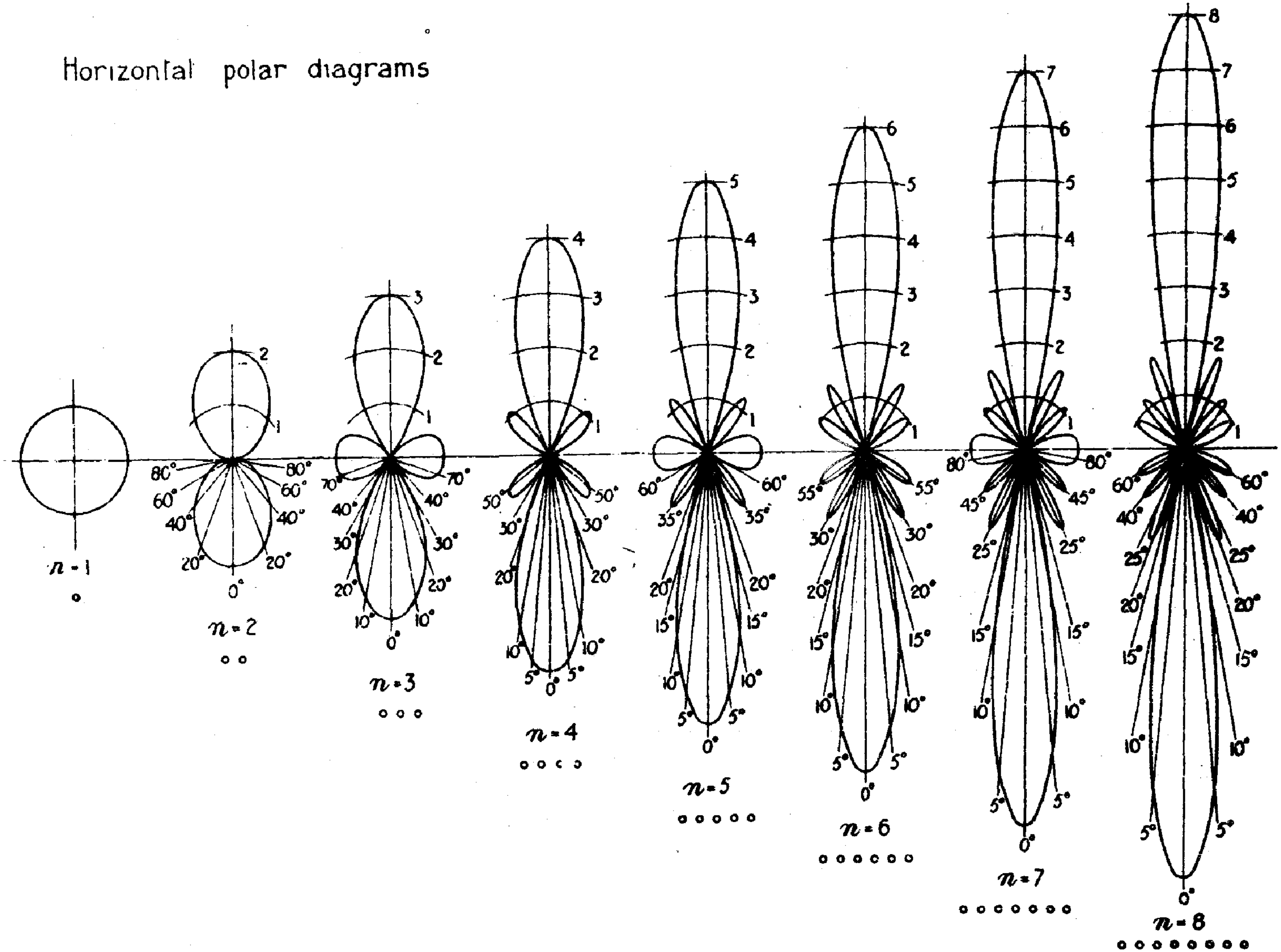
PRACTICAL TYPES OF AERIAL ARRAY

135. The simple broadside array consists of a number of aerials spaced at uniform distances along a horizontal line and fed in such a manner that all the currents are syn-phased. The width of the array, in wavelengths, is called its aperture. Fig. 59 shows the effect in the horizontal plane of an increase in the aperture of an array consisting of vertical dipoles spaced $\frac{\lambda}{2}$ apart. Each dipole is assumed to carry the same current, I, and the effect of mutual impedance between the various members is neglected. It will be observed that with n dipoles the field strength in the direction perpendicular to the line of the array is n times that given by a single dipole. To obtain this increase with a single dipole the current must increase to nI, and the power input would be proportional to $(nI)^2$. With the array of n elements, however, the power input to each is proportional to I^2, and the total power input to nI^2. The improvement of the array over a single dipole may be obtained from the ratio of powers and is obviously equal to n. In other words, to give a certain field strength in the required direction, an array of n elements requires only $\frac{1}{n}$ of the power which would be required by a single dipole to give the same field.

136. It must be emphasized that the improvement shown in fig. 59 can only be obtained by an appropriate increase in power supply. In order to bring out this point, fig. 60 has been prepared. This shows the horizontal polar diagrams of various arrays consisting of from one to eight elements as in the previous instance, but with the same power input in each array. The improvement is now proportional to $\sqrt{n}$ instead of to n. The effect of mutual impedance between the members may cause the improvement to be slightly less than $\sqrt{n}$, but the shape of the polar diagram is very little affected. The shape of the vertical polar diagram of such an array is given by the Current Distribution Factor for a single dipole, multiplied by the reflection diagram appropriate to the earth in the vicinity. The scale is dictated by the same considerations as that of the horizontal polar diagram.

137. The effect of a suitable reflector curtain is shown in the next diagram, fig. 61, in which each energized element is supposed to carry the same current. The effect of the reflector is to double the field strength in one of the two directions perpendicular to the array and to suppress the radiation in the other. This diagram is the theoretical one obtained with a reflector dipole placed $\frac{\lambda}{4}$ behind each energized dipole. Actually, it may be found desirable in practice either to detune the reflector wires by making them a little longer or shorter than the energized members, or to use a separation other than $\frac{\lambda}{4}$ between energized and reflector wires. Both methods may, of course, be used in conjunction.

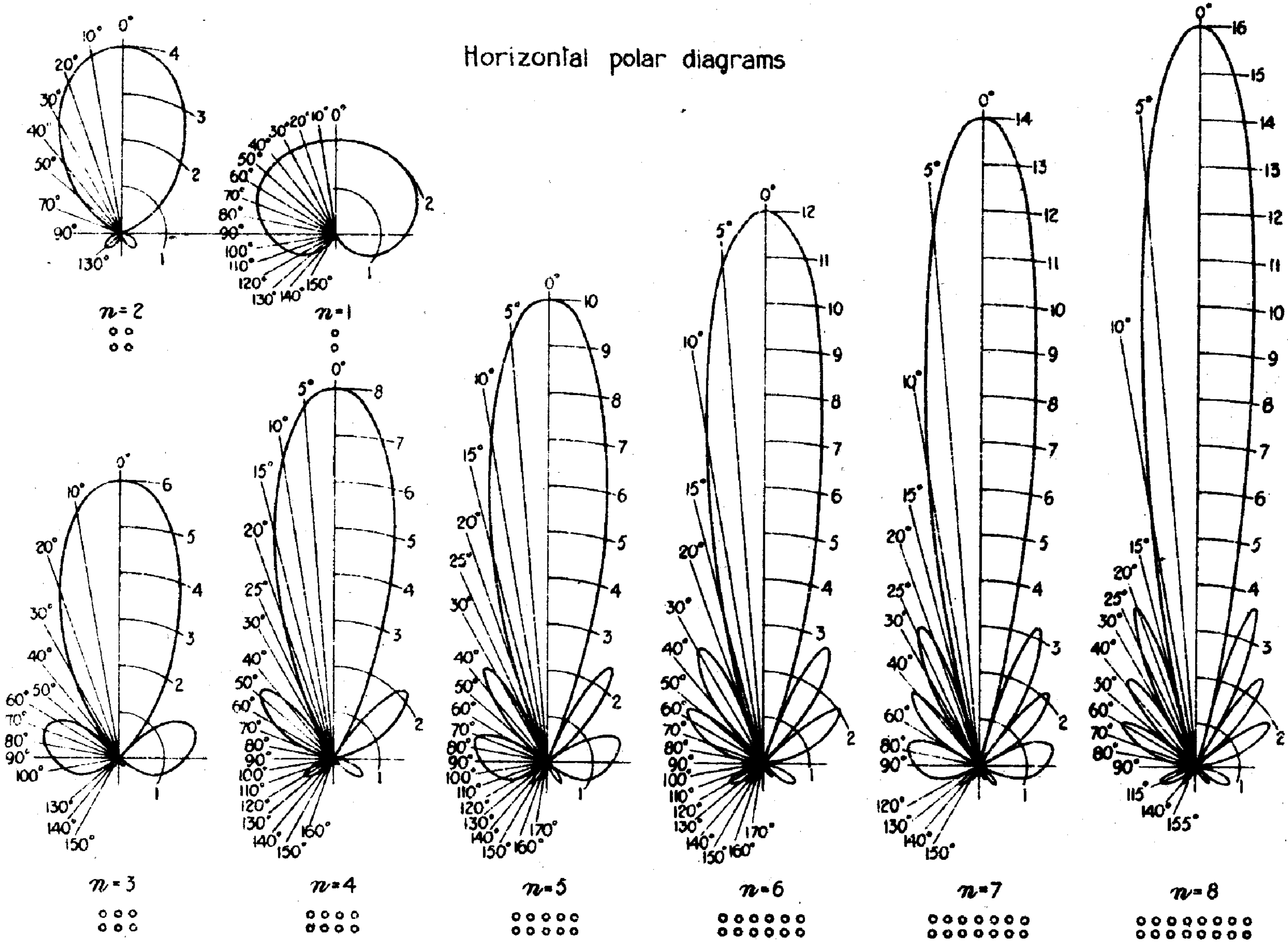
138. In order to obtain a low angle for the main beam, the lower ends of the vertical members should be as high above the ground as possible, and in any case not less than $\frac{\lambda}{2}$. Allowing for the

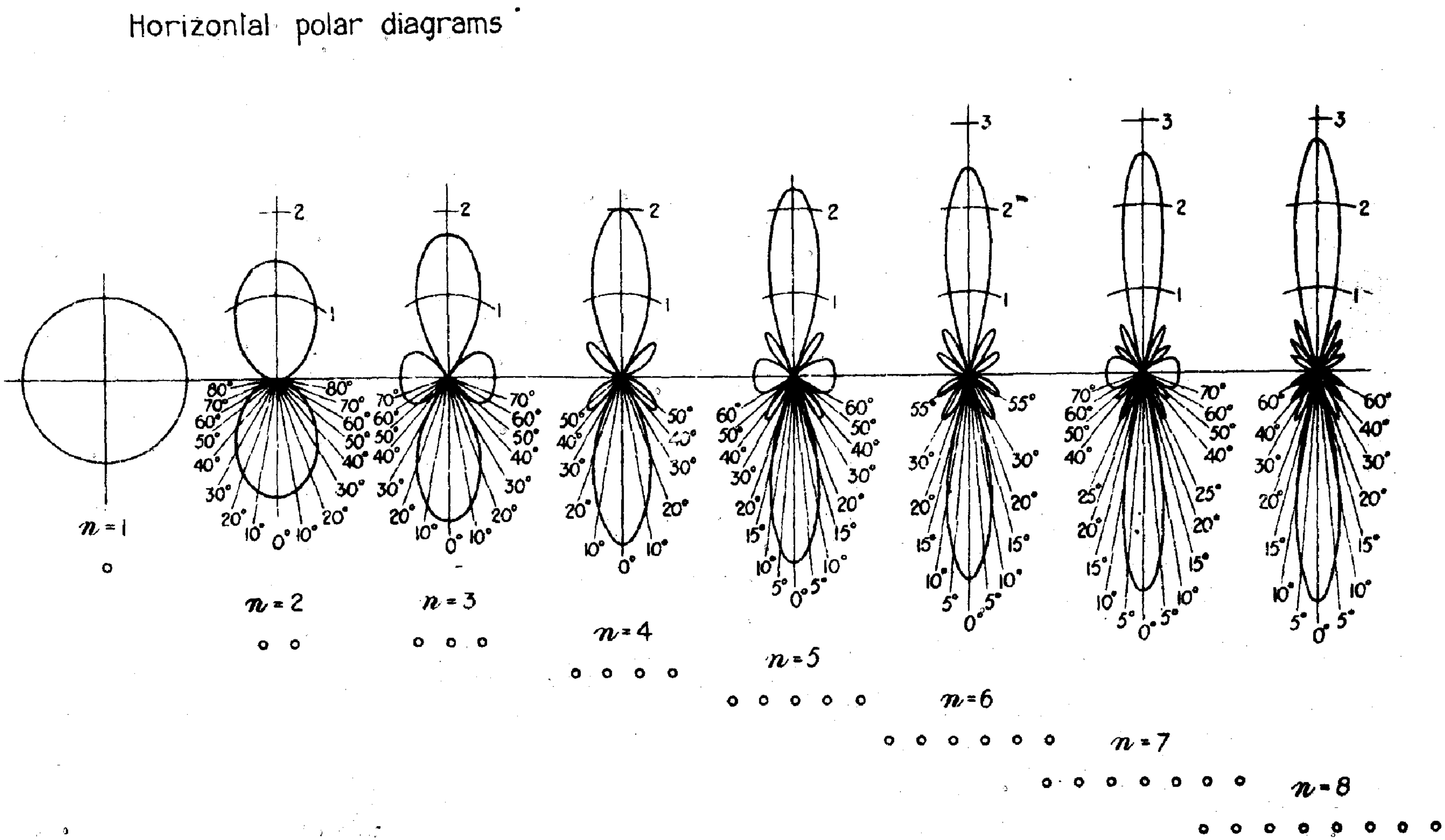


VERTICAL $\frac{1}{2}\lambda$ DIPOLES, UNIT CURRENT IN EACH ELEMENT

FIG. 59.
CHAP. XV

VERTICAL $\frac{\lambda}{2}$ DIPOLES WITH REFLECTORS, UNIT CURRENT IN EACH ELEMENT
FIG. 60.
CHAP. XV





VERTICAL $\frac{\lambda}{2}$ DIPOLES, UNIT POWER IN EACH ARRAY

FIG. 61, CHAP. XV

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sag in the triatic stay from which the wires are suspended, this means that the masts supporting an array of this type must be about $\frac{3}{2}\lambda$ in height. This is one of the practical disadvantages of this form of array for service purposes. Another disadvantage is the difficulty of feeding. If a feeder is attached to the lower end of each element of the energized curtain, i.e. at a voltage loop, the input impedance is of the order of 3,000 ohms, and an impedance matching device must be inserted between the feeder and the aerial element. The aerial itself is an unbalanced load, so that the alternatives presented are (i) to use a concentric feeder and some form of matching network (ii) to use parallel-wire transmission lines in conjunction with a transformer. As all the feeding points must be energized in phase, matching must be performed at a large number of points, or else a comparatively high degree of mis-matching accepted.

Tiered arrays

139. If masts of sufficient height are available, it is advantageous to arrange tiers of vertical radiators, one above the other. This results in an increase in field strength in the required direction and also gives increased directivity in the vertical plane. The problem of feeding the array also becomes somewhat simpler from the purely theoretical point of view, provided that the elements to be fed are $\frac{\lambda}{2}$ apart. It has previously been shown that a $\frac{\lambda}{2}$ length of loss-free transmission line acts as a perfect $-1:1$ transformer. It is therefore possible to arrange the feeder in the manner shown in fig. 62, alternate elements being voltage fed from opposite sides of the feeder line. The voltage distribution along the feeder is then as shown by the dotted lines, and the current distribution in the elements as shown by arrows. This method of feeding is an obvious development of fig. 58c, but in the present instance, instead of a single unbalanced conductor,

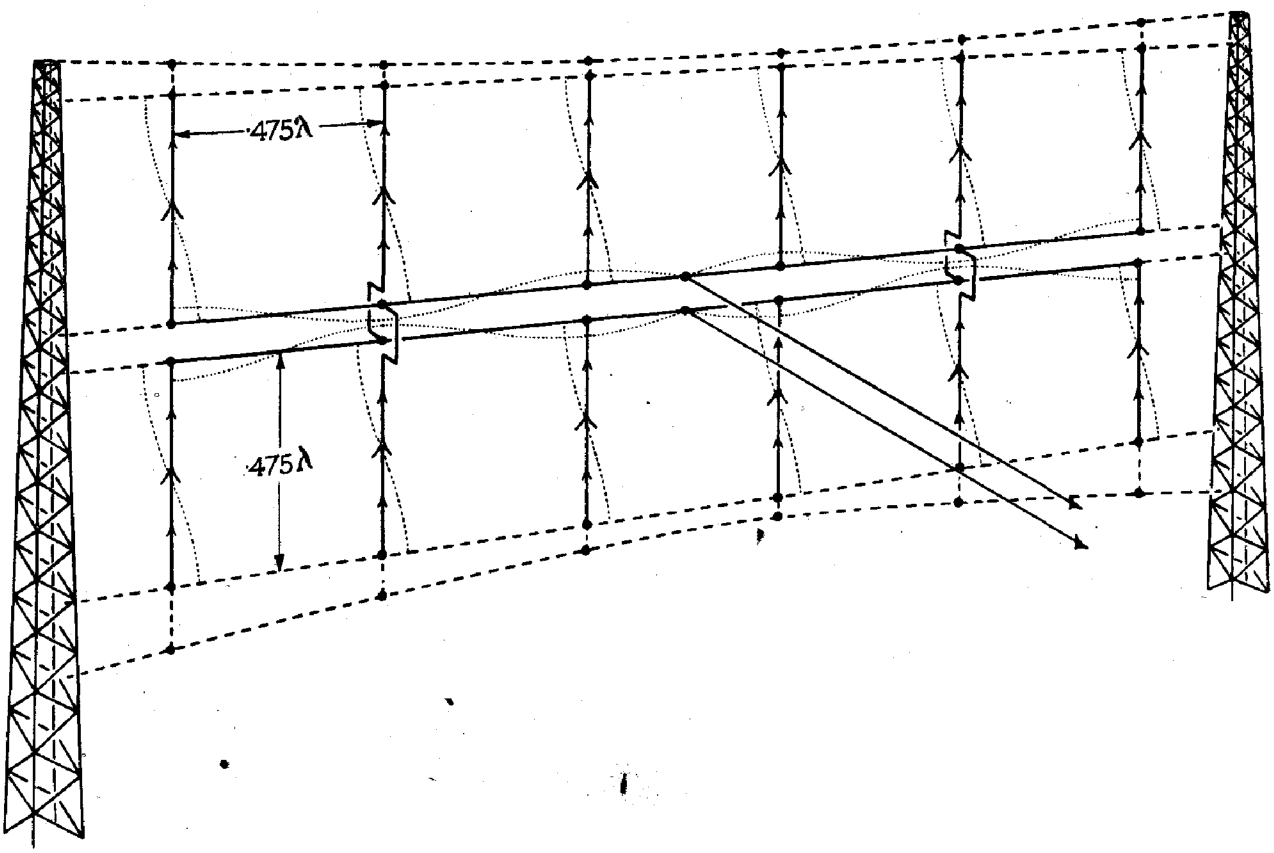


FIG. 62, CHAP. XV.—Array of vertical dipoles.

there is a twin resonant feeder line between each pair of feeding points. This feeder does not radiate appreciably and the load is very well balanced. If the bottom of the array is to be $\frac{\lambda}{2}$ above ground, the masts supporting it must have a height of about 2λ, and the feeding point will be one λ above ground. The mechanical difficulty of fitting and adjusting a matching device at this height above ground is such that the arrangement is rarely adopted.

Sterba array

140. The Sterba array is shown in fig. 63. Each unit consists of a single continuous conductor which is supplied with current at the appropriate frequency. The half-wavelength sections are arranged vertically and horizontally in an alternative manner, so that all the vertical wires carry current in phase, and set up radiation, while the horizontal sections are so arranged that

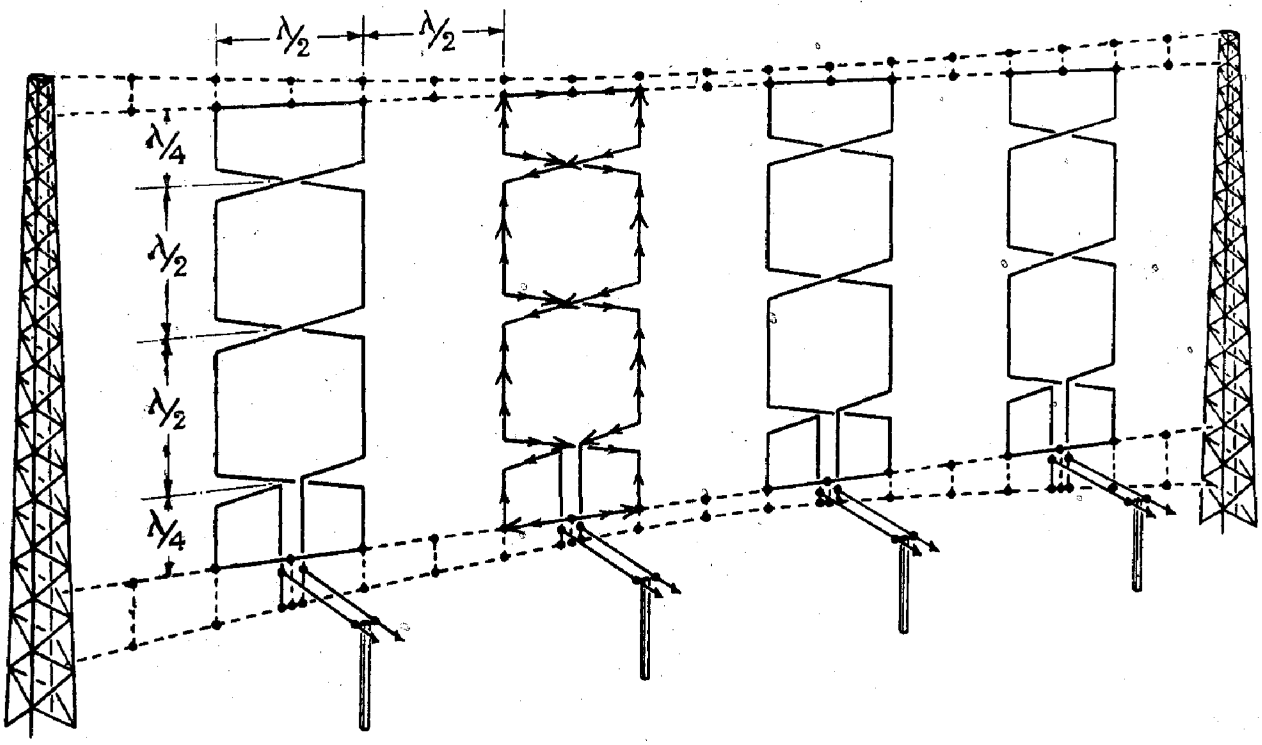


FIG. 63, CHAP. XV.—Sterba array.

they are non-radiative. The feeding points are at the current loop of one half-wave section, the unit is thus offering minimum impedance, i.e. it functions as an acceptor circuit. A number of units are erected side by side and fed with syn-phased currents by transmission lines *via* suitable matching devices. A similar array about a quarter of a wavelength behind the energized array will act as a reflector. The object of this arrangement is to allow a direct current to be fed through the radiator wires for the purpose of thawing any accumulation of snow or ice, suitable filtering devices being incorporated in the matching unit. Where " de-icing " is not necessary, the Sterba array offers no particular advantage over the simple broadside array.

End-fire array

141. An end-fire array differs from a broadside array in that there is a progressive phase difference between the currents in adjacent aerials. If β is the difference in phase and d the spacing, $\frac{d}{\lambda} = \frac{\beta}{2\pi}$. The effect of this phase progression is to cause the radiation in the horizontal

plane to be concentrated in a main lobe together with small subsidiary lobes, the main lobe being directed along the line upon which the aerials are situated. Whereas the broadside array is bi-directional, the end-fire array is unidirectional, so far as the main lobe is concerned, the radiation being directed towards the end at which the phase is lagging. The radiation in the vertical plane containing the array is more or less concentrated in a direction near the horizontal plane. The series-phase array described later is a particular example of this type.

Arrays of horizontal dipoles

142. Although the horizontal dipole gives no radiation along the earth's surface, it is found to be quite effective for long distance short-wave communication, and arrays consisting of horizontal doublets are now in extended use. They offer the theoretical advantage—which is

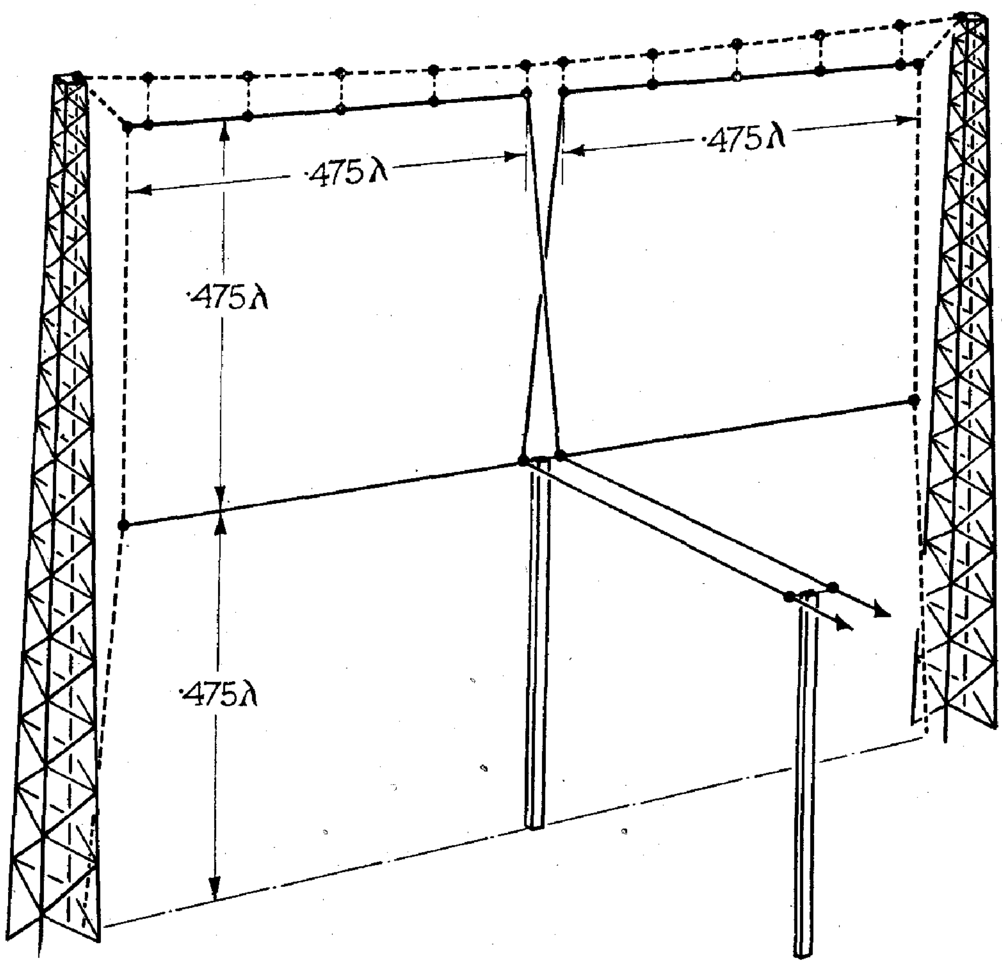


FIG. 64, CHAP. XV.—Array of horizontal dipoles.

borne out in practice—that the array is intrinsically much better balanced with respect to earth than an array of vertical elements. For the shorter wave-lengths, quite a serviceable array may be erected on 70 feet masts although, of course, higher ones are desirable for reasons already given. A very simple form consists of four $\frac{\lambda}{2}$ dipoles arranged as in fig. 64. The lower pair are connected directly to the terminals of the matching device, and the upper pair, which are $\frac{\lambda}{2}$

above the lower, are fed by means of a $\frac{\lambda}{2}$ length of twin transmission line operating as a $-1:1$ transformer. It follows, therefore, that the feeding points must be taken from the sides of the transmission line opposite to those from which the lower side is fed. As a rough approximation, the radiation resistance of the arrangement may be taken as 4 times that of a single member less about 17 per cent. due to the effect of mutual impedance between the various members. If the lower pair are one-half wavelength above ground, the radiation resistance of each member will be about 73 ohms, and the total radiation resistance of the order of 240 ohms. A reflector curtain may be used in conjunction with the energized curtain in order to concentrate the major portion of the radiation in the required direction. It is convenient to use a reflector aerial parallel to each energized member, the spacing being $\frac{\lambda}{4}$ and the length of the reflector about 8·5 per cent. greater than the energized member. The latter are usually 5 per cent. less than $\frac{\lambda}{2}$ so that the reflector wire has a length of about $\cdot 52\lambda$. If only 70 foot masts are available, allowing 10 feet for the sag in the triatic which supports the whole aerial, it is seen that the longest wavelength for which this aerial can be built is about 18 metres.

143. The effect of a reduction in the height of the lower members is of importance, and is very easily found to a good approximation by the use of fig. 8, and the methods explained in paragraphs 18 and 19. There is no need for extreme accuracy where only the angle and approximate magnitude of the main lobe is required. To illustrate the point, the vertical polar diagram of the four-element array has been derived, first, in fig. 65a, for the lower members $\frac{\lambda}{2}$ above earth and second, in fig. 65b, for the lower member at a height of $\frac{\lambda}{4}$. In the first diagram

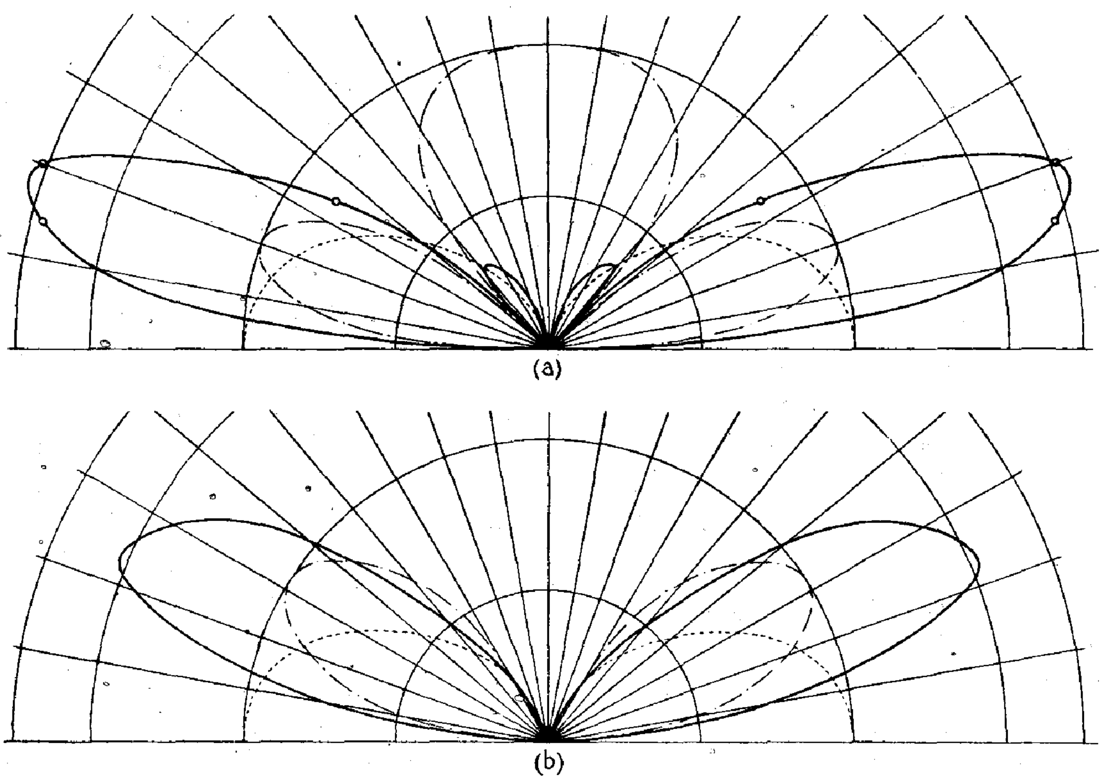


FIG. 65, CHAP. XV.—Vertical polar diagrams, arrays of horizontal dipoles.

the fine dotted-line curve corresponds to fig. 8 A 5 (parallel dipoles $\frac{\lambda}{2}$ apart with syn-phased current) and the chain-dotted curve to fig. 8 E 13 (parallel aerials $\frac{3}{2}\lambda$ apart, with currents in anti-phase) or to $h = 0{\cdot}75\lambda$ in fig. 21. The product has been obtained for only four or five points and the full-line curve drawn. In the second diagram the fine dotted line is diagram A 5 of fig. 8 as before, and the chain-dotted line is obtained from fig. 8 E 9 (parallel aerials λ apart, in anti-phase) or $h = 0{\cdot}5\lambda$ in fig. 21. The product is shown in full line. It is seen that in the first case the angle of the main lobe is about 18° to the horizontal, but that the field at an angle of only 4° is quite appreciable. With the lower aerial, however, the angle of the main lobe is about 22° and the field strength at angles less than 10° is very low. At a risk of over-emphasis, it is again pointed out that a few minutes study of fig. 8 and paragraphs 18 and 19, will give an approximate numerical solution of almost any example of this kind and is of greater value than many pages of purely

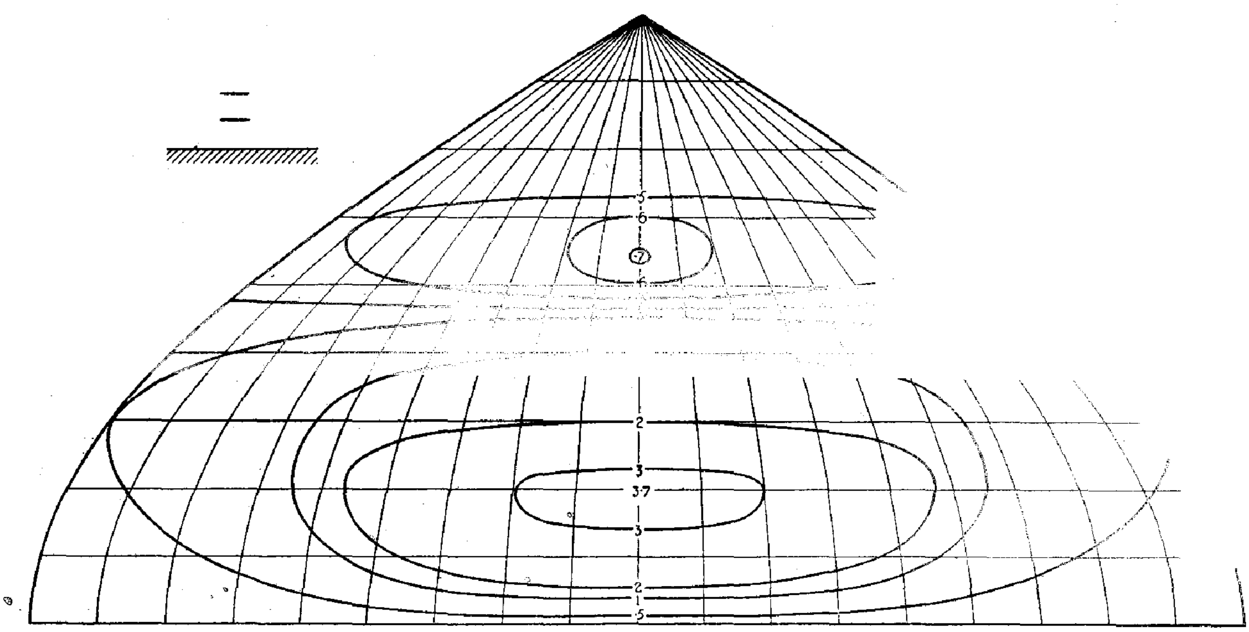


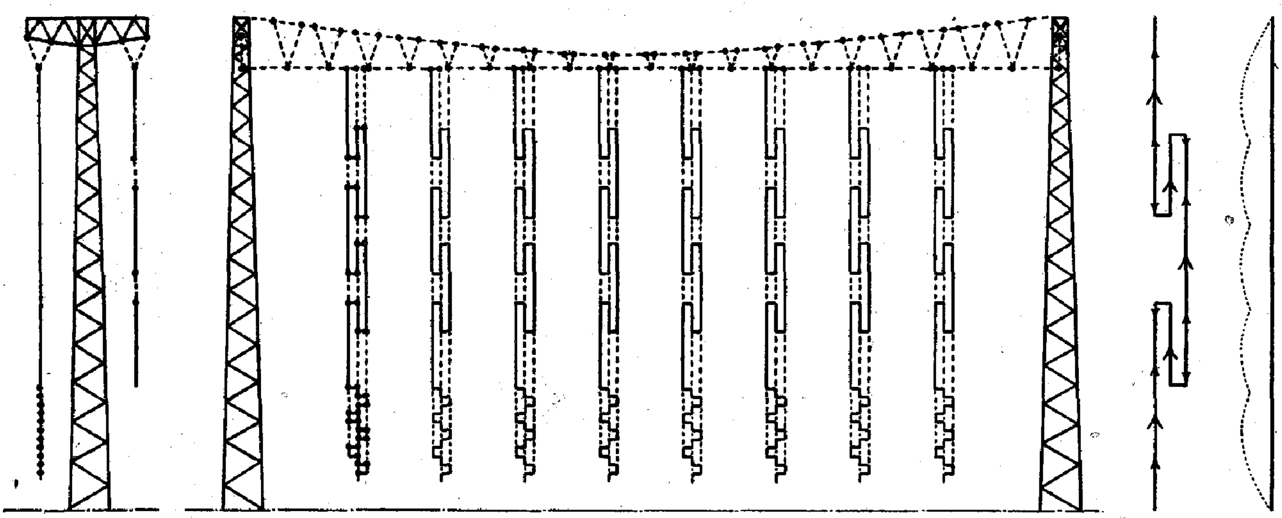
Fig. 66, Chap. XV.—Radiation in space; array of horizontal dipoles.

qualitative statements. In fig. 66 the distribution of the field is shown upon a sinusoidal graticule for the case where the lower members are $\frac{\lambda}{2}$ above ground, corresponding with the vertical polar diagram of fig. 65a. This diagram was obtained from the latter figure by rotating it about a vertical axis through the origin, and multiplying each radius vector by the appropriate value of the Current Distribution Factor for a $\frac{\lambda}{2}$ dipole, i.e. $\dfrac{\cos\left(\frac{\pi}{2}\sin\theta\right)}{\cos\theta}$, where θ is the angle in azimuth through which the vertical diagram has been rotated. The datum, $\theta = 0$, is the direction in azimuth in which the maximum radiation is produced.

The Franklin uniform array

144. This array is illustrated in fig. 67a in which the radiating members and reflecting members are shown separately. Each radiator is about 3λ in height and is doubled back upon itself in a sort of "Greek key" pattern, in order to obtain an approach to uniform current over the greater

part of the actual height. This point is further illustrated in fig. 67c, which shows the approximate current distribution, and it will be seen that the radiation from the ends of each element of wire cancels out. As the current at these ends is comparatively small, little energy is wasted in this way, but the whole available height is made to carry a nearly uniform current approaching the loop current in magnitude. Where the mast height is sufficient, the actual radiating members are located in the higher portion of each bay, and a folded, nearly non-radiative feeder is used to convey the current from ground level to the aerial feeding points. The reflector units are usually placed about $\frac{\lambda}{4}$ or $\frac{3}{4}\lambda$ behind the radiators as shown in fig. 67b, the length of each reflector wire being adjusted to give the best forward radiation. The aperture of the array depends upon the



(b) (a) (c)

FIG. 67, CHAP. XV.—Franklin uniform array.

nature of the service; two, four, six and eight wavelength arrays have been used in different circumstances. Although this form of aerial gives very good results, it is practically impossible to extemporize, and owing to its high cost, is now being superseded in commercial practice by the series phase array, at all events for shorter waves, i.e. 30 metres and below.

The series phase array

145. This form of end-fire array consists of a long wire, which is so bent that a series of vertical loops are formed. Each of these loops consists of a $\frac{\lambda}{2}$ section of wire doubled back upon itself, so that the height of each is $\frac{\lambda}{4}$. These loops are joined in series by horizontal portions and are separated in space by a distance of $\frac{\lambda}{4}$, the wire itself being thus continuous throughout its length. The action will be explained with reference to fig. 68. The arrangement of the wire is shown in fig. 68a, T_1, T_2, being the input terminals, to which the feeder line is connected. It will be observed that T_2 is actually the earth itself, and the array is of the unbalanced type. In contrast to most of the arrangements previously described, the array may be terminated at its distant end T_0 by a non-inductive resistance equal to the effective surge impedance of the aerial, considered merely as a current-carrying conductor; this is about 300 ohms. When so terminated, no standing

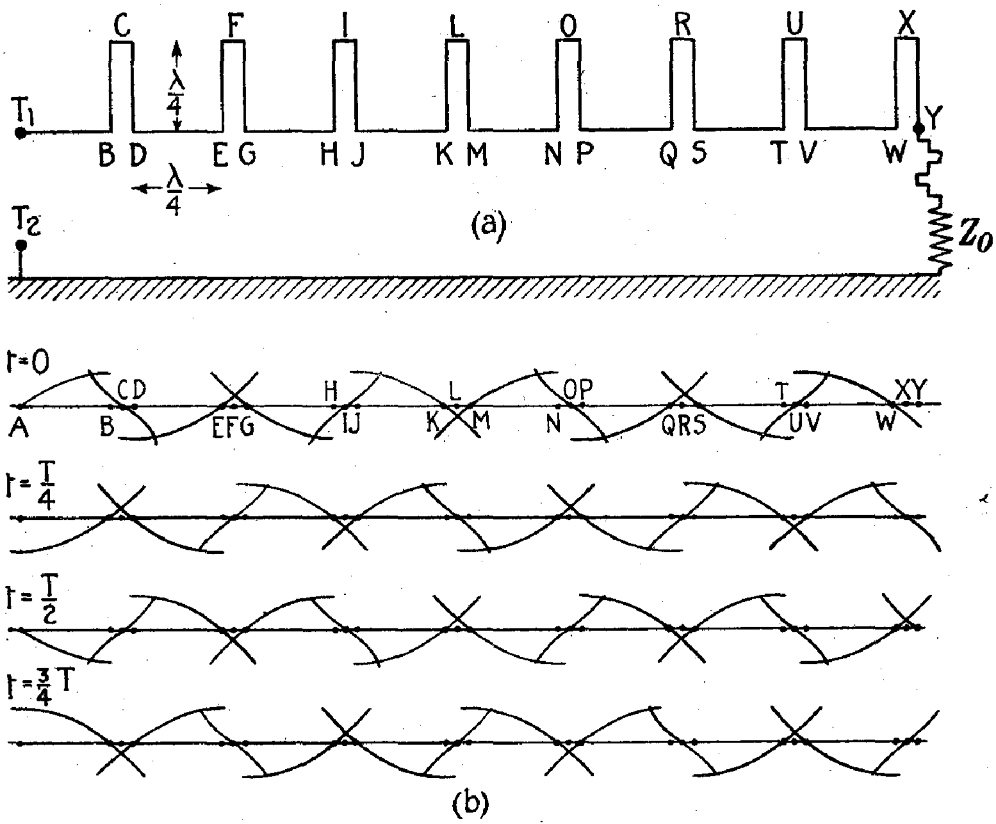


FIG. 68, CHAP. XV.—Series phase array.

waves are set up in any portion of the array—a point of primary importance. If a P.D. is applied to the terminals T_1, T_2, a travelling wave will be set up in the wire, moving from left to right. Considering only the vertical portions, it will be seen that at any given instant the current at all points in the section BC will be equal and opposite to that in the adjacent section CD. As these are so close together they may be considered as a single radiator carrying equal anti-phased currents, and we have seen that the effect of such currents in a $\frac{\lambda}{4}$ length of wire is to set up one quarter of a standing wave in the wire. Thus, in effect, the loop BCD acts as a $\frac{\lambda}{4}$ aerial. Similar considerations apply to the loops EFG, HIJ, etc. It must be understood, however, that in these successive loops the effective standing waves of current are not in phase with each other. The phase difference between any two successive loops will depend upon the length of horizontal connecting wire, and when this is $\frac{\lambda}{4}$, the standing wave in EFG will reach its maximum a quarter of a cycle earlier than that in BCD and so on. The current distribution at four successive intervals (of $\frac{1}{4f}$ seconds) is shown in fig. 68b. It is also seen that, at any given instant, the current in adjacent horizontal sections is in anti-phase, and consequently the total radiation from these portions is negligible. The radiation resistance R_r of a vertical $\frac{\lambda}{4}$ radiator is equal to about 36 ohms, and if I is the R.M.S. current at the base, the power radiated is $P = I^2R_r$. Now each vertical member of a S.P. unit acts as a $\frac{\lambda}{4}$ radiator, but its base current is effectively equal to twice the feed current. For a given feed current therefore, the power radiated is four times that which would be radiated by a $\frac{\lambda}{4}$ aerial with the same feed current, and the radiation resistance of each vertical member of a series phase array is therefore of the order of 144 ohms.

146. The horizontal polar diagram of a single-unit series phase array depends upon the number of vertical loops. If only two loops are used, the diagram approximates to fig. 8 A 3, i.e. a cardioid maximum radiation occurring toward the input end. If the length of the unit is extended with a corresponding increase in the number of verticals, the main lobe of the diagram becomes sharper, subsidiary lobes of small magnitude being developed. In practice the system is sometimes extended to a length of 4 to 6λ, i.e. from 17 to 25 verticals. When so extended, the attenuation of the current cannot be entirely neglected. It must be observed that since the radiating elements are in series, and each has a comparatively high radiation resistance, the attenuation is very much greater than in a non-radiating line of the same length. In the latter also, the whole of the power transmitted down the line is absorbed at the terminating resistance. A little reflection will show that if the attenuation is very great, the loops nearest the transmitter will radiate well, but the remote ones poorly, and the polar diagram will not be sharply directive, while if the attenuation is low, the majority of the power supplied to the array will be dissipated in the termination, and the efficiency will be very low. Thus, for transmission, there is an optimum length, which is of the order of 4λ to 5λ. Under these conditions, the ratio of currents in the first and last members may be of the order of 6 to 1, or a power ratio of 36 to 1. The terminating resistance is then practically unnecessary. It follows that the theory is more complex than was suggested above, in that, instead of a travelling wave, quasi-stationary waves will be set up in the system.

147. In the foregoing explanation, the lengths of the various vertical and horizontal elements were said to be $\frac{\lambda}{4}$. This is, of course, the electrical and not the actual length. Owing to the method of construction, in particular the large number of sharp bends, and to the effect of mutual impedance between the radiators, it is found that the verticals should be about ·225λ to ·23λ in height and the same distance apart. This is of importance in obtaining the desired "end-fire" polar diagram. Another point of practical significance is the attenuation of the current in successive radiators. If the feed current is I amperes, the ratio between the currents in successive radiators being x (<1), and the total resistance of a unit consisting of one horizontal and one vertical element is R ohms, the total power dissipated will be

$$\mathrm{R}\,\{I^2 + (xI)^2 + (x^2I)^2 + (x^3I)^2 \;.\;.\;.\;.\}$$
$$= RI^2\,\{1 + x^2 + x^4 + x^6 \;:\;.\;.\;.\;\text{to } n \text{ terms,}\}$$

the final term of the series representing the power dissipated in the terminating resistance. Hence the power input to the whole array is

$$\frac{1 - x^{2n}}{1 - x^2} I^2R \text{ watts,}$$

and the input resistance $\frac{1 - x^{2n}}{1 - x^2} R$ ohms. In practice x^{2n} is very much smaller than unity and the input resistance approaches the value $\frac{R}{1 - x^2}$ ohms. For instance, if $x = {\cdot}8$ the input resistance will be $\frac{R}{1 - 0{\cdot}64} = 2{\cdot}78\,R$. As R may be about 160 ohms, the input resistance is about 450 ohms for this particular attenuation. This calculation again ignores the effect of mutual impedance, which causes the radiation resistance of each successive member to differ from that of the preceding one.

148. The array was originally suspended with its horizontal members at a height of $\frac{\lambda}{4}$ above ground, but better results appear to be obtainable if this height is increased. The nature of the soil under the array is also of importance. Best results appear to be obtained when the ground is either very highly conductive or almost perfectly insulating but of low permittivity, and the moist earth of the average site in Britain appears bad. There are, however, little data yet available in these respects. The frequency toleration of the series phase array is only of the order of 2 per cent. This constitutes a considerable disadvantage for service purposes. The directivity of a series phase array consisting of 8 loops is shown in fig. 69a, and the vertical diagram in fig. 69b. The horizontal directivity can be improved by using two parallel arrays fed in

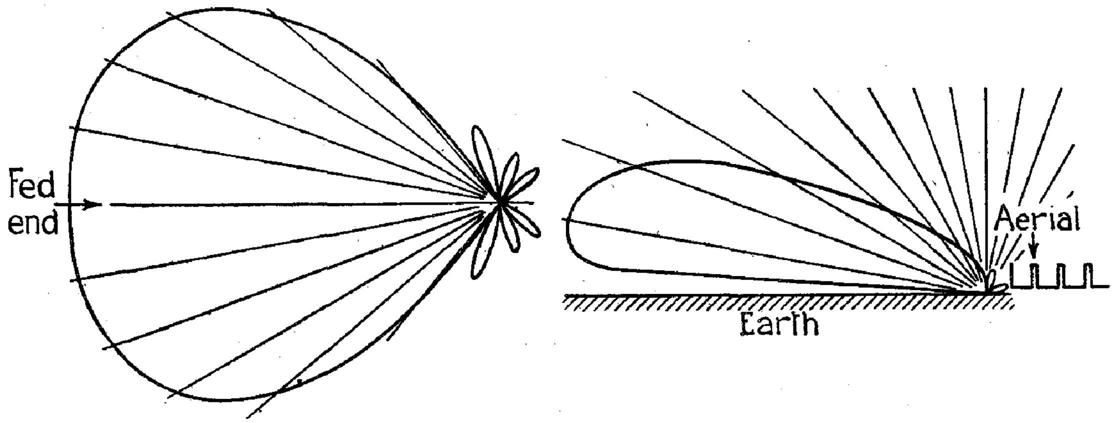


FIG. 69, CHAP. XV.—Polar diagrams of 2λ series phase array.

syn-phase. These may be $\frac{3}{4}\lambda$ apart, for convenience in feeding by means of a non-radiative feeder as described in paragraph 134. Two parallel arrays may then be connected, *via* a suitable matching device, to a twin wire feeder, and will constitute a balanced load.

Arrays used for reception

149. In general, any of the forms of array which have been previously described may be used for reception, the directional properties being practically the same for either purpose. Since, however, arrays are generally used where the traffic is continuous, it is rarely required to use a given array for alternate periods of transmission and reception ; in any case, the transmitter is usually remotely controlled and it is most convenient to erect an array for the sole purpose of reception. It is then obviously uneconomic to adopt arrangements which may be imposed by transmitting considerations, e.g. breakdown voltage does not enter into the design of a receiving array. On the other hand, correct termination is just as important if not more so than in the transmission case, and due attention must be paid to the nature of the aerial, balanced or unbalanced as the case may be, in designing the matching units. A single dipole opened at its centre for the connection of the feeders has a total resistance of the order of 100 ohms, and may be directly connected to a feeder of $Z_0 = 100$ ohms. Where the length of line is not too long a length of ordinary twisted flex may be used as a feeder, for its surge impedance is of this order. This arrangement is also suitable for transmission when the input does not exceed a few watts. As the insulating material between the conductors is partly air and partly of cotton, rubber, etc., the losses are rather greater than in an open line.

150. The series phase array is finding increasing favour for receiving purposes. Since the field is not uniform in phase over the whole of the array, it is rarely advantageous to extend the length beyond about $2\frac{1}{2}\lambda$, i.e. eleven vertical loops. In some cases the feeder end is elevated above the remote end in order to obtain additional vertical directivity. If two parallel arrays are used they

may be spaced $\frac{3}{4}\lambda$ apart and connected *via* a suitable matching device to each side of a twin wire feeder line through λ and $\frac{\lambda}{2}$ suppression units respectively. At the receiving end the received currents are then in the correct phase for connection to the input terminals of a balanced receiver, the line being also balanced and therefore practically non-radiative. The signal-noise ratio of this arrangement is found to be of a high order compared with that of a single dipole.

Rhombic array

151. This type of array has several useful forms, e.g. a single tilted wire, an inverted V, or a horizontal diamond shape. These are all classed together because the same principles are involved. The precise form adopted in any given case depends upon the polarization of the incoming wave, the direction, the wave tilt, the frequency, the available space and the material available for construction. The original form was the tilted wire aerial shown in fig. 70. First, suppose the wire

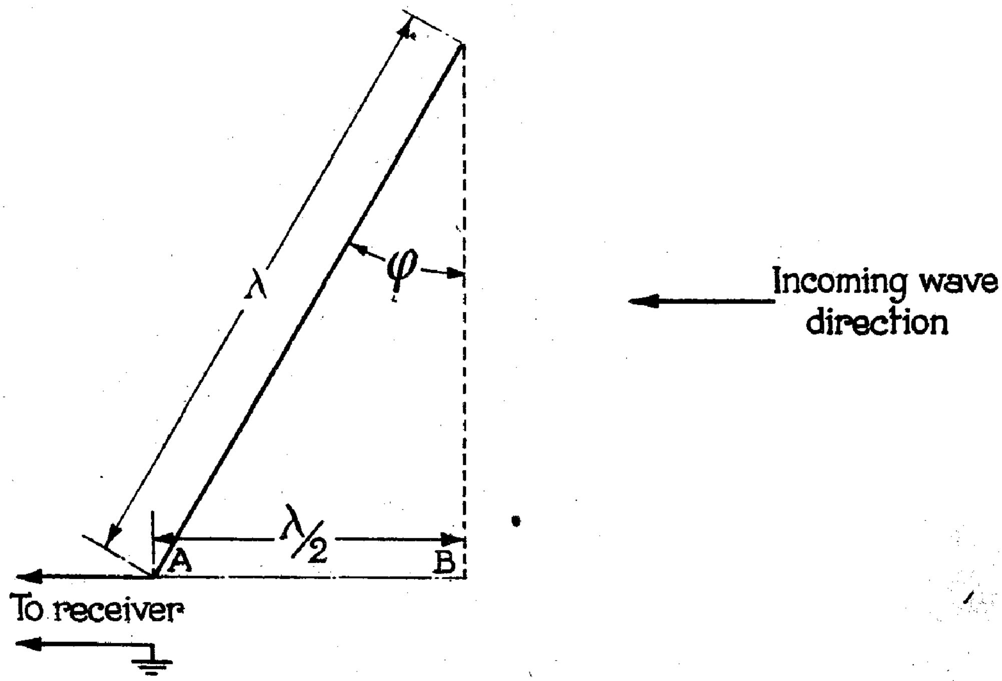


FIG. 70, CHAP. XV.—Tilted wire aerial.

to be several quarter-waves in length, erected vertically and connected to earth by a non-reflective terminating impedance. If the electric field vector Γ of an incoming wave is vertical, it will on arrival at the aerial induce in any element of length l an E.M.F. $l\Gamma$, and a voltage wave due to this will travel both upward and downward. The former wave will be reflected at the free end and will travel downward to the termination, so that in effect, there is chiefly a voltage wave downward. Since every elementary wave of strength l originates at a different point in the wire, they do not arrive at the termination in phase. The current in the terminating resistance is therefore due to the resultant of a number of elementary voltage vectors, and the magnitude of this resultant depends upon the length of the wire. If the length is $\frac{\lambda}{2}$, the resultant is a maximum, and if it is a whole wavelength the resultant is zero. If however this one-wavelength wire is tilted forward in the direction of the transmitter, any given phase of the electric field vector reaches the upper portion of the wire before the lower, and consequently the induced E.M.F.'s in the upper portion are advanced in phase with respect to the lower; this phase advancement is obviously progressive as we consider elements further from the termination. If then the tilt is such that the upper end is $\frac{\lambda}{2}$ nearer to the transmitter than the lower end, the current vectors due to the various voltage elements will all be in phase.

152. The angle which the tilted wire makes with the horizontal is thus very important; maximum energy is delivered to the receiver when the base A B of the triangle formed by the wire and the ground is $\frac{\lambda}{2}$ less than the length of wire. For a length l, $AB = l - \frac{\lambda}{2}$, e.g. if $l = \lambda$, the tilt angle φ, measured between the wire and the vertical, is $\sin^{-1} 0{\cdot}5$ or 30°. If $l = 2\lambda$, $AB = \frac{3}{2}\lambda$ $\varphi = \sin^{-1}\frac{1{\cdot}5\lambda}{2\lambda} = 49°$ and so on. As a result of this relation, the optimum tilt angle varies only very slowly, if the length is greater than about 4λ, and consequently the same aerial is effective over a fairly wide frequency range.

153. In practice, the above form is rarely used because it is possible to obtain better results without an appreciable increase in material. The simplest development is to place two similar tilted wires back to back forming an inverted V. If one end of this is open and the other connected to the receiver (the latter being properly matched to the aerial) the aerial has a broadly bi-directional response, but if the free end is earthed through a terminating resistance equal to the surge impedance of the aerial, its response becomes practically unidirectional, receiving mainly from the direction in which the termination is situated (fig. 71a). The forms shown in figs. 70 and

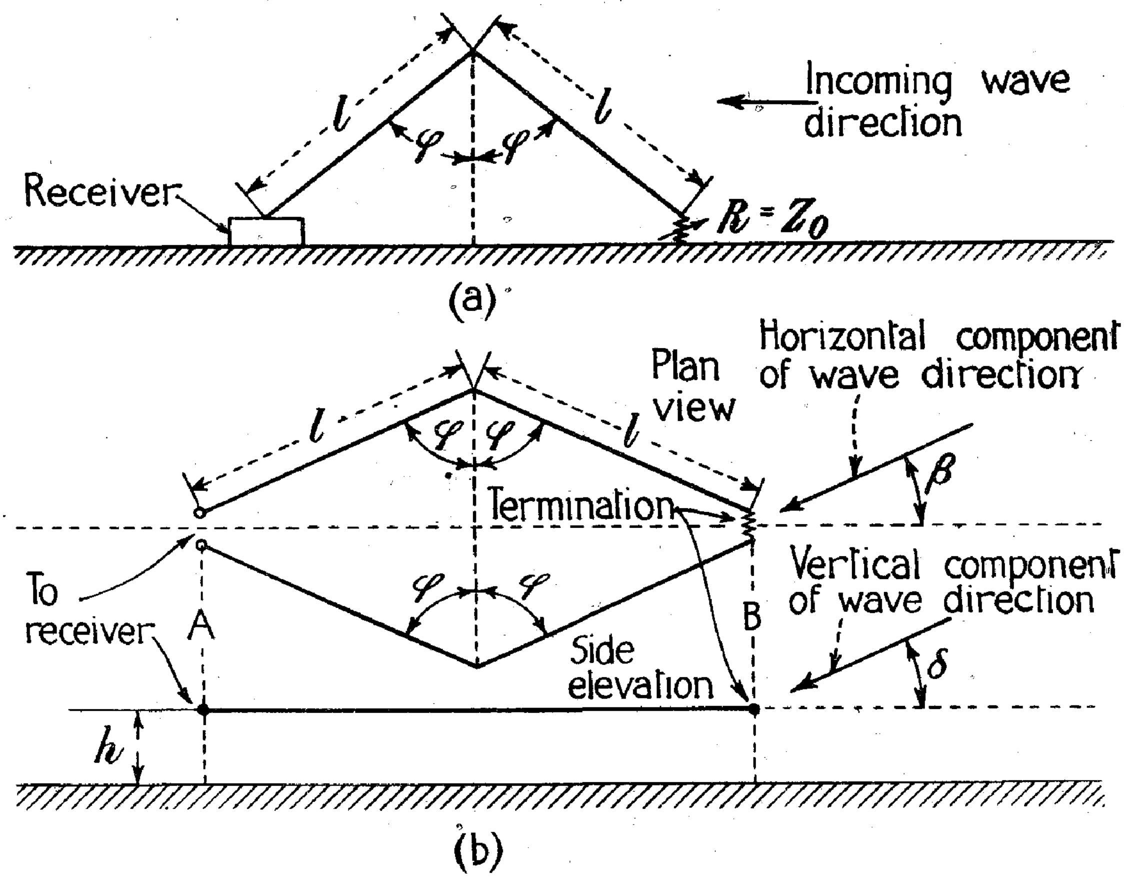


FIG. 71, CHAP. XV.—Inverted V and horizontal diamond arrays.

71a are intended for the reception of vertically polarized waves, but experimental results showed that the horizontally polarized component of down-coming waves would provide ample field strength for long distance reception, while an aerial suitable for such reception would have comparatively little pick-up of vertically polarized waves and might therefore be expected to give a high signal-noise ratio. As a result the horizontal diamond array was evolved. This is shown in plan and elevation in fig. 71b. In its simplest form it consists of two horizontally opposed V sections, similar to fig. 71a, one end of the array being connected to the receiver, and the other terminated by a suitable resistance. As in the tilted wire type, the length of each element decides the maximum gain of the array. The latter is obtained by making l exceed bv $\frac{\lambda}{2}$ the length of its projection upon the base A B.

154. In the design of a horizontal diamond array the three variables to be adjusted are l, φ and h (fig. 71b). The angles β and δ are regarded as constant. Then the lowest permissible height is

$$h = \frac{\lambda}{4 \sin \delta}\,,$$

while φ is given by

$$\sin \varphi = \cos \delta$$

and for maximum gain

$$l = \frac{\lambda}{2 \sin^2 \delta}.$$

155. In practice it is found that the greater l is, the wider is the efficient reception band of a particular aerial. The optimum value of the terminating resistance, for a high front to back reception ratio, is found by trial. The actual conductors forming each inverted V are frequently constructed of twin parallel wires, connected in parallel and spaced a few inches apart. By varying the spacing it is then possible to facilitate the matching between the aerial and the transmission line feeding the receiver. It is also possible by this means to make the surge impedance of the aerial uniform at all points throughout its length, and so to decrease the power loss in the aerial. To do this the twin wires are spaced apart at the apex of each V and close up towards the opposite ends of the wires. The diamond array may be used for transmission, but little data are available as to its performance. The directivity should, on theoretical grounds, be similar to that of the same array used as a receiver.