CHAPTER VII.—ELECTRICAL OSCILLATIONS AND ELECTROMAGNETIC WAVES

MECHANICAL AND ELECTRICAL OSCILLATIONS

Mechanical equivalents of inductance, resistance and capacitance

1. In the first two chapters the charge and discharge of a condenser through a resistance, and the growth and decay of current through a circuit containing inductance and resistance, were individually discussed. In radio circuits, the properties of resistance, inductance and capacitance are generally found in combination, and a knowledge of the phenomena associated with the discharge of a condenser in a circuit possessing both inductance and resistance is a fundamental requirement in the study of the principles of radio communication. The first portion of this chapter is therefore devoted to a consideration of the properties of oscillatory circuits, i.e. those containing inductance, capacitance and resistance, in which no continuously applied electromotive force exists.

2. An excellent grasp of the principles involved in the oscillatory circuit can be obtained by the study of a mechanical analogy, which unlike most analogies is practically perfect, namely, the mechanical vibration of a body or system of connected bodies, possessing the properties of mass, friction and elasticity. It has already been stated that the effect of inductance in an electrical circuit is to oppose any change in the value of the current flowing; in this respect inductance resembles that property of a body which we call its mass, for the distinctive property of mass is inertia, or opposition to any change in the motion of a body. If the body is at rest, it can only be set in motion by the application of a force, and if the force acts only for a short interval of time the body tends to continue in motion with the velocity it possessed at the instant at which the force is removed. An example of this may be seen when railway trucks laden with coal are being moved from a siding in the vicinity of the pithead to the railway proper, for which purpose horses are usually employed. Two or more horses may be required to urge one truck into motion, but when sufficient velocity has been attained the horses are dispensed with and the truck maintained in motion with nearly constant velocity by the unaided effort of one man. It will also be observed that to bring the truck to a sudden standstill considerable force must be applied. These results are very noticeable when trucks run on smooth well-laid lines, because the friction between wheels and rails is very small, and the effort required to maintain constant velocity is much less than is required upon a rough surface such as a highway.

3. The effect of friction in mechanics is generally similar to the effect of resistance in electrical circuits, but friction between solid surfaces is not an exact parallel to electrical resistance, the latter being more nearly analogous to the friction which exists between a smooth body and a viscous fluid when a body moves slowly through the latter, for in these circumstances the velocity produced is proportional to the force applied, or, if F is the force and u the velocity produced, $F \propto u$ or $F = Ru$. The constant of proportion R, which has been introduced to give equality to both members of the equation, may be called the coefficient of friction. This equation may be compared with Ohm's law which is $E = RI$, where E is the applied E.M.F., R the electrical resistance, and I the current. It will be noted that the current I is correctly compared with the velocity u, because the intensity of the current is the rate at which electrons move through the circuit, being measured in coulombs per second (one coulomb $= 6{\cdot}29 \times 10^{18}$ electrons). It is because of this analogy that practical men often think of the E.M.F. as the force acting upon the electrons, although we have seen in Chapter I that a more accurate conception of E.M.F. is based upon the conversion of energy into its electrical form.

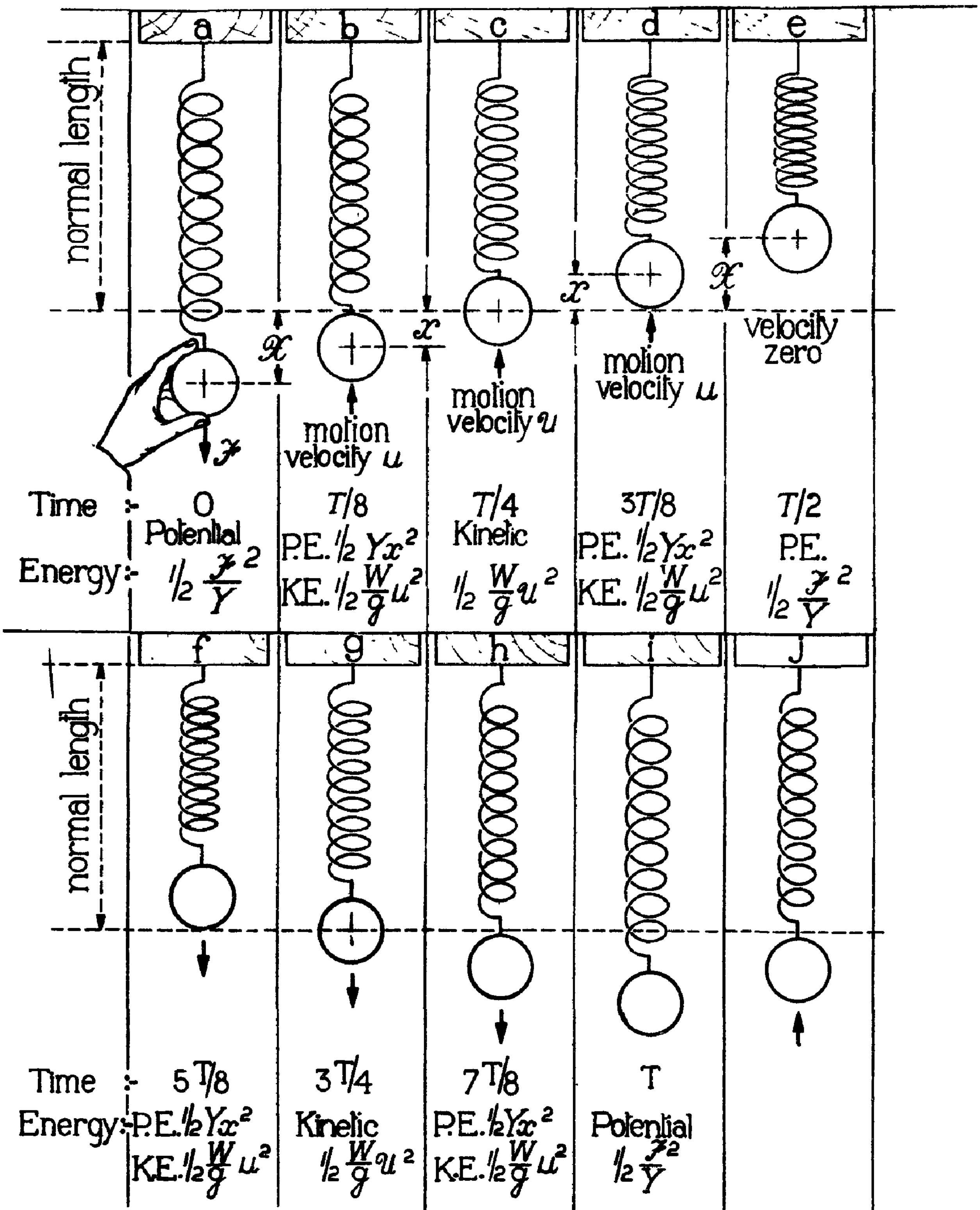
4. Reverting to the mechanical analogue of inductance, namely, inertia, it may be recalled that when a body is in motion it possesses kinetic energy, the quantity of energy being proportional to the mass of the body and the square of its velocity. Care must be taken not to confuse the mass of the body, which is merely the amount of matter it contains, with its weight, which is

the force with which it is attracted towards the earth by gravitation. In order that the reader may himself perform the mechanical experiment to be described and may easily make calculations regarding the results, the F P.S. (foot-pound-second) system of units will be adopted for the mechanical example, practical units being employed in the electrical analogue. The unit of mass is that upon which a force of 1 Lb. produces an acceleration of 1 ft. per second per second. Now a force of 1 Lb. produces in a mass of 1 lb. an acceleration $g = 32 \cdot 2$ ft. per second per second, and if the acceleration produced by a force of 1 Lb. is only 1 ft. per second per second, the mass acted upon must be $32 \cdot 2$ times as great; the unit of mass is therefore $32 \cdot 2$ lb., and is sometimes called a "slug." In this system both kinetic and potential energy are measured in foot-pounds (ft.-Lb.) and the quantity of energy stored in a body weighing W Lb., moving with a velocity of u ft. per second, is $\frac{1}{2}\frac{W}{g}u^2$ ft.-Lb. The convention of using the symbol Lb. to denote pounds force, and lb. to denote pounds mass, should be noted. Similarly the kinetic energy possessed by an inductance L in which a current of I amperes is flowing, is stored in the form of a magnetic field and its amount is $\frac{1}{2}LI^2$ joules. The electrical current has already been compared with the mechanical velocity, and it is apparent that the property of inductance in an electrical circuit enters into the expression for kinetic energy in the same manner as mass in the mechanical example.

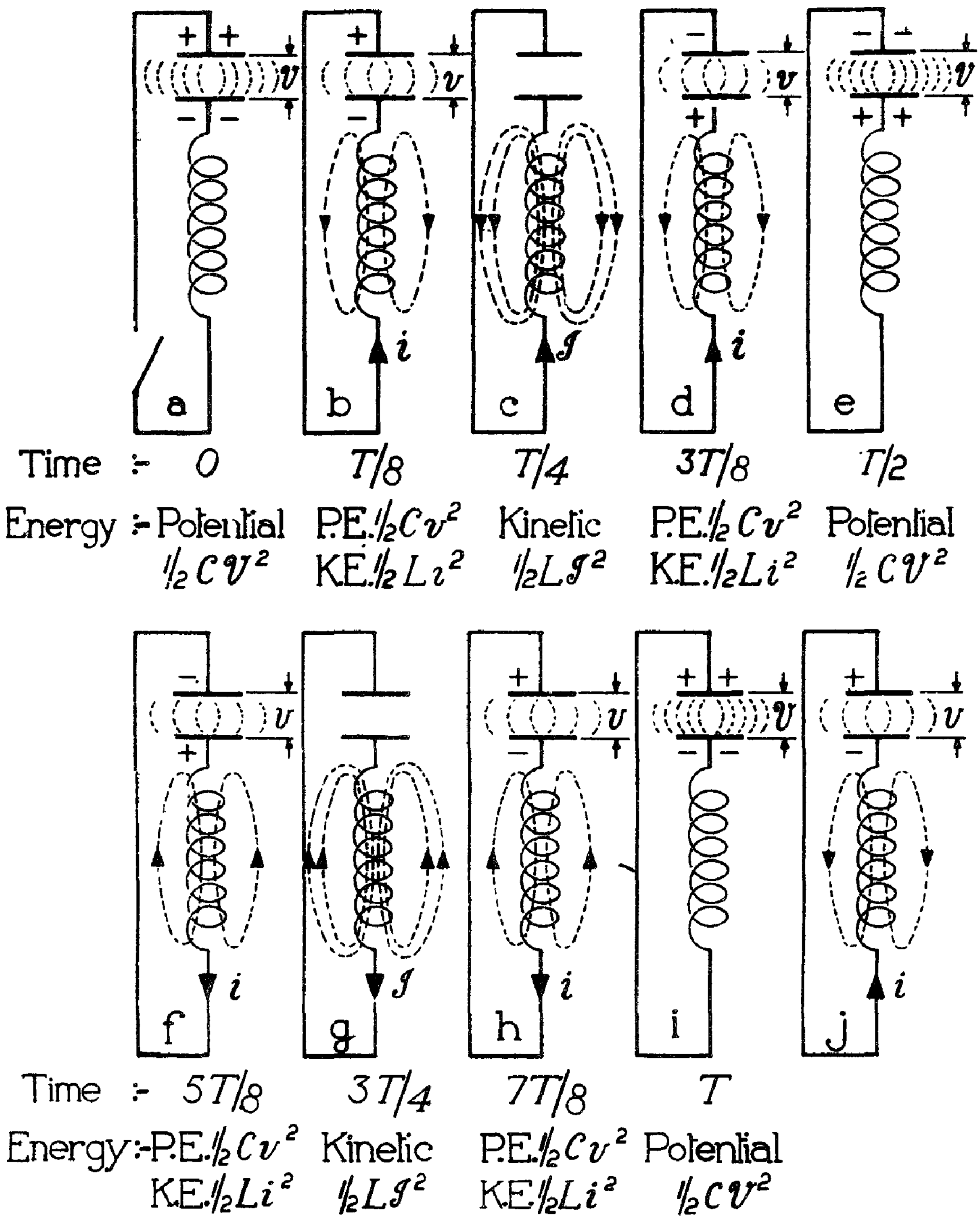
5. The mechanical analogue of electrical capacitance is ability to stretch or extend, or the converse, susceptibility to compression. The latter conception has already been used to illustrate the storage of energy in a condenser by comparison with the storage of compressed air in a gas cylinder; energy is also stored when an ordinary spring is compressed or extended, for example in the mainspring of a clock. The form of spring which best lends itself to actual measurement of its property of storing energy is the spiral spring, which can be made by winding a steel wire in a screw thread, the latter being removed when the winding is completed. The amount of extension obtained from a spring of this kind is directly proportional to the force with which it is extended, provided that the force applied is insufficient to produce a permanent elongation of the spring; when the latter takes place the elastic limit is said to be exceeded. If a constant force of F Lb. produces an extension of X feet within the elastic limit, then $F \propto X$ or $F = YX$, the constant of proportion Y being a property of the particular spring in use. It is called its stiffness, and is measured by the force (in Lb.) which will produce an extension of 1 ft. In the corresponding electrical example, the quantity of electricity Q, stored in a condenser of capacitance C farads, is proportional to the applied voltage, or $V = \frac{1}{C}Q$. As the formal analogue of force is V the mechanical quantity corresponding to quantity of electricity is X, the extension of the spring, and therefore its stiffness is analogous to the reciprocal of the electrical capacitance. This signifies that a large condenser, which will acquire a given charge with only a small applied voltage, is equivalent to a weak spring, which will acquire a given extension with only a small applied force. When the condenser is charged, its P.D. being constant and equal to V volts, it possesses a supply of potential energy in the form of electrical strain in the dielectric, the quantity being $\frac{1}{2}CV^2$ joules. Similarly the stretched spring possesses a store of potential energy, equal to $\frac{1}{2}YX^2$ or $\frac{1}{2}\frac{F^2}{Y}$ ft-Lb.

Oscillation of weighted spring

6. (i) The spring when supported at its upper end and extended by a given mass, e.g. an iron ball attached to its lower end, is said to be statically strained, and represents an electrical circuit possessing inductance, capacitance, and a very small resistance, the mechanical and electrical equivalents being shown in fig. 1. This state of static strain must be considered as the normal state of the system when the spring is arranged in the manner stated. The phenomena to be described would take place equally well if the ball and spring were arranged horizontally in such a way that the ball could slide freely along a perfectly smooth surface, but this is not practicable. The action of applying an external force to the mechanical system (fig. 2a) and



OSCILLATIONS OF WEIGHTED SPRING. FIG. 2 CHAP. VII



ELECTRICAL OSCILLATIONS

FIG 3
CHAP. VII

thus causing a further, or dynamic, extension of the spring, corresponds to the introduction of an electrical charge into a condenser (fig. 3a); in both cases, energy is stored in the system in potential form.

(ii) The action of releasing the ball corresponds to the closure of the switch S, by which the condenser is allowed to discharge through the inductance. Let us first observe the manner in which the spring loses its potential energy, neglecting the effects due to friction. Let the original applied force be $\mathscr{F}$ Lb., and the resulting displacement $\mathscr{X}$ feet. As soon as the ball is released, it is urged into motion in an upward direction by the action of the spring. Its inertia causes it to move slowly at first, but its velocity gradually increases (fig. 2b) and reaches a maximum at the exact moment when the spring has returned to its normal, i.e. statically strained, length. At this

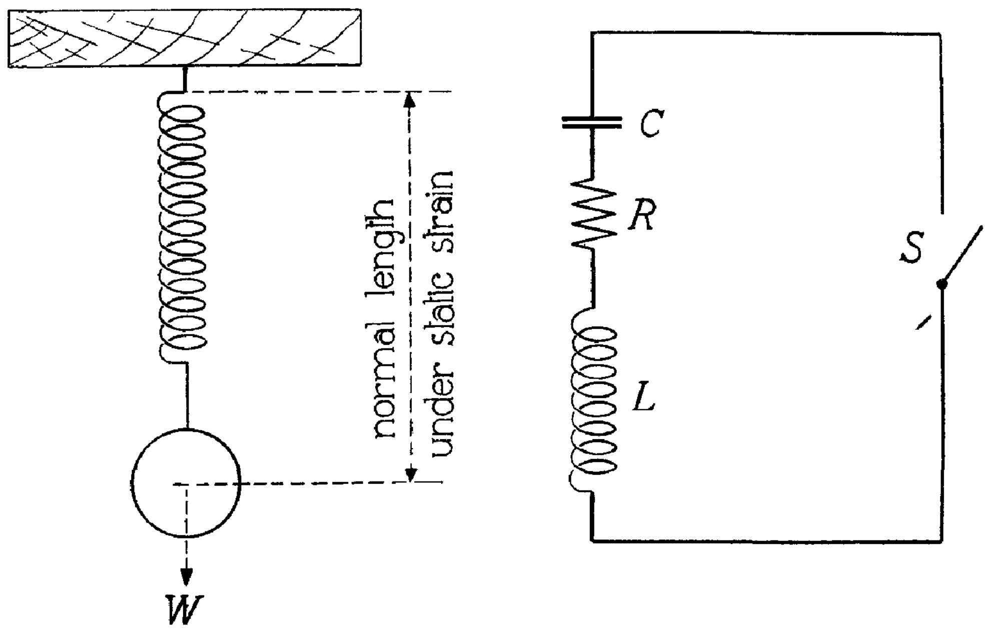


FIG. 1, CHAP. VII.—Mechanical equivalent of electrical oscillatory circuit.

moment the system possesses no potential energy, the latter having been converted into kinetic energy in the motion of the ball (fig. 2c). It is of interest to calculate the velocity of the ball at this instant. The original energy stored in the spring was $\frac{1}{2}\frac{\mathscr{F}^2}{Y}$ or $\frac{1}{2}\,Y\,\mathscr{X}^2$ ft.-Lb. and the energy stored in the moving mass is equal to the energy originally stored in the spring, hence if $\mathscr{U}$ is the velocity of the ball at the moment under consideration

$$\frac{1}{2}\frac{W}{g}\mathscr{U}^2 = \frac{1}{2}\frac{\mathscr{F}^2}{Y}$$

$$\mathscr{U}^2 = \frac{g}{WY}\mathscr{F}^2$$

$$\mathscr{U} = \mathscr{F}\sqrt{\frac{g}{WY}}$$

A numerical example will assist in making this clear. Suppose the spring has a stiffness of 2 Lb.

per foot, the mass of the ball to be 1 lb. and the initial extension to be 6 in. or ·5 ft. Then

$$\mathscr{F} = 1 \text{ Lb.}$$

$$Y = \frac{2 \text{ Lb.}}{\text{ft.}}$$

$$\mathscr{U} = 1 \text{ Lb.} \times \sqrt{\frac{32 \cdot 2 \frac{\text{ft.}}{\text{sec.}}}{1 \text{ Lb.} \times 2 \text{ Lb.}}}$$

$$= 4 \cdot 1 \frac{\text{ft.}}{\text{sec.}}$$

(iii) After reaching its normal position the ball continues to travel upwards (fig. 2d); in doing so it does work, for it is compelled to compress the spring during its motion, and energy is required for this operation. The velocity decreases as it loses energy, and eventually the ball comes to rest $\mathscr{X}$ feet above the point at which its velocity was a maximum (fig. 2e). The energy imparted to the spring is therefore $\frac{1}{2} Y \mathscr{X}^2$ ft.-Lb. which is the amount it originally possessed, the energy being stored by compression instead of by extension. The ball is now urged downwards by the force of the spring (fig. 2f), and again attains its maximum velocity (but in the downward direction) when the spring is dynamically unstrained (fig. 2g). The velocity then decreases (fig. 2h), and finally the ball comes momentarily to rest at the bottom of its travel, the spring being again extended $\mathscr{X}$ feet; it then commences to move upward for a second time (fig. 2j), and the whole cycle shown in fig. 2 is repeated. If no energy were converted into heat and into motion of the surrounding air, the oscillation of the system would continue indefinitely. This may be expressed by saying that once a certain amount of energy has been imparted to a loss-free system of this kind the energy contained in the system remains constant, although at any instant it may be partly possessed by the spring (potential energy) and partly by the mass (kinetic energy). The sum of these two energies at any and every instant is constant and equal to the energy originally imparted, or

$$\tfrac{1}{2} Y x^2 + \tfrac{1}{2} \frac{W}{g} u^2 = \text{constant}$$

x being the displacement of the mass from its normal position, and u its velocity, at any particular instant.

Electrical oscillations

7. (i) Now consider the electrical circuit, which consists of an inductance of L henries, and a capacitance of C farads, the condenser being charged to a voltage $\mathscr{V}$ volts, so that the energy stored therein is $\frac{1}{2} C \mathscr{V}^2$ joules. On closing the switch S (fig. 3a), the condenser commences to discharge, but the growth of the current causes an increasing magnetic flux round the inductance L (fig. 3b), and consequently a counter-E.M.F. which opposes the flow of current; as a result the latter does not attain its maximum value until the moment at which the condenser is wholly discharged (fig. 3c). The energy originally stored by the dielectric in potential form is now wholly stored in the magnetic field around the inductance, and its amount is $\frac{1}{2} L \mathscr{I}^2$ joules. Knowing the amount of energy originally stored, we can find the current at this instant, from the known conditions that $\frac{1}{2} L \mathscr{I}^2 = \frac{1}{2} C \mathscr{V}^2$. Solving this equation for $\mathscr{I}$ it is found that $\mathscr{I} = \mathscr{V} \sqrt{\frac{C}{L}}$. We may compare this with the expression obtained for the velocity of the mass in the mechanical system, i.e. $\mathscr{U} = \mathscr{F} \sqrt{\frac{g}{Y W}}$. The mass $\frac{W}{g}$ and inductance L occupy similar positions in the formulae, while $\frac{1}{Y}$ and C also correspond.

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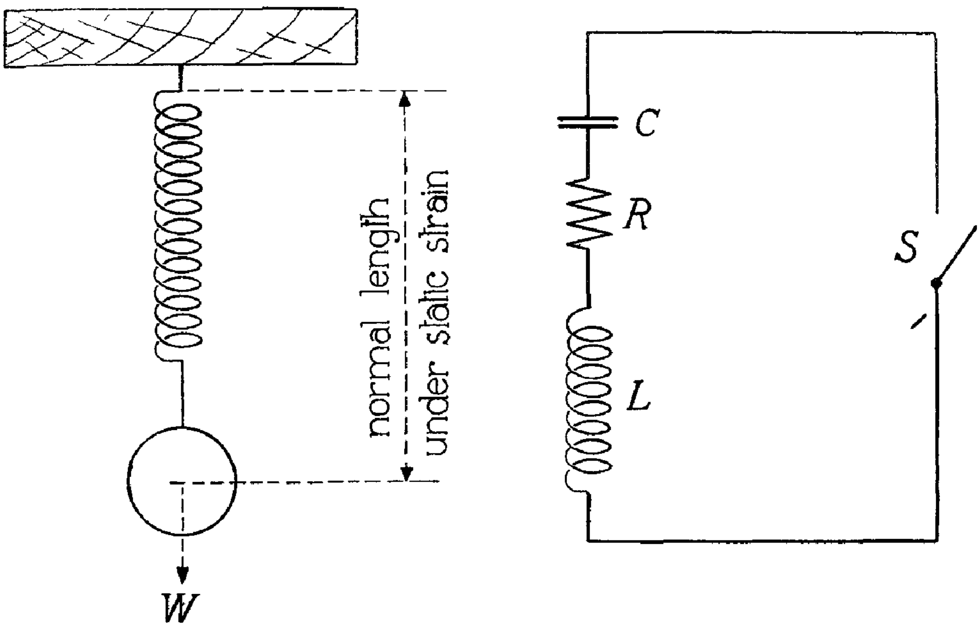

FIG. 1, CHAP. VII.—Mechanical equivalent of electrical oscillatory circuit.

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Electrical oscillations

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(ii) Owing to the counter-E.M.F. of self-induction caused by the changing magnetic flux, the current will continue to flow, charging the condenser with a polarity opposite to its original charge (fig. 3d). As soon as any charge is introduced into the condenser, it exerts a counter-E.M.F. which opposes the introduction of an additional quantity of electricity and therefore the current decreases with a consequent collapse of magnetic flux round the inductance. The change of magnetic flux in turn sets up a forward E.M.F. which tends to maintain the current against the back pressure of the condenser, but eventually the current falls to zero, together with the magnetic flux, and the whole of the energy which was stored in the inductance when the current was a maximum, is now again stored in the condenser (fig. 3e), its quantity being $\frac{1}{2} C \mathscr{V}^2$ joules as originally. The condenser will now discharge through the inductance once more (fig. 3f), the current flow being in the opposite direction. At the moment when the condenser is completely discharged (fig. 3g), the current in the circuit will again be a maximum, and the counter-E.M.F. caused by the collapse of the flux will cause the condenser to charge with its original polarity (fig. 3h). When this is accomplished (fig. 3i) one complete cycle of oscillation has been performed. The characteristic property of the oscillators we have described, both mechanical and electrical, is that at any instant/the total energy possessed by the circuit is constant and equal to that originally imparted since the effects of friction in the mechanical system and resistance in the electrical circuit have both been neglected.

Example 1.

A condenser of ·01 microfarad is given a charge of Q of 20×10^{-6} coulomb, and its plates then connected by a coil of 100 microhenries having negligible losses. Find (a) the initial voltage of the condenser, (b) the maximum current during discharge, (c) the total energy, W, stored in the circuit and (d) the current at the instant when the condenser P.D. is 1,500 volts

Since $Q = C\mathscr{V}$

$$\mathscr{V} = \frac{Q}{C} = \frac{20 \times 10^{-6}}{\cdot 01 \times 10^{-6}}$$

$$= 2{,}000 \text{ volts}$$

$$\mathscr{I} = \mathscr{V}\sqrt{\frac{C}{L}}$$

$$= 2{,}000\sqrt{\frac{\cdot 01}{100}}$$

$$= 20 \text{ amperes}$$

$$W = \tfrac{1}{2} C \mathscr{V}^2 \text{ joules.}$$

$$= \tfrac{1}{2} \times \cdot 01 \times 10^{-6} \times (2 \times 10^3)^2$$

$$= \tfrac{1}{2} \times \cdot 01 \times 4$$

$$= \cdot 02 \text{ joules.}$$

The energy w stored in the condenser when $v = 1{,}500$ volts, is $\frac{1}{2} Cv^2$ joules.

$$w = \tfrac{1}{2} \times \cdot 01 \times 10^{-6} \times (1 \cdot 5 \times 10^3)^2$$

$$= \tfrac{1}{2} \times \cdot 01 \times 2 \cdot 25$$

$$= \cdot 01125 \text{ joules.}$$

The energy stored in the inductance is ·02—·01125 or ·00875 joules, and is equal to $\frac{1}{2}Li^2$ joules.

$$L = 100 \times 10^{-6} \text{ henry.}$$

$$\tfrac{1}{2} \times 100 \times 10^{-6} \times i^2 = \cdot 00875$$

$$10^{-4} i^2 = \cdot 0175$$

$$i^2 = 175$$

$$i = 13 \cdot 2 \text{ amperes.}$$

Period and frequency of free oscillation

8. (i) The reader is strongly advised to perform the above experiment with a spiral spring and weight for himself. A spring which has been found very suitable for the purpose may be obtained from an old roller blind, this being generally of steel, about fifteen inches long when not extended, and having a stiffness of about 5 Lb. per ft. It will carry about double this weight without exceeding the elastic limit. If the duration of one complete oscillation is measured, it will be found to be given approximately by the formula $T = 2\pi\sqrt{\frac{W}{gY}}$, this time being called the period of oscillation. The stiffness Y can be measured by noting the initial extension given by the mass actually used for the experiment. If the stiffness is 5 Lb. per ft. and the weight of the ball is 2 Lb., the time of one complete oscillation will be approximately 0·7 second. In the electrical circuit which we have stated to be exactly analogous to the above, the duration of one complete oscillation can be found by substituting L, the value of the inductance in henries for the mass $\frac{W}{g}$, and C, the value of the capacitance in farads, for the reciprocal of the stiffness $\left(\frac{1}{Y}\right)$, giving $T = 2\pi\sqrt{LC}$.

(ii) The period of an electric oscillatory circuit can actually be measured, just as the period of a mechanical oscillation can, and this equation is found to be approximately true. It is not entirely so owing to the effect of friction in the mechanical oscillator and resistance in the electrical one. The true expression in the electric case is

$$T = 2\pi \frac{1}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}.$$

The frequency of the oscillation is the number of complete cycles executed in one second, and obviously if T is the duration of one cycle of oscillation, there are $\frac{1}{T}$ cycles per second. The frequency being denoted by f_n, $f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

(iii) When any system, whether mechanical or electrical, is set into oscillation by an initial stress or charge, and the frequency depends entirely upon the mass, stiffness and friction, or upon the inductance, capacitance and resistance, it is said to be in a state of free oscillation, and the frequency f_n at which it freely oscillates is said to be its natural frequency. Another type of oscillation may take place, for example, when a force of given frequency is applied to the mechanical system, forced oscillations take place at this frequency. The electrical parallel of this is the alternating current which is established when an E.M.F. of any frequency whatever is applied to an electrical circuit; this type of oscillation has been dealt with in Chapter V.

9. So far we have not considered the manner in which the displacement of the vibrating body varies with time. The motion of the ball undergoes the following variations. Starting at the bottom of its available travel, it commences to move upwards, and reaches its maximum velocity in the middle of its path, then receives negative acceleration because it is doing work (i.e. losing energy) in compressing the spring and temporarily comes to rest with maximum upward displacement. It then receives downward acceleration, possessing maximum velocity in the midpoint of its travel, and again reaches a position of momentary rest at its normal (i.e. statically strained) position. If the displacement of the ball in this path is plotted at every instant over a number of complete cycles it will be seen that the graph connecting displacement with time is of sinusoidal form; since the displacement has its maximum value at the time $t = 0$, the graph is actually a cosine curve, and the displacement x, after an interval of t seconds is

$$x = \mathscr{X} \cos \omega t.$$

In the electrical circuit, the P.D. between the condenser plates also follows a cosine law, i.e. $v = \mathcal{V} \cos \omega t$, but the current in the circuit is zero when the condenser is initially charged, and grows in value as the condenser P.D. falls, attaining its maximum value when the condenser P.D. is zero. The current continues to flow in the same direction, charging the condenser with reverse polarity. Thus the current variation obeys the law $i = \mathcal{I} \sin \omega t$; the relative phase between the condenser P.D. and current is therefore as shown in fig. 4, the loss of energy being assumed to be negligible. If the resistance losses are taken into account, the displacement curve will still follow the cosine law, but its amplitude will diminish in every succeeding half-cycle, signifying that a certain portion of the energy is converted into heat or some other unrecoverable form during every oscillation.

10. The acceleration of the vibrating mass may also be considered. We have seen that when the displacement is a maximum in the downward direction, the acceleration is upward and of maximum value, gradually decreasing in value as the velocity increases, and becoming zero

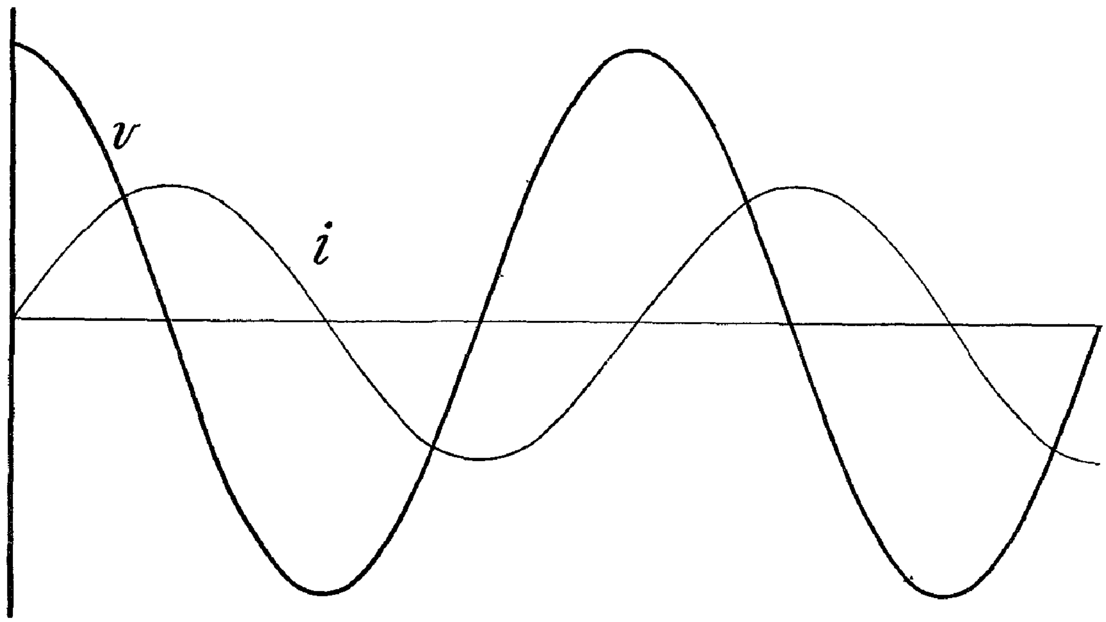


FIG. 4, CHAP. VII.—Relative phase of oscillatory current and condenser P.D.

when the velocity is greatest. The acceleration then becomes negative, reducing the velocity, and reaching a maximum negative value when the ball is at its highest point and its velocity zero, so that at every instant the acceleration is of the opposite sign to the displacement; if plotted on the same time scale as the displacement and velocity, the acceleration is seen to obey a cosine law, but is of opposite sign to the displacement. This relation between the phases of displacement, velocity and acceleration is a characteristic always possessed by a body or system which is executing the type of motion under discussion, i.e. simple harmonic motion. This is nothing more than another illustration of the law formulated in Chapter V for the rate of change of a sinusoidal quantity, i.e. if $x = \mathcal{X} \cos \omega t$, where x is the instantaneous and $\mathcal{X}$ the maximum displacement, and $\omega = 2\pi f$,

$$\text{the velocity, } \frac{dx}{dt} = -\,\omega \mathcal{X} \sin \omega t$$

$$\text{and the acceleration, } \frac{d^2 x}{dt^2} = -\,\omega^2 \mathcal{X} \cos \omega t$$

$$= -\,\omega^2 x.$$

Effect of resistance upon the natural frequency

11. (i) It has been stated that in an electrical circuit having such low resistance that its influence upon the period is negligible, the period of one complete oscillation is calculated from the equation $T = 2\pi\sqrt{LC}$, L being expressed in henries and C in farads. If the resistance is large, however, it may be necessary to take its value into consideration in calculating the period, and it becomes very desirable to appreciate the conditions under which it is permissible to use the approximate formula, and when it is necessary to employ the exact expression $T = \dfrac{2\pi}{\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}}$.

It is perhaps more convenient to deal with frequency instead of period, and the formulæ under discussion are then

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}, \text{ approximately} \quad \ldots \quad (a)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \ldots \quad (b),$$

which is rigorously true if the resistance term includes all sources of loss of energy. In order to appreciate the error which is introduced by using formula (*a*) let us calculate the natural frequency of a typical oscillatory circuit consisting of a coil of 200 μH inductance connected to a capacitance of $\cdot 0002 \mu F$, the resistance being variable. The natural frequency of the circuit neglecting the effect of resistance, is given by formula (*a*).

$$L = \frac{200}{10^6} \text{ henries} = \frac{2}{10^4} \text{ henries}$$

$$C = \frac{\cdot 0002}{10^6} \text{ farads} = \frac{2}{10^{10}} \text{ farads}$$

$$LC = \frac{4}{10^{14}} \text{ and } \sqrt{LC} = \frac{2}{10^7}$$

$$f_n = \frac{1}{2\pi} \times \frac{10^7}{2} = \frac{1}{\pi} \times 2\cdot 5 \times 10^6$$

$$\text{Now } \frac{1}{\pi} = \cdot 31831$$

$$\therefore f_n = \cdot 31831 \times 2\cdot 5 \times 10^6$$

$$= 795775 \text{ cycles per second.}$$

(ii) Now let it be assumed that the total resistance of the circuit is 40 ohms, and allow for this in calculating the natural frequency by formula (*b*).

The value of $\dfrac{1}{LC}$ is already known to be $\dfrac{10^{14}}{4}$ and we proceed to calculate the value of $\dfrac{R^2}{4L^2}$.

$$\frac{R}{2L} = \frac{40 \times 10^6}{2 \times 200} = 10^5, \ \frac{R^2}{4L^2} = 10^{10}.$$

From formula (*b*) therefore,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{10^{14}}{4} - 10^{10}}$$

$$= \frac{1}{2\pi}\sqrt{2500 \times 10^{10} - 1 \times 10^{10}}$$

$$= \frac{10^5}{2\pi}\sqrt{2499}$$

instead of $\frac{10^5}{2\pi}\sqrt{2500}$ which is the natural frequency if the resistance is zero. The error introduced by neglecting the resistance and using the approximate formula is in this instance only about 0·02 per cent. It may be taken as a general rule that no useful purpose will be served by calculating the frequency to a greater degree of arithmetical accuracy than the accuracy with which the values of inductance, capacitance and resistance are known. It was, of course, assumed above that the values of L and C were precisely those given, but in practice it is unlikely that either L or C will be known within one or two per cent. of the true values hence the labour expended in calculating the natural frequency by means of formula (b) is not repaid in the form of a more accurate answer.

12. (i) In certain circumstances, resistance may be deliberately introduced into the circuit, and it may then become necessary to calculate the natural frequency by the exact formula. Thus, if a total resistance of 707 ohms exists in a circuit having the above inductance and capacitance,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

$$\frac{R}{2L} = \frac{707}{2} \times \frac{10^6}{200}, \text{ and as } 707 = \frac{1000}{\sqrt{2}}$$

$$\frac{R}{2L} = \frac{10^7}{4\sqrt{2}}, \quad \frac{R^2}{4L^2} = \frac{10^{14}}{32} = 3{\cdot}12 \times 10^{12}$$

$\frac{1}{LC}$ remains as before, viz., 25×10^{12}.

$$\begin{aligned} \therefore \frac{1}{LC} - \frac{R^2}{4L^2} &= (25 - 3{\cdot}12) \times 10^{12} \\ &= 21{\cdot}88 \times 10^{12} \\ \text{and } f_n &= \frac{10^6}{2\pi}\sqrt{21{\cdot}88} \\ &= \frac{10^6}{2\pi} \times 4{\cdot}6776 \\ &= 744{,}470 \text{ cycles per second.} \end{aligned}$$

The natural frequency has been reduced by approximately 6 per cent. owing to the insertion of this comparatively large resistance.

(ii) Now suppose the resistance to be increased still further, say to 1,414 ohms. A repetition of the calculation then gives the natural frequency as 562,600, i.e. a reduction of about 30 per cent. As the resistance added to the circuit is still further increased, the natural frequency decreases rapidly, and if the total resistance of the circuit becomes 2,000 ohms, it is found that $\frac{R^2}{4L^2} = \frac{1}{LC}$, the quantity beneath the square root sign in formula (b) becomes zero, and the arithmetical value of the natural frequency also zero. It is important to understand the physical meaning of this, and here the mechanical analogy greatly assists. Suppose that the ball which is suspended on the spring, and possibly the spring itself, is immersed in a viscous fluid, e.g. treacle. When motion is taking place, the friction will be much greater than when the motion occurs in air, and in the extreme instance, it may be found that on pulling down the ball, it merely returns to its original position without oscillation, because the work done in overcoming the friction during the first upward motion is equal to the potential energy originally stored in the spring. The mass then possesses no kinetic energy at the instant when it reaches its normal position, and is therefore unable to do work on the spring by compressing it. The whole mechanical system, although it possesses both inertia and elasticity, is now non-oscillatory, because its friction is excessive. In an electric circuit possessing both capacitance and inductance, the introduction of a sufficient amount of resistance also renders the system non-oscillatory.

Damping

13. (i) The term $\frac{R}{2L}$, in the expression for the natural frequency of an electrical circuit, is called the damping factor. When the damping factor is exactly sufficient to prevent oscillation, i.e. when $\frac{1}{LC} = \frac{R^2}{4L^2}$ the circuit is said to be critically damped; the amount of resistance necessary to ensure this for any given values of L and C is called the critical resistance, and is derived thus

$$\frac{1}{LC} = \frac{R_c^2}{4L^2}, \; R_c \text{ being the critical value of R.}$$

$$R_c^2 = \frac{4L^2}{LC}$$

$$= \frac{4L}{C}$$

$$\therefore \; R_c = 2\sqrt{\frac{L}{C}}$$

Any value of resistance greater than this will cause the condenser discharge to be uni-directional, no oscillation being produced, and the larger the resistance, the longer will the condenser take to become fully discharged. A circuit which possesses an amount of total effective resistance which is sufficient or more than sufficient to prevent free oscillation is said to be aperiodic.

(ii) Although the influence of a small resistance upon the natural frequency of a circuit is negligible, the presence of resistance in an oscillatory circuit has another effect which is of the greatest importance. If set into oscillation, an electrical circuit without resistance, or a mechanical circuit without friction, would continue to oscillate with undiminishing amplitude for ever, for its total store of energy would never be depleted, although it would sometimes be in kinetic and sometimes in potential form, and during a large portion of each period it would be partly in one form and partly in the other. In practice a certain amount of energy is converted into heat and into motion of air in the mechanical system, while in the electrical circuit energy is converted into heat and is also expended in other ways to be discussed later. The energy stored at the end

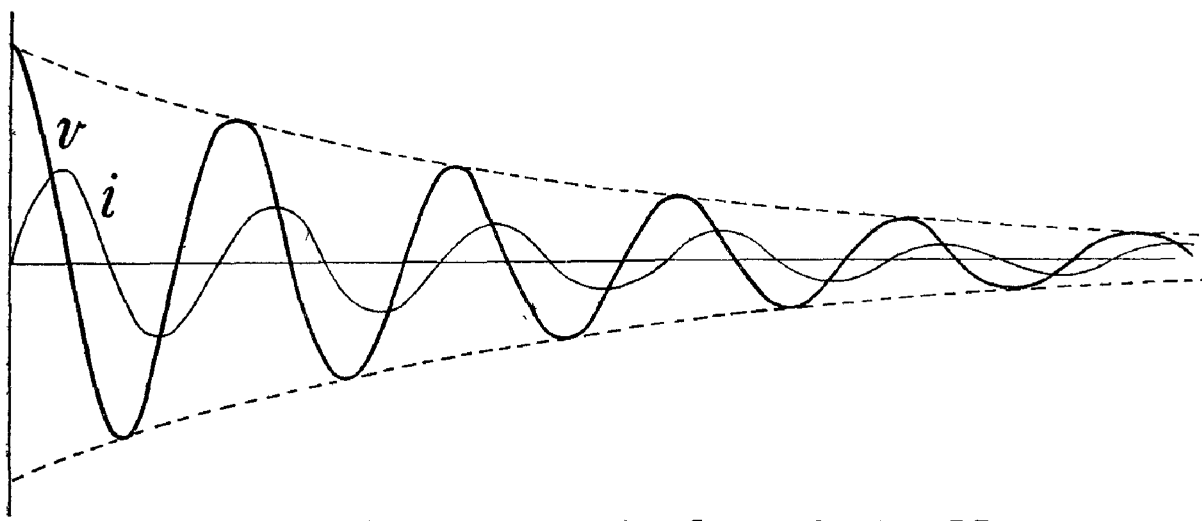


FIG. 5, CHAP. VII.—Damped oscillation. Current and condenser P.D.

of each succeeding half cycle becomes progressively less, the amount wasted during each cycle being a certain percentage of that which was stored at the commencement of the cycle. If the original energy is considered to be 100 per cent. and 20 per cent. is wasted in the first complete cycle, 80 per cent. of the energy is available for the second cycle, and during the second cycle 20 per cent. of 80 per cent., i.e. 16 per cent., of the original energy is lost, the energy remaining being 64 per cent. Both the peak value of the condenser P.D. and the peak value of the oscillatory current will decrease in amplitude during each successive half-cycle, and it is more usual to express the damping in terms of either of these amplitudes rather than in terms of total energy. Fig. 5 shows the manner in which the amplitudes of voltage and current decay with time, and it will be observed that each successive peak touches a curve (shown by the dotted line) which is similar in shape to the curve showing the discharge of a condenser through a resistance only (fig. 25, Chapter I).

Logarithmic decrement

14. The ratio of the amplitude of one peak to that of the same sign which follows it is called the decrement of the oscillation. If the successive amplitudes of positive condenser P.D. are 100, 90, 81, 72·9, 65·6, etc., the decrement is $\frac{100}{90} = \frac{90}{81} = \frac{81}{72 \cdot 9}$ etc. or 1.11............. It is more convenient however to use a quantity called the logarithmic decrement (which is the naperian logarithm of the decrement), the naperian logarithm being 2·3026 times the ordinary or common logarithm generally found in mathematical tables. This expression is often written and referred to as the log. dec., and is related to the magnification of the circuit. The latter term has been explained in Chapter V, and it will be recalled that the magnification χ is given by the ratio $\frac{\omega_r L}{R} = \frac{1}{\omega_r C R}$ where ω_r is the resonant frequency of the circuit, or the frequency at which an applied E.M.F. of given amplitude will cause maximum current to flow. The log. dec. may be derived from consideration of the oscillatory properties of the circuit by observing that it is the ratio of the energy expended in various losses during any one half-cycle to the maximum energy held in either kinetic or potential form during the same time. If the peak value of the current of a given half-cycle is $\mathcal{I}$, the heating effect during the half-cycle is $\frac{\mathcal{I}^2 R}{2} \times \frac{T}{2}$ joules, R being the total effective resistance and $\frac{T}{2}$ the duration of the half-cycle. The amount of energy stored in the circuit in kinetic form at the instant when the current reaches peak value is $\frac{1}{2} L \mathcal{I}^2$ joules, and the ratio referred to above becomes, using the usual symbol δ to denote the log. dec.,

$$\delta = \frac{\text{Energy wasted}}{\text{Energy stored}} = \frac{\frac{\mathcal{I}^2 R}{2} \times \frac{T}{2}}{\frac{1}{2} L \mathcal{I}^2}$$

$$= \frac{RT}{2L}, \text{ but since } T = \frac{1}{f_n},$$

$$\delta = \frac{R}{2Lf_n}$$

15. In circuits which are designed for the production of oscillations, i.e. in which resistance has not been deliberately introduced, the frequency f_n may be replaced with negligible error by the resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$ and the log. dec. becomes

$$\delta = \frac{R}{2L} \times 2\pi\sqrt{LC}$$

$$= \pi R \sqrt{\frac{C}{L}}$$

It has been previously shown that the magnification of the circuit is given by the expression

$$\chi = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is immediately apparent that $\delta = \frac{\pi}{\chi}$ and the importance of this relation lies in the fact that it is comparatively easy to measure the magnification of a circuit, at any rate up to frequencies of the order of a million cycles per second, and hence to calculate the log. dec. For example, if the magnification of a given circuit is found by actual measurement to be 157, the log. dec. is $\frac{\pi}{157} = \frac{3 \cdot 14}{157} = \cdot 02$, and this is the naperian logarithm of the ratio $\frac{I_1}{I_2}$. The common log of $\frac{I_1}{I_2}$ is therefore $\cdot 02 \times \cdot 4343$ or $\cdot 008686$, and reference to a table of common logs shows that the corresponding natural number is $1 \cdot 0202$. Hence $I_1 = 1 \cdot 0202\, I_2$. For some purposes it is more convenient to refer to the ratio $\frac{I_2}{I_1}$ which in this instance is $\cdot 9802$, and $I_2 = \cdot 9802\, I_1$. This inverted form of the decrement is called the persistency of the oscillation, and is conveniently expressed as a percentage of the initial amplitude. In the above example the persistency would be described as 98·02 per cent.

Example 2

In fig. 5 suppose the initial condenser P.D. to be 200 volts, and the amplitude of the next positive peak to be 131·2 volts. Find the log. dec. of the circuit. If $L = 100$ microhenries and $C = \cdot 001$ microfarad, find also the resistance of the circuit and its magnification.

The decrement is $\frac{200}{131 \cdot 2} = 1 \cdot 524$

Common log. of $1 \cdot 524 = \cdot 1830$

Naperian log. of $1 \cdot 524 = \delta = \cdot 1830 \times 2 \cdot 3026$

$$\delta = \cdot 4214$$

Since

$$\delta = \pi R \sqrt{\frac{C}{L}}$$

$$R = \frac{\delta}{\pi}\sqrt{\frac{L}{C}}$$

$$= \frac{\cdot 4214}{3 \cdot 1416}\sqrt{\frac{100}{\cdot 001}}$$

$$= \frac{\cdot 4214}{3 \cdot 1416}\sqrt{100{,}000}$$

$$= \cdot 4214 \times \frac{316}{3 \cdot 1416}$$

$$= 42 \cdot 2 \text{ ohms}$$

and

$$\chi = \frac{\pi}{\delta}$$

$$= \frac{3 \cdot 1416}{\cdot 4214}$$

$$= 7 \cdot 44$$

16. (i) The effect of resistance upon the oscillatory properties of a circuit is shown graphically in fig. 6. The oscillatory current in a perfectly loss-free circuit is shown by the sine curve of constant amplitude, while the current in a lightly damped circuit will have a slightly greater period as in the second curve. The third curve shows the growth and decay of the current when $R = 2\sqrt{\frac{L}{C}}$, i.e. when the circuit is critically damped; it will be seen that the current reaches its maximum value rather more rapidly than in an oscillatory circuit, but then dies away without reversal of sign. Finally, the growth and decay of current in a heavily damped circuit is shown. The current rises more rapidly and dies away more slowly than in a critically damped circuit. In discussing mechanical and electrical oscillations the decay of the mechanical oscillation of the spring and ball was ascribed to friction, and that of the electrical oscillation to resistance, without any consideration of the forms such friction and resistance may take. In the mechanical

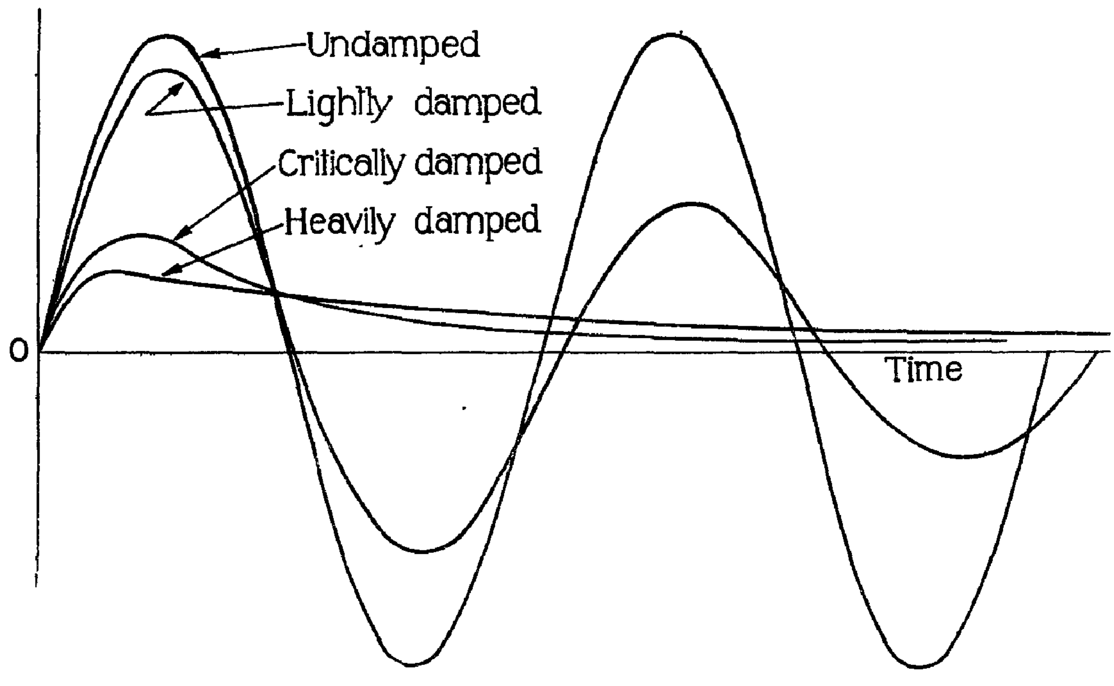


FIG. 6, CHAP. VII.—Effect of resistance upon condenser discharge.

system, the vibration is communicated to the surrounding atmosphere, and if the frequency of vibration is sufficiently high, results in the production of a sound wave in the air. The energy borne by this sound is, of course, supplied by the oscillatory system, and constitutes one form of damping. In the electrical circuit, the causes of energy loss may be divided into five components; these are :—

(i) Conductor resistance.
(ii) Eddy currents in neighbouring conductors.
(iii) Condenser losses in the actual electric circuit.
(iv) Dielectric losses in the surrounding insulators.
(v) Hysteresis losses, if iron is present.
(vi) Radiation of energy in the form of electro-magnetic waves.

(ii) The oscillatory circuit which has been discussed hitherto consists of a condenser, inductance and resistance in series, the condenser being tacitly assumed to be of the parallel plate type. Such a circuit is known as a closed oscillatory circuit in contrast with another type which is called an open oscillatory circuit. In this form the condenser plates are opened out as much as is practicable, in order that the greatest possible volume of dielectric may be embraced by the electric field of the condenser. Under these conditions the amount of energy converted into electro-magnetic radiation is much greater than in a closed oscillator. Before further discussion of the properties of open oscillators, it is necessary to explain what is meant by an electromagnetic wave.

ELECTROMAGNETIC WAVES

Wave motion

17. A disturbance which is propagated in any medium in such a manner that the shape, but not necessarily the magnitude, of the disturbance is repeated at regular intervals in space and time is called a wave, and the passage of such a disturbance through the medium is termed wave motion. A familiar example of wave motion is the surface wave caused by dropping a stone vertically into a pond. The water displaced downwards by the impact rises to occupy its former position, but owing to its inertia actually rises a little higher, forming momentarily a small mound of water upon the surface. This condition is evidently an unstable one, and the displaced volume of water immediately descends, passing its original position once more, continuing to oscillate up and down with decreasing amplitude until the energy imparted to the water by the falling stone has all been dissipated. The oscillation is communicated to the surrounding water in the immediate vicinity of the disturbance, and a wave spreads outward in ever increasing circles. The kinetic energy of the stone has been partly converted into wave motion, instead of being wholly dissipated in producing heat, as would have been the case if its fall had been arrested by a perfectly rigid earth. It must be fully realised that in no case of wave motion do the particles of the undulating medium travel forward with the wave but only move up and down, or to and fro, through a limited path. An illustration of this is found in the motion of a cork floating in the pond in which a surface wave has been produced, the cork moving up and down as the water beneath it becomes alternately the crest or trough of a wave, but possessing no average velocity along the surface of the water, although the energy imparted to the water is carried outward in ever widening circles.

Properties of waves

18. A wave can be described by means of four characteristic properties, (i) its velocity of propagation, (ii) its frequency, (iii) its amplitude and (iv) its wavelength. The *velocity of propagation* is the velocity with which the energy of the wave is conveyed from point to point in the medium, the *frequency* being the number of complete cycles of disturbance which pass a given point in unit time, i.e. one second. A *cycle* is one complete series of variations of displacement between adjacent repetitions of the wave in space, the *amplitude* or *peak value* is the maximum displacement of the medium from its normal position, and the *wavelength* is the distance between corresponding states of displacement in two adjacent repetitions of the wave form. It may be visualised as the distance between two adjacent crests or troughs. The most elementary form of wave motion is that in which each particle of the medium performs a cycle of displacement with simple harmonic motion, that is a " to and fro " or " up and down " reciprocating motion. When sinusoidal wave motion is taking place in a material medium each particle carries out this simple harmonic motion in succession. The term transverse wave motion is applied to a wave in which the particles execute their motion in a direction perpendicular to the direction of propagation of the wave, as for example, the water particles in the surface wave mentioned above. In many instances the particles vibrate to and fro about their mean position in the direction of propagation, and this mode of vibration gives rise to a longitudinal wave. The vibration of air particles when conveying sound energy is executed in this manner.

19. The relation between the frequency, velocity of propagation and wavelength of a wave in any medium depends upon the physical properties of the medium, sometimes in a complex manner because in some media the velocity of propagation is itself dependent upon the frequency. Such a medium is said to be dispersive, while a medium in which the velocity of propagation is

constant for all frequencies is called a non-dispersive medium. In non-dispersive media, to which the present discussion will be confined, there is a simple relation between the wavelength λ, velocity of propagation, u, and frequency f. Since one wavelength is the distance travelled in the periodic time, T, i.e. the time taken to execute one cycle of vibration,

$$\lambda = uT$$

and as T and f are in reciprocal relationship

$$\lambda = \frac{u}{f}$$

if u is in metres per second, λ is in metres. The mathematical expression for a sinusoidal wave moving through a medium with velocity u is

$$y = \mathcal{Y} \sin \frac{2\pi}{\lambda} (x - u\,t)$$

y being the displacement of a particle at time t, x the distance of the particle from the origin, and $\mathcal{Y}$ the maximum displacement of the particle. This equation may be deduced thus :— The expression for the motion of the particles about their mean position is

$$y = \mathcal{Y} \sin \omega t$$

whereas the equation of a sine wave, stationary in space on the axis OX (fig. 7) is

$$y = \mathcal{Y} \sin k\,x$$

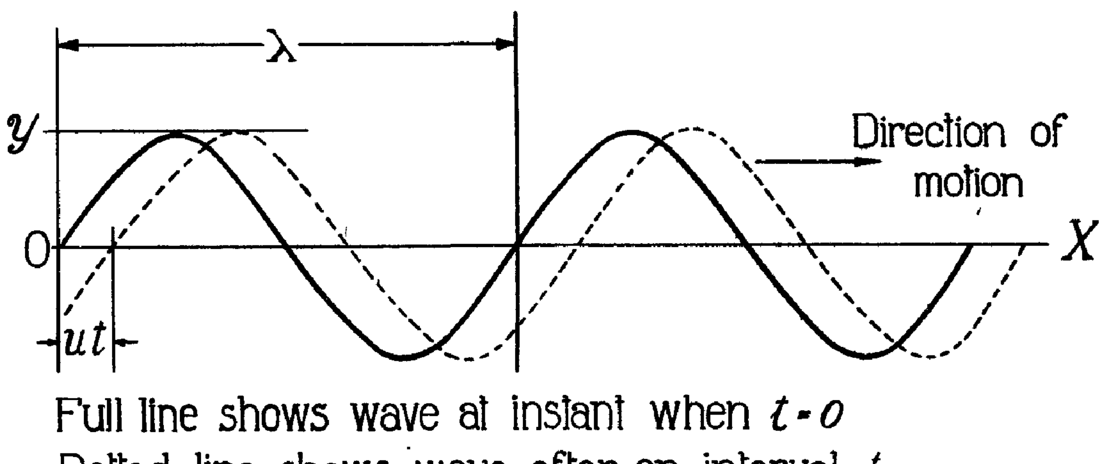


FIG. 7, CHAP. VII—Wave travelling with velocity u.

k being a constant, having dimensions "angle per unit distance" so that kx represents an angle although x represents distance. If the wave form moves forward in space with velocity u the distance passed through in t seconds is ut, and therefore

$$y = \mathcal{Y} \sin k\,(x - ut).$$

The value of the constant k can now be found ; when the wave has moved forward one wavelength, λ, the angle moved through is 2π radians and therefore $k\lambda = 2\pi$, or $k = \frac{2\pi}{\lambda}$. The complete equation is therefore as stated above.

The ether

20. In an earlier paragraph an allusion was made to the motion of a cork upon a water surface consequent upon the impact of a stone at some other point in the wave-supporting medium. The wave does work on the cork, and so the stone may be considered as the original

source of energy and the cork as a receiver of energy. In the solar system, energy is conveyed from point to point in this manner without the intervention of any material medium whatever, the term material medium signifying one which consists of matter in either of the three forms, solid, liquid or gas. For instance large quantities of energy are received by the earth from the sun, and we are conscious of its reception by the sensations of heat and light, although the space between the sun and the earth is to all intents and purposes devoid of matter. It is possible to prove that this energy is conveyed by wave motion, and the question arises, by what medium is the energy conveyed?

21. It must be admitted that no perfectly satisfactory answer to this question has yet been evolved, and it is therefore necessary to assume that some medium exists which is capable of and responsible for the conveyance of energy from point to point in the universe. To this hypothetical medium the name " luminiferous (i.e. light-bearing) ether " was originally given, nowadays generally shortened to " the ether ". A further essential assumption is that the ether penetrates all matter and pervades all space, so that it fills impartially the space between electrons and protons in the atom as well as between planets in the solar system and between stars in the universe. It is possible to prove by astronomical observation as well as by direct terrestrial experiment that wave motion, e.g. light, is propagated through this medium with the enormous velocity of $2{\cdot}998 \times 10^{10}$ centimetres per second (generally taken as 3×10^{10} centimetres per second). In order to appreciate the significance of this statement, this velocity should be compared with that of a compression wave in hard steel, i.e. approximately $5{\cdot}2 \times 10^5$ centimetres per second, which is almost the maximum velocity with which wave motion can be conveyed by matter. Sound waves in air travel much more slowly, a mere $3{\cdot}4 \times 10^4$ centimetres per second. Now it can be proved that in order to support wave motion, a material medium must possess two properties; first, a constraint tending to restore the displaced particles of the medium when the displacing force or stress is removed, which is called the elasticity of the medium, and second, inertia, the tendency of any element of the medium to continue in a state of rest, or of uniform motion in a straight line. The inertia of the particles causes them to pass through their original positions when returning thereto under the influence of the restoring force or elasticity.

22. The velocity u of wave motion in a material medium of density d and elasticity η is given by the equation $u = \sqrt{\frac{\eta}{d}}$. Since the ether has been postulated as the medium of wave motion in the conveyance of energy in the form of light and heat, it must be assumed to possess properties which are analogous to elasticity and density. It is neither necessary nor desirable to speculate upon the exact nature of these properties, but it is known that the magnetic permeability (μ) of the medium is connected with the etherial analogue of inertia and the dielectric constant or permittivity ($\varkappa$) with the property corresponding to elasticity. An attempt will therefore be made to exhibit the manner in which these two properties are involved in the velocity of propagation of an electro-magnetic wave. We have already suggested (Chapter I) that an electric field of strength Γ may be considered to represent electrical stress, the corresponding strain being the electric flux density D; the two quantities are related by the equation $\Gamma = \frac{4\pi D}{\varkappa}$. Similarly the magnetic field strength H and the resulting flux density B are related by the equation $H = \frac{B}{\mu}$. The occurrence of the constant 4π in the relation between the electric quantities but not in the corresponding magnetic equation, may be regarded as the result of an unfortunate departure from uniformity in the definitions of unit electric charge and unit magnetic pole.

Production of electric field by magnetic field in motion

23. Faraday's law may be taken as a convenient starting point in this discussion. This states that in an electric circuit, an induced E.M.F. results from any and every change in the flux linkage. The point to bear in mind here is that the word circuit does not imply that the current path is conductive throughout, for it may contain an electrical condenser. Once this

point is realised, there is no necessity for the circuit to contain any conductor whatever, a change of flux through a small area of dielectric, for example, will set up an electro-motive force round the line bounding that area. In Chapter II it was stated that a magnetic flux of density B (E.M. units) perpendicularly cutting a conductor with velocity u centimetres per second, generates in it an E.M.F. of Bu electromagnetic units per centimetre of conductor. As it has been inferred that the presence of the conductor is not essential for the production of an E.M.F., we may now vary this statement and say that a magnetic flux of density B moving with velocity u, produces an electromotive force of $Bu \times 10^{-8}$ volts per centimetre. The volt per centimetre is one of the units of electric field strength, hence the above argument can be expressed algebraically in the form

$$\Gamma = \frac{Bu}{c} = \frac{\mu H u}{c}$$

where c is a constant which has been introduced in order that the electric field strength Γ may be expressed in electrostatic units. It is the ratio of the electromagnetic unit of quantity to the equivalent electrostatic unit, or 3×10^{10}. The relative directions of B, u and Γ are shown in fig. 8a. The relation $\Gamma = \frac{Bu}{c}$ is sometimes called the second law of electro-dynamics.

Production of magnetic field by electric field in motion

24. Instead of commencing our reasoning from Faraday's law, it would be equally justifiable to commence by contemplation of an ordinary electric conduction current. This current is due to the movement of electric charges and its intensity is the rate at which the charges move. Now electric charges carry with them tubes of electric flux, one unit tube per unit charge. An electric current is, therefore, equivalent to a moving electric flux, and instead of the usual statement that an electric current produces a magnetic field, it may be asserted that an electric

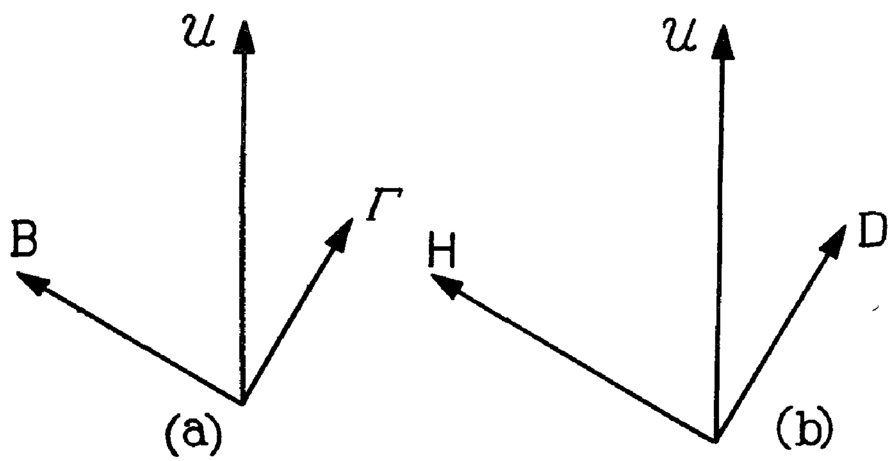


FIG. 8, CHAP. VII.—Laws of electro-dynamics.

flux in motion produces a magnetic field. We have previously also made use of the conception of magnetomotive force, the M.M.F. along a line of magnetic force being the work done in moving a unit pole along the line, that is to say, M.M.F. = force × distance, or M.M.F. = Hl. If this path encircles a conductor carrying a current of i E.M. units the M.M.F. is $4\pi i$ E.M. units. As was done earlier in the discussion, we may now substitute for the conduction current a displacement current in a small area A of dielectric material, and in this dielectric a small charge of Q units will carry Q tubes of flux, the average current during a time t being $\frac{Q}{t}$ electrostatic units. The area A may be considered rectangular and of dimensions l centimetres perpendicular to the direction of current and x centimetres in the direction of the current, in which circumstances the current is $\frac{Q}{t} = \frac{DA}{t} = \frac{Dlx}{t}$. But $\frac{x}{t}$ is the average velocity u of the charge in the direction x hence $i = Dlu$ electrostatic units. Collecting our equations we may now write :—

$$\text{M.M.F.} = Hl = 4\pi i$$

$$Hl = 4\pi Dlu$$

$$H = 4\pi Du$$

except that the constant c must be inserted in order that H may be expressed as usual in electromagnetic units. Hence $H = \frac{4\pi Du}{c} = \frac{\varkappa\, \Gamma u}{c}$ because $4\pi D = \varkappa\, \Gamma$. The relative directions of D, H and u are shown in fig. 8b.. The relation $H = \frac{4\pi Du}{c}$ is called the first law of electrodynamics.

Self-supporting electromagnetic field

25. We now see that a moving electric field of a given strength will produce a magnetic field of definite strength, and that the converse is also true, a moving magnetic field producing a corresponding electric field. If we commence by postulating an electric field, which appears to be the more fundamental one since it is inseparable from the electrons of which matter is partly composed, we see that the motion of an electron gives rise to a magnetic field, and many examples of the manner in which this magnetic field is detected and utilised have been given in the earlier chapters. If the velocity of our fundamental field is sufficiently great, say u, the magnetic field produced by its motion will produce a new electric field of intensity equal to the original, and any change in the electrical or magnetic state of the medium at one point must be communicated to all points in the medium with the velocity u, at which the electric and magnetic fields become self-supporting. This velocity can be found from the equations developed above, for the magnetic field strength is then

$$H_1 = \frac{\varkappa\, \Gamma_1 u_1}{c}$$

but $$\Gamma_1 = \frac{\mu H_1 u_1}{c}$$

$$\therefore H_1 = \frac{\varkappa\, \mu\, u_1^2 H_1}{c^2},$$

and this equation can only be true if

$$\varkappa\, \mu \frac{u_1^2}{c^2} = 1$$

$$\text{or } u_1 = \frac{c}{\sqrt{\varkappa\mu}}.$$

Now in ether unobstructed by matter, $\mu = 1$ and $\varkappa = 1$, and, therefore, $u_1 = c = 3 \times 10^{10}$ centimetres per second, which is identical with the velocity of light as determined by experiment. For this reason, among others, we conclude that light radiation is fundamentally an electromagnetic phenomenon.

The production of an electromagnetic wave

26. Consider a large rectangular loop of wire arranged in a vertical plane, say 100 yards in length and 10 yards high. If a source of unvarying E.M.F. is inserted in series with this loop,

a direct current will flow in it. The length of the loop has been made large in order that the mutual effects of the vertical portions of the wire may be neglected, and it is proposed to consider the current flowing in only one of the vertical portions, namely, that in which the electron flow has an upward direction. Let us further confine our observations to the phenomena associated with a single electron, which is an electric charge of e units, and may be considered to have a constant velocity of about one centimetre per second, which will be denoted by b. The electron is the focus of an electric field consisting of lines of electric force which converge upon it from all directions and extend to an infinite distance, this field moving with the electron and having the same velocity through space. At any point in this field distant x centimetres from the conductor the electric flux density will be $\frac{e}{4\pi x^2}$. (Chapter I.) In order to simplify matters still further, the behaviour of a single line of force will be observed. The relevant portion of the circuit is then as illustrated in fig. 9 where M represents the position of the electron at a certain instant, and the single line of force under observation is represented bv Mm.

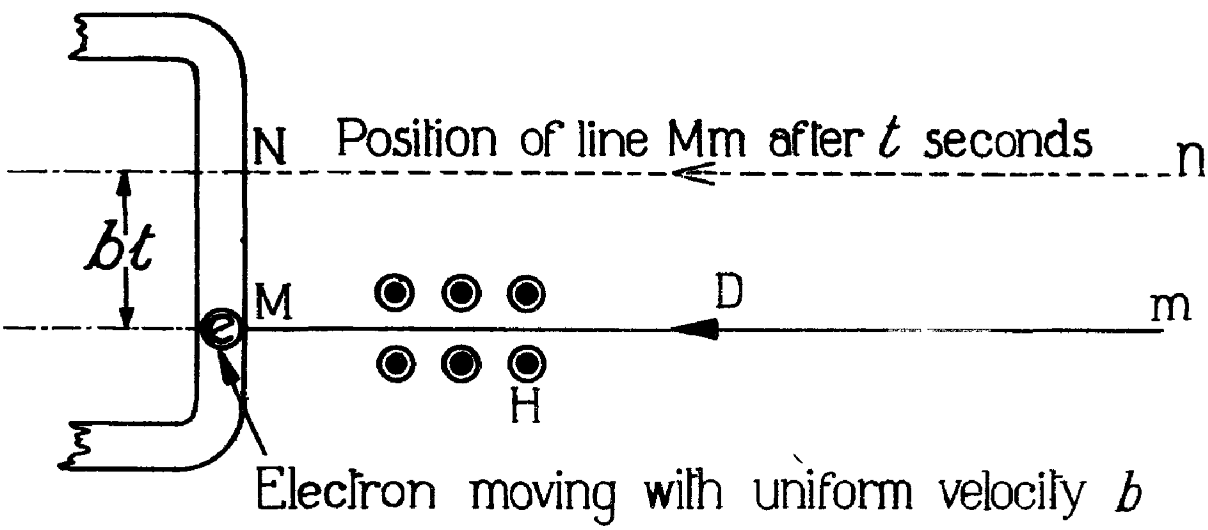


Fig. 9, Chap. VII.—Field due to electron moving with constant velocity.

27. Owing to the motion of the line of force, a magnetic field will be produced, as described in the preceding section. This magnetic field will be directed perpendicularly to the direction of motion of the line, and also perpendicularly to the direction of the electric field, hence by the first law of electro-dynamics the magnetic field strength H at all points in the line Mm is in the direction " out of the paper." This may be verified by the corkscrew rule, observing, however, that the conventional " direction of current " used in this rule is opposite to that in which the electron movement is taking place. In accordance with the foregoing analysis the strength H of this magnetic field is $4\pi D \frac{b}{c}$E.M. units; as $4\pi D = \frac{e}{x^2}$, $H = \frac{e}{x^2} \cdot \frac{b}{c}$, and it is demonstrated that the magnetic field strength caused by the moving line of force varies inversely as the square of the distance from the conductor, but is of constant magnitude at any point if the velocity b is constant. This moving magnetic field in its turn generates an electric field, and its intensity by the foregoing analysis is $\mu H \frac{b}{c}$ or $\frac{\mu e}{x^2} \cdot \frac{b^2}{c^2}$. Its magnitude is extremely small compared with the original electric intensity and may be disregarded.

28. The motion of an electron or any number of electrons with constant velocity in the conductor thus gives rise to a magnetic field in the surrounding medium, which forms concentric circles round the conductor, and occupies the same space as the radial electric field. These fields are called the induction fields and their intensity varies inversely as the square of the distance from the conductor. In accordance with the conclusions reached in paragraph 25, these fields would be self-sustaining if the velocity of the electron in the first place was sufficiently great, that is, if $b = \frac{c}{\sqrt{\kappa\mu}}$ or as the values of κ and μ for air are to all intents and purposes unity, if $b = c$. Thus it appears that if an electron were moving in the conductor with velocity c, and its motion were suddenly arrested, the fields would continue in motion in the upward direction, forming a single pulse which would travel into space. This speculation is an idle one, however, firstly, because the above reasoning is only strictly applicable when b is very small compared to c, and secondly, because in order to bring its velocity to zero the electron must receive acceleration. It will now be shown that the effect of such an acceleration will be to produce in the medium a self-sustaining electromagnetic field, or what is generally called an electromagnetic wave, which is propagated *radially outward* from the conductor.

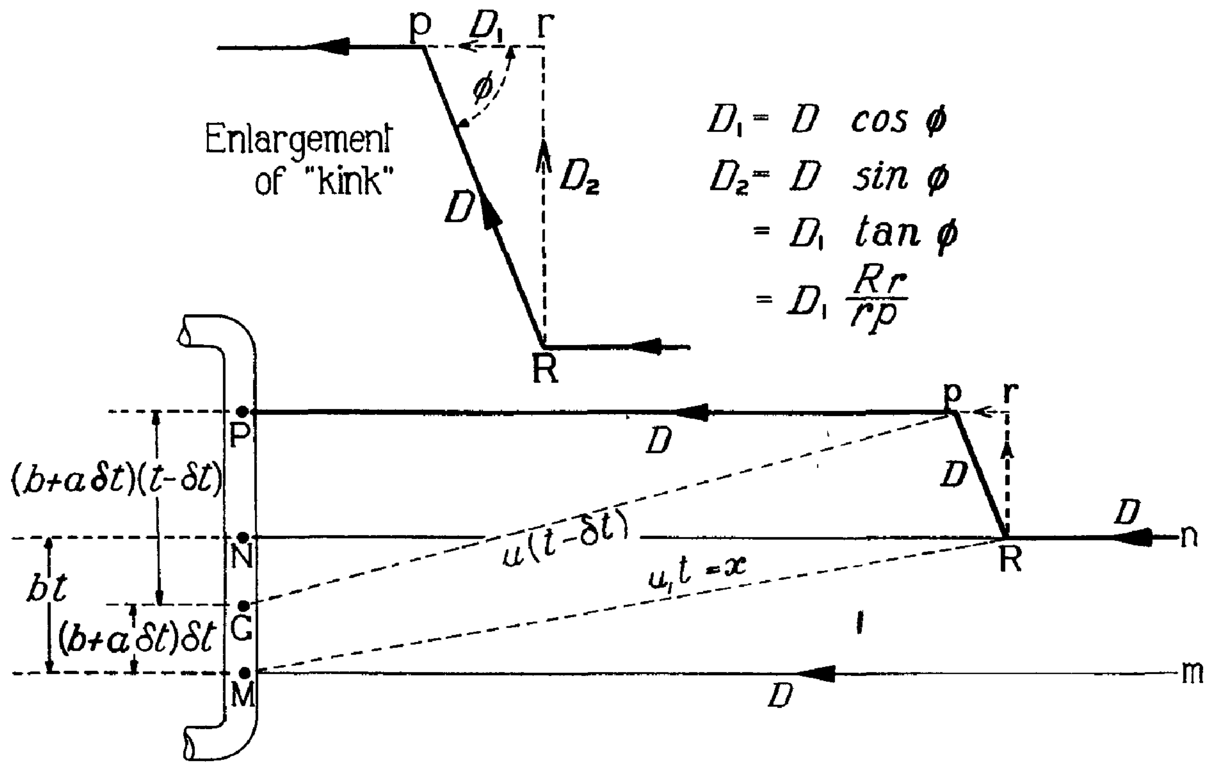


FIG. 10, CHAP. VII.—Field due to electron moving with accelerated velocity.

29. Let the conditions already laid down be repeated, the electron travelling upward, first with uniform velocity b from M, so that the electric line of force under observation is Mm. After t seconds the electron reaches the position N where MN $= bt$, and the line of force will move with it, reaching the position Nn. This repetition of the previous instance has been performed merely to serve as a frame of reference on the diagram, fig. 10. Now suppose that on arrival at the point M the electron receives an acceleration a which lasts for a short interval of time δt after which the electron continues to travel with its new velocity, $b + a\,\delta t$. In the time t, the electron will arrive at the point P instead of N, but as disturbances in the medium are propagated

with the finite velocity $u_1 = \frac{c}{\sqrt{\kappa\mu}}$, the occurrence of the acceleration is only effective upon that portion of the line of force which is within a radius $u_1\, t$ centimetres from the point M.

30. (i) The portion Rn of the original line of force, which is situated outside the radius $u_1\, t$, is still a portion of the line of force under consideration. Within the radius $u_1\,(t - \delta t)$ from the point G at which the acceleration ceased, however, the line of force assumes the position Pp, and the whole line of force instead of being Nn, has been changed by the acceleration into the line Pp Rn which is called a " kinked line " for the sake of brevity. The electric flux density D at the point p can be resolved into two perpendicular components, the first being in the radial direction r p and the second in the direction of motion of the line of force. Denoting the angle rpR by ϕ, and remembering that the radial electric flux density D_1 at the point p is $\frac{e}{4\pi x^2}$,

$$D_1 = D \cos\phi$$

$$\begin{aligned} D_2 &= D \sin\phi \\ &= D_1 \frac{\sin\phi}{\cos\phi} \\ &= D_1 \tan\phi \\ &= D_1 \frac{\mathrm{Rr}}{\mathrm{rp}} \end{aligned}$$

(ii) To obtain values for Rr and rp is must be observed that by hypothesis the velocity of the electron, b, is very much smaller indeed than the velocity u_1, so that in fig. 10, MR may be regarded as equal to NR and Gp equal to Pp. Then rp, which is actually equal to NR – Pp, may be said to be equal to MR – Gp, or since MR $= u_1 t$ and Gp $= u_1\,(t - \delta t)$, to $u_1 \,.\, \delta t$. Similarly Rr is equal to NP and NP = GP + GM – MN.

$$\begin{aligned} \text{Now GP} &= (b + a\,.\,\delta t)\,(t - \delta t) \\ &= bt - b\,.\,\delta t - a\,.\,\delta t^2 + at\,.\,\delta t \\ \text{GM} &= (b + a\,.\,\delta t)\,\delta t \\ &= b\,.\,\delta t + a\,.\,\delta t^2 \\ \text{MN} &= b\,.\,\delta t \\ \therefore\ \text{NP} &= bt - b\,.\,ot - a\,.\,\delta t^2 + at\,.\,\delta t + b\,.\,\delta t + a\,.\,\delta t^2 - bt \\ &= at\,.\,\delta t, \end{aligned}$$

$$\begin{aligned} \text{and } D_2 &= D_1 \frac{a\,t\,.\,\delta t}{u_1\,.\,\delta t} \\ &= D_1 \frac{a\,.\,t}{u_1} \\ &= D_1 \frac{atu_1}{u_1^2} \end{aligned}$$

$$\text{but } u_1\, t = x$$

$$\therefore D_2 = D_1 \frac{a\,x}{u_1^2}$$

(iii) The velocity of the component of flux D_2 is in the direction p r, i.e. radially outward, and is equal to u_1 centimetres per second, while the velocity of the component D_1 is equal to that of the electron, namely b centimetres per second, and is in the upward direction. The total electric flux density is the vector sum of these, and the corresponding electric field strengths can be calculated by the relation $\Gamma = \frac{4\pi D}{\varkappa}$ giving the following results, in dynes per unit charge :—

$$\Gamma_1 = \frac{e}{\varkappa\, x^2}$$

$$\Gamma_2 = \frac{a\, x}{u_1^2}\Gamma_1$$

$$= \frac{a\, x\, e}{\varkappa\, x^2\, u_1^2}$$

$$= \frac{a\, e}{\varkappa\, x\, u_1^2}$$

by noting that $\frac{1}{u_1^2} = \frac{\varkappa\mu}{c^2}$ this becomes

$$\Gamma_2 = \frac{a\, e\, u}{x\, c^2}$$

Γ_1, which varies inversely as the square of the distance from the conductor, denotes the strength of the induction field, while Γ_2, which varies inversely as the distance, is the strength of the radiation field. Each of these has an associated magnetic field, for the total magnetic field may also be resolved into two components, one caused by the movement of the electron with constant upward velocity b, the other caused by the motion of the electric radiation field with radial velocity u_1. These have the following magnitudes in dynes per unit magnetic pole :—

$$H_1 = \frac{\varkappa\, \Gamma_1 b}{c}$$

$$= \frac{e\, b}{x^2 c}$$

which is the induction field strength, while the radiation field has the strength

$$H_2 = \frac{e\, a}{x}\,\frac{\sqrt{\varkappa\mu}}{c^2}$$

In free space or any other medium of which the dielectric constant $\varkappa$ and magnetic permeability μ are unity.

$$\Gamma_2 = \frac{a\, e}{x\, c^2}$$

$$H_2 = \frac{a\, e}{x\, c^2}$$

or $\Gamma_2 = H_2$ numerically, although their units are different.

The above results are summarised in the following table.

Motion of charge	Radial electric field Γ_1	Electric field Γ_2 perpendicular to Γ_1 and x	Magnetic field H perpendicular to Γ_1 and Γ_2
Nil	$\frac{e}{\varkappa\, x^2}$	Nil	Nil
Constant velocity b	$\frac{e}{\varkappa\, x^2}$	Nil	$H_1 = \frac{e\, b}{x^2\, c}$
Velocity b, Acceleration a	$\frac{e}{\varkappa\, x^2}$	$\frac{e\, a\, \mu}{x\, c^2}$	$\frac{e\, b}{x^2 c} + \frac{e\, a\, \sqrt{K\mu}}{x\, c^2} = H_1 + H_2$

31. Now if instead of one electron undergoing a single acceleration, we consider the effect of the whole electron stream in the conductor, it is obvious that the intensity of the radiated field will be directly proportional to the current, and the effect of a single acceleration, e.g. that caused by switching the current on or off, will be to radiate a single pulse which can be detected by suitable apparatus at considerable distances. Such a pulse is of little value for ordinary signalling, however, and it is preferable to initiate a train of such pulses, similar to the waves in water to which reference has already been made. In order to produce electro-magnetic waves of this nature the electrons in the conductor must undergo sinusoidal acceleration and the resulting wave will then consist of sinusoidal magnetic and electric fields, which are perpendicular to each other in space. As already shown, the radiation electric field is parallel to the wire carrying the current to which the wave owes its existence, and the radiation magnetic field is perpendicular to the electric field, while both these fields are perpendicular to the direction in which the wave is travelling. This orientation of the fields, with respect to the conductor carrying the current by which the wave is caused, may be referred to as the natural polarisation of the wave. In the immediate vicinity of the conductor, the induction field is stronger than the radiation field; the former varies inversely as the square of the distance, and the latter inversely as the distance from the conductor, the total field being the vector sum of the two, so that very near to the conductor the total magnetic and electric fields are very nearly 90° out of phase with respect to time. As we go further and further from the conductor, the induction field strength falls off rapidly, and at a distance $\frac{\lambda}{2\pi}$ the induction field has fallen to equality with the radiation field. At still greater distances the induction fields become negligible and the magnetic and electric fields are in phase with each other.

Polarisation of wave

32. Suppose that an electromagnetic wave is originated as a result of the sinusoidal acceleration of electrons in a vertical wire, situated near the surface of the earth. The wave will be emitted with natural polarisation, and will travel over the surface of the earth with its magnetic field in the horizontal and its electric field in the vertical plane. From the point of view of reception of such waves, we are not particularly interested in the natural polarisation, but in the manner in which the wave is incident upon the receiving aerial. It is, therefore, usual to state the polarisation with reference to the earth's surface, and a wave which reaches the receiver in such a manner that the magnetic field is in the horizontal plane, and the electric field in the vertical plane is said to be normally polarised. If the orientation of the fields differs from this in any way whatever, the wave is said to be abnormally polarised, and the angle of polarisation is defined as the angle which the electric field makes with the vertical plane.

33. The practical production of an electro-magnetic wave suitable for radio-telegraphic purposes is contingent upon the sinusoidal acceleration of electrons and owing to the properties of simple harmonic motion (paragraph 10) this indicates the production of a sinusoidally varying current. We have seen that in simple harmonic motion the acceleration is proportional to the square of the frequency, hence in order to obtain appreciable radiation from the circuit, the frequency must be high. It is possible to provide such an alternating current, having a frequency of the order of 30,000 cycles per second, by means of an alternator constructed on the principles discussed in Chapter IV, but such machines have very little to commend them on practical grounds. The desired result can be obtained with much greater convenience by the use of an oscillatory circuit as described in the earlier paragraphs of this chapter.

Radiation from closed loop

34. It has already been stated that the type of oscillatory circuit which is best adapted for the radiation of energy in the form of electromagnetic waves is an open oscillator, which may be described as a circuit possessing inductance and capacitance and having an inherent resistance which is much lower than the critical value, the geometric dimensions of the circuit being of the same order as the wavelength of the oscillation; the last stipulation ensures that the inductance

and capacitance of the circuit are distributed in space and not localised as in the closed oscillator. In order to illustrate the effect of the geometry of the circuit upon the amount of energy radiated let us again contemplate the vertical rectangular loop of a preceding paragraph, but now consider two electrons, one being situated in each vertical side. In fig. 11 the point P is supposed to be situated at a considerable distance from the loop on the side opposite to the reader, who is supposed to observe the electric fields at P by looking through the loop. When a current is established in the circuit, the electron e_1 is moving upwards and the electron e_2 downwards, and if both receive equal positive acceleration, or increase of velocity, at the same instant, similar kinks will travel outward from each electron. Considering only the lines of force passing through P, it is seen that the kink emanating from the electron e_1 will reach this point at exactly the same moment as the kink originated by the acceleration of e_2, but as the latter acceleration is in the downward direction while that of e_1 is upward the outward-travelling flux-density (both magnetic and electric) of the two kinks are in antiphase and their combined effect is to annul each other, hence the effective travelling flux at the point P is zero, no matter how near to the loop P may be, provided it is equidistant from e_1 and e_2. At any point in the direction X, however, the kink due to the acceleration of e_1 is not received until a time $t = \frac{x}{c}$ after the effect of the acceleration of e_2, because the kink caused by e_1 has to travel a greater distance, i.e. the width of the loop,

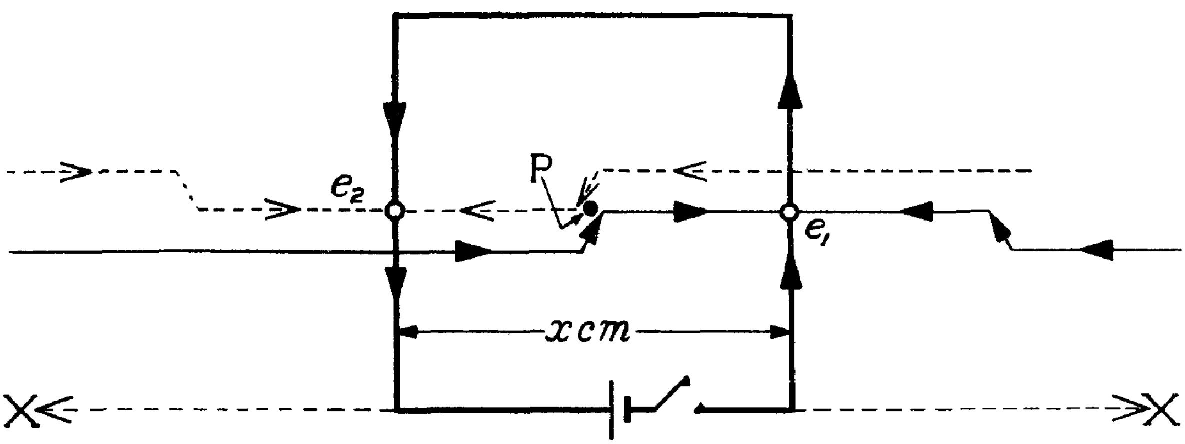


FIG. 11, CHAP. VII.—Radiation from closed loop.

which is x cms. (*see* fig. 11.). Similar considerations apply to the direction X^1, and it can be seen that a loop of this nature, i.e. in which the sides are an appreciable distance apart, will have directional radiating properties, which will receive further consideration in Chapter XVI. For the present, suppose that the width of the loop x is reduced. The difference in the time of arrival of disturbances due to e_1 and e_2 at any point in XX^1, will be less than before, and eventually as x approaches zero the amount of energy radiated along XX^1 becomes negligible. Thus, the effect of bringing the two sides into proximity so that the loop becomes non-inductive, is to render it non-radiative also.

Radiation of damped waves from dipole

35. Instead of a closed loop, let only a single vertical wire be used, which we will first suppose to be located well above the ground, in order that the capacitance of the wire with respect to earth shall have an inappreciable effect upon the electrical conditions. It will also be assumed that it is possible to locate a battery and switch in a position near the mid-point of the wire. Although a circuital conduction current cannot be established, it is still possible to cause an acceleration of the electrons in the vertical wire, because the latter possesses a certain amount of capacitance, the length connected to the positive terminal and that connected to the negative forming a kind of condenser. On closing the switch S (fig. 12) a momentary charging current will

flow in accordance with the principles explained in Chapter I, and consequently all the electrons in the conductor are momentarily displaced, i.e. they receive acceleration. This charging of the capacitance will be followed by a discharge which will consist of a damped train of oscillations as already explained. The vertical wire has, in fact, become a rudimentary form of transmitting aerial and will radiate equally well in all directions in the horizontal plane. If L is the effective inductance, C the effective capacitance and R the effective resistance of the wire the frequency of the oscillations will be given by the formula $f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$, and the log. dec. of the oscillations by $\delta = \pi R\sqrt{\frac{C}{L}}$. The effective inductance and capacitance obviously depend upon the

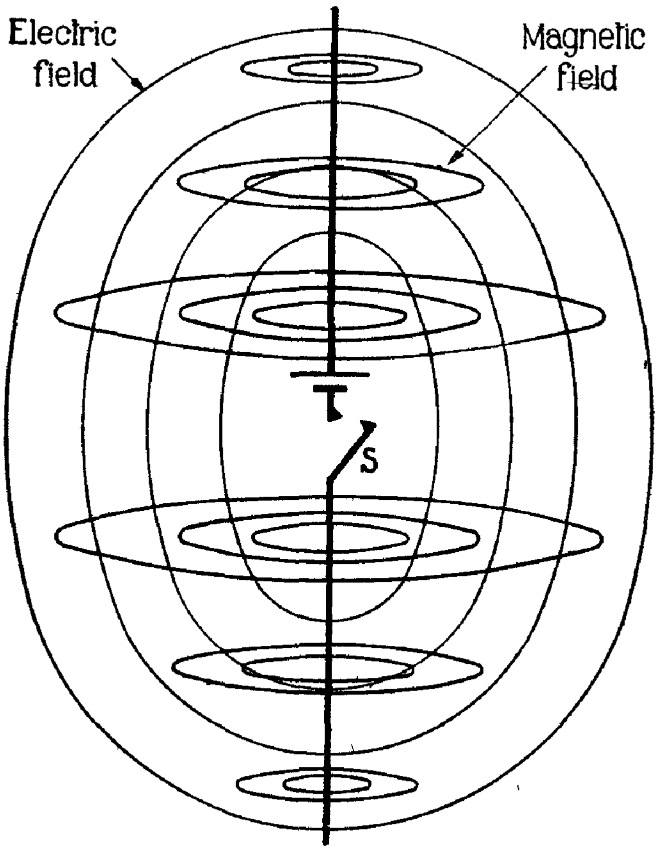


FIG. 12, CHAP. VII.—Electric and magnetic fields of dipole.

length of the wire, and it is found that under the conditions laid down, the wavelength of the emitted radiation is twice the length of the wire, which may be expressed algebraically as $\lambda = 2l$, where λ is the wavelength and l the total length of the wire. An aerial of this form is, therefore, called a half-wave aerial or a dipole. The latter term may be interpreted as signifying that the lengths of wire connected to each terminal are merely devices for increasing the inductance and capacitance of the two " poles " of the battery.

36. Single trains of waves produced by a battery in the manner described are, of course, useless for radio telegraphic communication, unless the wave trains are of comparatively long duration and are caused to succeed each other at very short intervals, while in order to initiate appreciable radiation, a much larger quantity of energy must be stored in the aerial capacitance than is practicable by means of a battery as envisaged above. As our present object is to discuss

the application of electromagnetic radiation to communication, it is of interest to calculate the duration of a wave train such as would be produced by the simple apparatus described above. For this purpose we shall require to know the decrement of the circuit. This has been defined as the ratio $\frac{I_1}{I_2} = \frac{I_2}{I_3} = \frac{I_3}{I_4}$, where I_1, I_2, etc., are the amplitudes of successive peaks of oscillatory current of the same sign. The wave train may be considered to carry negligible energy when the amplitude has fallen to ·01 of the amplitude of the first peak. Now $\frac{I_1}{I_2} \times \frac{I_2}{I_3} \times \frac{I_3}{I_4} = \frac{I_1}{I_4} = N^3$ where N is the decrement, and must not be confused with the log. dec. Similarly $\frac{I_1}{I_2} \times \frac{I_2}{I_3} \times \frac{I_3}{I_4} \times \frac{I_4}{I^5} = \frac{I_1}{I_5} = N^4$ and by extending this process to any number of successive amplitudes it is apparent that, if I_n represents the amplitude of the nth peak $\frac{I_1}{I_n} = N^{n-1}$, hence if $I_n = \cdot 01 I_1$, $N^{n-1} = 100$. This equation can be solved by taking the logarithms of both sides. If common logs. are used, $\log_{10} 100 = 2$ while $\log_{10} N^{n-1} = (n-1) \log_{10} N$. We do not yet know the value of $\log_{10} N$, but it can easily be found, if we know the naperian log. of N, i.e. δ; the common log. is $\frac{1}{2 \cdot 3026}$ or ·4343 of this, that is $\cdot 4343\delta$. Hence

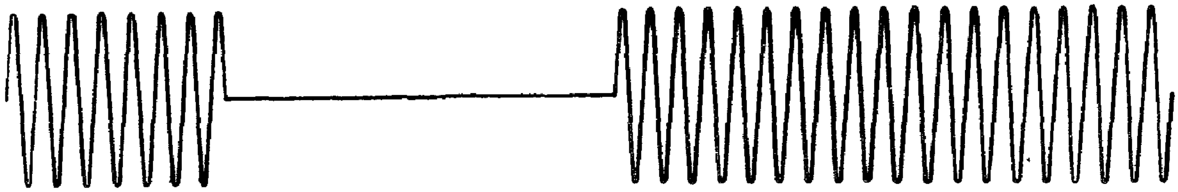
$$
\begin{aligned}
2 &= (n-1) \log_{10} N \\
\frac{2}{\log_{10} N} &= n - 1 \\
\therefore n &= 1 + \frac{2}{\cdot 4343\delta} \\
&= 1 + \frac{4 \cdot 605}{\delta}
\end{aligned}
$$

Taking typical values for the constants of a vertical aerial wire 10 metres or so in length, $L = 20\,\mu H$, $C = \cdot 00005\,\mu F$, $R = 100$ ohms

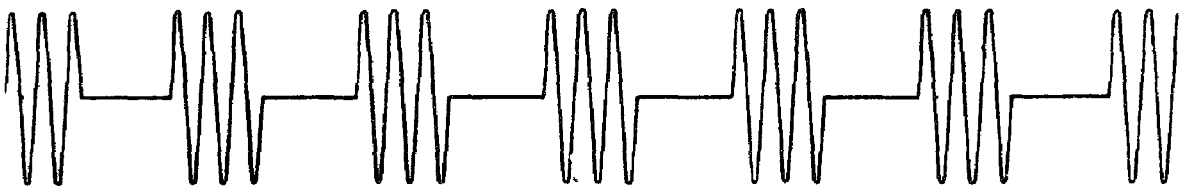
$$
\begin{aligned}
\delta &= \pi R \sqrt{\frac{C}{L}} = 314 \sqrt{\frac{\cdot 00005}{20}} = \cdot 496 \\
n &= 1 + \frac{4 \cdot 605}{\cdot 496} \\
&= 1 + 9 \cdot 3 \\
&= 10 \cdot 3
\end{aligned}
$$

The amplitude of the 11th wave will be less than ·01 of the initial amplitude. The number of waves, multiplied by the period of each wave will give the duration of the wave train. The period is $2\pi\sqrt{LC}$ (approximately) or $1 \cdot 985 \times 10^{-7}$ seconds, and, therefore, the whole wave train will last only about 2×10^{-6} seconds.

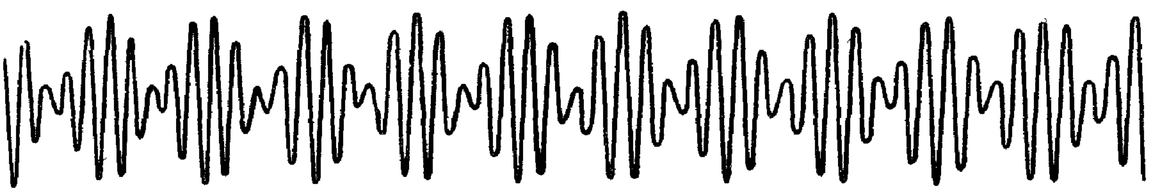
37. (i) If it is desired to signal with the morse code, using aural reception (i.e. by employing the telephone head set) the duration of a " short " must be not less than about $\frac{1}{25}$th second, and a " long " about $\frac{1}{8}$th second. Wave trains of such brief duration as 2×10^{-6} second can only be utilised if arrangements are made for the charging and discharging processes to take place with extreme rapidity, so as to produce, say 2,000 complete wave trains per second. Another reason why damped wave trains are unsuitable for communication purposes is the difficulty of radiating a large quantity of energy. The energy stored in the capacitance of the vertical aerial is $\frac{1}{2}C\mathcal{V}^2$ joules, and if arrangements were made to charge and discharge the aerial n times per second the total power expended would be $\frac{1}{2}Cn\mathcal{V}^2$ joules per second or watts.



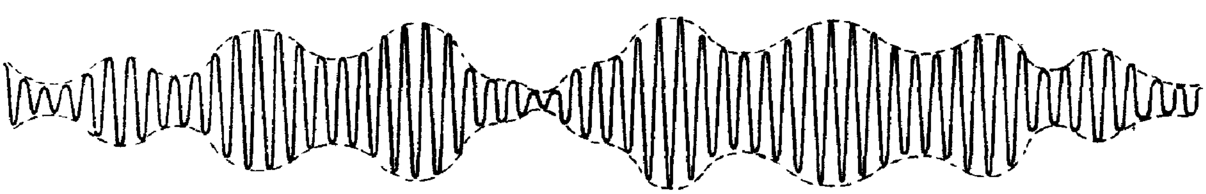
Type A1 wave



I.C.W. wave



M.C.W. or T.T.
Type A2 waves



Modulated wave

Type A3. modulation frequency within audible limit

Type A4. modulation frequency above audible limit

FIG. 13

CHAP. VII

TYPES OF RADIO WAVES

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Example 3.

An aerial of capacitance $\cdot 00005\,\mu F$ is charged to a P.D. of 2,000 volts and then allowed to discharge, the process being repeated 2,000 times per second. What is the power supplied to the oscillator ?

$$\begin{aligned} P &= \tfrac{1}{2}\, C\, n\, \mathcal{V}^2 \\ &= \tfrac{1}{2} \times 5 \times 10^{-11} \times 2{,}000 \times 2{,}000^2 \\ &= \tfrac{1}{2} \times 5 \times 10^{-11} \times 2 \times 2 \times 2 \times 10^9 \\ &= 20 \times 10^{-2} \\ &= \cdot 2 \text{ watt.} \end{aligned}$$

(ii) Of the power supplied, less than 50 per cent. may be converted into radiation (*see* paragraph 51). The amount of energy stored in the aerial can only be increased by increasing its capacitance, raising the voltage, or both. The capacitance must depend to some extent upon the frequency chosen for communication while the increase of voltage leads to other difficulties. Nevertheless the original " spark " or damped wave telegraphy was achieved in this manner, which however has been rendered quite obsolete by the development of continuous or undamped wave telegraphy, while the technique of radio-telephonic transmission depends essentially upon the production of an undamped oscillation.

38. In order to produce such an oscillation it is necessary to introduce into the oscillatory circuit during some portion of every cycle, an amount of energy equal to that which has been expended during the preceding cycle. If the amplitude of the first peak of oscillatory current is $\mathcal{I}$ amperes and the total losses are represented by an effective resistance of R ohms, the energy expended during the first half-cycle is $\frac{\mathcal{I}^2 R}{2 f_n}$ joules, and if an amount of energy equal to this is supplied to the circuit during the half-cycle immediately following the attainment of the first peak value, the next peak of current, which will be of negative sign, will reach the same amplitude as the first peak. If this addition of energy is performed at some period during every succeeding cycle an undamped oscillation will be produced, while if the energy added is in excess of the amount dissipated in various losses during the cycle, the amplitude of oscillation of each succeeding cycle will increase until some peak value is reached at which the energy supplied per cycle is just sufficient to make good the damping losses. A close analogy to this is the oscillation of a swing, which is maintained (and increased in amplitude if desired) by successive pushes at the moment when the swing has just passed the peak of its oscillation.

Wave form of electromagnetic waves

39. (i) Electromagnetic waves have been divided into several classes according to the waveform of the disturbance, which in turn depends upon the manner in which the oscillatory circuit is supplied with energy. There are two main classes :—

Type A waves, which are of constant frequency and amplitude and are therefore called " continuous waves ".

Type B waves, which are the damped waves emitted by an oscillator of which the capacitance is charged some 100 to 2,000 times per second and then allowed to discharge at its natural frequency, as described in paragraph 37. Type B waves are no longer used for communication purposes in the R.A.F.

Type A waves are only of use for such purposes as radio beacons, which enable an aircraft carrying the necessary apparatus to obtain its bearing from the transmitter. For signalling purposes, it is necessary to interrupt the emission in accordance with the morse code, or to vary its amplitude as the magnitude of the direct current in a simple telephone circuit is varied by the carbon microphone. In the former case, the waves are said to be key controlled, and in the latter to be modulated. The complete division of Type A waves is therefore as follows :—

*Type A*1 *waves*.—Continuous waves, unmodulated, key controlled. These are continuous waves of which the amplitude or frequency (or both) are varied by the operation of keying, for the purpose of telegraphic communication.

Type A2 waves.—Interrupted continuous waves (I.C.W.). Continuous waves in which a variation of amplitude is made in a periodic manner at audio frequency, and key controlled for the purpose of telegraphic communication (W/T). When the variation of amplitude is approximately sinusoidal, the emission is referred to as " Tonic train " (T.T.).

Type A3 waves.—Sound modulated waves, continuous waves in which a variation of amplitude or frequency is made in accordance with the characteristic vibrations of speech and music (R/T).

Type A4 waves.—Modulated waves. Continuous waves in which a variation of amplitude or frequency is made; the modulation frequencies may be very much higher than those required for the transmission of speech and music. This form of wave is utilised for television transmission.

(ii) Fig. 13 shows the various forms of electro-magnetic wave diagrammatically. In the case of Type A1 waves a morse " short " and " long " are shown, but it must be realised that many more waves are required to constitute each morse element than are actually drawn. If the " short " lasts $\frac{1}{20}$th second and the frequency of the oscillation is 10^6 cycles per second, the " short " would consist of 50,000 complete waves. In the representation of Type A2 waves, only a single " short " is shown. If the interruption takes place 200 times per second, the short will consist of ten groups of waves, and as a " long " has three times the duration of a " short " the " long " would consist of thirty groups. The representation of the Type A3 wave shows how the amplitude of the wave varies in accordance with the vibrations of speech or music. The dotted line which is touched by the peak of each successive wave must be regarded as merely a " construction line ". It is the wave form of the speech or music which is being transmitted, and is often referred to as the " modulation envelope " of the wave. The significance of this envelope will be dealt with in the chapter devoted to radio-telephony.

PRACTICAL RADIATING CIRCUITS OR AERIALS

40. In modern practice the introduction of energy into the oscillatory aerial circuit is accomplished by means of a thermionic valve, which is so connected as to draw a supply of energy from a source of constant E.M.F. and inject this energy into the oscillatory circuit during every cycle. The oscillatory current in the aerial circuit is thus maintained at a constant amplitude, and the valve, with its supply batteries, may be regarded simply as an alternator, the frequency of which accommodates itself to the natural frequency of the aerial circuit. The phenomena associated with the thermionic valve itself, and the manner in which the valve functions when acting as an energy converter, are discussed in Chapters VIII and IX. In the succeeding paragraphs, the source of energy in the oscillatory circuit will be considered to be an alternator of zero internal impedance, the frequency of which is coincident with the natural frequency of the circuit.

Stationary waves on conductors

41. Suppose, therefore, that a long wire is suspended vertically some distance above the ground as before, but with a source of alternating E.M.F. of any frequency whatever connected in place of the battery. The wire may be considered to have its inductance and capacitance equally distributed throughout its length, and its resistance will be neglected. The application of an alternating E.M.F. then causes an acceleration of the electrons situated in those portions of the wire which are immediately adjacent to the terminals of the alternator, the resulting movement of the electrons will cause repulsion of those in the vicinity, and the latter also receive acceleration. In a loss-free conductor the acceleration of electrons is, in fact, communicated from one to the other in the length of the conductor with a velocity which is found to be 3×10^{10} centimetres per second, precisely that with which electromagnetic disturbances are propagated in free space, although if the effect of the resistance of the wire is taken into consideration it may be somewhat less. The reader may appreciate a reminder that the velocity with which the

acceleration is communicated from electron to electron (3×10^{10} centimetres per second) is not the velocity with which the electrons actually move along the wire, the latter being only of the order of one centimetre per second, and if the two statements appear to conflict, the position may be cleared up by an analogy. When an engine starts to move a long train of wagons from a standstill the velocity with which the acceleration is communicated may be roughly estimated by measuring the time interval between the successive clanking sounds which accompany the acceleration of each wagon. If this is half a second, and each wagon is 22 ft. long, the acceleration is communicated from wagon to wagon with a velocity of $22 \div \frac{1}{2}$ or 44 ft. per second. The whole of the train is not in motion until the last wagon has received acceleration, and it is absurd to suppose that because the acceleration is conveyed with a velocity of 44 ft. per second the train is moving at 30 miles per hour!

42. We may, therefore, conclude that there is nothing incompatible in the statements that the average velocity of the electrons is about one centimetre per second, and that the occurrence of acceleration is communicated from electron to electron with a velocity approaching that of light. The fact that a variation of E.M.F. is occurring, then, is the cause of a progressive disturbance of electrons in the wire, and by our definition of wave motion we may say that an electric wave exists in the wire. This wave starts at the terminals of the alternator and travels outwards along each wire, but on reaching the ends of the latter, is reflected, and the wave travels back along the wire to the alternator, impressing an acceleration in the reverse direction upon the electrons. The acceleration of some electrons in the wire may, therefore, be the sum of two equal and opposite accelerations, or zero. Since the current is proportional to the velocity of the electrons, a current wave is also set up in the conductor. Confining our attention to the latter and assuming that no reflection occurs at the generator itself, the waves reflected at the ends of the wire travel back towards the opposite ends and a very complex electrical state may exist, but however complex it may be, the current at the ends of the wire remote from the generator is always zero, for no electrons can travel past these points. If the frequency of the supply is such that the length of wire attached to each terminal of the generator is an exact multiple of one-quarter of a wavelength (the wavelength being related to the frequency by the relation already given, viz., $\lambda = \frac{c}{f}$) the direct and reflected waves combine in such a manner that no travelling wave exists, but instead, what is called a standing or stationary wave is set up in the wire.

43. Without attempting academic accuracy, we can see that such a wave will be produced by reflection if the original current is assumed to be $\mathcal{I} \sin \omega (t - kx)$, that is to say, a current of the form $\mathcal{I} \sin \omega t$ which varies in phase from point to point in space, the latter being by definition a travelling wave. On reflection without loss of amplitude, the wave becomes $\mathcal{I} \sin \omega (t + kx)$, the positive sign signifying that the reflected wave is travelling in a direction opposite to the first, and the current at any point x is

$$\mathcal{I} \sin (\omega t - k_1 x) + \mathcal{I} \sin (\omega t + k_1 x),$$

the new constant k_1 being simply ω times the constant k previously used. The sum of these sinusoidal quantities, according to a formula developed in Chapter V, is

$$\mathcal{I} (\sin \omega t \cos k_1 x + \cos \omega t \sin k_1 x)$$
$$+ \mathcal{I} (\sin \omega t \cos k_1 x - \cos \omega t \sin k_1 x)$$
$$2 \mathcal{I} \sin \omega t \cos k_1 x,$$
$$\text{or } (2 \mathcal{I} \cos k_1 x) \sin \omega t.$$

This indicates that the amplitude of the current at any point in the wire is proportional to the current at the middle of the circuit and also varies with the distance from the mid-point. There is, however, no part of the expression which signifies a change of phase from point to point in the wire, and we conclude that the direct and reflected wave have combined to form the stationary wave. In the kind of circuit under discussion, that is one which has inductance and capacitance distributed along the length of the circuit in a more or less uniform manner, resonance is said to exist when the applied E.M.F. causes stationary waves to be set up in the wire. The wavelength corresponding to any resonant frequency depends upon the length of the wire, the lowest resonant frequency being that which makes the aerial a half-wave aerial, having a current maximum—which is termed a current loop—at the centre, while the current at either end is zero. Points at which the current is zero are termed current nodes, and so, in a half-wave aerial, we have a current loop at the centre and current nodes at either end.

44. From another point of view, the variation of current in different parts of the aerial may be attributed to the existence of distributed inductance and capacitance along the length of the conductor, as shown diagrammatically in fig. 14a. It must be appreciated that instead of the finite number of elements of inductance and capacitance which have been drawn, an infinite number really exist. Suppose the current flowing through the generator to be i amperes, and its direction upwards, at a given instant. A portion of this current, i, will leave the conductor at the point P in the form of a displacement current through the element of capacitance C_1, and

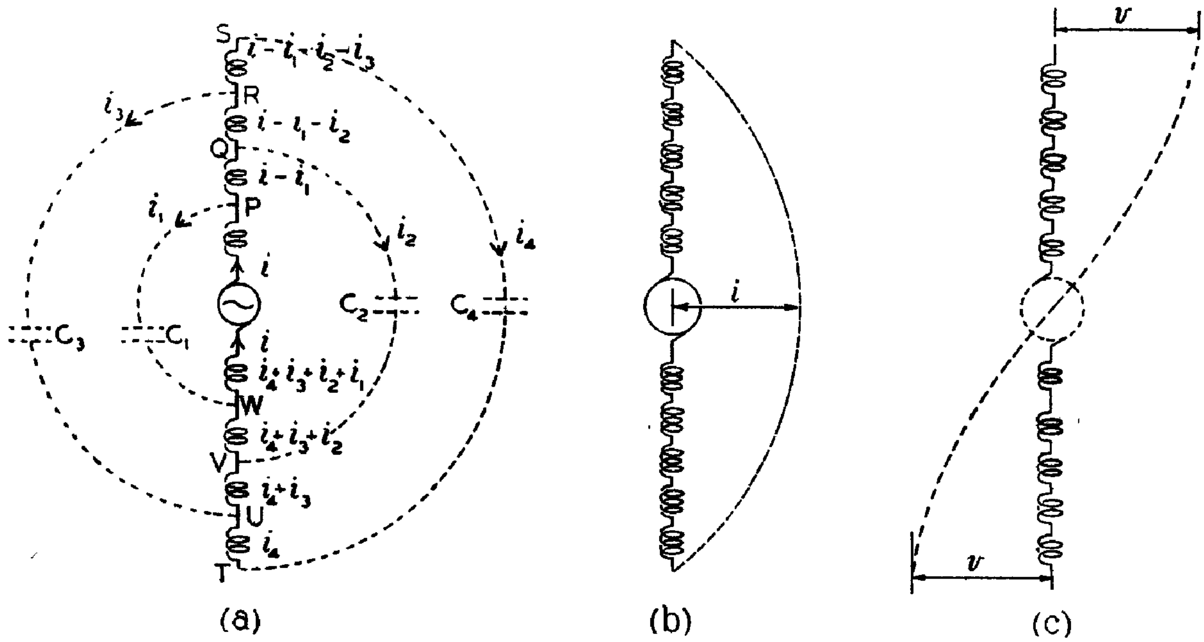

FIG. 14, CHAP. VII.—Current and voltage distribution of dipole.

the current flowing in the next element of inductance will be $i - i_1$. On reaching the point Q, a further displacement current i_2 will flow through the element of capacitance C_2, and the current in the wire itself will be $i - i_1 - i_2$. This reduction of conduction current continues at all points in the wire, and the current at the end of the wire is zero as already stated. Now consider the length of wire attached to the other terminal of the generator. At the instant under consideration, the current in this wire is also upward and is zero at the extreme end, but at the point T the displacement current i_4 enters the wire and flows toward the generator, being joined at U by i_3, at V by i_2 and so on. At the lower generator terminal therefore the current flowing upward is $i_1 + i_2 + i_3 + i_4 = i$ which is equal to that leaving by the upper terminal, and the amplitude of current in the aerial varies from a maximum at the centre to zero at the ends, fig. 14b. The P.D. with regard to the midpoint also varies from point to point in the wire, increasing as the distance from the midpoint increases, and being therefore greatest at the ends of the conductor. Thus all points which are loops of current are nodes of potential, while the positions of current nodes coincide with those of loops of potential. If then the conductor acts as a half-wave aerial, it possesses a potential node at its centre and potential loops at the ends (fig. 14c). Diagrams showing the amplitude of current and P.D. at all points in the aerial are termed current and voltage distribution diagrams respectively.

Quarter-wave aerial—the counterpoise

45. The form of aerial hitherto considered, which is energised at its midpoint, is frequently found to be impracticable, owing to its excessive length when low-frequency radiation is desired. By a simple expedient the length of aerial required to radiate a given frequency can be reduced by one half. This expedient is the replacement of one half of the dipole by a conductor of as large an area as possible, and the capacitance of the aerial is then that existing between this area of metal and the remaining half of the wire (fig. 15a). This is a form eminently suitable for use in

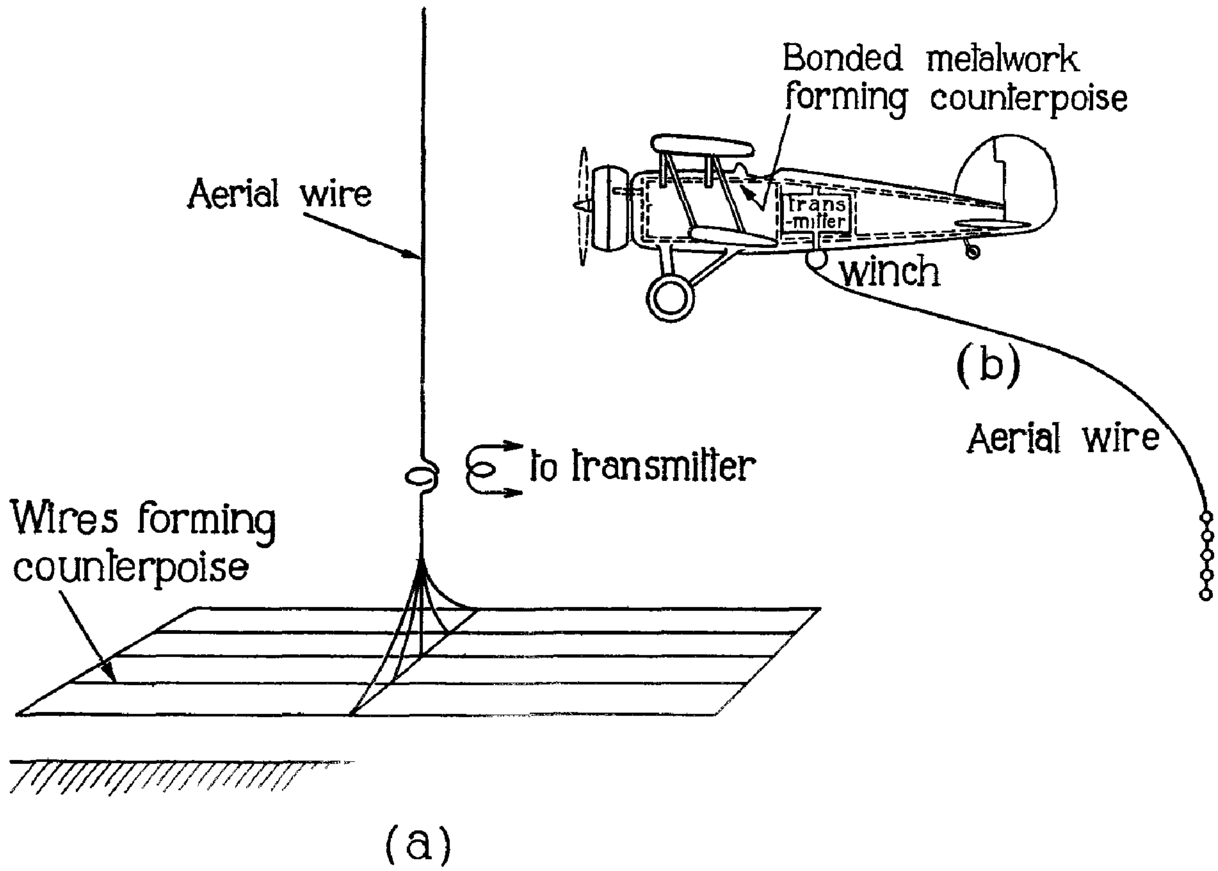

FIG. 15, CHAP. VII.—Counterpoise in ground station and aeroplane.

aircraft, for the actual aerial wire can be suspended from the aircraft and the " opposite plate " of the aerial capacitance is constituted by the whole of the metal work of the aircraft, which is held in electrical continuity by the use of bonding strips (fig. 15b). When operating at its natural frequency, one quarter of a stationary wave is set up in the aerial, a potential loop and current node existing at the lower end and a potential node and current loop at the point where it is connected to the bonding. The radio transmitter itself is connected between the latter point and the aerial wire. When used in the manner described, any conducting area is termed a counterpoise.

Earthed vertical aerial

46. When the radio transmitter is intended for use at a fixed location on the ground, an apparently obvious development is to make use of the earth itself as a counterpoise, and it now

seems hardly credible that ten years elapsed between the original production of electromagnetic waves by means of a dipole, which was achieved by Hertz in 1884, and the introduction of the earthed vertical aerial which was due to Marconi. The latter is the simplest form of ground station aerial, and the whole aerial system then consists of a vertical wire one quarter of a wavelength in height, earthed at its lower and insulated at its upper ends. The high frequency power supply device—which is usually a triode oscillator (Chapter IX)—is situated at the foot of the aerial and supplies energy to the latter by means of some form of inductive or capacitive coupling (fig. 16a). The distribution of current and voltage in such an aerial are shown in fig. 16b. If the earth is assumed to be perfectly conductive the distribution of the electric field around the aerial is exactly the same as round the upper half of a dipole, and the earthed vertical wire may be considered to form one half of a dipole, the other half of which is buried in the earth. The electric fields now terminate upon the surface of a theoretically perfect conductor instead of forming closed loops in free space, and consequently high frequency currents circulate in the earth between regions of positive and negative electrical sign. As in reality the earth is by no means a perfect conductor, the distribution of the field is not exactly as has been described, but a more important effect of its finite conductivity is that the earth currents give rise to a loss

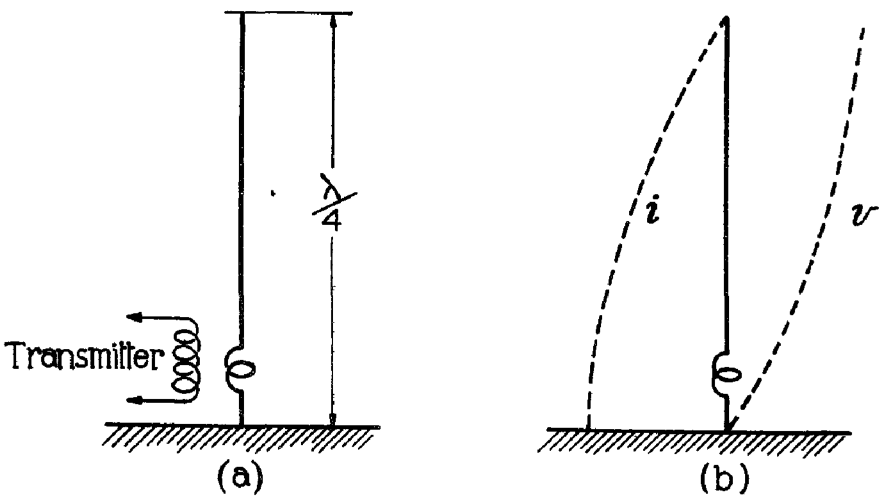

FIG. 16, CHAP. VII.—Current and voltage distribution, quarter-wave aerial.

of energy, the rate at which energy is expended being proportional to the square of the aerial current and to the specific resistance of the earth in the immediate vicinity of the station. A large counterpoise of low resistance, some six feet above the ground, will reduce these losses considerably and such a counterpoise when used in preference to an actual earth connection is called an earth screen, because its function is to prevent the flow of radio-frequency currents in the earth underneath the aerial. It should preferably extend a distance at least equal to one half the height of the aerial in all directions, but need not be a complete metallic area, a system of radial wires spaced 15 or 30 degrees apart being equally efficient. Even if the ideal earth screen cannot be adopted, a few wires arranged in this way will often reduce the energy wastage considerably.

L and T aerials

47. In order to increase the capacitance between aerial wire and earth or counterpoise, a conducting area is frequently added to the upper extremity. The advantages of this addition are twofold. First, the increase of capacitance allows a greater storage of energy for a given voltage and therefore a larger charging current in the vertical portion, and second, the current distribution in the vertical portion is more nearly uniform throughout its length. The significance of the latter will be appreciated when the " effective height " of the aerial has been defined. The portion

added usually consists of horizontal members, giving rise to the forms known as L or T-shaped aerials, which are shown in figs. 17 and 18. Where only a single mast is available, the so-called " umbrella " type may be employed, although its use has been largely discontinued owing to its rather poor radiating properties, while if more than two masts are permissible, a triangular or rectangular network may be employed. Radiation takes place not only from the vertical portion (usually called the " feeder ") but also from the upper capacitance area (or " roof ") but the radiation from the latter is not usually coincident in direction and polarisation with that from the former. The current and voltage distribution of L and T aerials is similar to that of a vertical aerial of the same overall length, measured from the base of the aerial to the extremity. The T aerial may be considered to consist of two L aerials placed " back to back ", and the current and voltage distribution of both types are shown in fig. 19. The distribution in other forms of aerial having added capacitance areas is similar to these, but do not lend themselves to diagrammatic representation because the aerial occupies three spatial dimensions.

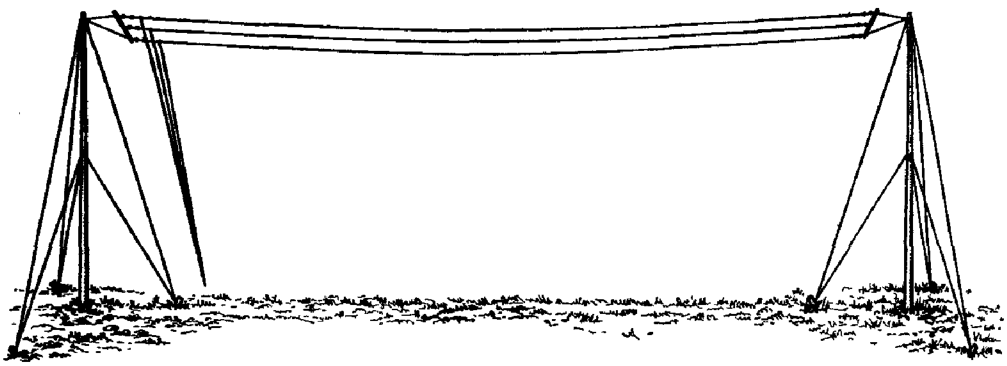

FIG. 17, CHAP. VII.—L-aerial.

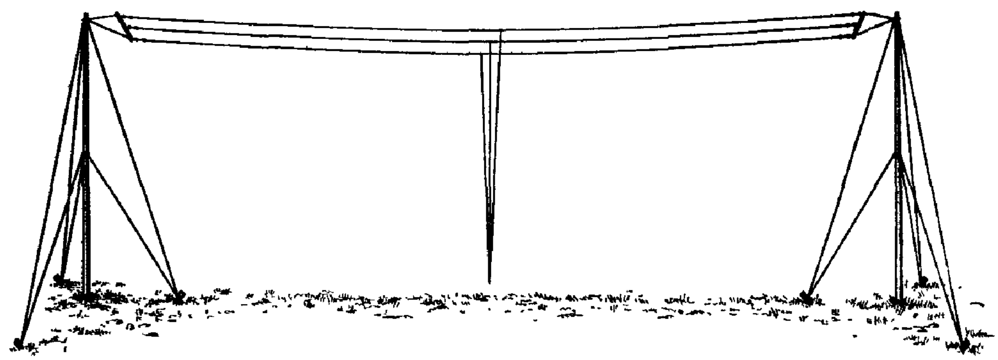

FIG. 18, CHAP. VII.—T-aerial.

Aerial tuning

48. Under certain conditions it is desirable to employ a single aerial to transmit any one of a large number of frequencies, e.g. the frequency found to be most effective for communication between aircraft and a ground station may not be equally effective for intercommunication between aircraft or between ground stations. This introduces the notion of artificially lengthening or shortening the aerial wire. Artificial lengthening, in order to decrease the resonant frequency, can be achieved by adding wire in the form of a coil, i.e. an inductance, at the most convenient point, which is where the transmitter is located, while if a condenser is similarly inserted the resonant frequency will be increased. The resonant frequency of the aerial circuit can be decreased to any extent by the introduction of a sufficiently large inductance, but it is important

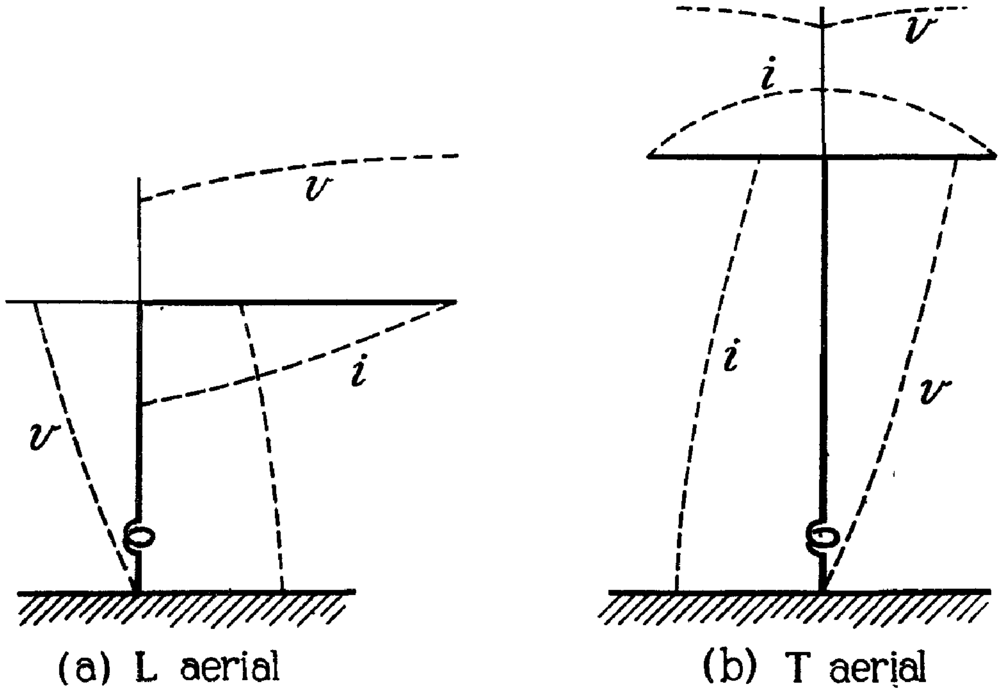

FIG. 19, CHAP. VII.—Current and voltage distribution, L and T aerials.

to remember that even the interposition of an infinitely small condenser, i.e. of zero capacitance, will tune the aerial to a frequency only twice the natural frequency, for a capacitance of zero value is equivalent to a complete break in the circuit and the aerial then becomes an isolated wire, i.e. a dipole or half-wave aerial. The current and voltage distribution is modified by the

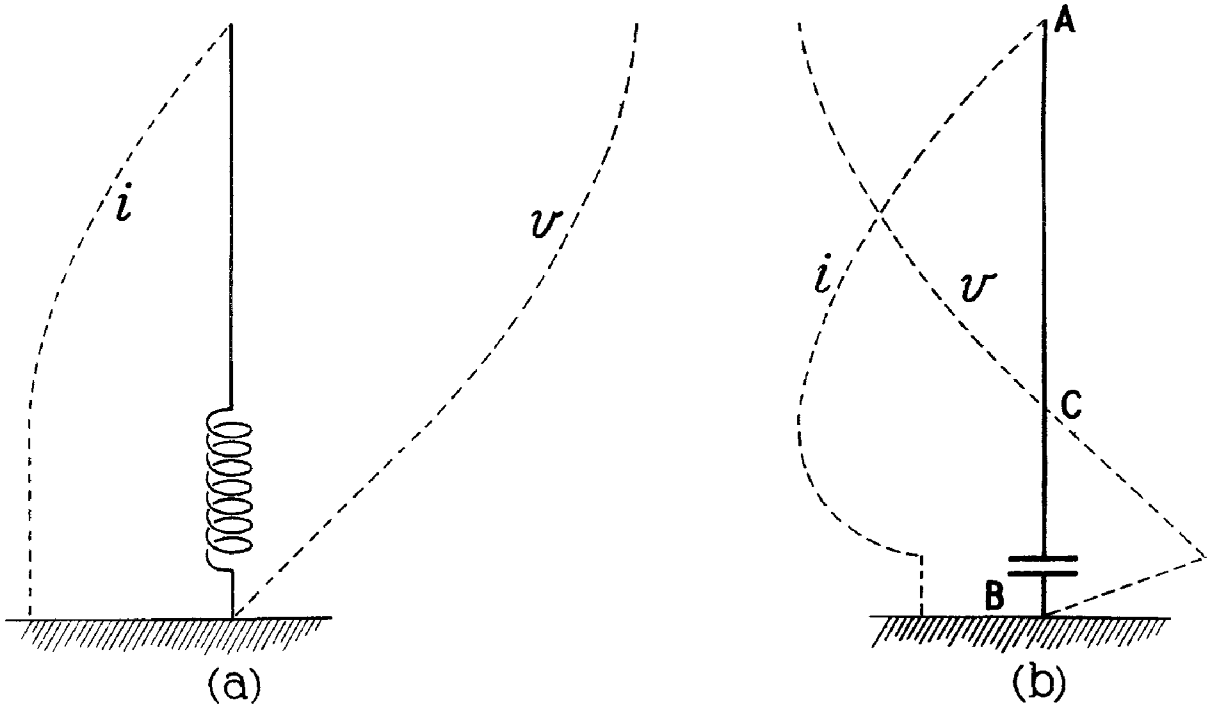

FIG 20, CHAP. VII.—Current and voltage distribution of vertical wire with added reactance.

insertion of these tuning devices. Consider the most usual case, which is the addition of inductance. The coil should have very small capacitance with respect to the earth or counterpoise, and little displacement current should occur in its vicinity, hence the current at both ends should be nearly the same. A considerable increase of voltage will take place between the same points, hence the current and voltage distribution will be somewhat as shown in fig. 20a. Let us now consider the effect of a series condenser. In fig. 20b, AB represents the aerial, and without added reactance it would be a quarter-wave aerial having the current distribution shown in fig. 16b. The insertion of the condenser reduces its effective length to AC (say), and therefore a current loop must exist at the point C, while below this point the current decreases. The current distribution is therefore that shown by the curve and the voltage distribution can be deduced from the current distribution, since a voltage loop must exist at the upper extremity and voltage nodes at the point C and at the earth connection. The current distribution diagram illustrates that when a series condenser is in use, the reading of a thermoammeter in the wire immediately adjacent to the earth connection may be a very unreliable indication of the radiating properties of the aerial.

Distribution of radiation in space

49. (i) Returning to a consideration of the vertical dipole situated far above the earth's surface, we may define the equatorial plane as an imaginary plane passing through the midpoint of the dipole and perpendicular to it. The point at which the dipole passes through this imaginary plane will be called the pole. At all points on this plane which are equidistant from the pole the radiation field will be of equal strength. This is not true for all forms of aerial, and it is usual to indicate the directive radiating properties of any particular type of aerial by what are called polar diagrams. The horizontal polar diagram shows the field strength in various directions in the equatorial plane, and theoretically is constructed as follows. From the pole are drawn straight lines on the equatorial plane, radiating in all directions, the length of each line being proportional to the field strength (or sometimes which is proportional to the square of the field strength) in that particular direction. The ends of these lines form the horizontal polar diagram. As the dipole radiates equally well in all directions in the equatorial plane, which is apparent from consideration of its symmetrical arrangement in space, the polar diagram will be a circle having the pole as its centre.

(ii) The horizontal polar diagram of a vertical earthed aerial, the height of which is one quarter of a wavelength or less, is also a circle having the aerial as its centre, the equatorial plane in this instance being practically coincident with the surface of the earth. In practice, the polar diagram of an aerial situated near the earth's surface is obtained by a method which is essentially as follows. Upon a suitable map is described a circle of given radius, say 100 miles, with the location of the aerial as its centre. A special form of radio receiver is carried from point to point round the circumference of this circle, and the strength of the radiation from the transmitter is measured at each point by means of this receiver. These measured " signal strengths " are then drawn to a convenient scale along the radii connecting the pole with the point at which each measurement was made, measuring outward from the pole, and the line joining the ends of these radii forms the polar diagram. It is obvious that the actual measurement takes considerable time, and the conditions at the transmitter must be kept constant during the whole of the time during which measurements are actually being undertaken. Fig. 21 shows a number of horizontal polar diagrams of a B.B.C. transmitter situated in London, each diagram being marked with the electric field strength, in millivolts per metre, the whole forming what is called a field strength contour diagram. The aerial in this instance was of the " T " type and was erected upon steel masts. The line joining these masts, if produced, was found to coincide, approximately, with the directions in which the field strength was least, i.e. roughly in the N.E. and S.W. directions. The marked reduction in field strength in these directions may therefore be attributed to losses in the steel masts. When, however, the masts are so designed that such losses are unimportant, it is sometimes found that an L or T aerial possesses directional properties, but these may not be very apparent unless the length of the horizontal portion is several times the vertical height, the field strength being greatest in the direction of the flat top. As a rough guide it may be assumed that no appreciable directional effect will be attained unless the ratio $\frac{\text{length}}{\text{height}}$ exceeds ten, and therefore when erecting an aerial of this kind for a given service there is nothing to be gained by siting the masts in any particular direction, unless the above ratio is fulfilled. Special aerial designs for directional transmission will be discussed in a later chapter.

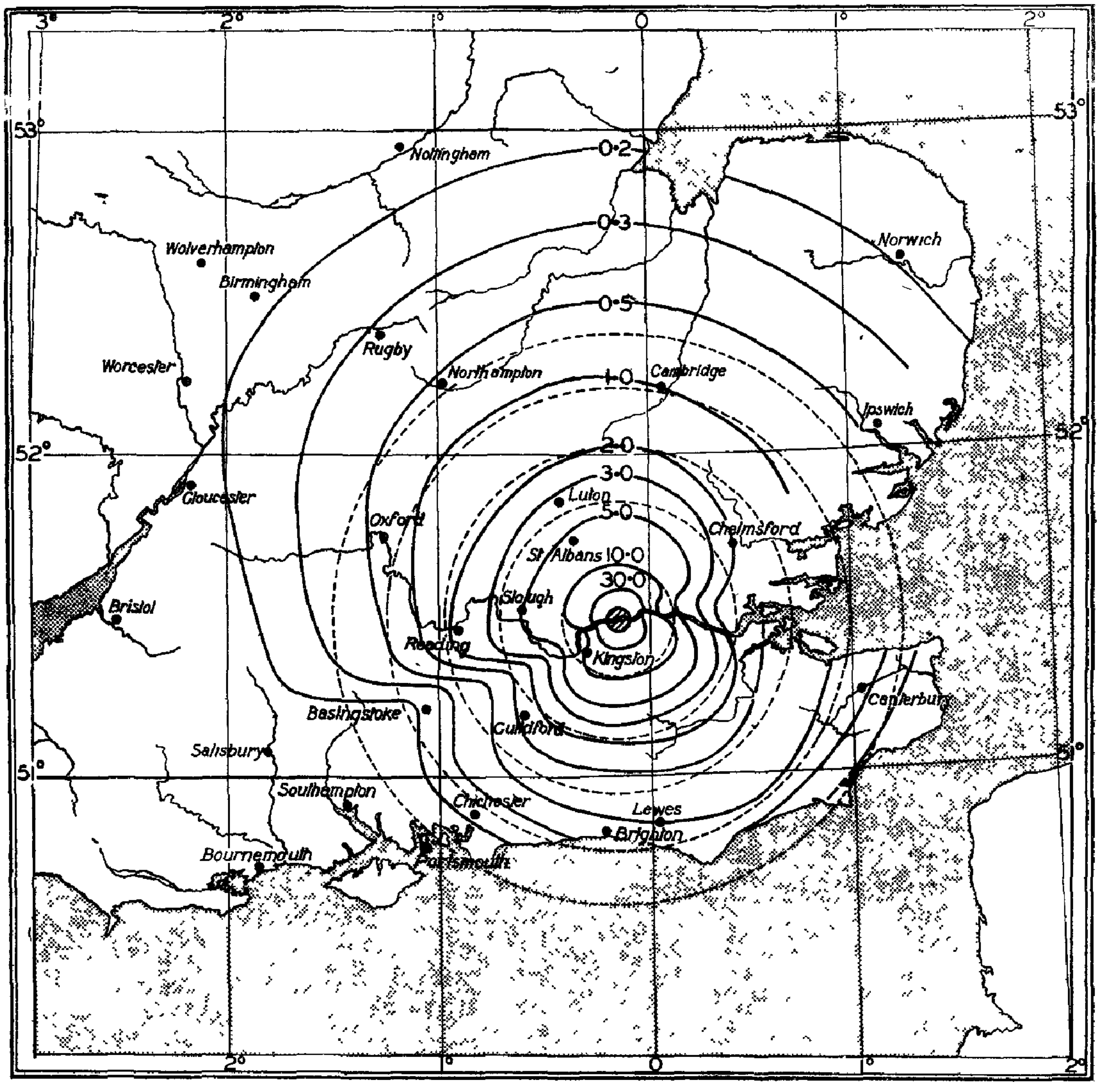

FIG. 21, CHAP. VII.—Field strength contour diagram.

Vertical polar diagram

50. (i) This diagram shows the distribution of field strength in any vertical plane through the aerial. Commencing as before with the dipole in free space the radiation fields are of maximum intensity in a direction perpendicular to the dipole, and of zero intensity along its axis. The vertical polar diagram consists of two circles, and is given in fig. 22a, while a quarter-wave aerial, situated upon a perfectly conducting earth, would have radiating properties identical with the upper half of this (fig. 22b).

(ii) When an aerial is situated near to the ground, its directional properties in the vertical plane are affected by the proximity of the earth, because the energy radiated towards the earth is reflected, and the reflected wave will either reinforce or detract from the energy received at points above ground level. As the earth is not a perfect conductor, some absorption of energy also occurs but no definite allowance can be made for this since the absorption depends upon local conditions. The vertical polar diagrams of a vertical half-wave aerial, situated at various distances above a perfectly conductive earth are shown in fig. 23 by the full lines, and the effect of finite conductivity of the surrounding soil is somewhat as shown by the dotted outline.

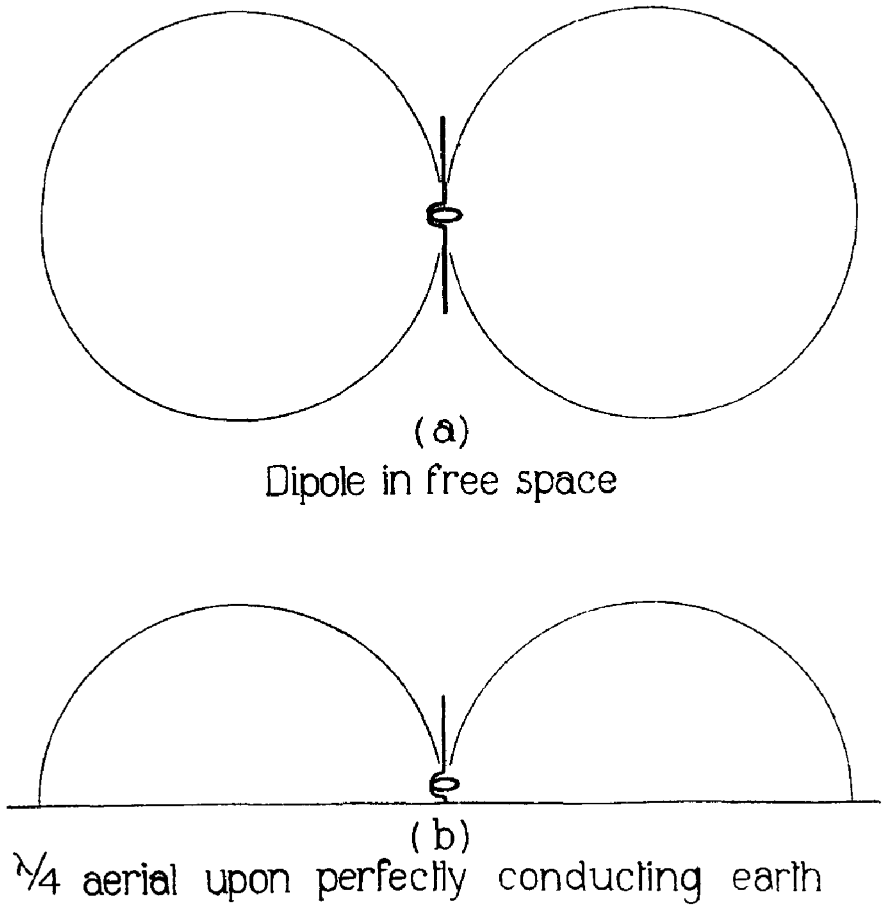

Fig. 22, Chap. VII.—Vertical polar diagrams.

Radiation resistance and aerial efficiency

51. Of the total energy supplied to a transmitting aerial a portion is transferred to the ether and is radiated into space, while the remainder is dissipated in the form of heat either in the aerial itself or in its immediate surroundings. The rate at which energy is supplied to the aerial, or input power P, must be capable of expression in the form I^2R, because the latter is a perfectly general form, but the terms current and resistance require special definition when used in connection with aerials. The current distribution is never uniform, and it is therefore usual to refer to the value of the current at the current loop nearest to the feeding point as the aerial current. The aerial resistance is then defined as that quantity which, when multiplied

by the square of the aerial current, gives the power supplied to the aerial circuit. Alternatively it may be stated that the aerial resistance R_A is defined by the equation

$$I_A^2 \quad R_A = \text{input power}$$

where I_A is the R.M.S. value of the current at the current loop nearest the feeding point. The value of resistance thus defined is called the total effective resistance of the aerial, and may be divided into two components corresponding to (i) the power dissipated in the form of heat in the aerial and its surroundings, which may be written $I_A^2\ R_h$ watts, R_h being the loss resistance of the aerial, and (ii) the power converted into radiation which may be written $I_A^2 R_r$, R_r being termed the radiation resistance of the aerial. This radiation resistance must be regarded as purely fictitious, and may be defined as the imaginary ohmic resistance which if supplied with a certain current would convert energy into heat at the rate at which energy is converted into radiation by the aerial when supplied with an equal current. The total effective resistance of the aerial is equal to the sum of the radiation resistance R_r and the loss resistance R_h, or $R_A = R_h + R_r$.

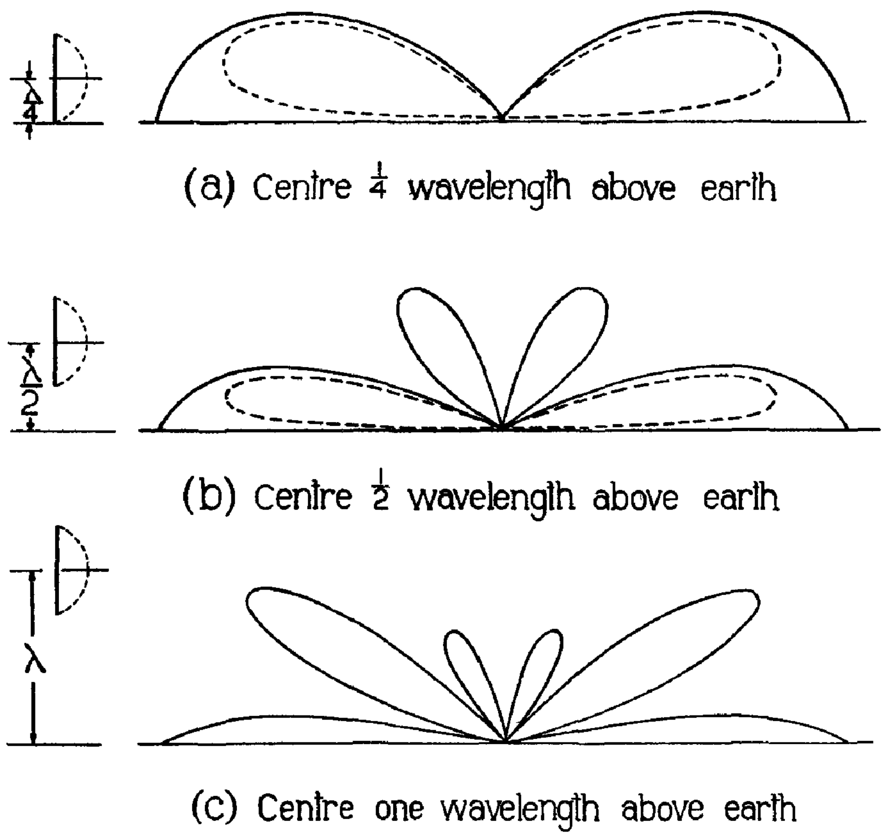

FIG. 23, CHAP. VII.—Vertical polar diagrams of half-wave aerial showing effect of elevation of aerial.

The efficiency of an aerial as a radiator can be expressed in the same manner as for all other electrical machinery, namely, efficiency $= \dfrac{\text{Power converted into useful work}}{\text{Total power supplied}}$ hence, if efficiency is denoted as usual by η

$$\eta = \frac{I_A^2\ R_r}{I_A^2\ (R_h + R_r)}$$

$$= \frac{R_r}{R_h + R_r}.$$

52. (i) The radiation resistance of an aerial depends upon its shape and its dimensions relative to the wavelength of the radiation emitted, while its surroundings affect both the radiation resistance and the loss resistance. It is possible to derive a theoretical expression for the radiation resistance on the assumption that the capacitance of the aerial is concentrated between the extreme ends of the wire, which is tantamount to the assumption that the current distribution is uniform over the whole length of the aerial. The imaginary radiator having this current distribution is shown in fig. 24. It is called a hertzian doublet, and must not be confused with the dipole or half-wave aerial in which it will be remembered a current loop exists at the middle point while a current node is found at each end. The radiation resistance of such a doublet in free space is given by the expression

$$R_r = \frac{80\,\pi^2 l^2}{\lambda^2}$$

where l is the length of the doublet.

For example, if the length of the doublet were $\frac{\lambda}{2}$,

$$R_r = \frac{80\,\pi^2}{\lambda^2} \times \frac{\lambda^2}{4} = 197 \text{ ohms.}$$

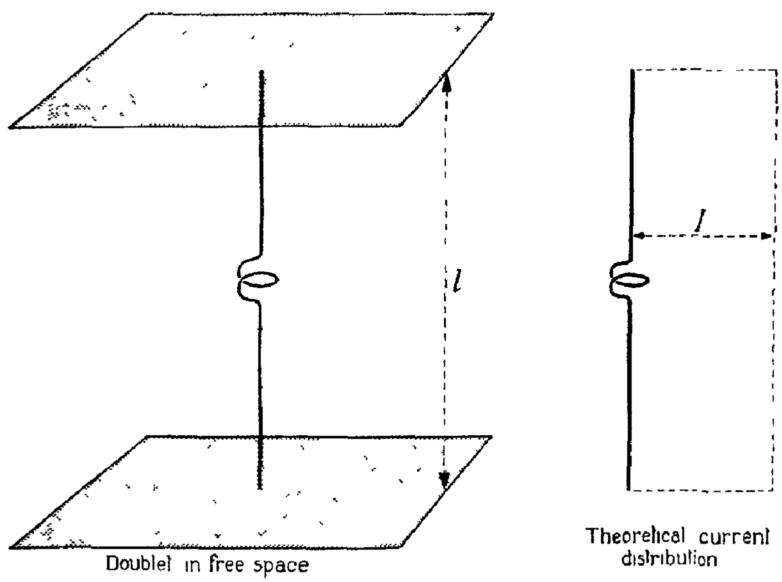

FIG. 24, CHAP. VII.—Hertzian doublet showing assumed current distribution.

(ii) The dipole differs from the doublet in that the current distribution is co-sinusoidal, being a maximum at the mid-point and varying in such a way that if I is the current at the middle of the dipole and i the current at a point x centimetres from the latter point, $i = I \cos \frac{2\pi}{\lambda} x$. At the end of the wire $x = \frac{\lambda}{48}$ and $i = I \cos \frac{\pi}{2} = 0$. The average value of the current over the whole length of wire is $\frac{2}{\pi} I$ and the dipole in free space will have a radiation resistance equal to that of a doublet of the same length but in which the current is only $\frac{2}{\pi} I$, or alternatively a doublet in which the current is I amperes throughout its length, but with the latter dimension reduced by a factor $\frac{2}{\pi}$. Hence the length of the doublet equivalent to a half-wave aerial is $\frac{\lambda}{\pi}$, and the radiation resistance is

$$R_r = \frac{80\,\pi^2}{\lambda^2} \times \left(\frac{\lambda}{\pi}\right)^2 = 80 \text{ ohms.}$$

This is the theoretical radiation resistance of the dipole in free space. Similarly the radiation resistance of a quarter-wave aerial earthed at its lower end is one-half that of the dipole in free space, i.e. 40 ohms. More accurate calculations show that the radiation resistance is actually rather less than the value derived from these formulae, which is partly due to the fact that the fundamental wavelength of a so-called quarter-wave aerial is not exactly four times its length, but is about 4·18 times. This " end correction " may be regarded in this way. Although it has been stated that no conduction current can flow past the end of the aerial wire, the variations

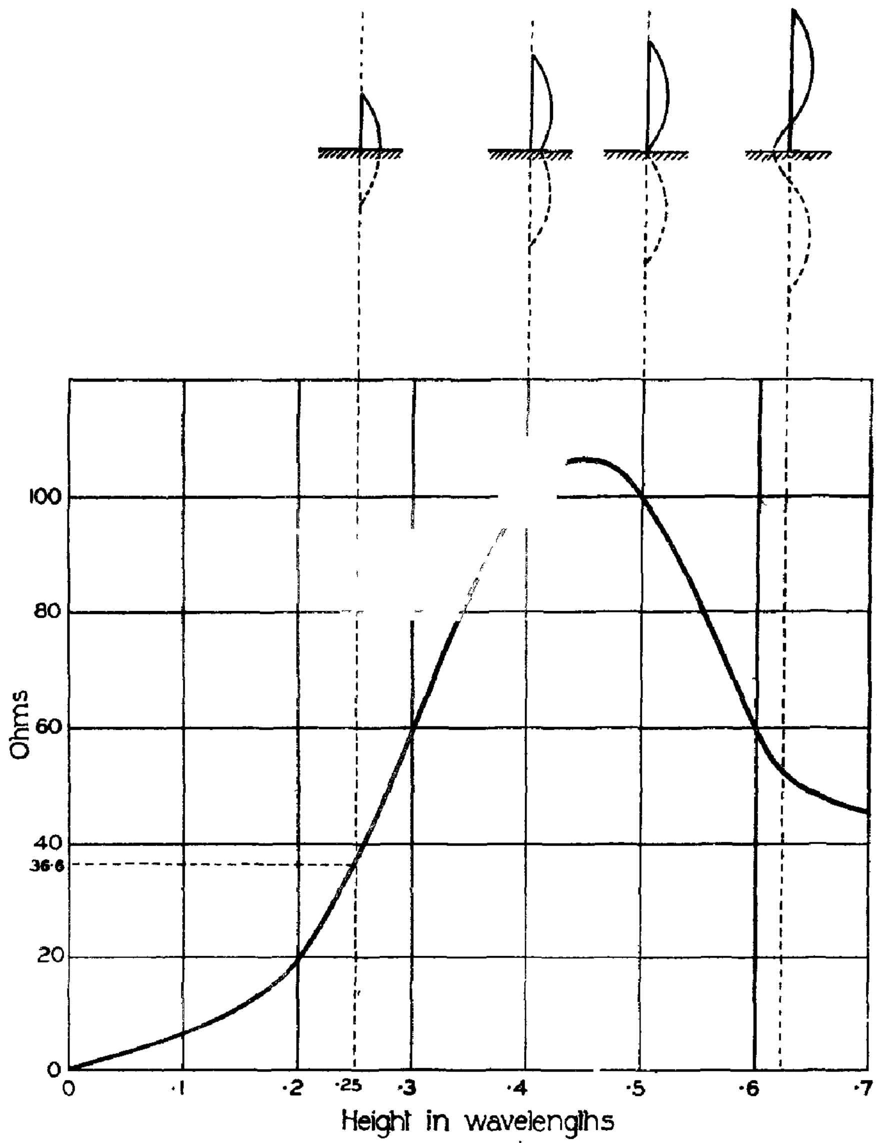

FIG. 25, CHAP. VII.—Radiation resistance of vertical aerials of height less than ·7λ.

of its potential cause displacement current to flow at this point, and some lines of electric strain must exist which are, geometrically, merely continuations of the lines of moving electrons constituting the conduction current. Hence the aerial wire may be regarded as being continued for a short distance into the dielectric. Applying a correction for this end effect the above resistances become for a dipole in free space $80 \times \left(\frac{4}{4 \cdot 18}\right)^2 = 73 \cdot 25$ ohms and for the earthed quarter-wave aerial $36 \cdot 6$ ohms. The radiation resistance of a vertical half-wave aerial, situated near the earth, is approximately equal to one half that of a doublet in free space, namely 98 ohms approximately.

(iii) The radiation resistance of vertical aerials having a height up to $0 \cdot 7\ \lambda$, the lower end being at or near ground level, is shown graphically in fig. 25, in which, for heights below $\cdot 25\lambda$, the power radiated is the product of the given radiation resistance and the square of the current at the base of the aerial, while for aerials having a height greater than $\cdot 25\lambda$, the power radiated is the product of the given radiation resistance and the square of the current at the current loop. The radiation resistance is said to be calculated with reference to base current or to loop current respectively.

53. The radiation resistance of L or T aerials less than a quarter of a wavelength long (i e. those used for the medium and lower radio frequencies), is usually computed by employing a formula derived from the conception of the doublet, but introducing the idea of " effective height ". For any given ratio of length of roof to height of an L aerial, a corresponding " form factor " is employed. The effective height h_e is then defined as the true height h_o of the aerial above the ground, multiplied by the form factor F corresponding with the ratio of length to height for that particular aerial. The effective height is then inserted in the formula

$$R_r = \frac{160\ \pi^2\ h_e{}^2}{\lambda^2}$$

which is derived from the doublet theory, the form factor being intended to correct for the distribution of current along the aerial. The value $\frac{2}{\pi}$ already used for this purpose is in fact the form factor for an earthed vertical aerial, for which the ratio $\frac{\text{length of roof}}{\text{vertical height}} = 0$. The form factor may be calculated from the empirical formula

$$F = \frac{2 + 8\frac{l}{h_o}}{\pi + 8\frac{l}{h_o}}.$$

where l is the length of the horizontal portion. This expression gives results within about 2 per cent. of the theoretical value.

Losses in aerials

54. The loss resistance may be divided into four components, namely:

(i) conductor losses in the aerial system, including the earth or counterpoise,

(ii) eddy current losses in adjacent conductors,

(iii) losses due to imperfect insulation,

(iv) dielectric losses.

Conductor losses are inherent in any electric circuit, but can be maintained at a minimum by attention to those factors which cause an increase of resistance at high frequencies. It is, however, always necessary in practice to compromise between electrical efficiency, mechanical robustness and cost. At fixed ground stations it is possible to reduce the conductor losses to a minimum by the use of high conductivity copper for aerial conductors, by employing several wires in parallel in the aerial, feeder and earth wires, and by utilising conductors of separately insulated stranded wire (litzendraht) for the tuning inductances. A point frequently overlooked is that the current distribution along the length of the aerial is not uniform, and it is therefore preferable that the current-carrying capacity should be greater in the vicinity of the current loop or loops than elsewhere. It is not usually feasible to arrange this except in the case of the lower radio frequencies, in which an earthed L-aerial is operated at a frequency below its fundamental. Under these conditions the number of wires used in the vertical portion or feeder should be at least equal to and preferably greater than the number of wires used in the flat top. This is hardly practicable when a T-aerial is employed owing to the increased sag set up in the " roof " by the additional weight of the feeder, tending to mechanical weakness and reducing the effective height. Another point of importance is the connection point of the feeder to the flat top. Unless it is possible to connect the feeder at the electrical centre of the span, and to lead off at right angles to the latter, the L-aerial will generally be found superior to the T-aerial. Finally, if it is necessary to join wires, such joints must be soldered, using as little heat as possible consistent with efficient soldering, and making the joint as quickly as possible. It should hardly

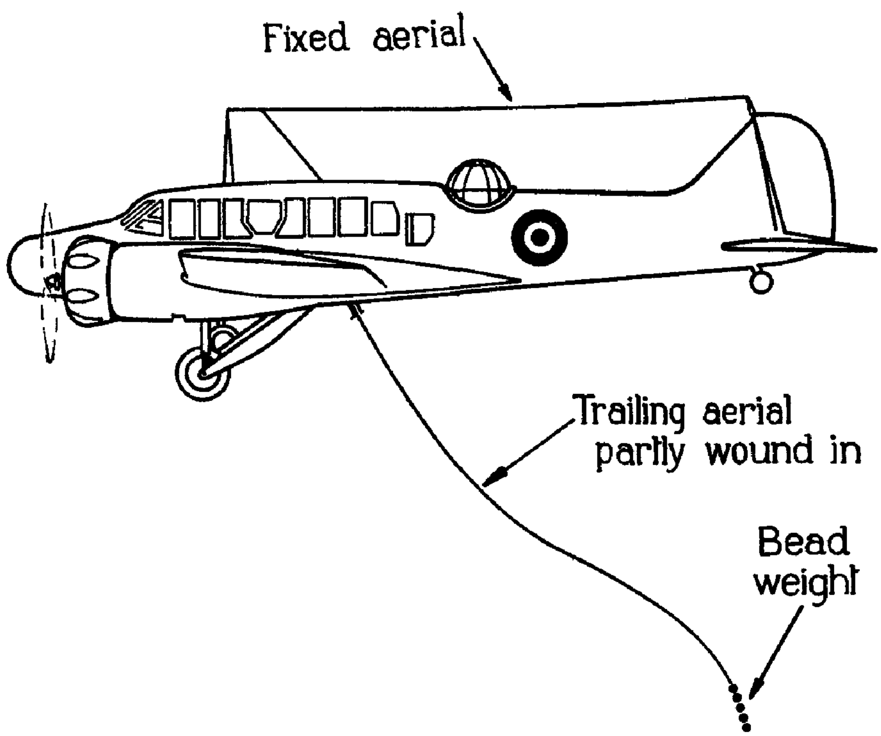

FIG. 26, CHAP. VII.—Arrangement of aerials on aeroplane.

be necessary to point out that where a specification is laid down for the erection of an aerial, it should be followed implicitly. In aircraft, it is often necessary to accept a type of aerial which is electrically far from ideal. Thus where the frequency of transmission renders a long aerial necessary, the only possible form is a trailing wire, fitted with a weight at its extremity. Either phosphor-bronze or stainless steel may be employed in different circumstances, the former combining moderate mechanical strength with fairly high conductivity, and the latter high mechanical strength and durability with rather lower conductivity at radio frequencies. At the higher radio frequencies used by aircraft it is possible to utilise fixed aerials which are erected upon the airframe according to specifications laid down for the particular aircraft. A typical example is shown in fig. 26. In aircraft, where the whole metalwork of the machine is utilised as a counterpoise, the bonding of these parts must be maintained at its highest possible efficiency. Conductor losses in the aerial and earth or counterpoise may be considered to vary as the square root of the frequency.

Eddy current losses in adjacent conductors are caused by induced currents in the latter. In ground stations, masts and stays are the principal causes of loss (cf. paragraph 49 and fig. 21) which is minimised by either ensuring that the masts and stays are well earthed, or alternatively highly insulated from earth. Eddy current losses vary with the square root of the frequency as do other conductor losses.

Losses due to imperfect insulation are reduced to a minimum by the employment of insulators of high efficiency and dielectric strength, such as glass or porcelain. These properties are very much impaired by atmospheric pollution, and the insulation can only be maintained at its highest possible value by frequent cleaning. While this process may be difficult in large ground stations where traffic is almost continuously handled, it is possible in aircraft to perform an inspection of all aerial insulation at frequent intervals, and to maintain this insulation at a high standard. Dielectric losses in the aerial may be assumed to vary inversely as the frequency.

Dielectric losses in adjacent insulating materials such as masonry, wood, trees, etc., are caused by dielectric hysteresis. The less efficient the dielectric, the higher these losses will be. They may be considered to vary inversely as the frequency.

55. The manner in which these components of aerial resistance vary with frequency may be exhibited in graphical form as in fig. 27 in which aerial resistance is plotted against frequency. Curve (i) shows the radiation resistance, and its shape indicates that this quantity varies directly as the square of the frequency. Curve (ii) depicts the nature of variation of the conductor losses, including those caused by eddy currents, and these are proportional to the square root of the frequency. Curve (iii) shows the manner in which the dielectric losses vary ; these are inversely proportional to the frequency as previously stated. The curve (iv) showing the total loss resistance is derived from these, by adding the value of curves (ii) and (iii) for several values of frequency and plotting this sum to give curve (iv). It will be observed that for any given aerial

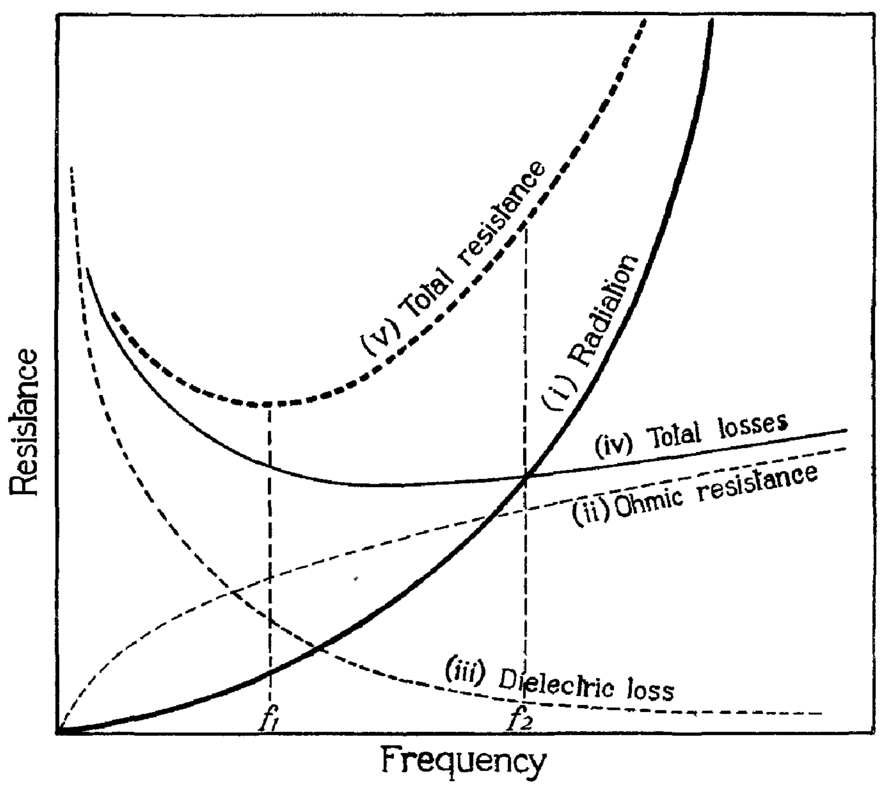

FIG. 27, CHAP. VII.—Components of aerial resistance.

there is always a particular frequency at which the total loss resistance is a minimum. The total effective resistance is given by curve (v) which is obtained by adding curve (i) (radiation resistance) to curve (iv) (total loss resistance). This curve also has a minimum value, showing that at one particular frequency a given aerial current can be obtained with smaller input power than at any other frequency. This is not the most economical frequency of operation however, for the efficiency under these conditions is low. In the diagram, the frequency at which the total effective resistance is least is denoted by f_1, and the radiation resistance is only about one-fifth of the total resistance, giving an aerial efficiency of 20 per cent. At the frequency denoted by f_2, however, the radiation resistance has increased to such an extent that it forms one-half of the total resistance giving an efficiency of 50 per cent.

High frequency aerials

56. It is now proposed to consider the principles governing the design of non-directive aerials for high frequencies, i.e. those lying in the band between 3 and 30 megacycles per second, the corresponding wave-lengths being 100 to 10 metres. It has hitherto been assumed that it is

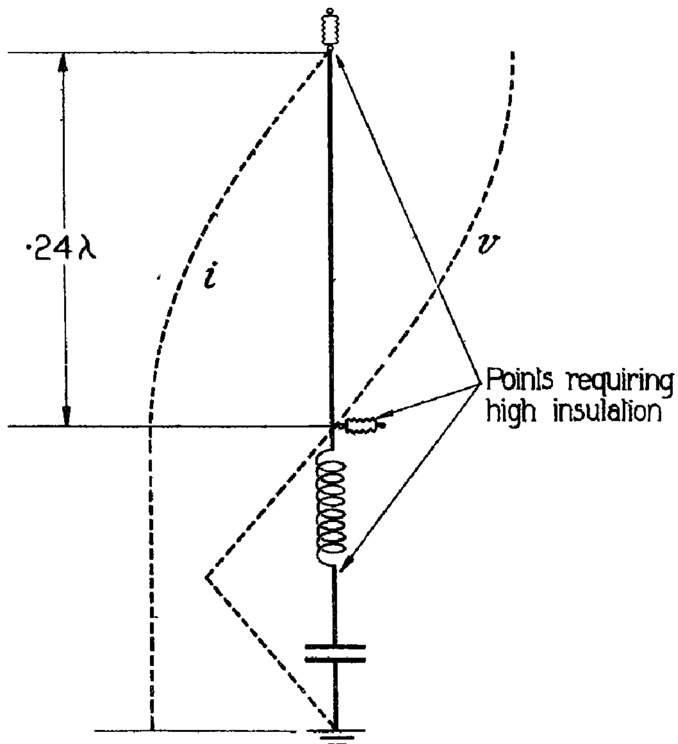

FIG. 28, CHAP. VII.—Quarter-wave aerial terminated by acceptor circuit.

desirable to erect a transmitting aerial in such a manner that radiation is normally polarised at the moment of its inception. Owing to the mechanism by which waves of high frequency are propagated, however, it is possible to utilise either vertical or horizontal radiating elements, at any rate for long distance communication. The height of the aerial, compared with the wave-length, is of great importance. If a horizontal dipole is erected very near to the earth's surface, the effect of reflection by the ground is to cause most of the energy to be radiated upwards, the vertical polar diagram being roughly a circle touching the ground at the point immediately beneath the aerial. To obtain appreciable radiation at an angle of 30° to the ground, it is necessary to elevate a horizontal radiator to a height of at least one half-wavelength above the ground. At the lower frequencies of the band under discussion, it may be necessary for extempore construction to accept a vertical quarter-wave aerial, e.g. for operation on a wavelength of 100 metres, the height of the aerial would be $\cdot 24\lambda = 24$ metres or 73 feet. The total aerial resistance will then be of the order of 100 ohms, and energy may be transferred to the aerial by connecting its lower end to an acceptor circuit tuned to the same frequency, the mode of connection and the current and voltage distribution being given in fig. 28.

High frequency feeder line

57. A half-wave aerial for this frequency would have an actual length of about 146 feet, and it will rarely be practicable to erect a vertical mast of such a height from service resources alone, but within the limitations suggested in the preceding paragraph, a horizontal half-wave aerial on 70-ft. masts is quite a feasible proposition, and again the method of transferring energy to the aerial of the greatest importance. As the aerial is about a quarter wavelength from the ground, where the transmitter must be situated, some form of feeder line must be employed. High frequency feeders may take one of two forms, namely the resonant and non-resonant types. Various forms of construction are possible, but the only kind of line which lends itself to extempore design and erection is that consisting of twin parallel wires, mounted upon poles if necessary, and well insulated from earth. It is generally most important that the two wires shall be absolutely symmetrically disposed with regard to the aerial, the ground beneath the aerial and the transmitter.

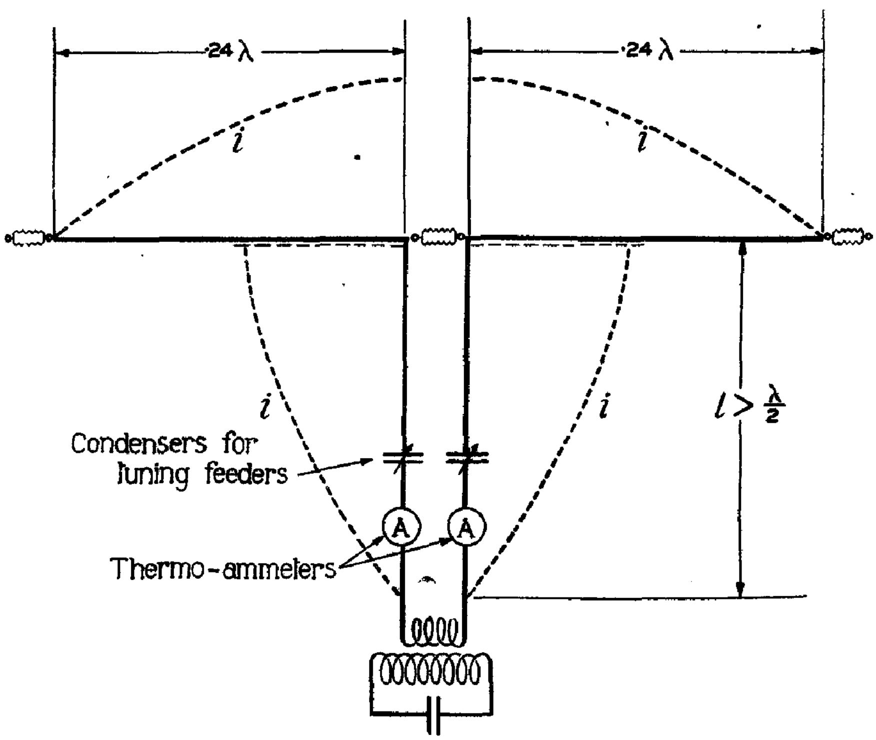

FIG. 29, CHAP. VII.—Current-fed half-wave aerial with tuned feeder.

Feeding by resonant line

58. The resonant line is only suitable for use when the distance between aerial and transmitter is not much greater than $\frac{\lambda}{2}$. This limitation arises because (i) a stationary wave is set up in each feeder and the peak value of oscillatory current rises to a value equal to that in the aerial; the ohmic losses are therefore high. (ii) It is difficult to ensure absolute symmetry

between the two feeders and this gives rise to radiation from the feeder, again leading to excessive power loss, for this radiation will not generally be in exact phase with that from the aerial and therefore will rarely act merely as a reinforcement of the latter. Assuming that the aerial is erected sufficiently near to the transmitter to admit the employment of a resonant feeder, the next point to be decided is whether it is desirable to feed at a current loop or voltage loop. This is merely a matter of practical convenience, and no electrical superiority exists in either method. If current loop feeding is adopted, the half-wave aerial is divided at its centre by a glass or porcelain strain insulator, each half being connected to one feeder. The electrical length of the feeders should be $\frac{\lambda}{2}$, i.e. either their actual length slightly greater than this, their effective length being adjusted by variable condensers connected in series with each line, or alternatively slightly less than $\frac{\lambda}{2}$, their effective length being increased by an adjustable inductance in each line. A

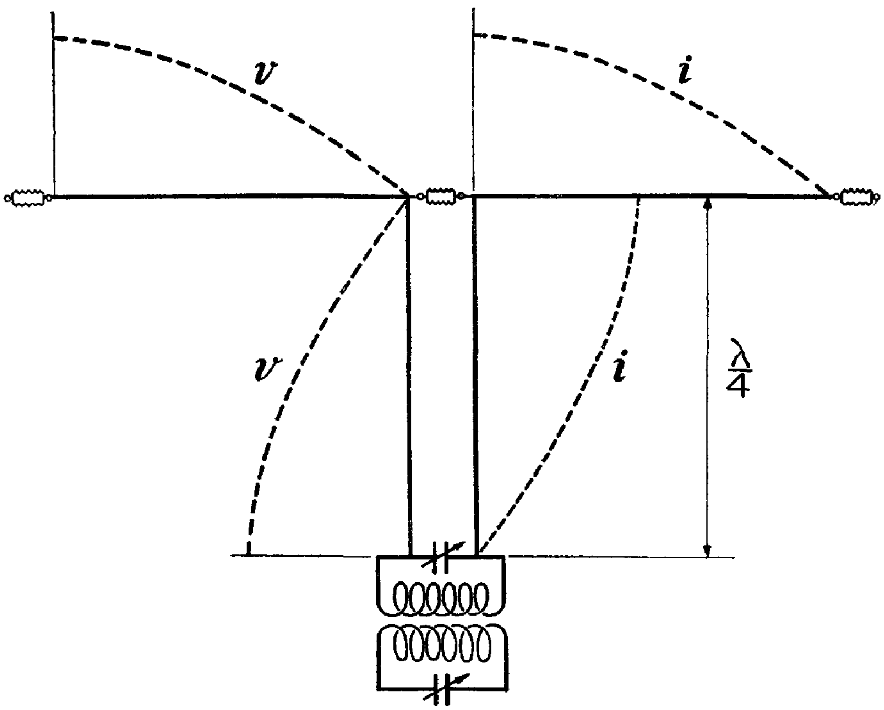

FIG. 30, CHAP. VII.—Current-fed half-wave aerial with $\frac{\lambda}{4}$ feeder.

thermoammeter in series with each will also enable a current balance to be achieved, and the feeder may be considered to be resonant when the sum of the two feed currents is a maximum. The effective length of the coupling coil by which energy is transferred from transmitter to line must also be taken into account. The current distribution of an aerial fed in this way is shown in fig. 29.

59. If the aerial and transmitter are so close that a $\frac{\lambda}{4}$ resonant feeder is preferable the desired current and voltage distribution can be achieved by inserting a rejector circuit, tuned to the desired frequency, in place of the coupling coil mentioned above. The circuit then becomes that given in fig. 30, and it will be observed that the rejector circuit replaces a length of feeder equivalent to one half-wavelength. Again, the relative positions of transmitter and aerial may render it more convenient to feed the half-wave aerial at one end. Now in the current feed arrangement, each half of the aerial is effectively a $\frac{\lambda}{4}$ aerial, having a radiation resistance of

about 36 ohms or 72 ohms in all, and the loss resistance may amount to a further 20 ohms in each half, so that the input resistance of each is in round figures 100 ohms. It will be noted that this figure is attained when feeding at a point where the current is large and the voltage small, i.e. the ratio $\frac{V}{I}$ is small (100 ohms). When feeding at the end of the $\frac{\lambda}{2}$ aerial, however, the current being a minimum amd the voltage a maximum, the ratio $\frac{V}{I}$ is large, being equivalent to about 2,500 to 3,500 ohms. It may be stated that whereas the load imposed upon the source by an aerial fed at a current loop resembles that of an acceptor circuit, the load imposed when fed at a voltage loop resembles that of a rejector circuit. The figure 2,500 to 3,500 ohms above mentioned is in fact calculated on the assumption that the average inductance of such an aerial is about $1{\cdot}85\ \mu H$ per metre and its capacitance about $\cdot 000006\ \mu F$ per metre, hence the dynamic resistance of the aerial is:

$$\frac{L}{CR} = \frac{1{\cdot}85 \times \frac{\lambda}{2}}{\cdot 000006 \times \frac{\lambda}{2} \times (72 + 40)}$$

$$= \frac{1{\cdot}85 \times 10^6}{6 \times 112}$$

$$= 2{,}750 \text{ ohms.}$$

In general it may be said that when a $\frac{\lambda}{2}$ aerial is fed at the end, the circuit by which energy is supplied should have the characteristics of a rejector circuit, while if fed at the centre, which is a current loop, the feeder should have the characteristics of an acceptor circuit. In this way the maximum transference of energy is necessarily obtained, and the feeders are said to be " matched " to the aerial. The importance of matching the various parts of the circuit is of even greater importance when a non-resonant line is employed.

60. One possible method of arranging the feeder is shown in fig. 31. It will be seen that although a twin-wire feeder is employed, only one of the wires is actually in use, the other being

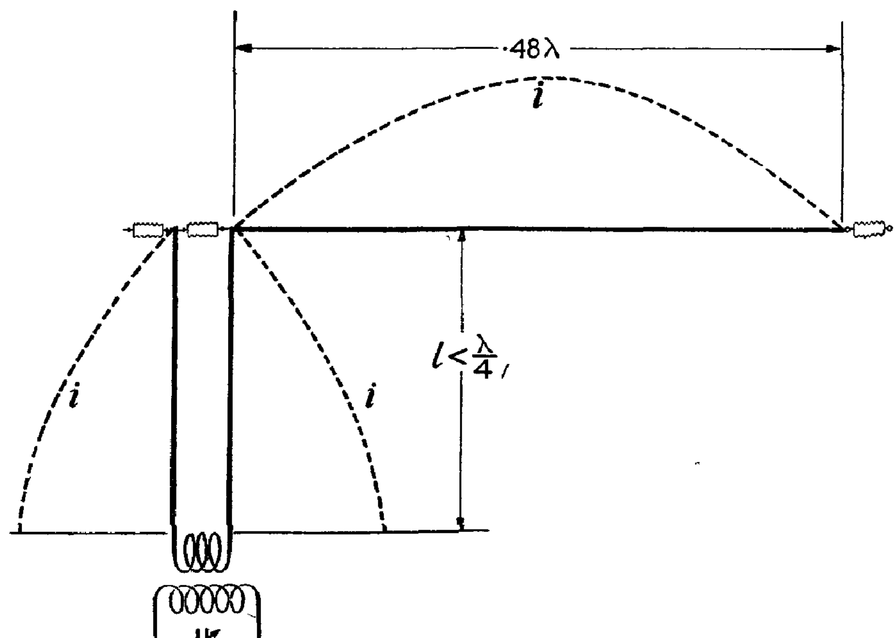

FIG. 31, CHAP. VII.—Voltage fed half-wave aerial.

used merely in an attempt to eliminate radiation from the feeder. This elimination cannot be complete because the current at the end of the inactive feeder is necessarily zero, but that at the end of the active feeder is the feed current of the aerial and will be of the order of one-fifth that at the current loop. The feeders themselves are approximately $\frac{\lambda}{4}$ in length, and are made electrically equivalent to $\frac{\lambda}{4}$ by adding either inductance or capacitance in each line at the transmitter end. A simple coupling coil serves to transfer energy from transmitter to feeder. The foregoing discussion is of course equally applicable to the higher frequencies of the 3—30 Mc/s band, half-wave or three-quarter-wave vertical aerials being usually employed. The latter form of aerial gives maximum radiation in a direction inclined about 30° to the horizontal, and this may be advantageous for long distance transmission in that less energy is absorbed by the earth in the neighbourhood of the transmitter. $\frac{3}{4}\lambda$ aerials are fed in the same way as $\frac{\lambda}{4}$ aerials.

Non-resonant transmission lines

61. The theory of the transmission line will receive more detailed consideration in Chapter XV to which the following may be considered an introduction. It has already been pointed out that any long length of conductor, an aerial being an example, has inductance, capacitance to earth and to adjacent conductors, and also resistance, distributed throughout its length, thus the resonant feeders and aerials previously considered may be regarded as transmission lines of such a length that the waves reflected from the ends of the conductor reinforce or cancel each other at certain points. If L' is the inductance and C' the capacitance, both measured in C.G.S. units per centimetre, the velocity with which an electromagnetic disturbance travels along the conductor is practically equal to the velocity in free space, namely 3×10^{10} centimetres per second, and the latter is also equal to $\frac{1}{\sqrt{L'C'}}$. Suppose that a twin wire transmission line is supplied with an alternating E.M.F. of any frequency whatever, and that the line is of infinite length, so that no reflected wave can occur. This absence of a reflected wave implies that the line is non-resonant, and behaves in the same way at all frequencies. Assuming that the inductance and capacitance are equally distributed along the whole length, and that the resistance is negligible, the current in the line will decrease gradually along the line owing to the displacement current between lines, hence the line possesses an effective impedance, called its characteristic or surge impedance, which can be shown to be equal to $\sqrt{\frac{L'}{C'}}$ ohms. The current and voltage in such a line are in phase with each other at all points and therefore the surge impedance is actually an effective resistance. It must be noted that as L' and C' are the inductance and capacitance per centimetre, any length of the same line has the same surge impedance, namely $\sqrt{\frac{L'}{C'}}$ ohms. Now in practice we cannot have a line of infinite length; if, instead, the line is of finite length, and the ends of the wires remote from the generator are connected to a non-inductive resistance equal to $\sqrt{\frac{L'}{C'}}$ ohms, the load imposed upon the generator by the finite line and its terminating resistance is exactly the same as the load imposed by an infinitely long line, for the current and voltage are in phase at all points and no stationary wave is set up. The physical meaning of this is that energy is converted into heat in the terminating resistance at exactly the rate at which it arrives at the end of the line. When a line is required to transmit energy from one point to another, therefore, it is highly desirable, and in fact essential if the utmost efficiency is to be obtained, that the device to which the energy is supplied shall be equal in effective resistance to the surge impedance of the line. Similarly, the internal impedance of the source of energy should be equal to the surge impedance of the line in order that maximum energy shall be transferred to the line by the source. The surge

impedance of a twin wire line is given with sufficient accuracy for practical purposes by the expression :

$$Z_0 = 276 \log_{10} \frac{2D}{d}$$

where Z_0 = the surge impedance

D = distance apart of centres of wires.

d = diameter of wires.

In consequence of the logarithmic relation the surge impedance is always of the order of 600 ohms for the lines used in practice. For example, if the wires are 0·1 in. in diameter spaced 5 in. apart, $Z_0 = 552$ ohms, and if the spacing is increased to 10 inches, or decreased to $2\frac{1}{2}$ inches (the diameter of wire remaining the same), the surge impedance becomes 636 ohms and 444 ohms respectively. For correct termination of a non-resonant line therefore, the impedance of the load should be of this order, while it is also necessary to couple the feeder to the transmitter in such a manner that the effective load imposed by the feeder upon the transmitter is of the same order as the effective resistance of the latter. A radio-frequency transformer with adjustable coupling will ensure that this matching is achieved.

62. A suitable radio-frequency transformer may also be used to couple a non-resonant line to the aerial, but this may not be practicable and is rarely the most convenient method. Fig. 32

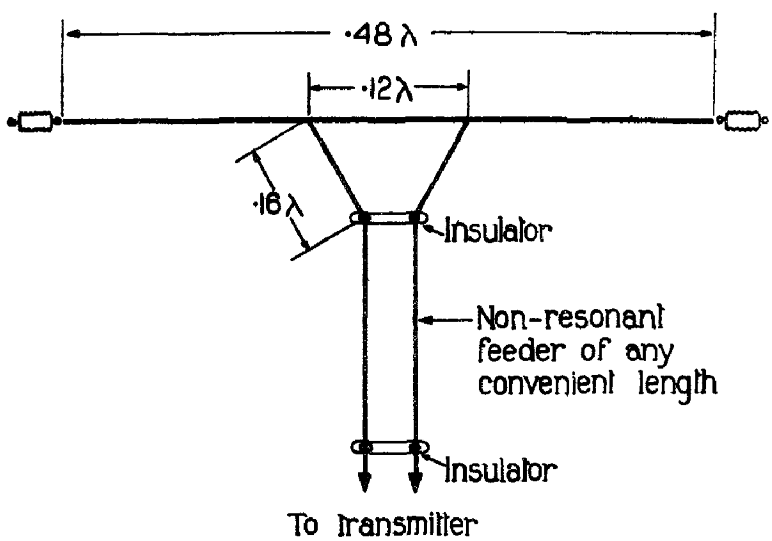

FIG. 32, CHAP. VII.—Auto-inductive coupling between half-wave aerial and non-resonant feeder.

shows a half-wave aerial coupled to a non-resonant line by a kind of auto-inductive coupling. The dimensions given in this figure are appropriate to a 600 ohm line. It is much easier to calculate the spacing necessary to secure this surge impedance than to calculate the tapping points and length of tapered feeder required for a line having a different value of Z_0. A feeder of length $\frac{\lambda}{4}$ may be employed as a radio-frequency transformer between the transmission line and a current-fed dipole. If V_i and I_i are the voltage and current at the input end of this " transformation feeder," as it is termed, and V_0 and I_0 the voltage and current at the output end,

$$V_i = I_0 Z_0$$

$$I_i = \frac{V_0}{Z_0}.$$

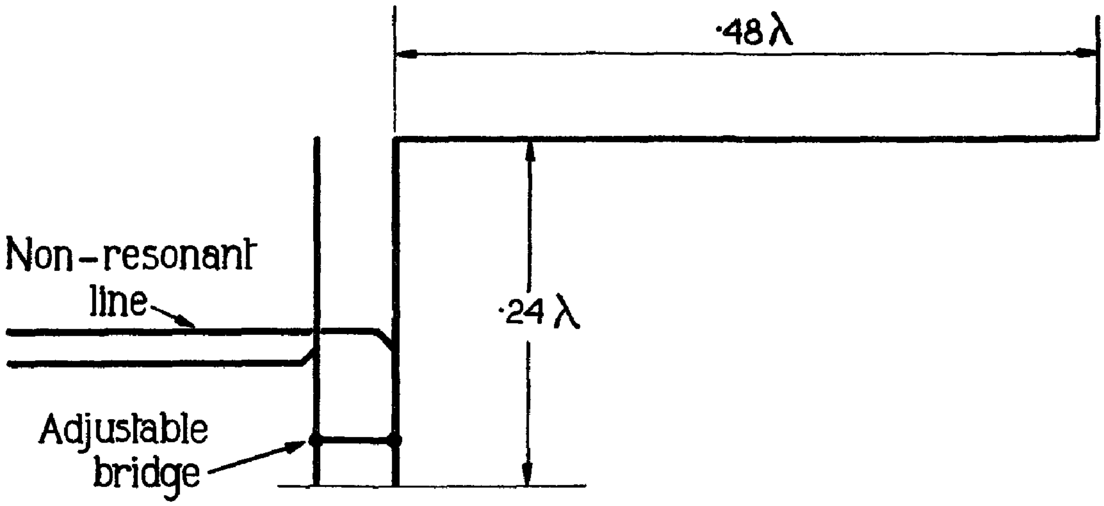

FIG. 33, CHAP. VII.—Voltage fed half-wave aerial with non-resonant feeder.

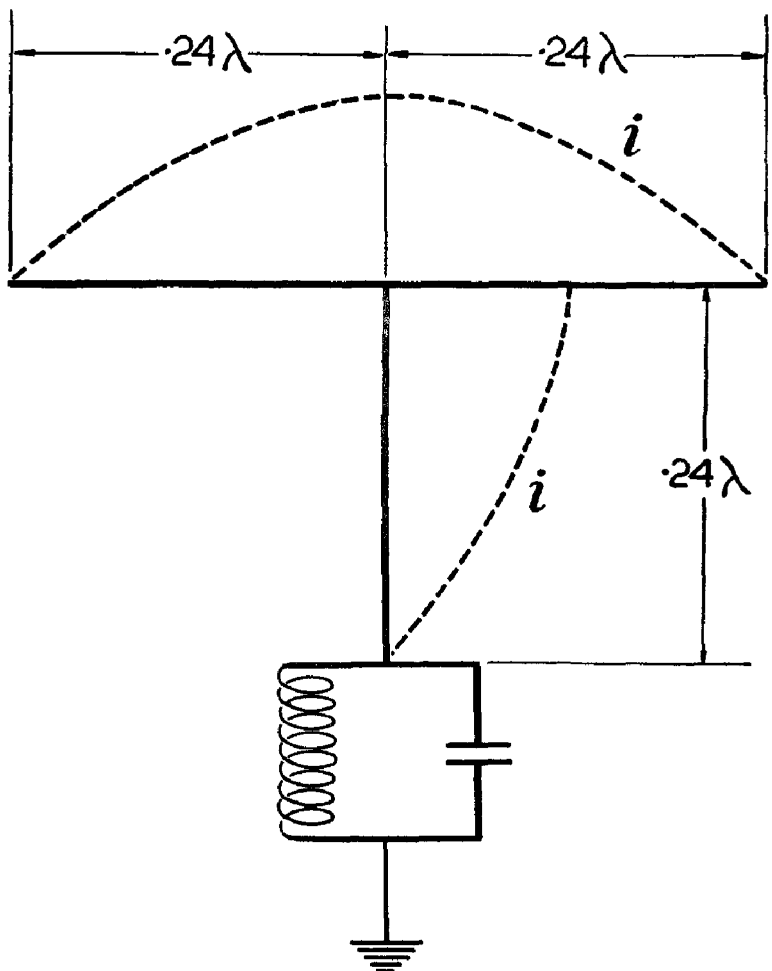

FIG. 34, CHAP. VII.—Basic form of inverted aerial.

Now if R_o is the actual effective resistance of the dipole, $V_o = I_o R_o$. The effective resistance of the line and the load R_o together, measured at the input terminals of the line, will be $\frac{V_i}{I_i}$ or $\frac{I_o}{V_o} Z_o^2$. But $\frac{I_o}{V_o} = \frac{1}{R_o}$, hence the input resistance of the combination, R_i, will be $\frac{Z_o^2}{R_o}$. This signifies that if the surge impedance of the non-resonant line is 600 ohms and the dipole has an effective resistance of 100 ohms, the transformation feeder should have a surge impedance of $\sqrt{R_i\ R_o}$ or $\sqrt{100 \times 600} = 245$ ohms. This can be achieved by using a feeder composed of four or six wires, i.e. two or three in parallel on each side, the wires being transposed in such a manner that all have uniform capacitance to earth and to every other wire.

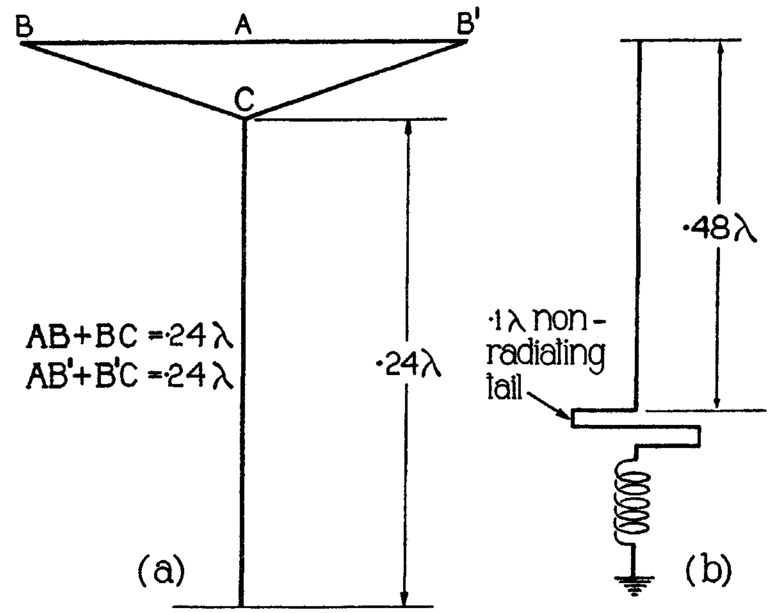

FIG. 35, CHAP. VII.—(*a*) Development of inverted aerial; (*b*) Tail feeder.

63. Another convenient method is shown in fig. 33. Here the aerial itself is fed by a resonant $\frac{\lambda}{4}$ feeder, the general arrangement being similar to that of fig. 31. Instead of being directly connected to the transmitter, however, the lower end of the resonant feeder is terminated by a short circuiting bridge, which can be moved from point to point for matching purposes, while the non-resonant line is connected at a point about two-thirds of the length of the resonant line, measured from the upper end. Matching is then carried out by adjustment of the position of the short-circuiting bridge. In order to carry out this adjustment, a low reading thermoammeter (e.g. reading 0–120 milliamperes) is fitted with a rectangular or triangular loop of stiff wire which is secured to a strip of insulating material, the latter being fitted with small hooks so that it may be hung on the feeder wire, the whole assembly forming the secondary winding of a radio-frequency transformer. The absence of stationary waves is indicated by the non-existence of current nodes and loops in the feeder; the current in the line should be practically the same at all points in its length. In the absence of a suitable thermoammeter, a 60 milliampere fuse bulb might be tried, fitting it to the loop in place of the meter. It would perhaps be preferable to remove the screw base and solder the lamp connections direct to the wire loop to avoid the capacitance shunt which would otherwise exist.

Inverted aerials

64. In ground to air working over comparatively short ranges it is desirable that the ground station aerial shall radiate most of its energy at a low angle, and it is desirable that the current loops shall be situated as high above ground as possible (cf. fig. 23). At the lower frequencies of the wave-band under consideration, vertical half-wave aerials are hardly practicable, but a similar vertical polar diagram can be obtained by what is called an inverted aerial, which is defined as an aerial less than $\frac{\lambda}{2}$ in height, but energised in such a manner that it functions as a half-wave aerial. Suppose the wavelength to be 100 metres, an inverted aerial might consist of a vertical wire 24 metres (i.e. $\cdot 24\lambda$) in height with a T or X shaped top each member of which is also 24 metres long (fig. 34). Since the ends of the horizontal members are at the same potential, they may be bent over and interconnected as shown in fig. 35a. The approximate current distribution is shown in fig. 34. The aerial should be fed via a rejector circuit in the same manner as a half-wave aerial. An alternative method of feeding a vertical half-wave or an inverted aerial is to connect a quarter-wave system to its lower end. This consists of a short length (about $\cdot 1\lambda$) of single conductor, which is folded upon itself so as to be non-radiating, which in turn is tuned by means of an inductance at the foot; this coil also serving for coupling to the transmitter. The general arrangement and current distribution are given in fig. 35b.


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