B.R.1600A
Code No.1563
A.P.1093E(2)

Chapter 5

WAVEGUIDES

List of Contents

WAVEGUIDES

INTRODUCTION

1. General

We have seen in Chap. 4 that electromagnetic waves may be propagated between a pair of parallel conducting cylinders or between a pair of coaxial cylinders and that the transmission line system so formed provides a convenient means for conveying high frequency power from a source to a load. It is also possible to pass an electromagnetic wave along the inside of a metal tube and, for various reasons which will be elaborated below, this is a desirable procedure at centimetre wavelengths. In place of the metal tube a dielectric rod is sometimes used. The tube or rod used for this purpose is called a waveguide. Whereas it was convenient to discuss propagation on transmission lines in terms of the voltages and currents associated with the wave disturbance it is more useful in the study of waveguides to concentrate attention on the electric and magnetic fields of the wave in the tube. It is useful, therefore, before proceeding to a detailed discussion of waveguide,propagation to summarise the principle features of the electromagnetic fields of waves in free space and on transmission lines.

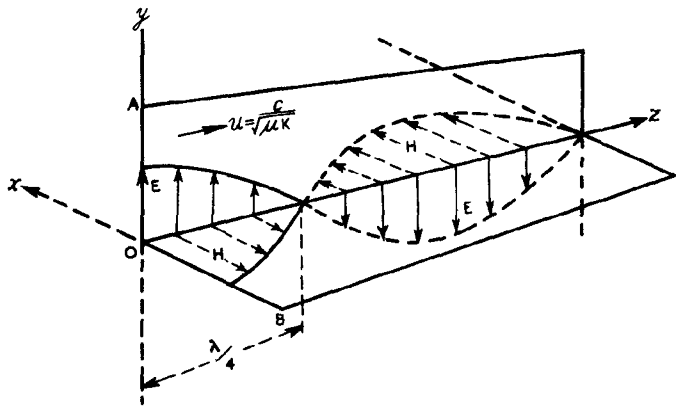

Fig. 195 - Field distributions in a plane polarised electromagnetic plane wave.

2. Properties of Electromagnetic Waves in Free Space

Fig. 195 depicts a sinusoidal plane-polarised electromagnetic plane wave travelling in an unlimited medium; its properties are as follows :-

(i) The wave comprises oscillations of an electric field E and a magnetic field H in directions at right angles to each other and to the direction of propagation,

(ii) the waves are transverse both in E and H and are, therefore, of the type called TEM (Transverse-Electro-Magnetic); that is, there is no component of E or H in the direction of propagation,

(iii) The velocity of propagation in free space is equal to c, the velocity of light (3×10^8 m/sec.) In a medium of dielectric constant K and permeability μ the velocity is

$$\frac{c}{\sqrt{K\mu}},$$

(iv) E and H oscillate in phase with each other when the medium is non-conducting,

(v) The amplitudes of E and H are constant all over a wave front.

3. Behaviour of an Electromagnetic Field at the Surface of A Conductor

The electromagnetic fields of waves on transmission lines and in waveguides are bounded by metal surfaces; consequently, to appreciate the forms of the field patterns it is essential to know how such fields behave at a metal surface at high frequencies.

It is assumed in the first instance that the metal has infinite electrical conductivity. It can be shown that the electric field E is perpendicular to the metal at its surface (as in electrostatics), or, in other words, the tangential component of the electric field vanishes at the surface of a perfect conductor. The magnetic field H of the wave is everywhere tangential to the surface; that is, there is no normal component of H at the surface. H is zero inside the conductor, (Fig. 196). To support the discontinuity in H at the surface, i.e., to allow the magnetic field to change suddenly from H to zero in crossing the conducting surface from without to within the conductor, there is a current sheet at the surface of the conductor.

Since any metal possesses a finite conductivity the above description of the electromagnetic field idealises the actual conditions, but the behaviour in the ideal case is a very close approximation to the actual behaviour at a metal surface when the frequency of the wave is high. At low frequencies electromagnetic fields penetrate deeply into the interior of a conducting medium and the corresponding currents are distributed throughout a volume of the medium. As the frequency is increased, however, the currents are crowded more and more into the surface of the conductor and the fields penetrate less and less deeply. This is the well known phenomenon of Skin Effect.

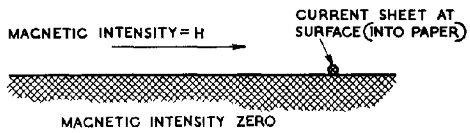

Fig. 196 - Behaviour of electromagnetic field at conducting surface.

4. Waves guided by Pairs of Conductors (Transmission Lines)

We will now discuss the properties of a wave travelling along a transmission line. They are very similar to those of the freely travelling waves in an unbounded medium already mentioned. Figs. 197 (a) and (b) represent sections of parallel pairs of long conducting cylinders whose contours are of arbitrary form but the same for each cylinder along its whole length.

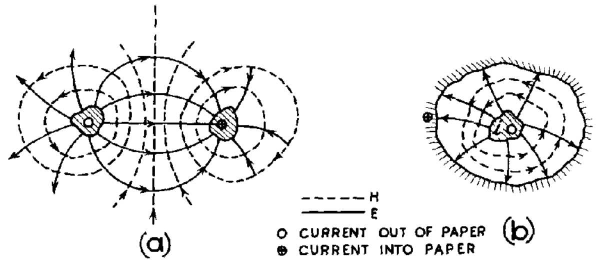

Fig. 197 - Cross-section of TEM-wave carried by conducting cylinders.

The figures indicate the patterns formed by the lines of E and H; (Since other types of wave can also be carried by the transmission lines the wave here discussed is usually called the Principal Wave). The wave system has the following properties :-

(i) The wave is propagated parallel to the axes of the cylinders, and is transverse both in E and H; i.e., the electric and magnetic fields are each perpendicular to the direction of travel and lie in a plane parallel to the wave front as shown in Fig. 197.

(ii) The pattern formed by the electric lines of force is that of the two-dimensional electrostatic field obtained by maintaining the cylinders (supposed infinitely long) at a suitable difference of potential. Hence, each line of force arises from a surface charge on one conductor and ends on a surface charge of opposite sign on the other conductor. The lines also cut the surface of the conductors at right angles.
(Boundary conditions, Sec. 3).

(iii) The lines of magnetic force surround one or both conductors and form a pattern which is the same as that obtained by a suitable steady current along the cylinders but in opposite directions in each. When a high frequency wave is transmitted the current in the conductor is entirely superficial, and the contour of each conductor in the section is followed by the neighbouring magnetic lines.

(iv) At all points the two fields are mutually perpendicular.

(v) The velocity of the wave is the same as that for the unguided wave and is the same for all frequencies. There is no limit to the frequency of the wave which can be propagated.

5. Limitations of Cables and Advantages of Waveguides at Centimetre Wavelengths

As explained in Chap. 4 Secs. 41 - 45, one limitation to the general use of coaxial cables at centimetre wavelengths is set by excessive absorption of power. This absorption arises both from ohmic heating of the conductors and from internal loss in the dielectric, the former loss increasing in proportion to the square root of the frequency and the latter to the frequency itself. In both cases the loss is accentuated by the presence of the inner conductor. The currents responsible for the ohmic loss are equal in magnitude in the inner and outer conductors, but since they are forced by the skin effect to flow in the surface layers of these conductors they dissipate most of their energy in the inner conductor which has the smaller surface. The dielectric loss increases with the field strength, and since the field is greater near the inner conductor these losses also are concentrated there.

It is to be anticipated, therefore, that when electromagnetic waves are propagated in tubes, in which both inner conductor and dielectric are absent, the attenuation is relatively small and comparable with that due to the outer conductor of a coaxial cable of the same dimensions. Such tubes are known as Waveguides and because of the small attenuation suffered by electromagnetic waves in them they are often preferred to cables in applications using centimetre wavelengths. A dielectric rod or tube may also be used as a waveguide, but in such a case the attenuation due to the presence of the dielectric is likely to exceed that due to the finite conductivity of the metal in an air-spaced coaxial transmission line. The precise attenuation coefficient in a waveguide depends on the wavelength, cross-section and form of wave, but whereas in a good cable the attenuation coefficient for a wave of length 10 cm. is of the order of 0·5 db/metre, in a waveguide it can be of the order of 0·025 db/metre. At wavelengths of 3 centimetres the attenuation coefficient in the cable would exceed 1 db/metre and that in the guide would have risen to 0.075 db/metres. It therefore follows that cables would not be used at wavelengths of 3 centimetres except in very short lengths.

Loss of energy from a wave as it travels along a transmission line is objectionable not only because it weakens the signal which arrives at the other end, but also because of the undesirable heating produced by the wasted energy. In the case of high power transmission this heating is often sufficient to soften the polythene (softening point about 100°C) in the cable and cause breakdown.

6. Waves in Waveguides compared with those in Free Space and on Transmission Lines

The salient features of waves in guides are as follows :-

(i) The oscillations of E and H are not both transverse and it is convenient to classify waves in guides into two principal types. Two different nomenclatures are in use for describing the waves. Sometimes the wave is known in terms of that field component which is entirely transverse; it is called, for example, a Transverse Electric, or TE wave. Sometimes the same wave is known in terms of the field component which has a longitudinal component, a wave with a longitudinal H-component, for example, being called an H-wave. The two principal types are therefore :-

(a) H-waves }
Transverse Electric }
(TE) waves, }

in which the electric field E is entirely transverse but the magnetic field H possesses a component H_z in the direction of propagation (longitudinal component) as well as a transverse component H_t, Fig. 198(a). H_t, and H_z oscillate in quadrature so that the resultant H vibration is elliptical. Ht and E vibrate in phase.

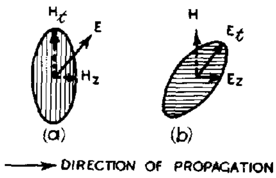

Fig. 198 - Component vectors of E-waves and H-waves.

(b) E-waves }
Transverse Magnetic (TM) }
waves,
characterised by entirely tranverse oscillations of H but in which E has a longitudinal component, E_z (Fig. 198 (b)). E_t and E_z oscillate in quadrature so that the resulting E vibration is elliptical.

(ii) The amplitudes of E and H are not constant over a wave front. In the simple case of a rectangular tube they vary sinusoidally across the tube.

(iii) The presence of the longitudinal component of either the E or the H field causes the wave front to travel faster than the energy of the wave. The velocity of the wave front is called the Phase Velocity u_p, whilst the speed with which energy is transmitted along the guide is called the Group Velocity u_G. It may be shown that in all cases :-

$$u_p \cdot u_G = c^2 .$$

The wavelength λg of a wave in a guide exceeds the wavelength λ of the free space wave of the same frequency since :-

$$u_p = f\lambda_g \quad \text{and } c = f\lambda .$$

(iv) Electromagnetic disturbances are propagated as waves inside a metal tube only if the free space wavelength ($\lambda = c/f$) is less than a critical value λ_c, called the Cut-off Wavelength. The corresponding frequency $f_c = c/\lambda_c$ is called the Cut-off Frequency. The cut-off wavelength is determined by the type of wave and the geometry and dimensions of the cross-section of the tube, but in a given tube the greatest value of λc is of the order of magnitude of the larger cross-sectional dimension of the guide.

(v) The following relation exists between the wavelength λ_g of a wave in a guide, its cut-off wavelength λ_c and the free-space wavelength λ :-

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2};$$

multiplication by $1/f^2$ gives :-

$$\frac{1}{u_p^2} = \frac{1}{c^2} - \frac{1}{f^2 \lambda_c^2}.$$

The phase velocity u_p in the guide is evidently a function of the frequency; whereas the velocity in free space is the same for all frequencies.

WAVES IN A RECTANGULAR TUBE

7. Synthesis of an H-Wave

Most of the important features of waves travelling in guides can be derived quite easily from a consideration of the case of a rectangular guide carrying a simple form of H-wave known as the H_{01} wave. This derivation is so instructive that it will be given in detail and the important points brought out. For more complicated cases results only will be quoted and no detailed derivation will be given. There are no important fundamental points which cannot be seen from the simple treatment now to be given, and it should not be thought that any complicated "mathematical" analysis can give any deeper insight into the phenomena.

The method of approach is as follows. We shall show that if a train of plane electromagnetic waves is incident obliquely on a perfectly conducting sheet of metal then the incident and reflected waves combine to produce outside the metal a composite wave in which the motion appears to take place in a direction parallel to the surface of the metal. This moving wave system can, if we wish, be thought of as a peculiar type of wave guided by a single infinite metal sheet. Precisely the same wave disturbance is produced when a wave is incident at an angle on to a plane surface of the earth which can be considered to be perfectly conducting. A detailed discussion of this case of an incident wave combining with its reflected wave after reflection from the surface of the earth is given in Chap. 17. We shall also show that in this type of wave the amplitudes of the electromagnetic fields alter as we cross the wave front and moreover the fields are not entirely transverse to the direction of advancement of the wave. If now we wish to insert in the guided wave another metal sheet parallel to the first to form an opposite boundary of a waveguide we find in general that it will disturb the wave pattern. It is found, however, that a metal sheet can be inserted without disturbing the wave system in certain planes where the magnitudes and directions of the fields are suitable. Under these conditions we have a wave system which is guided between two parallel planes which can be thought of as two opposite sides of a waveguide. We next show that the insertion of a second pair of plates perpendicular to the first pair will not disturb the wave, and thus we have built up a type of wave which can travel in a rectangular guide.

We now conduct the synthesis in detail. Let AB represent a metal sheet of infinite conductivity and suppose that a train of plane waves incident upon it in air (ideally, vacuum) dielectric, Fig. 199 (a), gives rise to an equal reflected wave (b).

The diagrams illustrate the distribution of fields in space at any chosen instant. The full lines are wave fronts over which E and H have maximum values; over the dotted lines mid-way between, E and H are everywhere zero. The section through the wave fronts is so chosen that the magnetic vectors H (represented by arrows on the full lines) lie in the plane of the paper, and the electric vectors E (represented by circles with a dot or cross inside) stand perpendicular to the plane of the paper. λ is the free-space wavelength, and the arrows c indicate the directions of the incident and reflected waves. Between one full line and the next there is a complete reversal of both E and H fields, corresponding to a phase shift of 180°. Consequently, the separation of the full lines is $\frac{\lambda}{2}$.

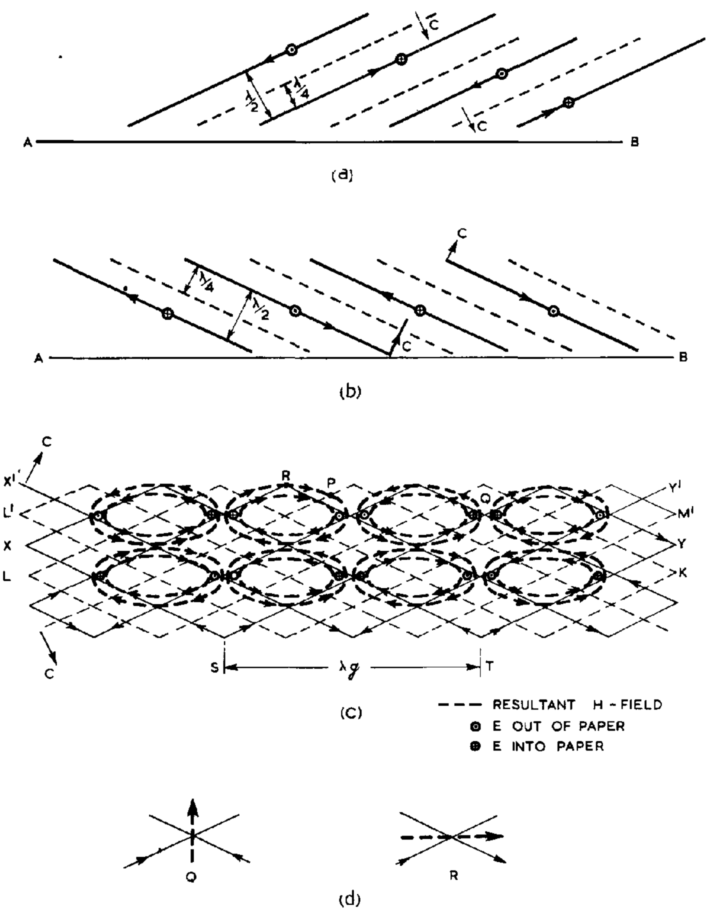

Fig. 199 - Synthesis of an H_{on}-wave.

As the time t changes the patterns move in the directions indicated by the arrows perpendicular to the wave fronts and with the velocity of light c. In Fig. 199 (c) the two wave trains are shown passing across the same region of space by superimposing the patterns shown at (a) and (b). The c- arrows are moved to the extremities of the wave fronts in order not to confuse the diagram. The resultant H-field at any point is obtained by adding the two vector magnetic fields of the original wave trains. The direction of the resultant H-field is indicated by the arrows on the heavy lines. At certain points of the pattern the direction of the resultant field may be seen by inspection. For instance, at all points on a dotted wave front the fields of that wave train are zero and the resultant field is that of the other wave train. Thus, along a dotted line the direction of H is parallel to the wave fronts of the other wave system, as for example at the point P. Where two dotted lines cross there is no field whatsoever. At a point of intersection of two full lines the resultant field H is directed along the bisector, either of the acute, or of the obtuse angle between the full lines according to the relative directions of the arrows appropriate to the intersecting wave fronts. The points Q and R of Fig. 199 (c), shown separately in Fig. 199 (d), provide examples.

When the directions of the resultant magnetic field are indicated by arrows at a sufficient number of points it is possible to sketch the lines of magnetic force in the pattern of the resultant field. These are seen to be closed curves whose precise form is determined by the angle of incidence of the original wave. As the centre of each magnetic loop is a point of intersection of two dotted lines there is zero resultant magnetic or electric field there.

Since the electric fields of the elementary plane waves shown at (a) and (b) are everywhere directed either up or down perpendicular to the plane of the paper, the resultant electric field in the pattern shown at (c) is also normal to the plane of the paper and attains a maximum strength twice that of the field in the separate wave trains. The circles with a cross or dot within indicate the sense of the local resultant electric field. It is to be understood that the pattern extends in depth into and out of the plane of the paper.

8. Pattern Velocity (Phase Velocity) of H-waves

A fixed feature of the pattern shown in Fig. 199 (c), such as the resultant magnetic field at the point Q, is associated with a point of intersection of two wave fronts of the wave trains shown at (a) and (b). The two wave fronts intersecting at Q are shown as QF and QG in Fig. 200.

As the wave fronts QF and QG travel in the ray directions FK and GK respectively, at velocity c, their point of intersection Q travels at velocity u_p in the direction L'M'; moreover Q moves from Q to K in the time taken for F to move from F to K or G from G to K. Let the angle FKQ (equal to GKQ) be α; then, since the angles KFQ and KGQ are right angles, it follows that :-

$$\frac{u_p}{c} = \frac{QK}{FK} = 1/\cos\alpha = \frac{\lambda_g}{\lambda} \quad \text{(Sec. 6 (iii))},$$

or

$$u_p = c/\cos\alpha = c\sec\alpha\,.$$

It is convenient at this point to summarise what has already been deduced in this section. We have seen that when a plane electromagnetic wave is incident obliquely on a metal sheet it combines with its reflected wave to synthesise a new type of wave in which the electric and magnetic field patterns are quite different from those of the constituent plane polarised waves. The direction of propagation of the new wave is parallel to the plane of the metal and the pattern velocity u_p is greater than the velocity c. It is shown that $u_p = c/\cos\alpha$, where α is the angle of incidence of the original wave on to the metal.

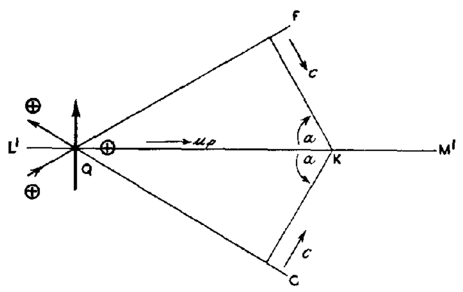

Fig. 200 - Illustration of phase velocity

Let us now examine in more detail the nature of the electric and magnetic fields over the wave system. Over planes such as XY and X'Y' Fig. 199 (c)) which separate two layers of magnetic loops, the magnetic field is everywhere tangential to the plane and the electric field is zero at all points of the plane. Such planes are nodal planes of E. Conversely, at the intervening planes such as LM, L'M' the magnetic field is perpendicular to these planes and the electric field reaches its maximum values. The distance ST, containing two complete loops as shown, is the wavelength λ_g of the composite pattern since the pattern repeats identically in a displacement of this amount parallel to XY. The composite pattern shown in Fig. 199 (c) refers to a chosen instant of time, but as the patterns of the elementary waves shown at (a) and (b) move obliquely with a velocity c, so is the composite pattern displaced without distortion in the direction of symmetry XY at another velocity u_p.

The magnetic field comprises a system of closed loops and consequently possesses in general a component H in the direction of propagation; the electric field E is entirely transverse but its amplitude varies in a direction perpendicular to the direction of propagation XY and lying in the plane of H. This wave is of the type called an H-wave in Sec. 6.

The H-wave is, as yet, of unlimited extent and it is necessary to discover whether such a wave can be caused to travel down a metal tube. We proceed in stages by considering first the possibility of propagating an H-wave between a pair of parallel metal plates of unlimited extent.

9. Propagation Between Parallel Plates and in Rectangular Tubes

As already mentioned, over the nodal E-planes XY, X'Y', etc. (Fig. 199 (c)) which separate adjacent layers of magnetic loops, the electric vector vanishes, and the magnetic vector is tangential to these planes. Reference to Sec. 3 shows that over these nodal planes, the behaviour of the electromagnetic field of the H-wave is the same as that at the surface of a perfect conductor. If, therefore, a perfectly conducting plate is inserted into the wave pattern of Fig. 199(c) to coincide with a nodal E-plane such as XY, the pattern above the plate is undisturbed and that below may be removed. The pattern above is supported by transverse surface currents i as

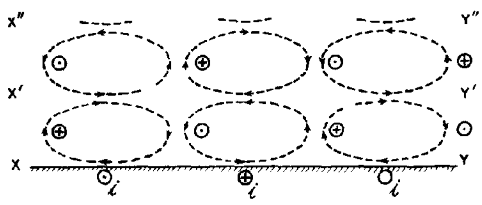

Fig. 201 - Positions of nodal planes.

indicated in Fig. 201.

It is, therefore, possible to insert a second metal sheet at any of these planes to form with the first a pair of sides of a waveguide. The second plate may be inserted so that it coincides with one of the more distant nodal planes such as X'Y' or X"Y". In this way any desired number of layers of loops of Fig. 199 (c) and 201, may be trapped between the plates to give an H-wave propagated at velocity u_p tangential to the plates but confined to the region between them. The elementary waves of Fig. 199 (a) and 199 (b) may now be considered to undergo successive reflections at the plates. Fig. 202 depicts a three-layer H-wave propagated between a pair of parallel plates at a separation b but of unlimited lateral extent.

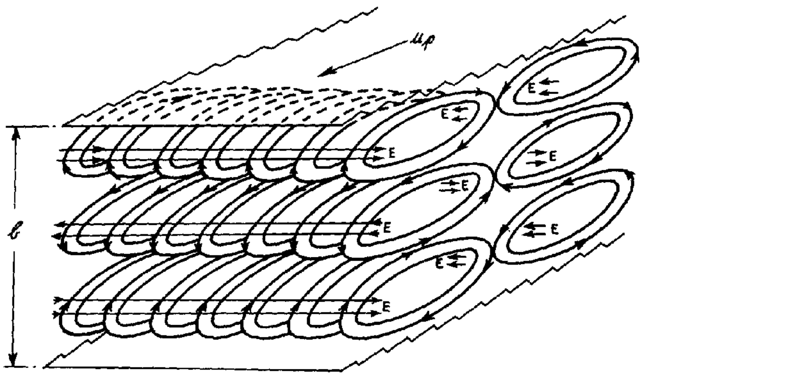

Fig. 202 - Three-layer H-wave.

To convert the parallel plate system into a rectangular tube it is necessary to insert side walls in such a way that the boundary conditions to be satisfied at the wall surface are consistent with the preservation of the wave pattern. The walls must evidently be inserted as shown in Fig. 203, that is, the wall surfaces are parallel to the planes containing the magnetic loops. Consequently H is everywhere tangential to these walls and, as indicated in the figure, E is perpendicular to them. The separation a in the present instance can be given any value.

An H-wave of the nature described above is not the only type of wave that can be propagated in a rectangular tube and it is therefore convenient at this stage to give the H-wave under discussion its usual designation. A wave of the type shown in Fig. 203, but with a number n of complete layers of magnetic loops in the 'b' dimension is called an H_{on}- (or TE_{on}) wave. That shown in Fig. 203 is, therefore, an H_{03} wave.

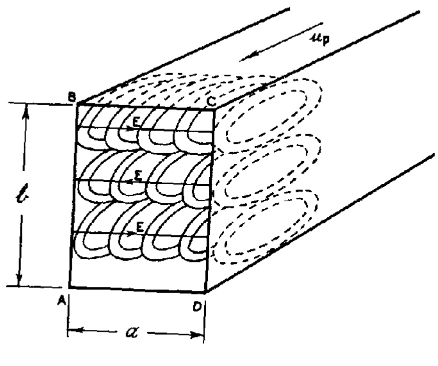

Fig. 203 - H_{03}-wave in rectangular guide.

The interpretation of the suffix zero is as follows :- if we choose any starting point on the face AB (Fig. 203) and move parallel to AD across to the opposite point on the wall CD, no variation in any component of H or E occurs. The suffix zero indicates this independence of E and H of displacements parallel to the edge 'a' of the tube section. On the other hand, in moving parallel to AB from the face AD to the face BC n layers of H-loops are crossed (or, more accurately, n half-cycles of variation in any one of the components of E or H are observed).

Of the possible H_{on}-waves in rectangular waveguides one is of outstanding practical importance; it is the H_{01}-wave and it will be further discussed below.

10. The H_{01}-wave in a Rectangular Guide

This wave, as the suffixes indicate, contains only a single layer of H-loops between the faces AD and BC (Fig. 203) and corresponds to the case in which conducting plates are inserted at adjacent nodal planes XY and X'Y' (Fig. 201). Fig. 204 represents a section parallel to both the H-field and the direction of propagation of an H_{01}-wave in a rectangular waveguide.

The wave fronts (e.g. AC) of the elementary plane waves reflected from plate to plate are shown, as well as a magnetic loop of the resultant H_{01}-wave. The ray direction corresponding to the wave front AC is BO and it is supposed that this ray makes an angle of elevation with the reflecting planes, as shown. The distance BO is $\lambda/4$ where λ is the free-space wavelength of the waves, and CO = $\lambda_g/4$, where λ_g is the wavelength of the H_{01}-wave in the guide.

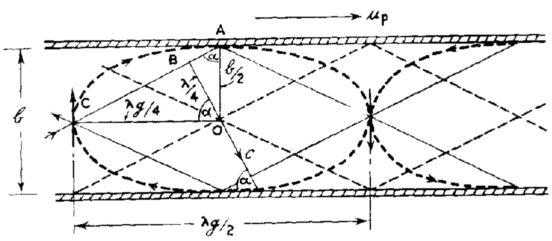

Fig. 204 - Section of H_{01}-wave in a rectangular guide.

From the diagram we deduce :-

$$\sin\alpha = BO/AO = \lambda/2b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)},$$

$$\cos\alpha = BO/CO = \lambda/\lambda_g = c/u_p \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}.$$

Squaring both (1) and (2), and adding, gives

$$(\lambda/2b)^2 + (\lambda/\lambda_g)^2 = 1,$$

so that

$$\left(\frac{1}{\lambda_g}\right)^2 = \frac{1}{\lambda^2} - \frac{1}{(2b)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}.$$

Equations (1), (2) and (3) are easily adapted to cover the case of an H_{0n}-wave by replacing b everywhere by b/n, since the distance b now spans n layers of magnetic loops instead of a single layer, and OA becomes b/2n.

Thus, $\sin\alpha = n\,\lambda/2b$ ....(1a),

$\cos\alpha = \lambda/\lambda_g = c/u_p$ ....(2a),

$1/\lambda_g^2 = 1/\lambda^2 - (n/2b)^2$ ....(3a).

11. Cut-Off Wavelength of the H_{01}-wave

If a wave which has a wavelength λ in free space is to be fitted into a tube by reflection back and forth between faces we have seen that it must be inclined so that the distance between nodal planes is equal to 1/nth of the distance b between the faces where n is any integer. Fig. 205 shows that if $\frac{1}{2}\lambda$ is less than b this can be done for n = 1. As λ decreases, the steepness of incidence of the wave will decrease and all wavelengths down to zero can be fitted into the tube. As λ tends to zero the original wave tends to travel along the surface of the metal. Incident and reflected waves tend to coincide and the pattern velocity approximates to that in free space. As the free-space wavelength increases the original wave has to be more steeply incident on the surface of the metal. Reference to Fig. 200 shows that the pattern velocity increases and finally, when the original wave is incident normally on the metal, the pattern velocity is infinite. Under these conditions the incident and reflected waves combine to give a standing wave system with nodal planes at the two metal surfaces. This limit is reached when the distance b is equal to $\frac{1}{2}\lambda$, and waves which have wavelengths greater than

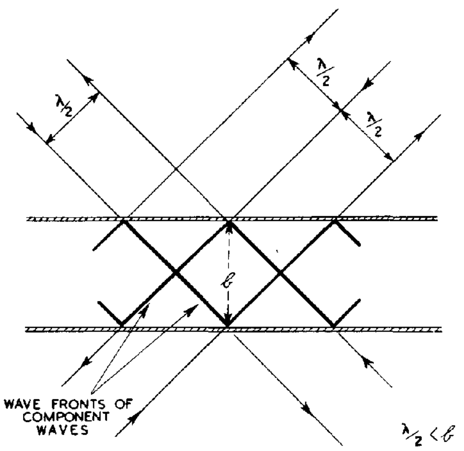

Fig. 205 - H_{01}-wave: relative dimensions.

$$\lambda_c = 2b \quad \text{.......................................} \quad (4)$$

cannot be propagated down the guide. The corresponding frequency, given by

$$f_c = \frac{c}{\lambda_c} = \frac{c}{2b}, \quad \text{..........................} \quad (5)$$

is called the Cut-Off Frequency for the guide and the corresponding free space wavelength λ_c is called the Cut-Off Wavelength. The cut-off wavelength for an H_{0n}-wave is given by

$$\lambda_c = \frac{2b}{n} \quad \text{.......................................} \quad (4a).$$

We have already deduced the following relation (3a) between the free-space wavelength and the pattern wavelength in the guide for an H_{on}-wave :-

$$\left(\frac{1}{\lambda_g}\right)^2 = \left(\frac{1}{\lambda}\right)^2 - \left(\frac{n}{2b}\right)^2,$$

and we now see that this can be written :-

$$\left(\frac{1}{\lambda_g}\right)^2 = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \qquad \text{.................................} \quad (6),$$

which is an expression of very general validity.

12. Evanescent Modes

It is of interest to enquire what kind of disturbance results if we put electromagnetic energy into a guide with a frequency corresponding to a free space wavelength of greater than λ_c. We have just seen that no wave-motion is possible. What then does happen?

In order to examine this problem let us write the equation for a wave of frequency f and wavelength λ_g in the complex exponential form where y is the magnitude of the disturbance at a distance x from the origin.

$$y = y_o \ \varepsilon^{2\pi j(ft - \frac{x}{\lambda_g})} \qquad \text{.........................} \quad (7).$$

In the case where the free space wavelength is greater than the cut-off wavelength $(\lambda > \lambda_c)$ the equation (6) shows us that $\frac{1}{\lambda_g^2}$ is negative so that $\frac{1}{\lambda_g}$ is a purely imaginary quantity which may be written:-

$$\frac{1}{\lambda_g} = \pm j\alpha.$$

The wave motion of equation (7) then becomes :-

$$y = y_o \ \varepsilon^{2\pi j(ft \mp j\alpha x)}$$

$$= y_o \varepsilon^{\pm 2\pi\alpha x} . \varepsilon^{j2\pi ft} . \qquad \text{..................} (8)$$

This expression (8) represents not a travelling wave motion but a disturbance in which the fields at all distances (x) oscillate in the same phase with the frequency f (factor $\varepsilon^{j2\pi ft}$) but their amplitudes die away exponentially with increasing distances (factor $\varepsilon^{\pm 2\pi\alpha x}$). A disturbance of this type is called an Evanescent Mode. Evanescent modes are important for the understanding of the behaviour of obstacles and irregularities in waveguides. The exponential diminution of amplitude with distance also finds an important practical application in the piston attenuator (Sec. 24).

13. Cut-Off Wavelength of Different Modes of H-Waves

Expression (4a) shows that the higher the order of mode (i.e. the larger n) the smaller becomes its cut-off wavelength λ_c in the tube of given dimension b. When λ exceeds λ_c for a certain mode in a given guide any electromagnetic disturbance in this mode must be of the evanescent type.

In a given tube there are no modes with cut-off wavelengths greater than that of the H_{01}-mode. The mode is therefore of great practical importance, for it is possible to convey power down a guide of suitable dimensions by the H_{01}-wave alone and to exclude other modes except where they may be excited in an evanescent form by obstacles or at the source. It is thus possible to radiate from the end of a guide into a mirror in a predictable manner because the distribution of field across the end of the guide is due to the H_{01}-wave alone. There are also other advantages, connected with switching and matching, in retaining only a single travelling wave in the guide.

As a practical example, we consider a waveguide which has been used in practice at a wavelength of about 9 cm. Its sectional dimensions are a = 1"; b = $2\frac{1}{2}$", (Fig. 203). Thus, corresponding to dimension a, the cut-off wavelength of the H_{01}-mode is ${}_a\lambda_c$ = 2" = 5·1 cm., and to dimension b, ${}_b\lambda_c$ = 5" = 12·7 cm. It is possible, therefore, to propagate in this guide an H_{01}--wave at λ = 9 cm. only if the plane of its magnetic loops lies parallel to the edge b, since ${}_a\lambda_c < \lambda < {}_b\lambda_c$. The polarisation of the E vector in the wave is not, therefore, ambiguous. With λ = 9·1 cm. and 2b = 12·7 cm. we find $\alpha = 45^\circ$ and λ_g = 13 cm. The corresponding cut-off wavelengths of the H_{02} mode are ${}_a\lambda_c$ = 1" = 2·54 cm. and ${}_b\lambda_c$ = $2\frac{1}{2}$" = 6·35 cm. This and higher modes are, therefore, evanescent for waves of $\lambda \doteqdot$ 9 cm.

14. Method of Launching an H_{01}-wave in a Rectangular Waveguide

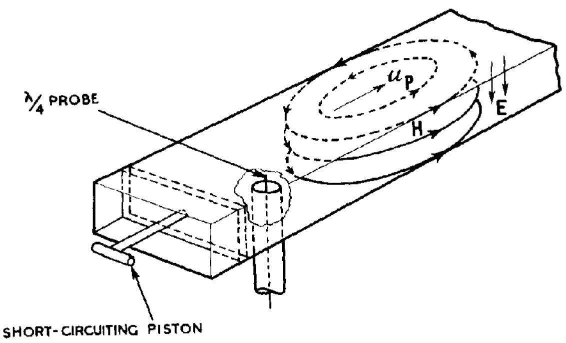

Fig. 206 - Launching an H_{01}-wave.

Fig. 206 indicates the commonest method of launching an H_{01}-wave in a rectangular guide. A probe, usually the extension of the inner conductor of a short length of coaxial transmission line, protrudes through the centre of one of the broad walls of the guide a distance $\lambda/4$ (not $\lambda_g/4$) into the guide. It then radiates as an aerial into the guide space. As it is not generally required to divide the stream of power, the guide to one side of the probe is closed by a short-circuiting piston whose position can be adjusted to cause the short length of guide between it and the probe to act as a matching reactance. The correct position of the piston will be roughly $\lambda_g/4$ from the probe. Near the probe the electromagnetic field is complicated and consists of an H_{01}-mode with evanescent modes superimposed, but at a sufficient distance down the guide the field simplifies to that of a travelling H_{01}-wave, as indication in Fig. 206.

15. Wall Currents in the case of the H_{01}-wave

Surface currents flow on the walls of the guide to support the tangential components of the magnetic field of the wave pattern at the metal surface. We consider the case of the H_{01}-wave.

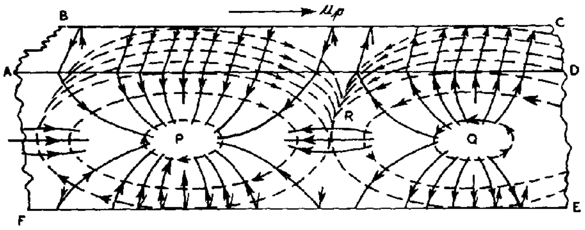

Fig. 207 - Wall currents: H_{01}-wave.

Fig. 207 illustrates the instantaneous directions of flow of the surface currents in the walls of a rectangular guide in relation to the magnetic field. The current flows everywhere at right angles to the magnetic field at the walls. Thus, on the face ADEF to which the magnetic loops are parallel, the currents converge on the area around P and diverge from Q. Thus positive charge is beginning to accumulate around P and negative charge around Q. There are no fields, however, at P and Q. The region around R carries negative charge and the opposite region on the opposite face of the guide carries positive charge. The transverse electric field of the wave is a maximum at the section R but is zero at P and Q. The currents on the wall ABCD and the opposite wall are entirely transverse, as shown, and the whole current pattern is carried along with the rest of the wave pattern at speed u_p.

It is important to appreciate the pattern of current flow in the walls when deciding where a hole or slot may be placed. In a standing-wave indicator, for instance, it may be necessary to cut a long slot in one of the walls parallel to the axis of the guide in order to insert a probe into the field. If the slot were placed in the wall ABCD or its opposite, and ran parallel to AD, it would clearly run across the currents in the wall. The flow of the currents would be disturbed and the slot would radiate into space. The slot may however be cut in the centre of the face ADEF along the line PRQ, so that there are no currents perpendicular to it, and if the slot is narrow the wave within the tube is unaffected by the slot. The radiation of energy from slots in waveguides is further considered in Chap. 17. Secs. 54 - 56.

16. E_{mn}-waves and H_{mn}-waves

By following the procedure of Sec. 7 we may also synthesise an E_{on}-wave capable of propagation between a pair of parallel conducting planes from a pair of simple plane polarised electromagnetic waves. It will suffice to indicate the procedure.

We begin with a plane wave incident obliquely on a metal as in Fig.199 but with the E and H positions of the lines interchanged. Thus, in Figs. 199 (a) and (b) we suppose the vectors E to lie in the plane of the paper and H to stand perpendicular to the plane of the paper. Consequently, in the modified diagram the arrows along the wavefronts represent the direction of E and the circles with a cross or dot within represent H. With the directions of c as shown, however, when the arrows are also drawn as shown, it is necessary to reverse the sense of the H vectors from those indicated by the circles.

In the composite diagram Fig. 199 (c) the closed loops now represent the E-lines and the circles (with their senses reversed) the H-lines. The wave, is, therefore, an E-wave (TM wave). To imprison n loops of this wave pattern between a pair of parallel conducting plates it is again necessary to insert the plates into the pattern in positions such that the electromagnetic boundary conditions are not infringed. Since the closed loops are now E-lines and the transverse lines those of H, the correct positions for the plates are any two of the planes LM, L'M', etc., over which the tangential component of E vanishes and H is entirely transverse. A cross-section of an E_{on}-wave between two plates is shown in Fig. 208 (a), and a view into the advancing wave in Fig. 208 (b).

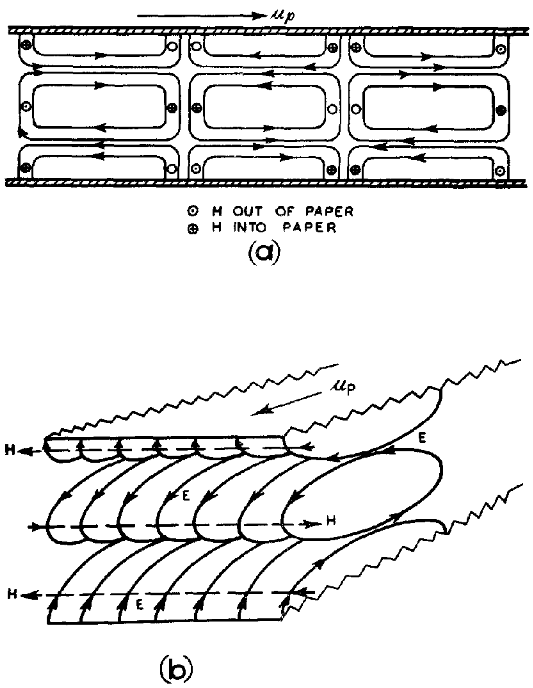

Fig. 208 - E_{on}-wave between parallel plates.

The formulae for cut-off wavelength λ_c and wavelength λ_g between the plates are the same as those for the H_{on}-wave, i.e., equations (1a) to (4a) in sections 10.

When, however, an attempt is made to transform the pair of plates into a tube by putting in side walls the wave system is disturbed since H is normal to these walls and E entirely tangential. Thus, there is no E_{on}-wave in a rectangular tube. The simplest E-wave in a rectangular tube is the E_{11}-wave whose pattern is shown in Fig. 209, but is not derived. Fig. 209 (a) represents a central section of the guide containing the axis and parallel to one of the walls. The electric lines of force are in the form of loops which meet the walls perpendicularly. Along the axis the lines are grouped into bundles. Fig. 209 (b) is a transverse section of the wave pattern at the position P in (a) with the wave approaching the observer. The centre of the section is a source of electric lines which diverge from an axial bundle and terminate on the walls.

The magnetic field is entirely transverse and consists of closed loops surrounding the central bundle of electric lines as shown. The pattern in the section at Q (a) is the same as that shown at (b) but with the directions of the fields everywhere reversed. The central region of the section, therefore, is a "sink" of electric lines instead of a source.

One method (but not the simplest) of deducing the form of the E_{11}-wave pattern is to build up the wave from an original plane wave sent into the tube in a suitable direction so that successive reflections occur from all four walls. The plane of polarisation of the original wave must also be chosen appropriately.

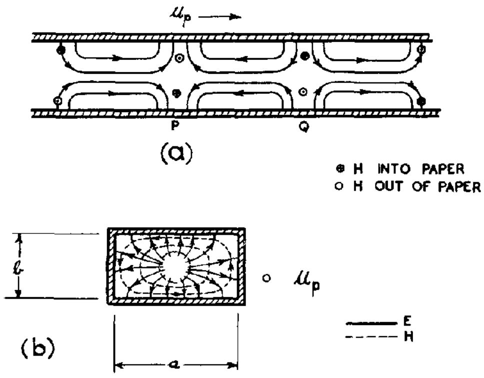

Fig. 209 - E_{11}-wave in a rectangular tube.

It can be shown that if a and b are the cross-sectional dimensions of the guide then the wavelength λ_g is given by :-

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \left\{\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{2b}\right)^2\right\} \quad \dots\dots\dots\dots\dots\dots (9).$$

The cut-off wavelength may be deduced from equation (1), since when $\lambda = \lambda_c$ the wavelength in the guide is infinite.

Hence $$0 = \frac{1}{\lambda_c^2} - \left\{\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{2b}\right)^2\right\},$$

i.e. $$\frac{1}{\lambda_c^2} = \left(\frac{1}{2a}\right)^2 + \left(\frac{1}{2b}\right)^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots (10).$$

The general E_{mn}-wave (TM_{mn}) may be considered as a number mn of E_{11}-waves squeezed into a single tube so that individual patterns fit together to form a single large pattern. For instance, Fig. 210 shows a transverse section of the field pattern of an E_{32}-wave at a position where the E-lines are diverging from the axial bundles. This takes the form of an overall pattern in which the unit is the E_{11} distribution. Further, in any pair of adjacent unit patterns the one has a source at its centre P and the other a sink Q. By interposing metallic partitions these patterns could be isolated into mn independent E_{11}-waves. The suffix n is usually associated with the horizontal dimension a, and m with vertical dimension b. It may be inferred from formula (2) above that, since the equivalent sectional dimensions of one of the constituent E_{11}-waves are a/m and b/n, the cut-off wavelength of the E_{mn}-wave is given by :-

$$\frac{1}{\lambda_c^2} = \left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 \quad \text{.........................} \quad (11).$$

Like the E_{mn}-wave, the H_{mn}-wave can also be considered as an assemblage of unit patterns of a simpler wave, in this case the H_{11}-wave whose sectional pattern is shown in Fig. 211. The corners of the section are sources and sinks of H-lines which run down the tube as bundles located in the corners. Each magnetic line forms a closed loop. The electric lines are entirely transverse as shown.

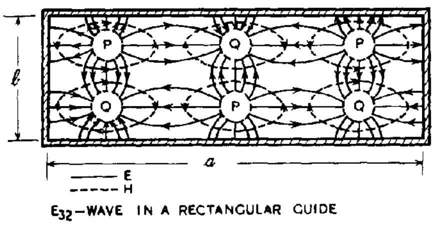

Fig. 210 - E_{32}-wave in a rectangular guide .

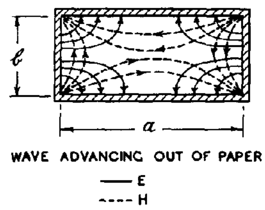

Fig. 211 - H_{11}-wave in a rectangular guide.

The transverse pattern of the H_{mn}-wave comprises mn units each like Fig. 211, in which the fields in adjacent units are reversed. The resultant pattern is not shown.

The formula for the cut-off wavelength of an H_{mn}-wave is the same as that for an E_{mn}-wave, viz., equation (11).

Of the possible E_{mn}- and H_{mn}-waves in rectangular guides only the H_{01} has found extensive practical application, for reasons already given in Sec. 13. Consequently the properties of the general E_{mn}- or H_{mn}-waves will not be further discussed.

WAVES IN CIRCULAR TUBES

17. General

Waves can be propagated in guides whose cross section is other than rectangular. The most commonly used of these other types has a circular cross section. A more detailed treatment of the possible modes of propagation in a circular guide is complicated, but a good insight into the main phenomena involved is obtained by imagining the circular guide to be built up from a deformed rectangular guide of the type previously considered.

18. H_{01}-wave in a Circular Guide

We begin with an H_{01}-wave propagated between a pair of parallel conducting plates as shown in Fig. 212 (a). The plates are next

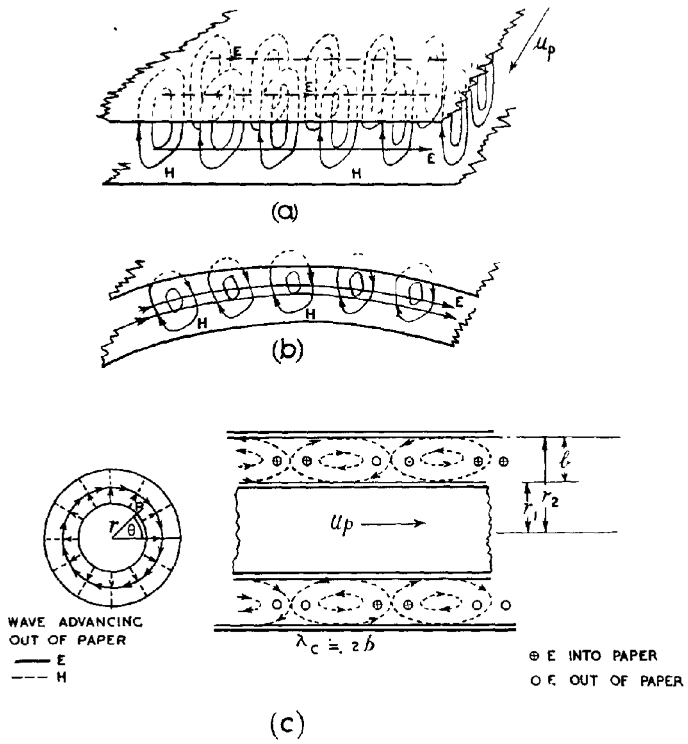

Fig. 212 - H_{01}-wave on a coaxial line.

curved to form a coaxial transmission line, the one plate forming the inner cylinder and the other the outer as shown at (b) and (c). The H_{01}-wave pattern between the plates transforms, on curvature, to an H_{01}-wave between the coaxial cylinders as depicted at (c). This figure represents a type of wave which can be propagated along a coaxial line used as a waveguide. It should not be confused with the entirely different pattern of the TEM wave (Sec. 2), or the principal mode of propagation in a coaxial line.

Suppose the inner cylinder of the coaxial system of Fig. 212 (c) were to shrink, so that it becomes an axial wire. The magnetic loops now virtually touch on the axis but are in fact separated by the wire. Since there is no discontinuity in H at the surface of the wire, no current is needed to support the H-field so that the wire is superfluous and may be removed. There remains a hollow tube with a wave travelling along it. This is the H_{01}-wave for a circular waveguide (Fig. 213). We can obtain a rough idea of the magnitude of its cut-off wavelength as follows. The cut-off wavelength of the wave in the original rectangular guide is 2b and as a result of the deformation of the rectangle the dimension b becomes approximately equivalent to the radius r_g of the circular guide.

Hence we expect the cut-off wavelength to be approximately $2r_g$. Exact calculation shows it to be $1{\cdot}64\ r_g$.

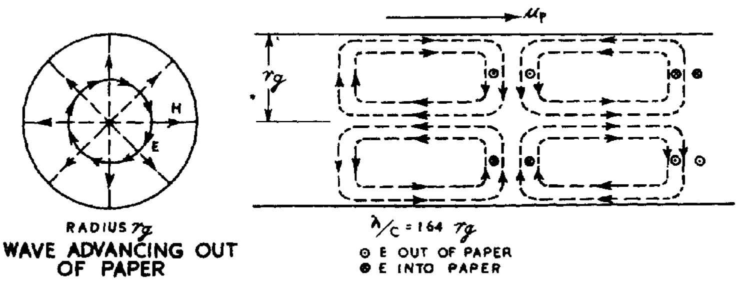

Fig. 213 - H_{01}-wave in a circular guide.

19. Classification of Waves in Circular Guides

It is convenient at this point to remark on the significance of the suffixes in the appellation H01 of the wave which has just been described, and to generalise the nomenclature to deal with H_{mn}- and E_{mn}-waves.

In rectangular guides the suffix zero indicates that none of the field components was changed in a displacement parallel to one of the edge (edge a) of the section (horizontal displacement in Fig. 211). In general, each suffix indicates the number of repetitions of the unit pattern (apart from reversal of field directions) included in the appropriate sectional dimension.

In the case of waves in circular tubes, cylindrical co-ordinates are required. Thus, the position of a point P, Fig. 212(c), within a section of the wave pattern is given in terms of its polar co-ordinates (r, θ). A horizontal displacement across the pattern between the plates of Fig. 212(a) transforms to a circular displacement of P on a circle of constant radius r. Thus, the suffix zero indicates no variation in any component due to a change in θ at a fixed r.

An H_{on}-wave between coaxials is similarly obtained by transforming an H_{on}-wave between plates (Fig. 202) into a cylindrical pattern as described above. This becomes an H_{on}-wave in a circular guide when the radius of the inner cylinder is reduced to zero. The pattern of this wave also exhibits axial symmetry, but contains n layers of H-lines in the radial direction.

We note that the suffix zero of an H_{on}-(or E_{on}-) wave between cylinders or in a circular tube denotes that the wave possesses axial symmetry.

As for waves in a rectangular guide, the symbol H indicates that there is a longitudinal component of H in the wave propagated along the guide, and similarly the symbol E indicates a longitudinal component of E.

An E_{mn}-wave, or an H_{mn}-wave has 2n units of the basic pattern in the variation of θ from 0 to 2 π (i.e. it is divided by n

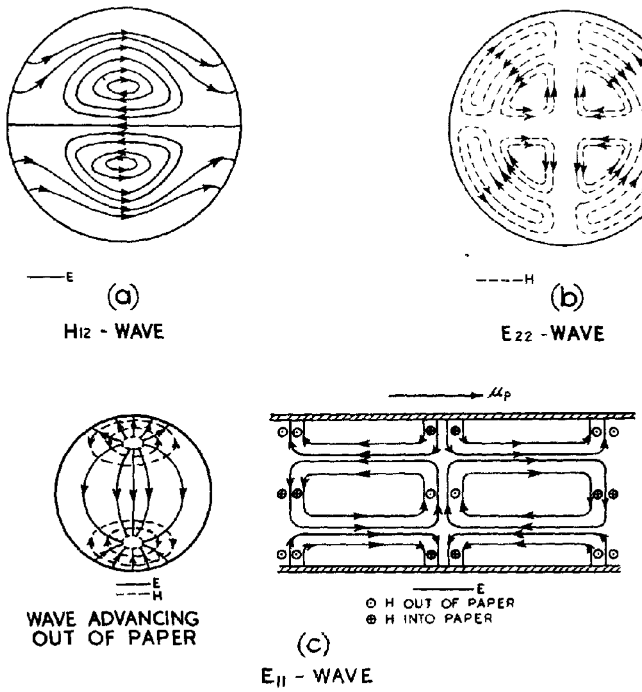

Fig. 214 - Waves in circular guides.

diameters) and m units in the variation of r from 0 to r_g, where r_g is the radius of the guide. As examples of the system of classification Fig. 214 shows cross-sectional drawings of an H_{12}-wave (a) and an E_{22}-wave (b) in a circular guide. Fig. 214 (c) shows transverse and longitudinal sections of an E_{11}-wave.

20. The E_{01}-wave in a Circular Guide

This wave is the analogue of the E_{11}-wave in a rectangular guide shown in Fig. 209. If the guide be given a square section which is then distorted into a circular section, the pattern of Fig. 209 transforms into that of Fig. 215.

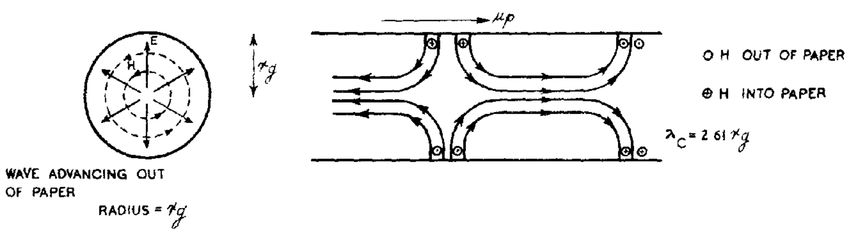

Fig. 215 - E_{01}-wave in a circular guide.

The electric lines are loops whose longitudinal portions form axial bundles which diverge radially to the walls as shown. The magnetic field is entirely transverse and in the form of circular lines about the source of divergence of the E-lines.

The cut-off wavelength of the E_{11}-wave in a square section tube of side b is $\lambda_c = b\sqrt{2}$. If we make the crude identification of radius r_g of the circular section with b/2, half the side of the square, the following approximate formula for the cut-off in the circular guide is obtained :-

$$\lambda_c = 2r_g\sqrt{2} \doteqdot 2{\cdot}8\ r_g\ .$$

The correct formula is $\lambda_c = 2{\cdot}61\ r_g$.

A method of launching the E_{01}-wave in a circular guide is indicated in Fig. 216. The wave is radiated from a probe through the centre of a terminating plate at the end of the guide. The E_{11}-wave in a rectangular guide may also be launched in this fashion.

Fig. 216 - Launching an E_{01}-wave in a circular guide.

The E_{01}-wave is of considerable practical importance since it is commonly employed in rotating joints for use with scanners at wavelengths of 3 cms. Its importance lies in the fact that of the waves with axial symmetry in a circular guide it possesses the greatest cut-off wavelength. Further, it is easily excited by an H_{01}-wave in a rectangular guide feeding through a junction in the wall of the circular guide. It is, however, more convenient to discuss the details of rotatable joints in a separate section (Sec. 42).

21. The Septate Guide

We consider an H_{01}-wave in a rectangular guide polarised as shown in Fig. 217 (a).

The guide is again curved into a coaxial system but this time it is the edge b which is curved. The two original side walls are retained and merged into a single metallic septum which supports the inner cylinder. This structure is called a Septate Waveguide. The field of the wave is distorted as indicated in the figure. The original flat magnetic loops are bent over so that their longitudinal portions run in opposite senses one on either side of the septum. There is thus a discontinuity in H in crossing the septum which therefore carries surface currents to support this discontinuity. The electric field in the wave is entirely radial. It vanishes on each face of the septum and is a maximum diametrically opposite the septum.

The feature of the septate guide that recommends it for practical employment is its relatively large cut-off wavelength. The cut-off wavelength of the prototype guide of Fig. 217 (a) is $\lambda_c = 2b$. The dimension b transforms into the mean circumference of the septate whose cut-off wavelength is therefore given by :-

$$\lambda_c \doteqdot 2\pi\ (r_1 + r_2).$$

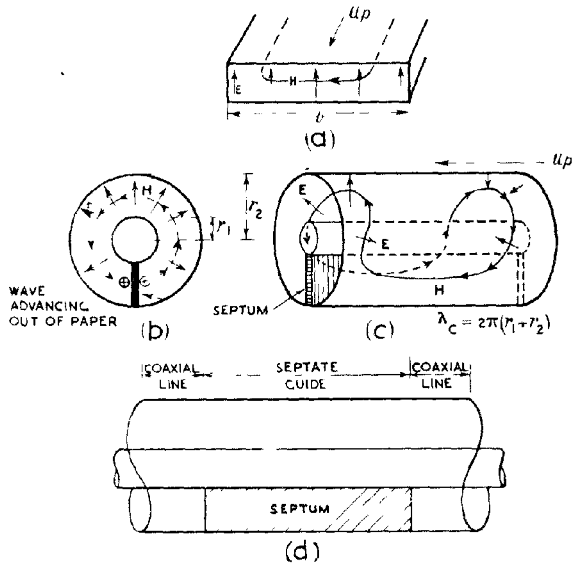

Fig. 217 - Septate guide.

This cut-off wavelength is approximately twice as great as the longest cut-off wavelength of any wave in a circular guide of the same outside radius $r_g = r_2$.

The similarity of the sectional pattern (Fig. 217(b)) of the septate wave to the pattern in the principal wave of a coaxial line permits the septate wave to be easily fed from a coaxial transmission line formed as an extension of the inner cylinder without the septum. The septate pattern also feeds efficiently into a coaxial line (d). Since the principal wave in a coaxial line possesses axial symmetry, the combination of a septate guide with short lengths of coaxial line at each end may be used to provide a rotatable joint at wavelengths of the order of 10 cms because the E_{01}-wave requires a waveguide whose radius is inconveniently large at these wavelengths.

The combination of septate guide and coaxial line is virtually a coaxial line in which the dielectric support for the inner is replaced by the septum. The combination is capable of handling higher powers than a polythene cable.

22. The H_{11}-Wave in a Circular Guide

The stages in the development of the H_{11}-mode, by the method of the preceding sections, are shown in Fig. 218. A pair of rectangular guides carrying identical H_{01}-waves (a) are bent until they merge into a coaxial line with two septa, (c). The progressive distortion of the field may also be followed from the diagrams. Since the magnetic lines on opposite sides of each septum are in the same direction and of equal density, the septa are superfluous and may be removed without affecting the field. The field pattern of this coaxial mode is that shown at (c).

The value of λ_c for each of the prototype guides of (a) is $\lambda_c = 2b$.

The analogue of 2b in (c) is the mean circumference. Thus for this coaxial mode we have $\lambda_c \doteqdot \pi (r_1 + r_2)$. The electric field is entirely transverse and oppositely directed at diametrically opposite points of the section. Of supplementary modes in coaxial cables this mode possesses the greatest cut-off wave-length and is consequently the one that is most readily excited.

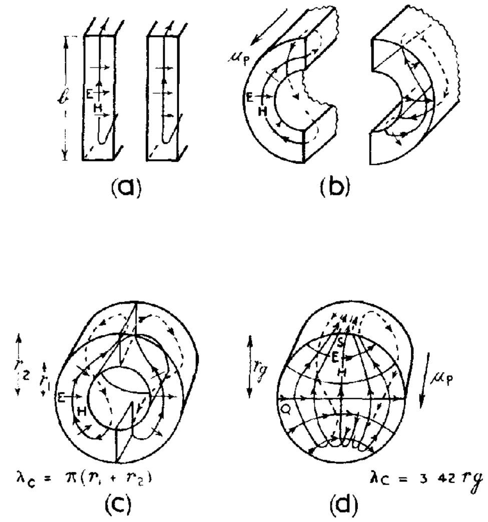

Fig. 218 - H_{11}-wave in a circular guide.

To obtain an H_{11}-wave in a circular guide the inner cylinder of the coaxial is allowed to shrink to zero. The lines of electric force join to bridge the section and the magnetic lines move inwards towards the diameter to give the pattern of figure (d). The inner of the coaxial may be dispensed with since there is no discontinuity in the H-field at the centre and no currents are required there to support the wave. The magnetic lines are loops whose longitudinal portions are collected into bundles above and below but which spread across the section where the magnetic field is transverse.

According to the formula for λ_c in the coaxial line we should expect to obtain λ_c in the guide by putting $r_1 = 0$.

Thus, $\lambda_c \doteqdot \pi r_2$.

The correct formula is $\lambda_c = 3{\cdot}42\ r_g$ where r_g is the radius of the guide.

Fig. 219 illustrates one method of launching an H_{11}-wave in a guide. The inner conductor of the coaxial line is carried across the guide and terminates in another coaxial line with an adjustable tuning piston. This piston together with the movable terminating piston in the guide itself serves to match the guide to the line. Another method employs a simple $\lambda/4$ probe as indicated in Fig.206.

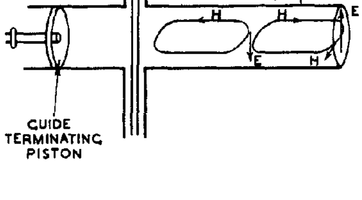

Fig. 219 - Launching an H_{11}-wave.

The H_{11}-wave is the

analogue of the H_{01}-wave in a rectangular guide. It is often used because it possesses the largest value of λ_c of any mode in a circular guide.

ATTENUATION IN WAVEGUIDES

23. Attenuation of Propagated Modes

It has been supposed, in what has preceded, that the walls of the waveguide were perfectly conducting, and consequently that progressive waves were propagated in it with no loss of power. In practice, however, the metal walls possess a large, but finite, electrical conductivity and the currents that flow in them when an electromagnetic wave is propagated along the guide are accompanied by the generation of heat. The energy transformed into heat is abstracted from the power carried by the wave whose field components are therefore attenuated with axial distance z away from the source. In the expressions for the amplitude of each component of the electric field E and of the magnetic field H there must now be included a factor $\varepsilon^{-\alpha_\ell z}$. The ratio of the field amplitudes at two corresponding positions on the cross-section at distance z and (z + d) is therefore $\varepsilon^{\alpha_\ell d}$ and the logarithm of this ratio to the base ε, is $\alpha_\ell d$; this is the loss, measured in nepers, suffered by the wave. The loss per unit length is therefore α_ℓ nepers or $a_\ell = 8{\cdot}686\,\alpha_\ell$ decibels. The loss coefficient depends on the electrical conductivity of the material of the walls, the frequency, the mode of wave being transmitted and the dimensions and geometry of the waveguide. When the loss ($\alpha_\ell \lambda_g$ nepers) suffered in a distance equal to the wavelength λ_g is very much less than unity, as occurs in practice in waveguides with silver, copper or brass walls, the appropriate formulae for α_ℓ may be derived by the following method. It should be noted that much larger losses may arise if a film of moisture is allowed to collect on the walls.

The field pattern is assumed to be the same as for a waveguide formed of perfectly conducting material, with the exception that the amplitudes of all field components are exponentially attenuated in passage down the guide. In other words, apart from this longitudinal attenuation, it is assumed that in waveguides with highly conducting metal walls, the field components in the wave are the same as those in a hypothetical waveguide with perfectly conducting walls; whereas in fact there is a small component of the electric intensity E tangential to the metal surface.

The components of E and H each contain the factor $\varepsilon^{-\alpha_\ell z}$, consequently the power W transmitted over a cross section of the waveguide, which is obtained by integrating the product of the transverse components of E and H over the cross section, contains the factor $\varepsilon^{-2\alpha_\ell z}$

Thus,

$$W = W_0\,\varepsilon^{-2\alpha_\ell z}$$

where W_0 is the power transmitted and

$$\frac{dW}{dz} = -2\alpha_\ell W.$$

But the loss of power per unit length, $-\frac{dW}{dz}$, is the energy dissipated in the walls of the waveguide in ohmic heating. This energy loss per unit length can be calculated separately from the currents in the walls which are directly proportional to the tangential components of the magnetic field at the wall, and in terms of the conductivity of the metal and the frequency. Calling this energy loss per unit length A, we have

$$A = -\frac{dW}{dz} = 2\alpha_{\ell} W.$$

or,

$$\alpha_{\ell} = \frac{A}{2W} \quad \text{nepers per unit length.}$$

Thus, the loss coefficient is found theoretically by calculating the energy lost per unit length in ohmic heating, and the flux of power W over the corresponding cross section. This procedure leads to the following formulae for attenuation coefficients of progressive waves in rectangular waveguides.

The symbols which appear in these formulae have the following interpretations :-

f is the frequency of the wave in cycles per second.

f_c is the cut-off frequency for the wave mode concerned.

a and b are the dimensions of the waveguide cross section measured in metres; the dimension a is associated with the mode integer n and b with m.

R_s is the surface resistance in ohms per unit square of the metal surface; that is, power is dissipated in unit area of the surface at the rate $\frac{1}{2} R_s I^2$ watts, where I is the RMS value of the total surface current in ampères per unit length. R_s is found from the following formula:

$$R_s = 2\pi (10^{-7} \rho . f)^{\frac{1}{2}} \quad \text{ohms per unit square....(1)}$$

where ρ is the specific resistance of the wall metal in ohms per metre cube.

For copper, $\rho = 1{\cdot}6 \times 10^{-8}$ ohms per metre cube; consequently, with copper walls,

$$R_s = 8\pi \times 10^{-8} \sqrt{f}.$$

The surface resistance of any other metal may be obtained by multiplying this value by the factor

$$N = \sqrt{\frac{\rho \text{ metal}}{\rho \text{ copper}}}.$$

α_{ℓ} is the loss coefficient in nepers per metre. To obtain the loss in decibels per metre it is necessary to multiply the value in nepers by 8·686.

Formulae for loss coefficients in rectangular air-filled Wave Guides :

The H_{10}-Wave

(Electric Field parallel to the edge b of the cross section)

$$\alpha_\ell = \frac{R_s}{120\pi b} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}} \cdot \left[1 + 2\frac{b}{a}\left\{\frac{f_c}{f}\right\}^2\right]$$

nepers per metre .... (2).

To obtain α_ℓ for the H_{01}-mode, interchange a and b in this formula.

The E_{mn}-Wave

$$\alpha_\ell = \frac{2R_s}{120\pi b} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}} \cdot \left[\frac{m^2\,(b/a)^3 + n^2}{m^2\,(b/a)^2 + n^2}\right]$$

nepers per metre .... (3).

The H_{mn}-Wave

$$\alpha_\ell = \frac{2R_s}{120\pi b} \left[\frac{\frac{b}{a}\left(m^2 \cdot \frac{b}{a} + n^2\right)}{\left(m^2\left(\frac{b}{a}\right)^2 + n^2\right)} \cdot \left\{1-(f_c/f)^2\right\}^{\frac{1}{2}} + \left(\frac{b}{a}+1\right)\left(\frac{f_c}{f}\right)^2\right] \times \frac{1}{\sqrt{1-(f_c/f)^2}}$$

nepers per metre .... (4).

According to these formulae the attenuation in a rectangular guide depends on

(a) the size of the tube,

(b) the ratio of the cross-sectional dimensions,

(c) the resistivity of the wall ,

(d) the frequency.

We shall not, however, discuss the influence of these factors in detail, because in radar practice the only wave that is employed to carry power through a long run of waveguide is the H_{01}-wave in a rectangular guide and it will suffice to discuss the attenuation of this wave in the actual 2½" x 1" and 3" x 1" waveguides used in service equipments.

The losses in copper waveguides with these standard internal dimensions are calculated as a function of frequency from formula (2). The results are exhibited in the curves of Figs. 220 (a) and (b) which give the loss a_ℓ in decibels per metre suffered by an H_{10}-wave, as a function of frequency f.

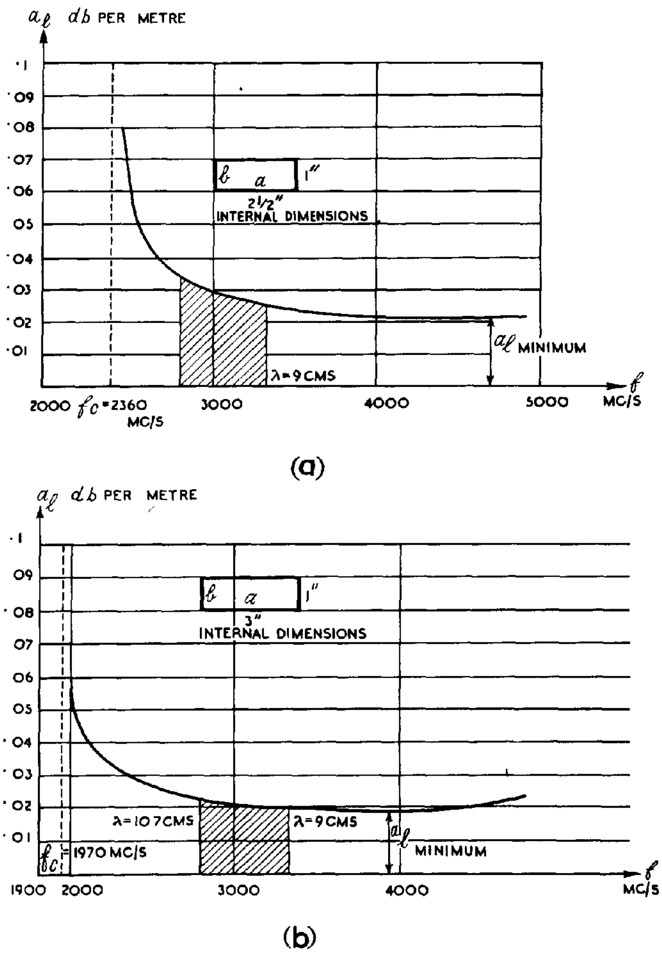

Fig. 220 - Loss coefficient a_ℓ in standard S-band copper waveguides as a function of frequency.

According to formula (2) the loss becomes infinite at the frequency $f = f_c$ due to the factor $1/\sqrt{1 - (f_c/f)^2}$ and also at $f = \infty$ due to the factor $\sqrt{f}$. We may therefore expect a to reach a minimum value, in a given waveguide at some frequency greater than f_c. These properties of the curves are shown Figs. 220 (a) and (b). The asymptotic approach of a_ℓ to infinity as f approaches f_c and the minimum for each curve are indicated.

The values for a_ℓ relate to waveguides with copper walls, but to obtain the loss when another metal is used it is necessary only to multiply the ordinates of the curve by a factor $N = \sqrt{\frac{\rho \text{ metal}}{\rho \text{ copper}}}$, where ρ is the specific resistance.

Thus :-

METAL	BRASS	SILVER	ALUMINIUM
N	2·1	0·97	1·27

With iron, the factor N is

$$N = \sqrt{\frac{\rho \text{ iron}}{\rho \text{ copper}} \times \text{permeability of iron}}$$

Because of the high permeability of iron and steel the attenuation in tubes of these metals is large.

The position of the band of frequencies corresponding to wavelengths from 9 - 10·7 cm. is indicated on each curve.

The smaller $2\frac{1}{2}$" x 1" waveguide, to which Fig. 220 (a) relates, is used at a wave-length of about 9 centimetres, that is at the right hand end of the band shown in the figure, where the attenuation is about 0·027 N db, per metre. For equipments working at wavelengths at the other end of the band, from 10 - 11 cms, this waveguide would be operated on the rising portion of the curve (a). Consequently it is desirable to employ a guide of larger dimensions for which the 10 - 11 cm band is nearer the loss minimum. Curve (b) shows how this is achieved by the use of a 3" by 1" guide. The attenuation at the operating frequencies in both waveguides is therefore approximately 0·025 N db. per metre and in a brass waveguide this is 0·025 x 2·1 =·0525 db. per metre.

The corresponding loss in the standard polythene cable, Uniradio 21, is about 0·6 db per metre; that is, about ten times as great.

Since the attenuation is proportional to the surface resistance it is important to avoid corrosion of the interior surface of the guide due to weather or salt spray. For this reason waveguides are often plated with silver or cadmium internally, or hermetically sealed at their free ends by celophane diaphragms, the air within being kept dry with silica-gel cells which communicate with the interior through small holes in the walls.

24. Attenuation of Evanescent Modes

Consider an arbitrary disturbance, sinusoidal in time, to be excited in a tube whose dimensions in cross-section are small enough to make the longest cut-off wavelength of possible modes much less than the free-space wavelength of the disturbance. The arbitrary disturbance may, in principle, be represented as a set of co-existent evanescent modes whose amplitudes and phases are appropriately chosen. Each mode (mn) decays in amplitude with distance z according to the factor

$$\varepsilon^{-\delta_{mn} z} \quad \text{where } \delta_{mn} = 2\pi\left[\frac{1}{(\lambda_{c,mn})^2} - \frac{1}{\lambda^2}\right]^{\frac{1}{2}}$$

and is the loss coefficient of the mode of order mn. We have postulated however that λ greatly exceeds $\lambda_{c,mn}$; consequently,

$$\delta_{mn} \doteqdot \frac{2\pi}{\lambda_{c,mn}}$$

Thus, the smaller the cut-off wavelength $\lambda_{c,nm}$ the more rapidly does the mode attenuate with distance. At a sufficient distance z from the source of the disturbance only the mode with the largest cut-off wavelength persists, provided its relative amplitude is appreciable at the source. This means in practice that the modes of higher order disappear first.

We conclude, therefore, that at a sufficient distance z, depending on the precise circumstances, the amplitude of all the field components of the residual disturbance decay with distance according to a simple exponential law, i.e.

$$\varepsilon^{-\delta z},$$

where $\delta = 2\pi/\lambda_c$, λ_c being the cut-off wavelength of the most persistent mode. Further, the coefficient δ is independent of the free-space wavelength λ. If the magnitude of the EMF induced by the field in a loop feeding into an output coaxial cable is V_1 at $z = z_1$, but V_2 at $z = z_2$, with the loop similarly situated with respect to the field in the two cases, then

$$\frac{V_2}{V_1} = \varepsilon^{-\delta(z_2 - z_1)}.$$

The reduction of signal strength in distance $(z_2 - z_1)$ is:

$$\log_\varepsilon\left(\frac{V_1}{V_2}\right) = \delta(z_2 - z_1) \text{ nepers},$$

or:

$$20 \log_{10} \frac{V_1}{V_2} = 8{\cdot}686\,\delta(z_2 - z_1) \text{ db}.$$

Thus, reduction in signal strength measured in decibels (or nepers), depends only on the displacement $(z_2 - z_1)$ of the loop in a given tube, δ being a constant, $2\pi/\lambda_c$, independent of frequency. This result is in effect correct for all frequencies which are low compared with the cut-off frequency.

We have, therefore, a method of reducing a signal, say that at $z = z_1$, by any desired number of decibels by a simple linear displacement of a pick-up loop.

This method therefore provides a simple means of producing a voltage which is a given fraction of the input voltage, and this fraction can be altered by a linear displacement of the pick-up loop. Moreover, for wavelengths about 10 cms or less such an attenuator is compact and is suitable for inclusion in test equipment. A device of this nature is called a Piston Attenuator.

CAVITY RESONATORS

25. General

It is found that electromagnetic oscillations can be excited within an empty cavity bounded by conducting walls in much the same way as hollow gas-filled vessels can be excited into acoustical resonance. A given cavity resonator will resonate at a number of discrete frequencies each corresponding to a particular mode of oscillation with its own characteristic electromagnetic field pattern. These field patterns, like those of progressive waves in waveguides, can be classified into E and H types.

Since the free-space wavelength that corresponds to the mode with the lowest frequency is of the order of magnitude of the greatest linear dimension of the cavity, it follows that at centimetre wavelengths a cavity resonator has a small physical size which renders it convenient for laboratory use and for incorporation in equipments. The principal uses for cavity resonators are :

(i) tuned elements in oscillators in place of the conventional L-C-R circuits which are physically unrealisable at centimetre wavelengths.

(ii) accurate wave metres.

(iii) echo boxes, which are equivalent to ringing circuits.

The electro-magnetic oscillations in cavity resonators in the form of hollow rectangular boxes or cylinders have field patterns, with one exception, that are the same as those belonging to standing waves in rectangular and circular waveguides and it is therefore possible to employ elementary methods to deduce many of the important features of cavity resonators.

We begin with the simple case of a rectangular resonator.

26. Stationary Waves in a Waveguide

Consider a rectangular waveguide in which a complete standing wave is produced by allowing two trains of H_{01}-waves with the same wavelength λ_g and field strengths, to be propagated in it in opposite directions. The progress of the individual waves is indicated successively in the first two of each of the Figs. 221 (a), 221 (b) and 221 (c). The third diagram in each case gives the resultant field derived by superimposing the field patterns of the two waves. We take the time t = o to correspond to the instant at which the magnetic loops in the two oppositely travelling patterns superimpose exactly, so that at t = o in the standing wave the magnetic field strength is a maximum at all points as indicated in the third of Figs. 221 (a).

Because the constituent waves travel in opposite senses, the electric fields are in opposition when the magnetic fields are additive. Consequently, the electric fields cancel at t = o, and at this instant the field in the standing wave is entirely magnetic. Fig. 221 (b) illustrates the situation at time $t = T/4$, one quarter of a cycle later, T being the period of the oscillations. The travelling wave patterns in Fig. 221 (b) are displaced one quarter wavelength, $\lambda_g/4$, to the right and left respectively, as can be seen from the displacement of the pair of loops which has been distinguished by a horizontal arrow at the centre. When the patterns are superimposed the magnetic fields cancel but the electric fields add to produce a maximum. Thus at $t = T/4$ the field in the standing wave is entirely electric. Similarly, from Fig. 221 (c) we deduce that at $t = T/2$ the electric field vanishes and that the field is again entirely magnetic but reversed in direction, compared with the field at t = 0. At $t = 3T/4$ we should again find the resultant field to be entirely electric but reversed in direction relative to that at $t = T/4$. At time t = T the field is again that of Fig. 221 (a). At other instants in the cycle the electric and the magnetic components are both present. Further, whatever the field intensity at any instant, the directions of the E and H fields remain fixed and the pattern does not progress to right or left as in the constituent progressive H_{01}-waves. We therefore note the following features of the electromagnetic field for a standing wave.

(i) The field patterns of the magnetic and electric fields are individually the same as those in the progressive waves.

(ii) Whereas in the progressive wave the field patterns are propagated along the axis of the waveguide at speed u_p, but the field intensities in the moving patterns are unchanged, in the stationary wave the patterns are fixed in position but the field intensities oscillate harmonically between maximum positive and negative values.

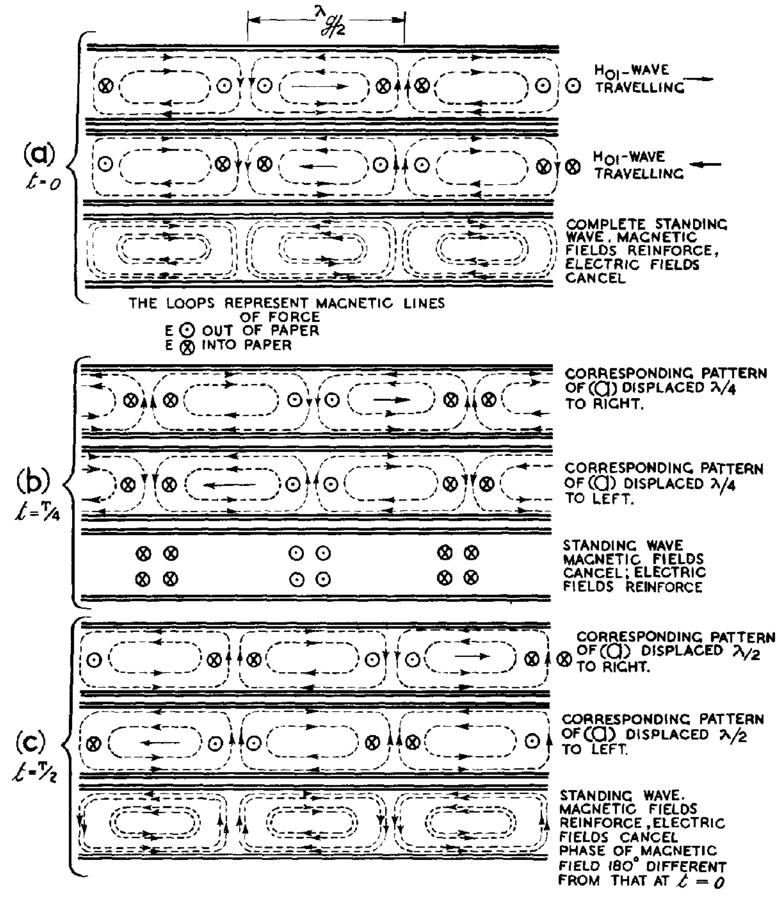

Fig. 221 - Superposition of two oppositely travelling H_{01}-waves of equal strength to produce a complete standing wave.

(iii) In the progressive waves the transverse components of the electric and magnetic fields coincide (near the ends of the largest loops), but in the stationary wave the positions at which the transverse components of the magnetic and electric fields have maximum amplitudes are separated by a quarter of a wave-length ($\lambda_g/4$).

(The electric field is concentrated around the centre of a magnetic loop).

(iv) The resultant electric and magnetic fields within each $\lambda_g/2$ cell of the pattern oscillate in quadrature.

There is no mean flow of power along the axis of the waveguide, but the energy stored in each cell of the pattern is transformed every quarter-period from the magnetic to the electric form, and back again in the succeeding quarter-period. The oscillations of the electro-magnetic field are therefore analogous to the mechanical vibrations of a pendulum or an escapement wheel in which the energy is transformed alternately from one to the other of the kinetic and potential forms.

27. Field Patterns in Cavity Resonators

We have obtained a stationary but oscillating electromagnetic field; we now consider whether this field is one that can exist within a closed cavity with conducting walls. At the walls the field must satisfy the following conditions: the magnetic field cannot intersect any portion of the conducting surface of the cavity; i.e. where it does not vanish it must lie tangentially against the surface: the electric field on the other hand cannot lie along the boundary but where it does not vanish must stand at right angles to the surface. Evidently, as appears from Fig. 222, the standing-wave pattern of Fig. 221 can be fitted into a rectangular box, formed by placing conducting partitions across the waveguide a distance apart equal to $p\lambda_g/2$ where p is an integer. It is then possible to fit p cells of the pattern into the box provided the end walls touch but do not cut a magnetic loop. A possible disposition is shown in Fig. 222 for the case p = 2. Figs. 222 (a) and (b) illustrate respectively the fields at times t = o, when the field is entirely magnetic, and $t = T/4$, when the field is entirely electric.

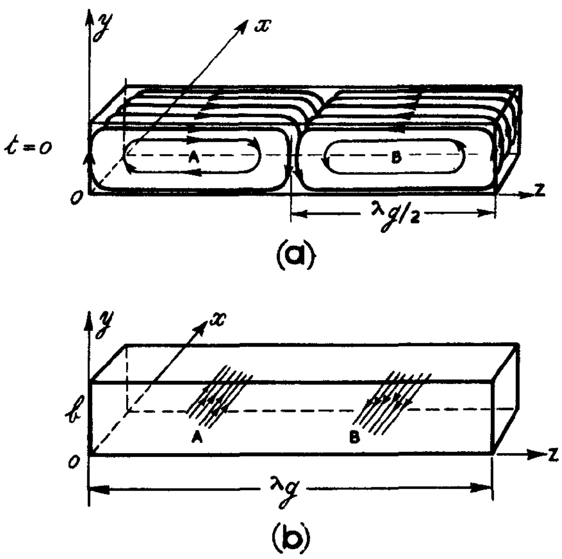

Fig. 222 - Field patterns of the H_{012} mode in a rectangular cavity.

If we choose a set of cartesian axes of reference with the origin O at the near left-hand corner, as shown, then with respect to these axes the constituent travelling waves of Fig. 221 that combine to give the stationary wave are H_{01}-waves. Since p = 2, i.e., two cells of the pattern are fitted into the resonator, and since the mode is derived from progressive H_{01}-waves, it is designated H_{012}.

It is evident that the same procedure will lead to the field patterns of more general modes of oscillation both in rectangular and in cylindrical resonators. We first find the standing wave pattern in the unclosed guide corresponding to the general E_{mn} - or H_{mn} - wave. The guide is then converted to a resonator by the introduction of conducting partitions, at positions such that p cells of the pattern are enclosed within the resonator, without violation of the conditions imposed on the behaviour of the electromagnetic field at the cavity surface. The resulting mode is then designated, according to the type of its constituent progressive waves, an E_{mnp} - or an H_{mnp} - mode.

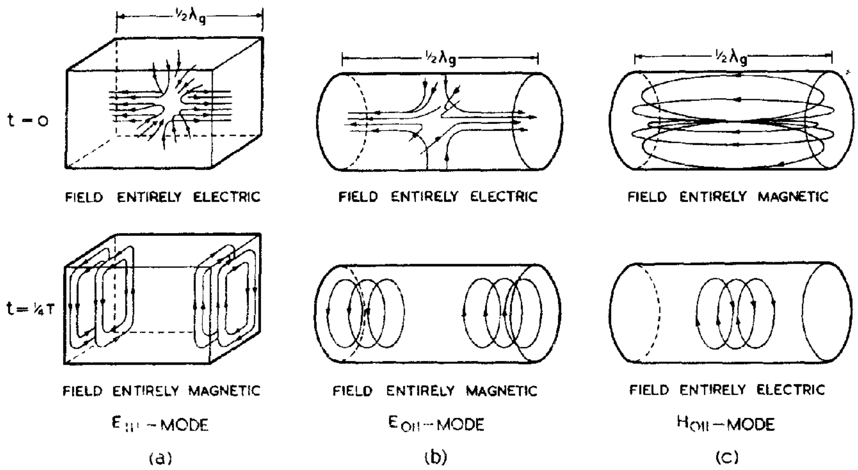

Fig. 223 - Examples of modes of oscillation in cavities.

Examples are shown in Figs. 223 (a), (b) and (c) respectively, of the E_{111} - mode in a rectangular cavity and the E_{011} - and H_{011} - modes in a cylindrical cavity, at times t = o and t = $^{T}/_{4}$. As shown in the earlier example, the magnetic and electric fields oscillate in quadrature and are displaced relatively by a distance $\lambda_{g}/_{4}$ in comparison with their positions in the corresponding patterns of the travelling waves.

28. Resonant Wavelength of the Cavity

Since the E_{mnp} - and H_{mnp} - modes have field patterns the same as those of sets of p elementary cells of the corresponding E_{mn} - and H_{mn} - standing waves in a waveguide, it follows that the length of the resonator must be p $\lambda_{g}/_{2}$, each cell occupying a length $\lambda_{g}/_{2}$ of the guide. If, therefore, the length of the resonator is d, resonance occurs for an n m p - mode when

$$d = p\,\lambda_{g}/_{2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where λ_g is the guide - wavelength of the associated E_{nm} - or H_{nm} - wave in a waveguide whose cross section is identical with that of the resonator. Since, the wavelength λ_g is related to the free-space wavelength λ and the cut-off wavelength λ_c by the equation

$$\frac{1}{\lambda^2} = \frac{1}{(\lambda_g)^2} + \frac{1}{(\lambda_c)^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2),$$

it follows from (1) and (2) that the resonant free-space wavelength of the cavity is given by

$$\frac{1}{\lambda^2} = \left(\frac{p}{2d}\right)^2 + \frac{1}{(\lambda_c)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}.$$

We consider in turn rectangular and cylindrical resonators.

Rectangular Resonators

The linear dimensions of the resonator shown in Fig. 222 along Ox, Oy and Oz are respectively a, b and d. The cut-off wavelength λ_c of an E_{mn} - or an H_{mn} - wave in a rectangular guide whose cross section has linear dimensions a and b is given by

$$\left(\frac{1}{\lambda_c}\right)^2 = \left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}.$$

Finally, the resonant free-space wavelength λ_{nmp} of the E_{nmp} - and H_{nmp} - modes is, from (3) and (4),

$$\left(\frac{1}{\lambda_{mnp}}\right)^2 = \left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2 \quad \ldots\ldots\ldots\ldots\text{(5)}.$$

The resonant frequency is

$$f_{mnp} = \left(\frac{c}{\lambda_{mnp}}\right) = \frac{c}{2}\left\{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right\}^{\frac{1}{2}}, \quad \ldots\ldots\ldots \text{(6)}$$

where c is the velocity of propagation in free space.

(We have assumed throughout that the cavity is empty
(or air-filled)

For example, the resonant frequency of the H_{011}- mode is

$$f_{011} = \frac{c}{2}\left\{\frac{1}{b^2} + \frac{1}{d^2}\right\}^{\frac{1}{2}},$$

and if b > a this is the lowest frequency at which the cavity can resonate. Since there is no E_{om} - or E_{no} - wave in a rectangular guide the E - mode of the resonator with lowest frequency is the E_{111}-mode whose frequency f_{111} is also that of the H_{111}-mode,

$$f_{111} = \frac{c}{2}\left\{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{d^2}\right\}^{\frac{1}{2}}.$$

The resonant wavelength of the H_{011} - mode when the resonator is a cube (a = b = d) is given by

$$\frac{1}{\lambda_{011}} = \frac{1}{2} \cdot \frac{\sqrt{2}}{a}$$

or $\lambda_{011} = \sqrt{2}\,a$, the diagonal of a face. This shows, as mentioned in Sec. 25, that the fundamental wavelength is of the order of magnitude of the linear dimensions of the cavity.

The smallest cube that will resonate at a wavelength of 10 cms (f_{011} = 3,000 Mc/s) has an edge length of 7·07 cm.

Cylindrical Resonators

The cut-off wavelengths λ_c of waves in circular guides are not given by a simple formula such as (5) but depend on the roots of Bessel functions. The cut-off wavelengths in a waveguide of radius r_g, for some of the lower order modes are :-

Mode	H_{11}	E_{01}	H_{01} & E_{11}
λ_c	$3{\cdot}42\ r_g$	$2{\cdot}61\ r_g$	$1{\cdot}64\ r_g$

The resonant wave-lengths of these modes in a cylindrical cavity of radius r_g and length d, are therefore given by

$$\frac{1}{\lambda^2_{mnp}} = \begin{cases} \left(\frac{1}{3{\cdot}42\ r_g}\right)^2 + \left(\frac{1}{2d}\right)^2 \ \ldots\ldots\ldots\ldots \ H_{111}. \\ \left(\frac{1}{2{\cdot}61\ r_g}\right)^2 + \left(\frac{1}{2d}\right)^2 \ \ldots\ldots\ldots\ldots \ E_{011} \ldots (7). \\ \left(\frac{1}{1{\cdot}64\ r_g}\right)^2 + \left(\frac{1}{2d}\right)^2 \ \ldots\ldots\ldots\ldots \ H_{011} \text{ and } E_{111}. \end{cases}$$

29. Charges and Currents on Internal Surface of Resonator

When electric lines of force begin on one portion of the boundary and end on another, they terminate on electric surface charges of opposite sign. For instance, in the resonator shown in Fig. 222 (b) positive charge resides on the region around A and negative charge on that around B, with corresponding compensating charges on the opposite wall. When the magnetic field is present skin currents flow on the interior surface everywhere at right angles to the contiguous tangential magnetic field. Where there is no surface field there is no current. We consider the simple case of the H_{011}-mode in a rectangular resonator at the moment when the field is entirely magnetic (Fig. 224 (a)) and later when it is entirely electric (Fig. 224 (b)). The lines of current flow are shown running perpendicular to the magnetic field at the surface. The current is shown converging towards the central region of face ABCD (Fig. 224 (a)) and away from face A'B'C'D'. These faces become fully charged one quarter of a cycle later, as shown in Fig. 224 (b), and electric lines of force run from the positive to the negative charges.

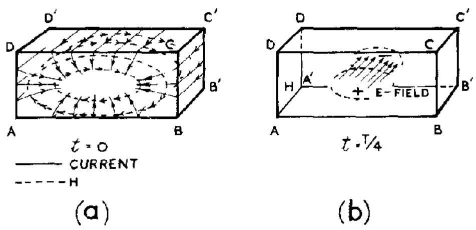

Fig. 224 - Wall currents in a cavity resonator.

30. Methods of Excitation of a Cavity Resonator

A cavity resonator may be excited either by a loop (Fig. 225 (a)) or a probe (Fig. 225 (b)). In the former case the loop must be so introduced that lines of magnetic force can thread through it. Fig. 225 (a) shows two of several possible positions for the loop. As shown, one loop could be an input loop and the other an output loop. The degree of coupling can be controlled by rotation of the plane of the loop.

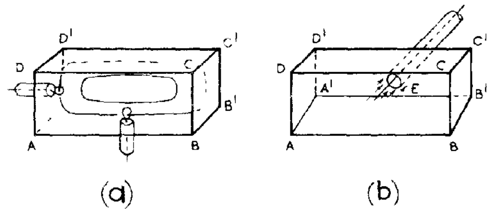

Fig. 225 - Exitation of the H_{011}-mode in a rectangular resonator.

When a probe is used it is introduced into a place of maximum electric field strength and is set parallel to the electric field. Thus in Fig. 199 (b) the probe is shown projecting into the cavity from the face A'B'C'D'.

Excitation to resonance can also be made by means of a slot cut in a face such as to interrupt the flow of current. This face could be made common with the wall of a waveguide from which current could be fed into the cavity.

31. The Q-factor of a Cavity

Because of the finite conductivity of the walls, power is dissipated as heat in the walls, and free oscillations decay exponentially. The Q-factor of the cavity is defined by the expression

$$Q = 2\pi \left(\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}\right) \qquad (6).$$

In practice the walls are made of copper or silver and the energy dissipated per cycle is only a small fraction of the stored energy; consequently Q is very large.

Let W be the energy stored; then the energy dissipated per cycle is $\left(\frac{dW}{dt}\right)T$ and expression (6) may be written:

$$Q = \frac{2\pi}{T} \cdot \frac{W}{\left(\frac{dW}{dt}\right)} = \omega \cdot \frac{W}{\left(\frac{dW}{dt}\right)} \qquad (7).$$

Whence,

$$\frac{dW}{dt} = \frac{\omega}{Q} W \qquad (8).$$

Thus,

$$W = W_o \, \varepsilon^{-\frac{\omega}{Q} t} \qquad (9),$$

where W_o is the stored energy at t = 0.

If, therefore, the resonator is shock-excited and left to oscillate freely, the stored energy is reduced to $1/\varepsilon$ of its initial value in a time $t' = \frac{Q}{\omega} = \frac{QT}{2\pi}$,

so that $$Q = 2\pi\frac{t'}{T} \quad \text{.................................... (10).}$$

According to (10) an alternative interpretation of Q is that it is 2π times the number of cycles required for the stored energy to decay to $1/\varepsilon$ (approximately one-third) of its initial value.

Since Q values of 10^4 and greater are easily achieved, it is evident that a cavity will ring for a great many cycles before the stored energy is reduced to a small fraction of its initial value. A useful approximate rule which gives the order of magnitude of Q in terms of the skin depth δ of the wall currents, and the dimensions of the cavity is given by :-

$$Q \doteqdot \frac{\text{Volume of Cavity}}{\delta \times \text{surface of Cavity}} \quad \text{.................. (11).}$$

A formula for δ in terms of the wave length and wall conductivity is

$$\delta = 2{\cdot}82 \,.\, 10^{-2} . \sqrt{\frac{\lambda}{\sigma}} \quad \text{metres.}$$

where δ and λ are in metres and σ is in mhos per metre cube.

Suppose the resonator to be made of copper for which $\sigma = 5{\cdot}8 \times 10^7$ mhos per metre cube. Then $\delta = 3{\cdot}7 \times 10^{-6}\sqrt{\lambda}$ metres.

According to equation (6) the Q-factor for a cubical resonator of edge a metres, is

$$Q \doteqdot \frac{a}{6\delta} = \frac{a \,.\, 10^6}{6 \,.\, 3{\cdot}7\sqrt{\lambda}} \quad \text{................ (12).}$$

For the H_{011}-mode, $\lambda = a\sqrt{2}$, then,

$$Q = \frac{10^6\sqrt{\lambda/2}}{22{\cdot}2} .$$

For λ = 10 cm (= $\frac{1}{10}$ metre), this becomes

$$Q = 1{\cdot}1.10^5$$

Such a cavity, if shock-excited, would ring for $\frac{Q}{2\pi} \doteqdot \frac{10^5}{6} = 1{\cdot}7 \,.\, 10^4$ periods before the stored energy was reduced to $\frac{1}{\varepsilon}$ of its initial value.

Each period (λ = 10 cm) is $\frac{1}{3000}$ microsecond so that the time of ring is about 5 microseconds (or about $\frac{1}{2}$ mile of radar range).

Because they are highly selective, cavity resonators are used as wave-meters at centimetre wavelengths. This, and other applications of importance in radar are described below.

32. Applications of Cavity Resonators

(i) Wave Meters

Coaxial Line Wave Meter   A convenient wave meter for use at wavelengths of 9-11 centimetres is the coaxial line wavemeter shown in Fig. 226. It comprises a cylindrical cavity into which a rod can be intruded axially to any desired extent by means of a rack and pinion. The metal block serves as a guide for the rod and as a short-circuit to the coaxial transmission line system formed by the cavity and the rod.

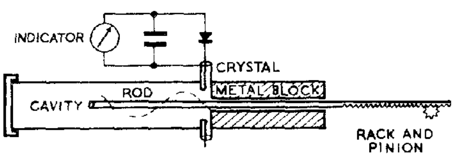

Fig. 226 - Coaxial line wave-meter.

The cavity is excited by injecting an EMF into the input loop from the source whose wavelength is required, and the rod is moved by means of the pinion until a position of resonance is indicated by the detecting crystal and microammeter fed from the output loop. The shortest resonant length ℓ of the rod within the cavity is slightly less than $\lambda/4$ since the transmission line system is open-circuited but with some small end-capacitance. Other resonant positions correspond to $(\ell + n\lambda/2)$ where n is an integer, since the end capacitance remains the same independent of the position of the end of the rod. This is because the end of the rod never closely approaches the closed end of the cavity.

The rack and pinion carry a scale and vernier from which the displacement of the rod can be measured directly in centimetres. The displacement of the rod between successive resonances is equal to $\lambda/2$ and gives directly the wavelength on the coaxial line, which is the same as the free-space wavelength of the source.

The diameter of the cavity is small enough to ensure that hollow cavity modes of oscillation cannot occur.

Resonant Cavity Wave Meter   A wavemeter suitable for measuring changes in wavelength with great accuracy is the cavity wavemeter shown in Fig. 227 (a). It is a metal cylinder whose length can be adjusted by rotation of the screwhead to which the upper end of the cylinder is attached. The resonant mode employed is the H_{011}-mode which is excited by an input loop near the middle of the cavity, as shown, and resonance is indicated by means of an output loop, at the same height but shifted through a quadrant, and a crystal detector and microammeter combination. The magnetic lines of force at resonance are indicated in Fig. 227 (a). To prevent the excitation of the E_{11}-mode, which has the same resonant frequency as the H_{011}-mode, the movable end clears the cylinder wall with a small gap. The currents in the H_{011}-mode flow on the cavity surface in circles about the axis (Fig. 227) and no flow occurs from the flat ends to the curved surface. The current distribution is therefore indifferent to the presence of the gap between the movable flat end and the curved wall. In all other modes, except H_{omn}-modes, current is required to flow from the flat ends to the curved wall; consequently the introduction of the gap effectively suppresses these unwanted modes. Wire filters of suitable form can also be used as suppressors but they are less convenient. In principle the resonant wavelength could be obtained from the dimensions of the cavity at resonance, but in practice it is more

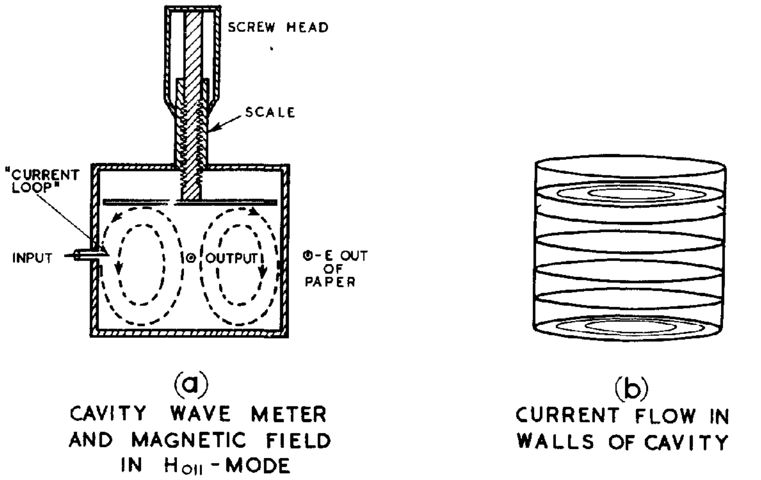

Fig. 227 - **Cavity wave-meter and magnetic field in H_{011}-mode (a). Current flow in walls of cavity (b).**

accurate to measure displacements of the lid by means of a micrometer screw and scale, as indicated. This scale is then calibrated against the harmonics of a crystal oscillator, or, more crudely, against the coaxial line wavemeter described above, using a tunable source of EMF. The wavelength scale is very open and a small change in wavelength corresponds to a relatively large displacement of the movable end. The interior of the cavity is usually silvered so that a large Q and sharp resonance results. The bandwidth of the wavemeter then may be so narrow as to permit examination of suitable RF spectrum of a magnetron pulse (Fig. 228).

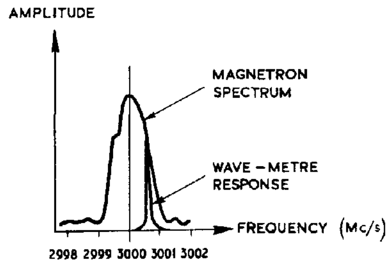

Fig. 228 - **Use of wave-meter for examining magnetron frequency spectrum.**

(ii) <u>Echo Boxes</u>

It is often necessary to check the overall performance of a centimetre wave radar equipment in situations where it is difficult to obtain echoes from objects at suitable ranges. For instance a radar equipment in an aircraft may have its scanner directed downwards so that it is impossible to obtain echoes when the aircraft is on the ground. The echo box is a simple device for checking roughly the overall performance of a set. It is merely a resonant cavity designed to possess a high Q. The cavity is shock-excited by the transmitter pulse and continues to ring and emit a signal which is spread along the time base for an appreciable distance after the cessation of the transmitter pulse. A possible arrangement is shown in Fig. 229. The echo box is fed via a screened low-loss cable from a pick-up probe fixed near the edge of the mirror. The energy abstracted from the transmitted pulse is stored as a resonant mode in the box and is re-radiated to the receiver as an exponentially decaying signal. For the greater part of its duration the signal saturates the receiver but finally decays to a level at which it no longer does so. The appearance on a Type A display is illustrated in the figure. The range at which the echo box response disappears into the noise gives an indication of the overall performance of the set.

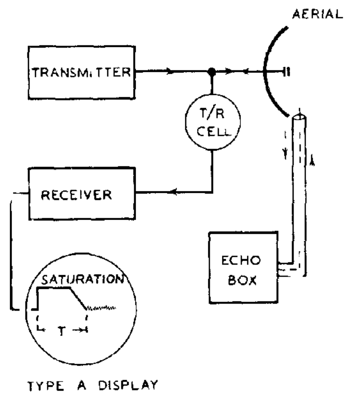

Fig. 229 - Arrangement of radar system with echo box.

Echo boxes are of two types, tuned and untuned. Tuned echo boxes are the same as the wave meter already described and illustrated in Fig. 227. They require tuning to the frequency of the transmitter which then shock-excites the H_{011}-mode. The output loop is not used.

Untuned echo boxes are very large cavity resonators whose lowest modes, H_{111}, E_{111} etc., correspond to wavelengths much greater than the wavelength of the radar set. They are usually hollow cubes with copper walls.

The modes that are excited by the radar transmitter are therefore higher order E_{mnp}-and H_{mnp}-modes where m, n and p are relatively large integers.

Consider a cubical resonator whose edge a comprises several wavelengths.

It can be shown that the spacing of the higher modes is such that the number of modes comprised within the wavelength range λ to $(\lambda + \Delta\lambda)$ or frequency range f to $(f - \Delta f)$ in a rectangular resonator, is

$$N \doteqdot 8\pi . \frac{V}{\lambda^3} . \left(\frac{\Delta\lambda}{\lambda}\right) = 8\pi \frac{V}{\lambda^3} . \left(\frac{\Delta f}{f}\right)$$

where V is the volume of the resonator. Thus, for a cubical resonator of edge a

$$N \doteqdot 8\pi \left(\frac{a}{\lambda}\right)^3 \left(\frac{\Delta\lambda}{\lambda}\right) = 8\pi \left(\frac{a}{\lambda}\right)^3 \left(\frac{\Delta f}{f}\right)$$

Consider the case of a resonator of edge a = 1 metre excited by a magnetron pulse for which f = 3000 Mc/s, (λ = 10 centimetres) and the bandwidth is 1 Mc/s. The formula gives, for the number of modes covered by the bandwidth,

$$N \doteqdot \frac{8\pi . 10^3}{3 . 10^3} = \frac{8\pi}{3}$$

i.e. there are 8 modes. Thus, the transmitter pulse is able to excite a number of resonant modes whatever its frequency f and it is not necessary to tune the resonator. In the case discussed, the mean frequency separation of the modes is $\frac{1}{8}$ Mc/s = 125 kc/s.

Since the ratio of volume to surface of a cubical resonator is proportional to the edge a, these large untuned resonators possess very high Q values and will ring for many microseconds.

(iii) Cavity Resonators in Centimetre Wave Oscillators

The usual resonant circuits, comprising lumped inductances, capacitances and resistances, cannot be constructed in a useful form at wavelengths of 10 centimetres and less, and it is necessary to replace them by other resonant systems such as resonant lengths of coaxial transmission line or resonant cavities. In the klystron oscillator, whose main application is as low-power local oscillator in centimetre wave radar receivers, a resonant cavity is used in which a mode of oscillation is maintained by a bunched electron stream. In order to bring about bunching and to abstract power from the bunched stream, the electrons must pass through the oscillating fields within the cavity against or parallel to the electric field where it is most intense. Further, the time of transit must be small compared with the period of oscillation. It is not possible to accomplish this in any of the resonators described so far because their dimensions are comparable with the wavelength λ and it would be necessary for the electrons to move at speeds comparable with the velocity of light in order that the transit time should be less than the period of oscillation. What is required therefore is a resonant cavity (Rhumbatron) in which an intense oscillating electric field is concentrated across a short path so that electrons can travel the whole extent of a line of force in a time short compared with the period.

In one type of reflector klystron, the Sutton Tube, an example of which is the CV67, the resonator (rhumbatron) assumes the form indicated in Fig. 230 (b). This rhumbatron may itself be regarded as a distorted form of a the prototype shown in Fig. 230 (a). In Fig. 230 (a), A and B represent a pair of coaxial conducting cones (or dimples) with their tips removed so as to form a small gap between them. It is known that when an alternating EMF is applied across the gap the pair of cones forms a transmission line system and that a principal (TEM) wave is guided along them. The lines of electric and magnetic force run as shown in the figure. If a conducting spherical surface C of radius $\lambda/4$, concentric with the middle of the gap, is used to close the cones then the TEM wave is reflected without distortion and a complete standing wave is produced on the transmission line which then forms a resonant system. The equivalent twin transmission line system is shown in Fig. 230 (c). We may, however, regard the system of Fig. 230 (a) as a hollow spherical cavity resonator with a pair of conical projections. It follows from what has been said that the fundamental resonant wavelength of this resonator is four times the radius of the sphere. A voltage antinode is located at the gap AB where the electric field attains its greatest intensity. It is possible therefore to maintain such a rhumbatron in resonance by passing a bunched electron stream across the gap AB. In practice, in the reflector klystron used as a local oscillator, it is necessary to control the resonant frequency by means of external tuning screws; consequently, the rhumbatron is divided in two by a glass tubular envelope which is evacuated and contains the electron gun assembly and the reflecting electrode. The portion of the resonator external to the glass envelope carries the tuning screws. To introduce the glass envelope it is necessary to distort the shape of the rhumbatron to that

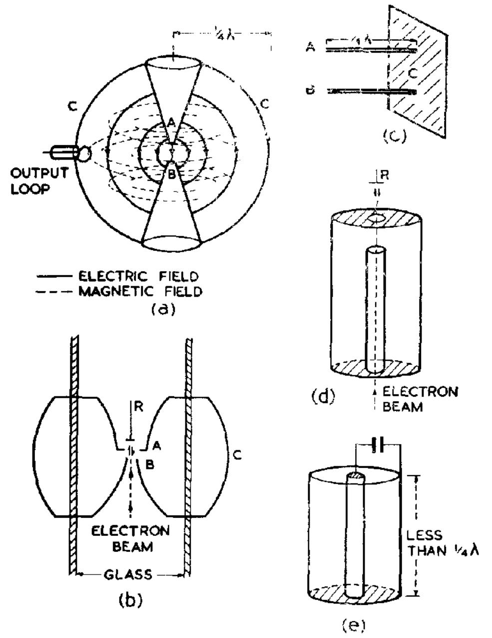

Fig. 230 - Common types of resonators.

shown in Fig. 230 (b). One of the cones is also distorted to bring the reflector close to the gap. Power is abstracted through a loop placed, as shown in Fig. 230 (a) with its plane parallel to the axis of the cones.

The input impedance to the transmission line system shown in Fig. 230 (a) is large at resonance, and it is necessary to drive the rhumbatron from a high impedance source. Thus the power supplied by the electron stream must be in the form of relatively high voltage electrons and relatively small current. Thus, in the CV67, the accelerating voltage is 1200 volts and the electron current is 6 milliamps. The maximum power output at $\lambda = 9$ centimetres is 200 milliwatts which is ample for a local oscillator.

The high operating voltage is an inconvenience and a more convenient form of klystron operates on a voltage of 300 and is therefore able to use the same power pack as the radar receiver. The resonator here comprises a short - circuited coaxial transmission line with a gap between the inner and the end plate of the outer at the top, as shown in Fig. 230 (d). This end plate has a hole in its centre, and both this hole and the end of the inner are covered by a wire gauze through which the electron stream passes into and out of the gap. The length of the transmission line is less than $\lambda/4$ and it is brought to resonance by the capacitance between the free end of the inner conductor and the end plate.

The equivalent resonant system is shown in Fig. 230 (e). The shunting impedance of this system is much lower than that of the double cone system and the power carried by the electron stream can be supplied at lower voltage and larger current.

The operation of klystrons is discussed more fully in Chap.8, Secs.22 and 23.

Various types of cavity resonators which may be employed in the Resonant Cavity Magnetron are illustrated in Chap. 8 Secs. 27 and 31, where the many possible modes of oscillation are discussed.

CHOICE OF WAVEGUIDE SHAPE AND DIMENSIONS

33. General Considerations

In foregoing sections we have reviewed the principal features of electromagnetic disturbances in metal tubes. It remains to describe how this basic knowledge is applied to useful ends.

The primary function of a waveguide is to convey electromagnetic power from a generator to an aerial or from an aerial to a receiver, at wavelengths of ten centimetres or less. In fact, waveguides are required to perform the same functions at these wavelengths as are performed by transmission lines at longer wavelengths.

The principal problems of transmission line practice are, to produce a reflectionless termination (matching), to achieve aerial switching and a satisfactory system of common aerial working (common T/R) and to measure standing wave ratios (impedance measurements). To achieve these objectives certain ancillary devices such as reactive stubs, quarter-wave insulators, quarter- and half-wave transformers and the like, are employed. In what follows we shall discuss the parallel problems of waveguide practice and investigate the character of the analogues of the ancillary devices.

When waveguides were first recognised as providing the most suitable method of conveying large powers from a source to an aerial at wavelengths of 10 cms and less, it became a matter of practical importance to answer correctly the following questions :-

(i) what is the most suitable size for a waveguide operating on a specified wavelength ?

(ii) what is the best geometrical form to give it ?

We shall consider these queries in the following sections.

34. Choice of Waveguide Dimensions

Transmission line practice is based on the fact that at the usual frequencies progressive waves exist only in the principal or TEM mode, all other modes being evanescent. In order to adapt transmission line practice to waveguides it is therefore necessary to ensure that progressive waves are present in one mode alone, all other modes in the waveguide being evanescent. This result is achieved by so choosing the cross-sectional dimensions of the guide that the free-space wavelength of the wave to be propagated is less than the greatest of the cut-off wavelengths of the various modes in the waveguide, but greater than those of all other modes. In this way the mode with the greatest cut-off wavelength is present as a progressive wave, but all other modes are evanescent.

For instance, in the example given in Sec. 13, the longer cross-sectional dimension of a rectangular waveguide designed to carry waves whose free-space wavelength λ is 9 cms, is $b = 2\frac{1}{2}$ inches. The cut-off wavelength of this guide for the H_{01}-mode is $\lambda_c = 2b = 5$ inches $= 12{\cdot}7$ cm. The H_{01}-wave is therefore propagated in this waveguide, but as the cut-off wavelengths of all other E_{mn}- or H_{mn}-modes, given by :-

$$\frac{1}{\lambda_c^2} = \left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2$$

when $a < b$, are less than $\lambda = 9$ cms, these modes are evanescent.

35. Geometrical Form of Waveguide Section

The choice, in practice, lies between circular and rectangular sections and it is found that, except in specialised devices such as rotary joints, circular sections are unsatisfactory.

For instance, the wave in the circular waveguide would normally be an H_{11}-wave since this mode possesses the greatest cut-off wavelength. Should, the tube however, be slightly elliptical over an appreciable length the H_{11}-wave would then resolve into two waves with different values of u_p, so that on recombination where the tube again assumes a circular section, the plane of polarisation in the resulting H_{11}-wave is rotated with respect to the original plane. This difficulty of preserving the polarisation of the field pattern over a long run of waveguide is a disadvantage of circular guides. Consequently, when circular waveguides are employed they appear only in short and straight lengths, and in practice long runs of waveguide are almost invariably rectangular in cross section.

The dimensions a and b of the rectangular cross section are always made unequal in such guides. The smaller dimension a is such that 2a is less than the wavelength of the H_{01}-mode, but the dimension b accepts the H_{01}-mode as a progressive wave but no other modes. Thus the field pattern preserves a unique sense and can be propagated only with the transverse electric field parallel to the shorter edge a. The following are dimensions of typical waveguides:-

Free-space wavelength	Internal Guide Dimensions
10 cm.	3" x 1"
9 cm.	2½" x 1"
3·3 cm.	1" x ½" (British)
	0·9" x 0·4" (American)

It is undesirable to operate the waveguide near the cut-off frequency of the H_{01}-wave since dispersion (that is, the dependence of u_p on frequency) is very marked near the cut-off frequency (Sec. 6). Also there is a marked increase in attenuation as the cut-off frequency is approached (Sec. 23).

In a long run of waveguide dispersion can affect the shape of a pulse. The dimensions given in the table above remove the cut-off wavelength well away from the operating wavelength; for instance, in the 9 cm. guide whose dimensions are 2½" x 1", the angle of incidence $(90^\circ - \alpha^\circ)$ (Fig. 204) of the waves successively reflected between the walls is 45°, whereas at cut-off it is zero.

STANDING WAVES IN WAVEGUIDES

36. Wave Impedance

It is shown in Chap. 17 Sec. 7 that when a plane-polarised electromagnetic plane wave is propagated in free space, the wave front is presented with the equivalent of a resistance whose magnitude is given by

$$R_o = 120\,\pi \text{ ohms.}$$

This is called the Wave- or Field-impedance. If E is measured in volts per metre and H in amps. per metre then this may be written

$$\frac{E}{H} = R_o = 120\,\pi \text{ ohms.}$$

A similar result holds for waves propagated in waveguides. Considering the H_{on}-wave of Sec. 7, depicted in Figs. 199 and 200, there are seen to be two components of the H-field, one along the guide and the other transverse (vertical) whilst there is a single transverse component of the E-field perpendicular to the plane of the paper. If axes x, y, and z, are taken as indicated in Fig. 195 these components are given by:-

$$H_y = 2H \cos\alpha \sin\left(\frac{2\pi y}{\lambda} \cdot \sin\alpha\right) \cos\left(\omega t - \frac{2\pi z}{\lambda}\cos\alpha\right)$$

$$H_z = 2H \sin\alpha \cos\left(\frac{2\pi y}{\lambda} \cdot \sin\alpha\right) \sin\left(\omega t - \frac{2\pi z}{\lambda}\cos\alpha\right)$$

$$E_x = 2E \cdot \sin\left(\frac{2\pi y}{\lambda} \cdot \sin\alpha\right) \cos\left(\omega t - \frac{2\pi z}{\lambda}\cos\alpha\right)$$

H, E, are the amplitudes of the H- and E-fields of the component waves ; α is the angle between the direction of propagation of the component waves and the reflecting plates (Fig. 200).

Whereas H_y and E_x oscillate in phase, H_z is in quadrature with the other two, and may be considered as supplying the reactive or storage component of the resultant fields in the guide. The other two components convey the energy along the guide. By division we obtain

$$\frac{E_x}{H_y} = \frac{E}{H} \sec\alpha.$$

Since $\frac{E}{H}$ is the ratio of E-field to H-field in a plane polarised electromagnetic plane wave in free space, of the type considered in Chap. 17 Sec. 7, its value is $120\sqrt{\pi} \doteqdot 377$ ohms.

Hence

$$\frac{E_x}{H_y} = 120\sqrt{\pi} \sec\alpha \doteqdot 377 \sec\alpha \text{ ohms.}$$

Since, for an H_{on}-mode, $\sec\alpha = \frac{\lambda g}{\lambda}$, this equation may be written

$$\frac{E_x}{H_y} = 120\sqrt{\pi}\,\frac{\lambda_g}{\lambda}.$$

In general the equation $\frac{\text{E transverse}}{\text{H transverse}} = 120\sqrt{\pi}\,\frac{\lambda_g}{\lambda}$ holds for all H_{nm}-waves propagated in rectangular guides. The corresponding result for E_{nm}-waves is

$$\frac{\text{E transverse}}{\text{H transverse}} = 120\sqrt{\pi}\,\frac{\lambda}{\lambda_g}$$

(In this case it is the E-field which is resolved into two components, whilst the H-field is entirely transverse, so that the effective ratio $\frac{E}{H}$ is less than that for free space).

This ratio $\frac{\text{E transverse}}{\text{H transverse}}$ is called the Wave Impedance for the mode considered, and has a uniform value at all points of the wave front of a progressive wave.

If a waveguide is terminated in a resistive film whose resistance per unit area is equal to the wave impedance the guide is properly matched for the mode considered (see Sec. 37 and 38). For any other termination some reflection occurs and standing waves are developed.

Whereas it is not usually practicable to measure the effective impedance of the termination, the standing wave ratio may be determined by means of a standing wave indicator, and various means may be adopted of eliminating the standing waves on the main run of the guide. Since for a single mode in a given guide the SWR and the position of nodes and antinodes gives a sufficiently complete description of the standing wave pattern for practical purposes, it is possible to apply the same techniques for dealing with standing wave problems in waveguides as are used for transmission lines. In particular, circle diagrams may be used, the value of the normalised impedance (Chap.4, Sec.48) indicating the position in the guide relative to the nearest E-node or antinode for a given SWR. Since we are not interested in the actual impedances in the guide it is not necessary to use any but normalised impedances and admittances, for which the same symbols will be used as for transmission lines.

Where circle diagrams are used to illustrate waveguide properties it is assumed that the reader is familiar with the treatment of circle diagrams given in Chap. 4.

37. Reflection from the Waveguide Termination

As in the case of transmission lines, unless a waveguide is properly terminated, energy will be reflected and standing waves will arise. The ideal termination must have an impedance equal to the characteristic impedance of the guide for the mode considered. If the normalised impedance presented by the termination is $\boxed{z_\ell}$ the fraction of the wave reflected is given by :-

$$\rho = \frac{\boxed{z_\ell} - 1}{\boxed{z_\ell} + 1} .$$

ρ is called the reflection coefficient (compare Transmission Lines, Chap. 4, Sec. 9). If $\boxed{z_\ell} = 1$ the waveguide is properly matched and no reflection occurs. If the guide is terminated in a perfectly conducting metal sheet, $\boxed{z_\ell} = 0$ and $\rho = -1$, so that the termination acts as a short-circuit and at the termination the E-field of the reflected wave is in opposition to the E-field of the direct wave. If $\boxed{z_\ell} = \infty$, the termination acts as an open-circuit, and at the termination the H-field of the reflected wave is in opposition to the H-field of the direct wave. In waveguides, however, if the end of the guide is left open, the terminating impedance is not infinite but possesses a resistive component, due to the radiation from the end of the guide, and a reactive component due to evanescent modes excited at the discontinuity; (see Sec. 42).

The equivalent of an open-circuit termination is provided by a short-circuited $\frac{\lambda g}{4}$ - section of the guide (compare Chap. 4, Sec. 26). The input impedance of such a section, if losses are neglected, is infinite.

A theoretically perfect match is obtained for any mode if the guide is terminated in a resistive film whose resistance per unit area is equal to the wave-impedance for that particular mode in the guide. The film must be backed by a short-circuited $\frac{\lambda_g}{4}$-section of the guide so that the finite impedance at the open end does not appear in parallel with the terminating impedance. Other methods of correctly terminating a waveguide are discussed in the next section.

Where the guide is improperly terminated, so that a standing wave occurs, the standing wave ratio is given by :-

$$S = \frac{\hat{E}}{\check{E}} = \frac{\hat{H}}{\check{H}} = \frac{1 + |\rho|}{1 - |\rho|}$$

where $\hat{E}$, $\check{E}$ and $\hat{H}$, $\check{H}$ are maximum and minimum values of the transverse components of electric and magnetic fields respectively, measured at corresponding positions of the guide cross-section.

38. Practical Methods of Achieving a Reflectionless Termination

In order to test a waveguide run for reflections at joints and bends and to measure the impedance of obstacles, stubs and slots, it is necessary to be able to terminate the waveguide in a reflectionless termination. Any reflected wave then appearing in the guide must be due to some cause other than the termination.

A practical form of termination that gives negligible reflection and is suitable for work at low powers is the wooden load. Wood, as a material, strongly absorbs electromagnetic radiations at centimetre wavelengths; consequently if a wooden wedge of either of the forms of Fig. 231 is inserted into the end of a rectangular waveguide which it fits closely, the two component plane waves of the H_{01}-wave in the guide enter the wood at the sloping face and penetrate into its interior where they are absorbed so that there is little reflection at the surface.

The sloping faces of the loads are parallel to the electric vector of the incident H_{01}-wave, consequently Fig. 231 represents a view of the broad face of the waveguide. That portion of the wave which is reflected from the wood surface continues to travel along the guide in the same direction, obeying the optical law

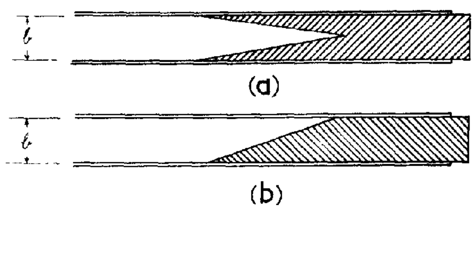

Fig. 231 - Reflectionless terminations.

Angle of Incidence = Angle of Reflection,

and undergoes successive reflections until the energy is almost completely absorbed. To avoid appreciable reflection the sloping faces should not be less than $2\lambda_g$ in length; the SWR in the guide is then of the order 1·05:1.

In a circular guide the reflectionless load is a long cone with a cylindrical butt-end. As an alternative to wood, a composition of bakelite and graphite is sometimes used, and for work at high powers a mixture of graphite and sand is employed.

In both rectangular and circular guides there is an optimum value for the angle between the waveguide walls and the sloping surface of the load.

39. Attenuating Section

In measurements of standing wave ratios, in order to determine the reflection coefficients and impedances of waveguide elements and loads it is essential to isolate the generator from the standing wave indicator and the termination in order to avoid variations in the load on the generator which could affect both its frequency and power output. This is achieved by the inclusion of an attenuating length of waveguide which is merely a length of normal rectangular guide with a thin strip of absorbing material, wood or bakelite impregnated with graphite, fixed internally to one of the narrow faces. The attenuation of some 10 db. in the signal which is thus achieved is sufficient to isolate the generator. No reflected wave of any appreciable amplitude then reaches it.

40. Reflection from an Irregularity in a Waveguide

If an obstacle or irregularity is introduced into a waveguide, electromagnetic energy is reradiated or "scattered" from it so that a wave is reflected back along the guide (Fig. 232). Another wave is generally reradiated from the obstacle away from the source, but this does not interfere with the input admittance of the guide and is, for the moment, ignored. The obstacle may thus be regarded as introducing an additional impedance or admittance either in series or in parallel with the impedance already present. Whether this impedance must be considered as a series or as a shunt impedance depends on the shape and position of the obstacle in relation to the fields and walls of the guide.

A theoretical investigation, into which we shall not enter here, leads to the conclusion that when the discontinuity scatters symmetrically (that is, the transverse electric vectors in the wave fronts of the two waves scattered in opposite directions are equal and similarly directed at equal distances from the scattering section) then the discontinuity behaves like an impedance (or admittance) placed in shunt across a transmission line. If, however, the scattering is anti-symmetrical (amplitudes at equal distances are equal but the electric vectors are oppositely directed) then a series representation is required.

Obstacles possessing geometrical symmetry when placed with an axis of symmetry parallel to the electric vector, scatter symmetrically and therefore behave as shunt elements.

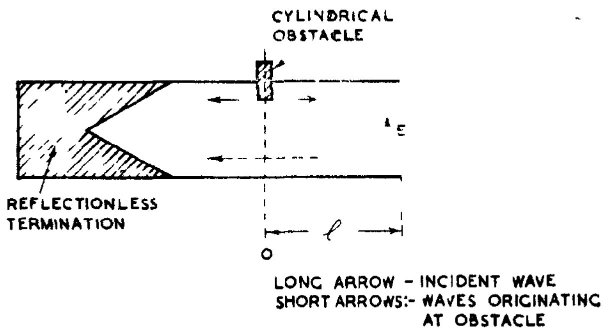

Fig. 232 - Scattering of waves by an obstacle in a rectangular guide.

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Determination of the Admittance of an Obstacle

Consider a rectangular waveguide (Fig. 232) terminated in a reflectionless load, within which a metallic obstacle has been placed. The obstacle may be, for instance, a thin metal cylinder that projects a short distance into the waveguide, with its axis parallel to the short edge of the cross-section. When the cylinder is withdrawn the electromagnetic field in the guide is merely that of the H_{01}-wave travelling from the generator to the load.

When the obstacle is introduced at the dotted section it distorts the field of the H_{01}-wave in its vicinity because the component of the electric field tangential to the surface of the obstacle must vanish. In the particular case of the cylinder the electric lines of the H_{01}-wave which themselves run parallel to the short edge of the cross-section are also parallel to the axis of the cylinder. The distorted field near the dotted section can be resolved into the following constituents :-

(i) The original H_{01}-incident wave (indicated by the long arrow in Fig. 232).

(ii) A pair of H_{01}-waves originating at the obstacle, the one propagated towards the generator and the other towards the load (these waves are indicated by short arrows in Fig. 232).

(iii) A series of evanescent modes which are prominent near the obstacle but disappear at a sufficient distance from it. These form the storage field.

The relative amplitudes and phases of the components (i), (ii) and (iii), of the field are such that the tangential electric field is zero both over the surface of the obstacle and at the walls of the waveguide. Take the cross-section of the waveguide at which the cylinder is introduced as the section $\ell = 0$. At another section at a sufficient distance ℓ away towards the generator, the evanescent waves will have become unimportant and the electromagnetic field reduces to that of the incident wave, $A \cos(\omega t + k\ell)$ with that of the reflected wave $B \cos(\omega t - k\ell + \phi)$ superimposed. The resultant field E is given by the equation

$$E = A \cos(\omega t + k\ell) + B \cos(\omega t - k\ell + \phi),$$

$$= A \left[\cos(\omega t + k\ell) + |h| \cos(\omega t - k\ell + \phi)\right],$$

where, $B/A = |h|$ and $k = 2\pi / \lambda_g$. Or in complex notation,

$$E = A\, \varepsilon^{j\omega t} \left(\varepsilon^{jk\ell} + h\varepsilon^{-jk\ell}\right),$$

where $\quad h = |h|\, \varepsilon^{j\phi}.$

Thus from the standpoint of the section at position ℓ, the section at $\ell = 0$ possesses a reflection coefficient $h = |h|\, \varepsilon^{j\phi}$, when the obstacle is in position, and an associated admittance $y_t = (1 - h)/(1 + h) = G_t + jB_t$.

When the obstacle is removed the reflection coefficient of the section $\ell = 0$ becomes zero, and its normalised admittance becomes unity.

The coefficient h = $|h| \epsilon^{j\phi}$ is the reflection coefficient of the obstacle for a single progressive wave incident on it.

To find the individual admittance $\boxed{y_1}$ of an obstacle of arbitrary shape it would be necessary to measure it experimentally. The most direct method is to terminate the waveguide in a metal plate and to introduce the obstacle at an E anti-node in the standing wave pattern. At this place the admittance $\boxed{y_t}$ of the section is zero and becomes $\boxed{y_t} = \boxed{y_1}$ when the obstacle is introduced. The standing wave pattern between the obstacle and the generator is now investigated with a standing wave indicator and the admittance $\boxed{y_1}$ determined from it.

Alternatively, the waveguide can be terminated in a reflectionless termination and the obstacle may be introduced at any convenient position. The impedance of the section then becomes $\boxed{y_t} = 1 + \boxed{y_1}$.

Obstacles that do not themselves absorb power possess admittances $\boxed{y_1}$ that are pure susceptances, $\boxed{y_1} = \pm j \boxed{B_1}$. When the susceptance is positive it is termed capacitive and when negative, inductive.

41. Standing Wave Indicator

The deleterious effects of standing waves in transmission lines have been discussed in Chap. 4, Sec. 32. Whilst standing waves may be introduced deliberately into short sections of waveguide for matching purposes, their presence in general is equally undesirable, and for similar reasons, as in the case of transmission lines. It is therefore usual to incorporate in the guide some device for detecting the presence of standing waves so that matching systems can be adjusted to minimise them.

As in coaxial lines, the measurement of the standing wave ratio in a waveguide normally necessitates the insertion of a probe or loop into a slot or series of holes in the guide. Alternatively apertures may be made in the guide so that small, evanescent waves are formed outside the guide, indicative of the field strength inside. Care must be taken that the energy radiated through the slot or aperture is negligible, and that the irregularity introduced into the waveguide does not appreciably disturb the mode of propagation (see Sec. 15). The field strength is indicated by a suitable form of galvonometer, such as a neon lamp, valve voltmeter or crystal detector and microammeter. The variation in field strength at different points along the guide indicates the presence of standing waves, and the ratio of maximum to minimum amplitudes gives the standing wave ratio,

If suitably designed, the standing wave indicator may be calibrated so that it fulfils the dual role of measuring both standing wave ratio and also absolute field strength.

A novel form of standing wave indicator, suitable for operation at wavelengths of the order of three centimetres, is illustrated in Fig. 233.

A straight length of rectangular waveguide has two narrow transverse slots cut in the broad face $\lambda_g/4$ apart. A piece of curved waveguide is fixed to the straight waveguide so that the slots are common to both guides where the walls are in contact. Thus power can be radiated from the straight waveguide through the slots into the arms A and B of the curved guide. By this arrangement the radiations from the slots which are excited by the direct wave in the main guide (travelling from left to right in Fig. 233) reinforce to give a wave in the arm B, but cancel to give no radiation in the arm A (compare End-fire Array, Chap. 17, Sec. 39). Conversely, the reflected wave in the main guide causes a wave to travel in A but not in B.

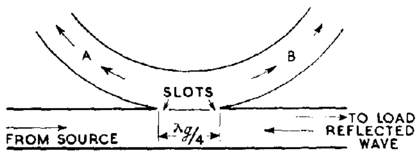

Fig. 233 - Fixed frequency standing wave indicator.

Thus, waves travel in the arms A and B whose amplitudes are proportional to the reflected and incident waves in the main waveguide. In principle, when A and B are terminated in similar field-strength meters the reflection coefficient of the load in the guide may be obtained directly. In practice it is difficult to obtain meters of equal sensitivity and a better arrangement is to terminate B in a reflectionless load (Sec. 38) and to calibrate the output from A against standard mismatches in the main guide. Then, provided the power in the direct wave does not vary, the standing-wave ratio may be read directly from the meter at A. This device can be used at only one wavelength since the slots must be $\lambda_g/4$ apart.

42. Elimination of a Reflected Wave

To eliminate a reflected wave the standing wave pattern is first found by means of a standing wave indicator, and the section at which the admittance assumes the form $\boxed{y_t} = 1 + j\boxed{B}$ is located from a circle diagram. If now, an obstacle whose normalised susceptance is $\boxed{B_1} = \boxed{-B}$ is introduced at this section the admittance of the section becomes $\boxed{y_t} = 1 + j\boxed{B} - j\boxed{B} = 1$. (It is assumed that the obstacle may be represented as a shunt impedance (see Sec. 40)). The reflection coefficient $\rho t = (1 - \boxed{y_t} \ / (1 + \boxed{y_t})$ therefore becomes zero and no reflected wave returns to the generator.

Fig. 234 indicates how the correct position for the obstacle may be found relative to that of an E antinode in the standing wave. An E antinode is a position of minimum admittance $\check{Y}$. The point K on the circle diagram corresponds to this position along the guide. The point L corresponds to the position where the admittance is $\boxed{y} = 1 + j\boxed{B}$ It is a distance $\ell_1 = n_1 \lambda_g$ away from the E antinode in the direction of the generator where n_1 is the n-value of the arc ML on the circle diagram.

It is required to add the susceptance $-j\,\boxed{B}$ at this point in order to match the waveguide. Alternatively an admittance $+j\,\boxed{B}$ could be added at a distance $\ell_2 = n_2 \lambda_g$ which corresponds to the point N on the diagram with n_2 as the n-value of the arc MN.

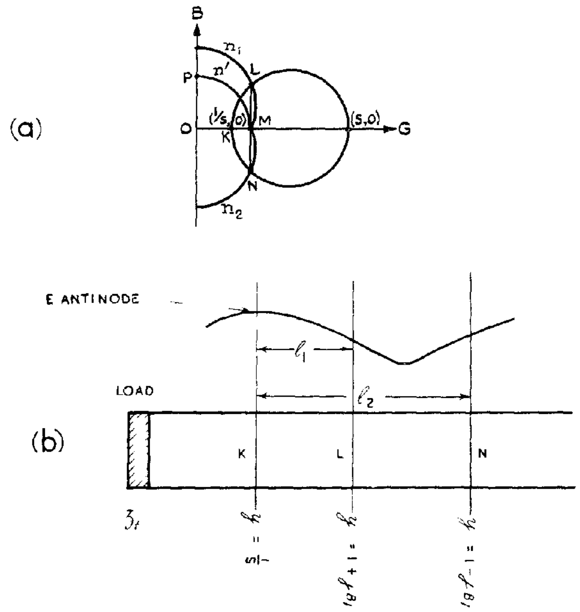

Fig. 234 - Elimination of reflected wave.

It is useful to illustrate what has been said above by actual experimental results, obtained using a standing wave indicator. A $2\frac{1}{2}$" x 1" rectangular waveguide with an open end was fed by a 9 cm generator under the following conditions :-

Wavelength λ_g in guide = 13·8 cm (twice distance between adjacent minima).

Standing wave ratio S = 2·83.

Distance ℓ_1 of nearest E antinode from the open end

= 6·6 cm = 0·478 λ_g.

From the circle diagram we find:-

$$\boxed{z_t} = 2{\cdot}46 - 0{\cdot}89j \text{ or } \boxed{y_t} = 0{\cdot}36 + 0{\cdot}13j\ .$$

Thus the impedance $\boxed{z_t}$ is, clearly, not infinite but a resistance and a capacitance in series, or, regarded as an admittance, a conductance and a capacitive susceptance in shunt.

The equivalent circuit representation of the waveguide and its termination $\boxed{z_t}$ is a transmission line of characteristic impedance R_o terminated in a load comprising a resistance 2·46 R_o and a reactance (condenser) = $j.0{\cdot}89R_o$ in series, or alternatively in a resistance whose conductance is $0{\cdot}36/R_o$ and a condenser whose susceptance is $j.0{\cdot}13/R_o$ in shunt.

These loads possess the same reflection coefficient and produce the same standing wave pattern on the line as does the open end in the waveguide.

We suppose next that it is required to eliminate the reflected wave in the major portion of the waveguide. We have :-

$$\text{Standing wave ratio } S = 2{\cdot}83 = \frac{1}{0{\cdot}354}.$$

The admittance at an E antinode is $\frac{1}{S} = 0{\cdot}354$ consequently, OK in Fig. 234 (a) is equal to 0·354. From a circle diagram it is found that, for the point L, $\boxed{y} = (1 + j\boxed{B})$, $n_1 = 0{\cdot}163$ and ML = 1·07.

We conclude that at a section which lies at a distance $\ell_1 = n_1 \lambda_g = 0{\cdot}163 \times 13{\cdot}8 = 2{\cdot}25$ cm away on the generator side of any E antinode, the admittance of the guide is there equal to $\boxed{y} = (1 + j.1{\cdot}07)$. At the point N we find $n_2 = 0{\cdot}337$ and MN = $-j.1{\cdot}07$.

The admittance at a distance $\ell_2 = 0{\cdot}337 \cdot 13{\cdot}8 = 4{\cdot}65$ cm is therefore $\boxed{y} = (1 - j.\,1{\cdot}07)$.

It was found that a complete elimination of the reflected wave could be achieved by protruding a metal screw into the guide through a longitudinal slot in the broad face at the position N (Fig. 234 (b)). The screw was mounted in a holder so that by turning it slowly the extent to which it projected into the guide parallel to the electric field could be controlled.

For a certain length of screw within the guide the standing wave indicator showed that the reflected wave had disappeared.

It follows that, if $\boxed{y_1}$ is the normalised admittance of the screw itself then, since,

$$\boxed{y} = (1 - j.1{\cdot}07) + \boxed{y_1} = 1$$

$$\boxed{y_1} = +j \,.\, 1{\cdot}07.$$

This length of screw therefore possesses a capacitive susceptance $\boxed{B_1} = 1{\cdot}07$. To check the value of this susceptance the end of the waveguide was closed by a metal plate and the screw removed. A standing wave indicator located an E antinode and the screw was placed at this position projecting to the same extent into the guide as before.

The standing wave pattern was found to be displaced as a whole a distance 1·8 cm. away from the generator by the introduction of the screw; that is, the E antinode lay 1·8 cm away from the screw on the side away from the generator.

If we form $n' = 1{\cdot}8/\lambda_g = 0{\cdot}13$ and locate the point P (Fig. 234 (a)) on the real axis at the end of the n-arc whose n value is n = n' = 0·13 we obtain the normalised susceptance of the screw as OP = j.1·07. This follows because a displacement of $(0{\cdot}5 - 0{\cdot}13)\lambda_g$ from the screw towards the generator leads to a position of zero susceptance (E antinode). The corresponding motion on the circle diagram is along the imaginary axis from P upwards to plus infinity and then from minus infinity to O.

It is found that when the screw is short its susceptance is positive (capacitive) but when it extends across the guide almost to the opposite face its susceptance becomes negative (inductive). There is an intermediate resonant length (approximately $\lambda/4$) at which the susceptance is zero. A wire extending across the centre of the waveguide with its length parallel to E behaves as an inductance in shunt.

43. Matching Devices

Various matching devices are employed in waveguides, most of them being of the symmetrical type which in effect introduces a susceptance in parallel with the guide. The only series obstacles in common use are slots in the walls, used for purposes of radiation and switching and these are dealt with elsewhere (Aerials, Chap. 17 Sec. 54).

Although in general, the only method of finding the susceptance of an obstacle is to measure it, yet the principles of electromagnetism have been applied to calculate the susceptances of several obstacles possessing simple geometrical forms. This makes it possible to design a structure to have a specified susceptance. A common form of obstacle is the diaphragm which is formed by a thin metal strip or a pair of strips lying in the cross section of the waveguide and stretching from one wall to its opposite. Such devices are commonly called irises, but this term should be reserved for diaphragms which can be varied mechanically.

The principal types are :-

(i) Capacitive Diaphragm (Fig. 235)

(ii) Inductive Diaphragm (Fig. 236)

(iii) Resonant Diaphragm (Fig. 241)

The Capacitive Diaphragm (H_{01}-mode)

In this case the strips are normal to the electric field as shown in Fig. 235. The formula for the normalised susceptance of this Iris is :-

$$\boxed{y_1} = j\boxed{B_1} = j \frac{4a}{\lambda_g} \log_\varepsilon \operatorname{cosec} \frac{(\pi d)}{2a} .$$

The field in the vicinity of the edges of the strips resembles the electrostatic field that would exist were the side walls of the guide to be removed and the upper and lower faces to become a parallel plate condenser.

This quasi-electrostatic storage field can be represented mathematically as a set of evanescent E-waves. However, under the influence of the incident

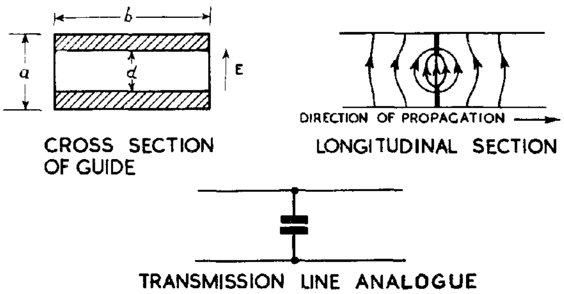

Fig. 235 - Capacitive diaphragm (H_{01}-wave).

H_{01}-wave, oscillatory charging currents flow into and out of the strips from the upper and lower walls of the waveguide and these currents radiate the scattered H_{01}-waves. The presence of the storage field introduces a phase difference between these oscillatory currents and the oscillations of the incident E-field, so that a phase shift ϕ is produced in the reflected wave relative to the incident wave.

The capacitive diaphragm is not employed where high powers are to be handled since the concentration of electric field at the edge tends to cause breakdown in the dielectric (air).

The Inductive Diaphragm (H_{01}-mode)

Here the strips run parallel to the electric field, as illustrated in Fig. 236.

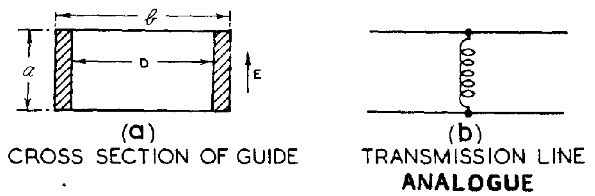

Fig. 236 - Inductive diaphragm (H_{01}-wave).

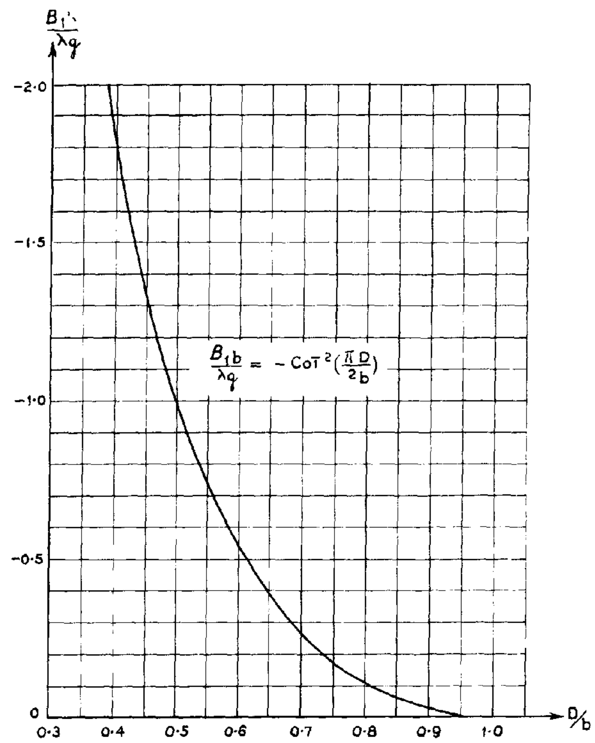

Fig. 237 - Design curve for inductive diaphragm in rectangular guide (H_{01}-wave).

The formula for the susceptance of this diaphragm is

$$\boxed{y_1} = -j\boxed{B_1} = -j\,\frac{\lambda_g}{b}\,\cot^2\frac{(\pi D)}{2b}\ .$$

Fig. 237 is a design curve in which $\frac{\boxed{B_1}\,b}{\lambda_g}$ is plotted as a function of D/b. The currents driven in the strips by the incident E vector excite a storage field resolvable into evanescent H-modes and at the same time radiate an H_{01}-wave, part of which is reflected back along the guide. This diaphragm behaves like an inductance shunted across a transmission line. It is used in practice to eliminate the wave reflected from the open end of a waveguide.

Inductive Wire (H_{01}-mode)

A cylindrical wire stretched across the waveguide parallel to the E field behaves as an inductive shunting susceptance. The susceptance of the wire shown in Fig. 238 is, when the wire is thin ($r \ll \lambda_g$)

$$\boxed{y_1} = -j\boxed{B_1} = \frac{-j\,\frac{2\lambda_g}{b}}{\left(\log_\varepsilon\frac{2b}{\pi r}-2\right)}\ .$$

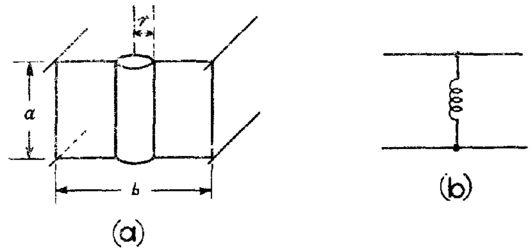

Fig. 238 - Inductive wire (H_{01}-wave).

Diaphragms in Circular Waveguides (H_{11}-mode)

When a circular waveguide is designed to carry an H_{11}-wave but no other, then suitable diaphragms can be used to eliminate reflections as in rectangular waveguides.

Figs. 239 (a) and 239 (b) illustrate respectively the forms assumed by a capacitive and an inductive diaphragm in a circular waveguide carrying an H11-wave. The shaded regions represent thin metal diaphragms that lie in the plane of the cross-section.

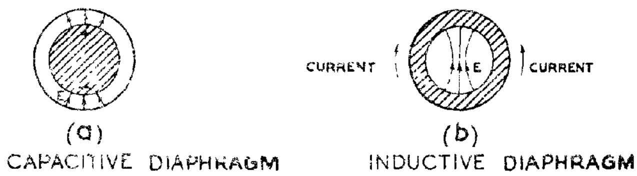

Fig. 239 - Diaphragms in circular guides (H_{11}-wave).

Resonant Diaphragms

We next consider how it is possible to combine a capacitive and an inductive diaphragm to produce a composite diaphragm whose admittance is zero and which therefore does not reflect the wave. Consider first the arrangement of irises indicated in Fig. 240, for an H_{01}-wave in a rectangular guide.

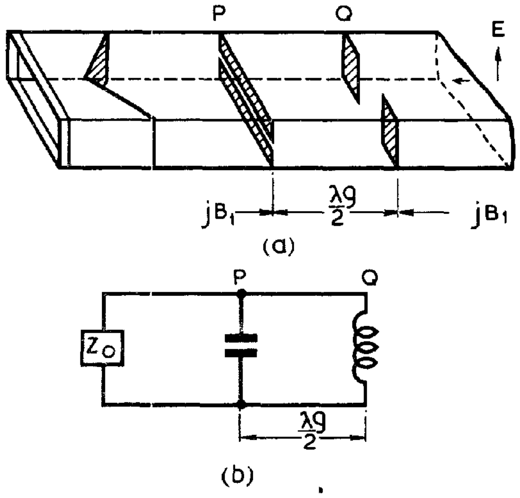

Fig. 240 - Resonant diaphragms (H_{01}-wave).

The waveguide, which ends in a reflectionless load, carries a capacitive diaphragm at the section P. The admittance of this section is $(1 + j\boxed{B_1})$ where $\boxed{B_1}$ is the susceptance of the diaphragm. The admittance of the waveguide section at Q a distance $\lambda_g/2$ nearer the source than P is also $(1 + j\boxed{B_1})$ before the introduction of the inductive diaphragm. When an inductive diaphragm whose susceptance is $-j\boxed{B_1}$ is introduced at B the admittance of the section at Q becomes

$$\boxed{y} = 1 + j\boxed{B_1} - j\boxed{B_1} = 1$$

Its reflexion coefficient is zero and no reflected wave returns to the source. In fact the diaphragm at B has been used to match the waveguide at this point. Fig. 240 (b) shows the analogous transmission line system.

There is clearly an infinity of pairs of diaphragms $(\pm j\boxed{B_1})$, that can be combined in this way to produce a reflectionless combination. It would clearly be more convenient however if the two diaphragms could be combined at the same section, Q for instance, instead of at separate sections P and Q. It is, in fact, found possible to superimpose a capacitive and an inductive diaphragm to produce a reflectionless rectangular structure such as that shown in Fig. 241 (a). Such is known as a Resonant Diaphragm. Its admittance is zero. It is found that the widths of the capacitive and inductive portions of the resonant diaphragm are not the same as those required to give no reflection with the arrangement of Fig. 240 (a). This is because the storage fields of the two diaphragms are now intermingled and the inductive and capacitive portions do not behave independently of each other. A useful, but approximate, design procedure is illustrated in Fig. 241 (b), where PQRS represents the section of the rectangular waveguide. The curves PLS and QMR are the two branches of a hyperbola that passes

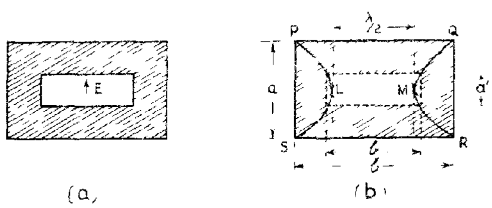

Fig. 241 - Resonant diaphragm (H_{01}-wave).

through the corners PQRS and such that the distance between the poles LM is $\lambda/2$ (λ = free-space wavelength). If the corners of the aperture of the composite diaphragm are made to fall on this hyperbola then the diaphragm is approximately resonant. The dotted rectangle in Fig. 241 (b) is an example. There is an infinity of such resonant structures for any waveguide section.

If b and a, and b' and a' are the dimensions respectively of the waveguide cross-section and of the diaphragm, then the geometrical construction given above is equivalent to the following relation between these dimensions.

$$\frac{b}{a}\left[1 - \left(\frac{\lambda}{2b}\right)^2\right]^{\frac{1}{2}} = \frac{b'}{a'}\left[1 - \left(\frac{\lambda}{2b'}\right)^2\right]^{\frac{1}{2}}$$

It may be seen from Fig. 241 (b) that when a'/b' is small, then b' is approximately equal to $\lambda/2$; that is, the resonant length of any narrow slot centrally placed in a diaphragm, with its length perpendicular to the electric field, is very nearly $\lambda/2$.

The transmission line analogue of a resonant diaphragm is shown in Fig. 242. At resonance the L-C circuit becomes a rejector circuit that places zero shunt admittance across the line. Consequently a progressive wave passes the structure without reflection. A thin $\lambda/2$ slot cut in a metal diaphragm across a circular guide carrying an H_{11}-wave and placed with its length at right angles to the electric field behaves as a resonant iris that transmits the incident wave completely.

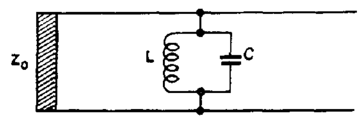

Fig. 242 - Analogue circuit for resonant diaphragm.

Fig. 243 - Transparent structures.

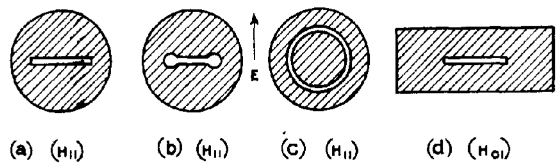

Fig. 243 shows a set of resonant structures that transmit the progressive wave without reflection. The resonant length of the slot in Fig. 243 (a) was found at λ = 9·1 cm. to be about 4% shorter than $\lambda/2$. For the composite L-C resonant iris of Fig. 243 (c) the inner circumference of the gap at resonance, when thin, is almost equal to λ .

The Q-Factor of Resonant Diaphragms

Although the diaphragm at resonance is almost perfectly transparent to the mode for which it is designed, changes in the frequency of the wave propagated along the guide cause changes in the admittance of the diaphragm and partial reflection occurs. The behaviour is similar to that exhibited by a parallel resonant circuit shunted across a transmission line.

As shown in Chap. 1., Sec. 19 , the amplitude $\hat{v}$ of the voltage developed across a parallel resonant circuit when supplied with a constant feed current is given by

$$\hat{v} = \hat{v}_r \cos \phi,$$

where $\tan \phi \doteqdot 2Q\delta$, δ being the fractional detuning $\pm \frac{\Delta f}{f_r}$ and $\hat{v}_r$ the value of $\hat{v}$ at resonance. It is thus possible to measure the Q of a parallel resonant circuit in terms of the reduction in the amplitude of $\hat{v}$ for a given value of δ.

A similar procedure enables the Q of a resonant diaphragm to be determined experimentally. The quantity measured is the magnitude of the voltage or current induced in a circuit coupled to the fields developed in the guide on the side of the diaphragm farther from the generator, by means of a cable terminated in a non-resonant loop or probe. Thus, when Q is large the diaphragm begins to give rise to serious reflections for a small fractional frequency shift $\pm f/f_r$, but if Q is small then it remains transparent over a reasonably large range of frequencies.

As example , consider the resonant structures shown in Fig. 243. At a wavelength of 9·1 cm. it was found that the Q of the slot of Fig. 243 (a) was 25 for a slot width of 0·5 mm but equal to 50 for a width of 0·1 mm. In general, the narrower the slot the larger is the value of Q.

The value of Q may be reduced by using the structure of Fig.243(b) instead of a slot. With the straight section 4·2 mm. wide and the radius of the end circles equal to 1·4 mm the Q was reduced to the value 9. Such a structure therefore is transparent over a relatively wide range of frequencies.

The Q values in terms of ring width for the diaphragm of Fig. 243 (c) were as follows :-

Ring width in mm.	0·1	0·5	0·8
Q	40	20	16

In all these examples the diaphragms were made of foil 0·004" thick. When brass of thickness $\frac{1}{8}$" was used the Q factors were approximately doubled.

44. Switches and Protective Devices.

The electric field strength at the centre of a resonant slot is many times greater than that in the progressive waves at some distance from the slot. Consequently, when high powers are transmitted an electrical discharge may occur in the dielectric across the gap. This fact is made use of in the gas-filled resonant cell. Here the slot is enclosed in a glass capsule which is filled with commercial neon at a low pressure.

Fig. 244 (a) represents a sectional view of Fig. 243 (c), and Fig. 244 (b) a sectional view of Fig. 243 (a), each representing a resonant diaphragm enclosed in a gas-filled cell. These cells with the diaphragms

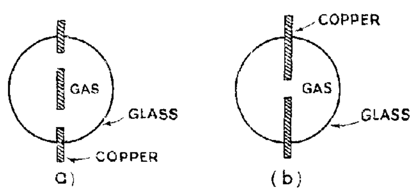

Fig. 244 - Simple T/R cell.

projecting as rims can be fitted into circular waveguides. They are quite transparent to waves of low power, such as the signal to the receiver, but spark over and become completely reflecting when a powerful wave from the transmitter strikes them.

A cell containing a resonant diaphragm of the form shown in Fig. 243 (b) is used at 3 cm wavelengths under the name CV.115. In it, however, the circles are larger and the intervening straight portion of the window is much shorter than as shown in the figure.

Since cells are used to provide automatic switching of power in common T/R systems with waveguides, they are often referred to as T/R cells. A disadvantage of the simple cell of the type CV.115 is that when used for receiver protection the spark gap cannot be relied upon to strike immediately. When lag in striking occurs enough power may pass to the receiver to burn out the crystal.

A modified form of cell (American type 1B.24, British type CV.221) which affords satisfactory protection of the crystal is shown, diagrammatically, in Fig. 245. It is used at 3 cms wavelengths and breaks down at weaker field strengths than those required for the CV.115 and with no appreciable lag.

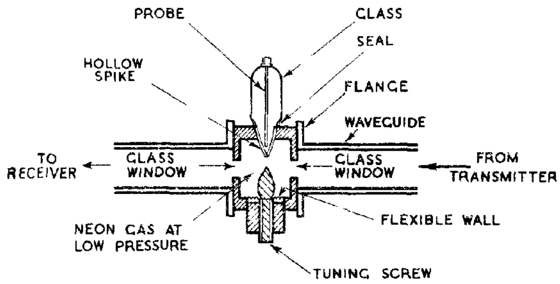

Fig. 245 - **Adjustable T/R cell.**

The cell is a gas-filled resonant cavity placed in series with the waveguide to which it is coupled at each end by slots. The cell cavity is separated from the two portions of the waveguide by glass windows. It is brought to resonance by adjusting the separation of the two spikes that project into it as shown, the upper spike being hollow. This adjustment is accomplished by means of a screw which works against a flexible area of the wall that carries one of the spikes. To avoid lag in the striking of the discharge when a power wave enters the cell, a glow discharge is kept running between the probe and the inner wall of the upper spike. This spike has a hole in the end and electrons diffuse from the glow discharge into the space between the spikes to provide the initial ionisation from which the high frequency discharge is able to build up without lag. A high resistance is included in series with the probe to limit the glow discharge, but its value must be chosen to avoid an intermittent discharge, for then the protective action might be vitiated; (compare Flashing Neon, Chap. 10 Sec. 1 and Chap. 19 Sec. 10). The cell permits weak signals to pass unaffected but blocks strong signals.

A soft rhumbatron may also be used as a T/R cell, in a manner similar to that employed in transmission line circuits. Instead of the resonant cavity being coupled by means of pick-up loops, as described in Chap. 4, windows are used, the whole cell being tuned by the cavity tuning adjustment. The arrangement is further described in Sec. 50.

45. Reflectors in Waveguides

Various reflecting diaphragms are depicted in Fig. 246.

When each is compared with the corresponding resonant diaphragm of Fig.243, it will be observed that the reflectors are obtained by an interchange of the metal and the open portions of the diaphragms followed by a rotation through a right angle. A pair of diaphragms related in this way are said to be complementary. Not only are their geometrical properties complementary, but it is found both by theory and practice that their electromagnetic properties are complementary also. Thus,

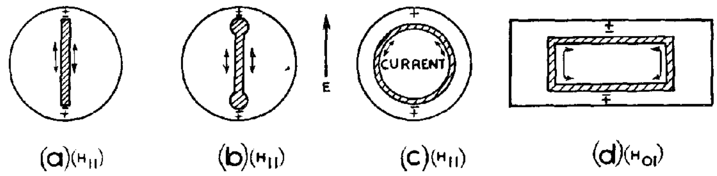

Fig. 246 - Reflecting diaphragms.

the electrical behaviour of the thin ring in Fig. 246 (c) may be compared with that of its counterpart, the thin circular slot of Fig. 243 (c). When the circular slot has an inner circumference approximately equal to the wavelength λ it is transparent and has zero susceptance. It is found, on the other hand that when the ring of Fig. 246 (c) is made slightly greater than λ in circumference its susceptance becomes infinite and the section containing it becomes completely reflecting. Similarly, the halfwave strips of Fig. 246 (a) and (b) are reflectors to an H_{11}-wave.

Reflecting rings form very convenient mechanical switches for diverting power alternately from one branch in a waveguide to another. The small inertia of the switches allows them to be turned at high speed. Two examples of the use of ring switches are given in Fig. 247. The reflectors must be positioned as shown in Fig. 246 in order to reflect almost completely the incident wave and must be turned through a right angle so that they are perpendicular to the electric field if they are to allow the incident wave to be propagated undisturbed. This may be achieved if the iris is rotated about the horizontal transverse axis. The mechanical simplicity of this arrangement makes it particularly suitable for use as a reflecting switch. Sometimes in rectangular waveguides a reflecting switch is made in the form of a rectangle (Fig. 246 (d) to fit into the waveguide cross section. Its total perimeter is still of the order of one wavelength.

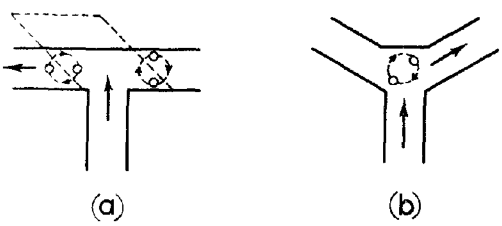

Fig. 247 - Use of rotating diaphragm for switching.

REFLECTIONS FROM JOINTS AND BENDS IN WAVEGUIDES

46. General

In radar a magnetron oscillator is often connected directly to a waveguide so that the circuit load on the magnetron is the impedance which the waveguide presents at its input end. It is explained in Chap. 8 Sec. 41 that the frequency of oscillation of a magnetron is peculiarly sensitive to the nature of the load. If therefore any irregularity is present in a waveguide, so as to cause a reflected wave, it may produce an effective impedance at the magnetron end which will cause the oscillation to take place at some undesirable frequency. This control of the frequency of oscillation by the attached waveguide is called Frequency Pulling. When a long run of waveguide is employed an additional effect called Frequency Splitting is observed when the termination reflects a wave down the guide. This effect does not appear when l/λ_g is small.

For these reasons it is desirable to keep all waveguide runs short and to design all joints and bends so that they cause the minimum of reflection.

47. Waveguide Joints

Flanges. At wavelengths of 9 and 10 centimetres coupling between two sections of waveguide is achieved through flat end-flanges, as shown in Fig. 248 (a) and (b).

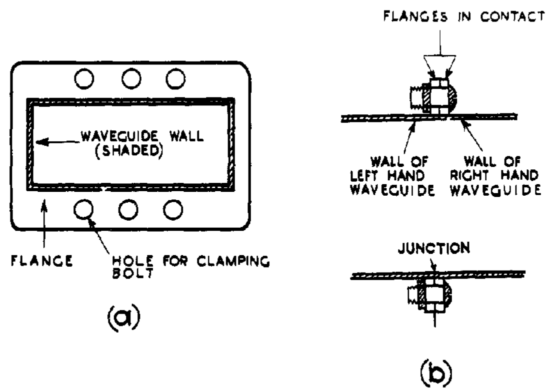

Fig. 248 - Flanges for connecting waveguide sections.

Fig. 248 (a) shows the face of the flange fitting flush with the end of the guide and 248 (b) two waveguides clamped together by their flanges.

The principal causes of reflection at these junctions are :-

(i) Misalignment of the walls at the junction causing a step in the walls of the guide. To avoid this the bolt holes must be located accurately.

(ii) Gaps between the waveguide walls across the junction either due to imperfections in the plane surfaces of the flanges or because they are not flush with the ends of the guides. Fig. 249 indicates what may occur when the flange faces are not accurately plane. The flanges are in contact at C but have left a gap at G between the waveguide walls. The two flange faces between G and C may be regarded as a pair of transmission lines of length ℓ and short-circuited at C, so that if by chance ℓ is approximately equal to $\lambda/4$ (λ = free-space wavelength) an E antinode and an H-node occur at the gap G. There is therefore a discontinuity in the transverse component of H in the H_{01}-wave in the guide at G, and therefore also in the longitudinal wall current. Such a gap causes a serious reflection. Consequently, the faces of the flanges should be made as nearly as possible accurately plane and parallel. Alternatively a copper gasket fitted between the flanges may be used to ensure good contact.

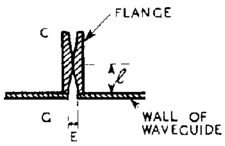

Fig. 249 - Irregularity due to flanges.

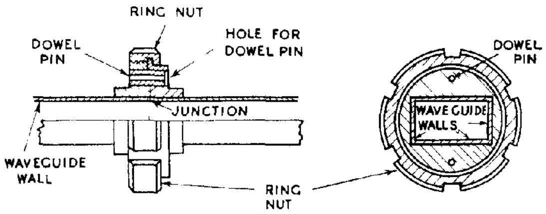

Fig. 250 - Coupling unit for λ = 3 cms.

At a wavelength of 3 centimetres these effects (i) and (ii) are more difficult to avoid than at the longer wavelengths and the design of the coupling unit is somewhat more complicated than that described above. A suitable device is shown in Fig. 250. The two flanges shown here are of more solid construction and are clamped together by a ring nut which presses on the one (female) and screws on to the other (male). A step at the junction is avoided by accurately placed locating pins.

As mentioned above and illustrated in Fig. 249 a common source of reflection at a joint is the existence of a gap between the walls of the two lengths of waveguide. A form of coupling which is finding increasing favour uses the electrical properties of a space between the flanges to provide a theoretically perfect electrical union between two lengths of waveguide.

The principle of these double quarter-wavelength joints, commonly called choke or capacity joints, is shown in Fig. 251. An L-shaped recess GBA is formed by suitably shaping the surface of one of the flanges, the other remaining plain. This recess follows the contour of the section of the waveguide wall and may be employed with either circular or rectangular waveguides.

The lengths BG and BA are each $\lambda/4$ (λ is the free-space wave-length) as indicated. The portions of the recess GB and BA each form quarter-wavelengths of transmission line; consequently, the short-circuit at A is transferred to G where the standing wave in the recess produces an H-antinode. The transverse component of H in the wave thus remains continuous across the gap. Hence, equal longitudinal currents flow into and out from the gap as indicated, and no reflected wave is generated in the waveguide at G.

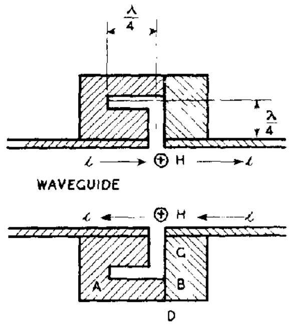

Fig. 251 - Double $\lambda/_4$ coupling unit.

The same principle is used to approach the ideal of a perfectly reflecting piston or plunger in circular and rectangular waveguides. It is difficult to ensure, with the normal piston, that good surface contact exists around the whole perimeter without at the same time rendering the piston stiff in action.

The Double $\frac{\lambda}{4}$ Tuning Plunger indicated in Fig. 252 achieves both easy movement and good electrical termination. The recesses AB and BC are both $\lambda/4$ lengths of transmission line systems, and as before ensure that across the gaps between the face of the piston and the waveguide wall (sections at A and D) there is an antinode of H and no component of E tangential to the face. The whole face, including the gap, behaves therefore as a perfectly fitting and reflecting disc which reflects the wave in the guide without generating evanescent modes.

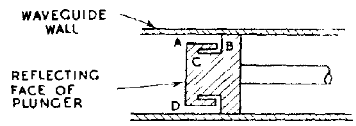

Fig. 252 - Double $\lambda/_4$ tuning plunger

48. Bends in Waveguides

In practice it is necessary to introduce corners and bends into waveguide runs by the insertion of sections with the forms illustrated in Fig. 253. The bend shown at (a) is known as an H-bend and that of (b) an E-bend. Fortunately, it is found that provided the inner radius of the bend exceeds λ_g, and the section of the guide is not distorted in the process of bending, the reflection produced as an H_{01}-wave enters is very small. The standing wave ratio associated with a good bend is of the order of 1·05.

The wave in the bend resolves into two waves, and in the case of waveguides with a circular section carrying an H_{11}-wave these waves travel in the bend at different speeds; consequently when they emerge to recombine into an H_{11}-wave, the plane of polarisation of the issuing wave is rotated with respect to that entering. This is a disadvantage of circular waveguides.

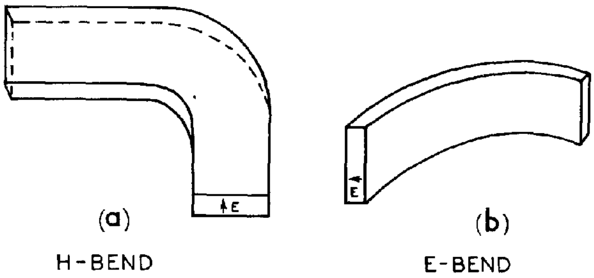

Fig. 253 - **Smooth bends in waveguides.**

When it is necessary to economise in space, the smooth bends discussed above are sometimes found to be inconvenient, especially at the longer wavelengths; consequently the sharp corners depicted in Fig. 254 are commonly employed. Fig. 254 (a) depicts an H-corner and (b) an E-corner, and it is to be noted that in each case the outer corner of the bend has been sliced off and the hole formed in the walls closed by a flat plate.

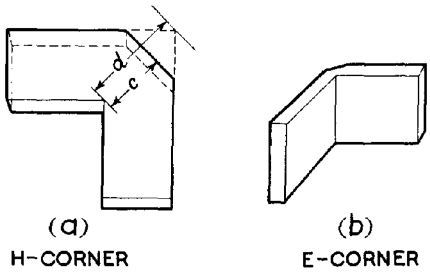

Fig. 254 - Right-angle corners.

Experiment shows that for each bend there is an optimum value of the ratio c/d that gives negligible reflection. This value for an H-corner was found to be

$$c/d = 0{\cdot}65,$$

and for an E-corner $c/d = 0{\cdot}61$,

with a wavelength of 10·8 cm. in a 7 cm. x 3·25 cm. waveguide.

Twists A rectangular waveguide may be twisted so that the plane of the E-vector is rotated through an angle (usually 90°), and little reflection arises provided the length of twist is not small compared with λ_g. Such a twist is shown in Fig. 255.

Fig. 255 - Twist in wave-guide.

49. Rotating Joints

The aerial systems of most centimetre wave equipments comprise a reflector which is a portion of paraboloid or parabolic cylinder, and is capable of motion about horizontal and/or vertical axes. Power is fed to the aerials from the end of a waveguide which in some instances is terminated in a horn. The waveguide feed must move with the mirror but must also be coupled to the fixed waveguide run from the transmitter. The aerial feeder must be coupled to the main waveguide so that no variation in power or polarisation of the radiated pulse occurs as the mirror moves. This problem is solved by use of special rotary transformers. These transformers take advantage of the fact that an E_{01}-wave in a circular guide is readily excited by an H_{01}-wave in a rectangular guide which feeds into the wall of the circular guide so that the E vector in the H_{01}-wave is parallel to the axis of the circular guide.

Fig. 256 illustrates the principle of this device. At (a) we have an H_{01}-wave in a rectangular guide, running into the lower end of a closed circular guide whose radius is large enough for the guide to accept a progressive E_{01}-wave. It is arranged that the electric vector in the H_{01}-wave is parallel to the axis of the circular tube and readily excites in it the E_{01}-mode as a progressive wave which travels to the top of the circular guide as shown. The longitudinal E-vector in the circular tube excites an H_{01}-wave in the second rectangular guide leading to the aerial.

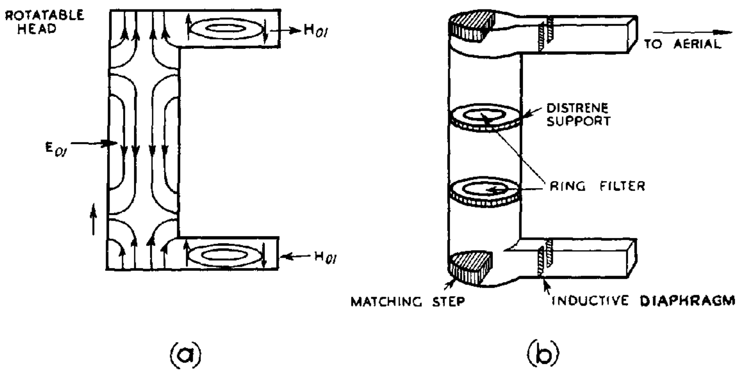

Fig. 256 - Rotary E-H transformer.

At (b) certain ancillary devices are also shown; they are, a pair of ring filters to reflect the H_{11}-mode which can also travel in the circular tube, and matching steps and inductive diaphragms to eliminate reflections at the junctions of the circular guide and the rectangular guides, both in transmission and in reception.

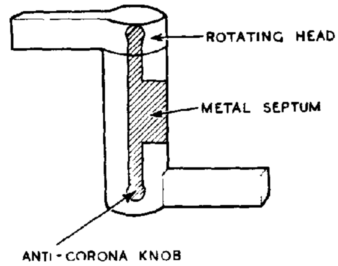

Fig. 257 - Rotating joint, embodying two septate-coaxial mode transformers.

If the top of the circular tube is made rotatable then the rectangular guide can be swung in azimuth. Since the E_{01}-wave possesses axial symmetry it will feed equally well into the rectangular guide at all azimuths.

An alternative rotary joint employs a septate-coaxial combination which permits a considerable reduction in the overall diameter of the circular tube (Sec. 21). A diagrammatic representation of this form of rotary joint is given in Fig. 257.

50. COMMON T/R WITH WAVEGUIDES

In Chap. 4 Common T/R circuits are described, applicable to transmission lines. In radar systems employing waveguides, the corresponding waveguide connections must be introduced in order to provide common T/R working. To reduce the waveguide problem to the same basis as that of the transmission line, consider the currents, at any instant, in the walls of the guide (Fig. 207), which is taken to be rectangular and carrying an H_{01}-wave.

Referring to the figure there are two current flows to be distinguished. Some current-flow lines are entirely confined to the wide walls of the guide and are directed, generally speaking, in the direction of propagation. These currents are strong along the centre line of the wide sides and weaken considerably for small deviations from the centre. Other current-flow lines start near the centre of a wide side and trace out a path over the narrow side and on to the wide side opposite to that from which they started. These current-flow lines are, generally speaking, transverse to the direction of propagation of power down the guide. The first type of current line corresponds to the current in a twin balanced transmission line, which may in this case, be taken as a pair of parallel ribbons. The second, transverse, current line corresponds to the current in a shunt short-circuited stub across a twin transmission line. The circuit equivalent of a rectangular waveguide carrying an H_{01}-wave is, therefore, a twin transmission line modified by the addition of a large number of shunt stubs. This is shown in Fig. 258. If the stubs are not to upset the transmission, they must be $\lambda/4$ long.

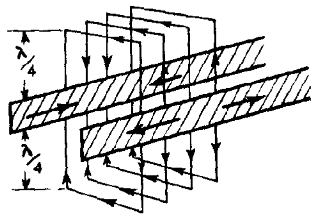

Fig. 258 - Transmission line analogue of waveguide walls.

It is now possible by analogy to work out the common T/R connections. A soft rhumbatron (Chap. 6) is employed as a switch. This acts like a parallel tuned circuit of fairly high Q and will spark over readily. In Chap. 4 (Fig. 182 (a)) the rhumbatron is shown with loop coupling, but for coupling to waveguides windows or irises are used. A hole is cut in the side of the rhumbatron and a similar hole in the guide wall; the rhumbatron being then clamped firmly against the guide (Fig. 259). With a suitable size of hole the coupling is practically 1:1 and the rhumbatron, or parallel tuned circuit, may then be considered as directly connected. Thus if a rhumbatron, with window coupling, is placed on the narrow side of a rectangular guide (Fig. 259), the equivalent circuit is as shown in Fig. 260, the parallel tuned circuit and switch being $\lambda/4$ away from the transmission line and shunt connected. The shunt arrangements of Figs. 180, 181 and 182 (Chap. 4) can thus be developed.

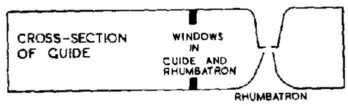

Fig. 259 - Shunt window coupling between rhumbatron and guide.

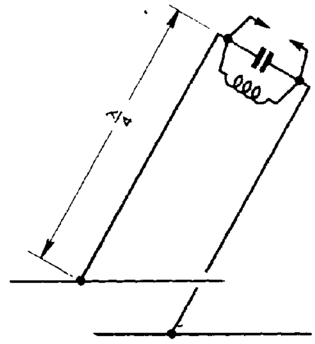

Fig. 260 - Circuit analogue for rhumbatron switch.

A typical T/R connection with rectangular waveguide is shown in Fig. 261. Two rhumbatrons are employed with window coupling to the guide. The receiver feeder is loop-coupled into the rhumbatron nearest the aerial, and the equivalent circuit is shown in Fig. 262. During transmission, the gas in the rhumbatrons ionises. The tuned circuits are thus short-circuited, giving rise to open circuits $\lambda_g/4$ away on the main line and allowing the transmitter power to pass to the aerial. During reception the gases are deionised. The rhumbatron nearer the transmitter acts like an open circuit and presents zero impedance $\lambda/4$ away, on the main line. This apparent short-circuit seen from the receiver junction $3\lambda_g/4$ away, looks like an open-circuit. The received signals therefore find a very high impedance, looking towards the transmitter, at the receiver junction, and proceed through the receiver rhumbatron and into the receiver feeder.

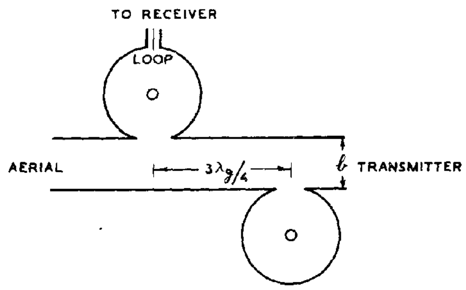

Fig. 261 - Side view of common T/R switch, employing two soft rhumbatrons shunt-connected.

On very short wavelengths, the feeder connection from the rhumbatron to the receiver may be undesirable owing to excessive attenuation in the feeder. In this case a waveguide replaces the feeder and there is window coupling both into and out of the rhumbatron. Series connection to the waveguide may also be used instead of shunt. In this case the rhumbatron is connected to the broad face of the guide and is usually off-set effectively $\lambda/2$ from this face by interposing another section of waveguide (Fig. 263). This section and the guide leading to the receiver are often circular. They carry an H_{11}-wave and so are similar to a rectangular guide carrying an H_{01}-wave. The

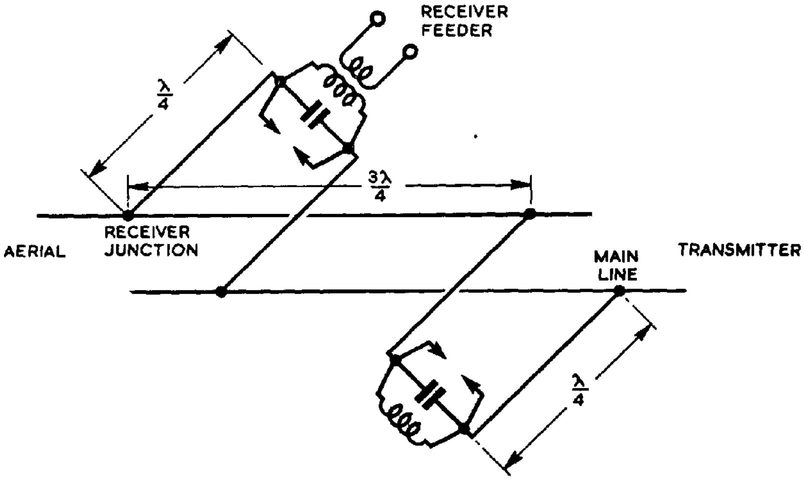

Fig. 262 - Circuit analogous for fig. 261.

circular guide is wider than the rectangular and so it is easier to place a crystal across it, to form the crystal converter which is the initial part of the receiver.

It is usually found in practice that the tuning of the rhumbatron nearer the transmitter is very broad and hardly affects the performance of the set. On very short wavelengths this rhumbatron is sometimes replaced by a resonant iris of the dumbell type (Fig. 243 (b)). This has a low Q and would not be suitable as a replacement for the receiver rhumbatron.

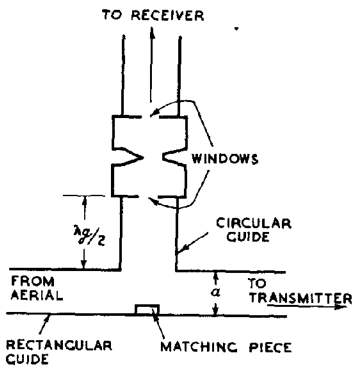

Fig. 263 - Series connection of soft rhumbatron switch.


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